LBL-35525
UC-413

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Presented at the International Conference on Nuclear Physics
and Related Topics, Hanoi, Vietnam, March 14–18, 1994,
and to be published in the Proceedings

Hadronic Matter and Rapidly Rotating Compact Stars

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March 1994

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Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

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HADRONIC MATTER AND RAPIDLY ROTATING COMPACT STARS *

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Invited Paper:
International Conference on Nuclear Physics and Related Topics
Hanoi, Vietnam
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To be published by World Scientific

ABSTRACT

In part one of this paper we review the present status of neutron star matter calculations, and introduce a representative collection of realistic nuclear equations of state which are derived for different assumptions about the physical behavior of dense matter (baryon populations, pion condensation, possible transition of baryon matter to quark matter). Part two deals with the theoretical determination of the minimum possible rotational periods of neutron stars, performed in the framework of general relativity, whose knowledge serves to distinguish between pulsars that can be understood as rotating neutron stars and those that cannot. Likely candidates for the latter are hypothetical strange stars. Their properties are discussed in the third part of this contribution.

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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1. Introduction

In the course of this conference, S. Nagamiya (see his contribution contained elsewhere in this volume) presented an overview of the enormous efforts that are made at various physics laboratories, like BNL or CERN, toward exploring the behavior of nuclear matter that is under extreme conditions of temperature and density. On earth, relativistic heavy-ion colliders provide the only tool by means of which such matter can be created and its properties studied.

On the other hand, however, it is well known that nature has created a large number of massive stellar objects, i.e. white dwarfs and neutron stars, which contain matter in one of the densest forms found in the universe. Neutron stars, for example, which are associated with two classes of astrophysical objects – pulsars and compact X-ray sources –, contain matter in their cores which possess densities ranging from a few times the density of normal nuclear matter (2.5×10^{14} g/cm^3) to about an order of magnitude higher, depending on mass. It is the purpose of this contribution to outline the present status of the investigation of the structure and stability of such massive stars, and what can be learned from such an attempt about the behavior of super-dense nuclear matter.

2. Equation of State of Neutron Star Matter

The equation of state (pressure as a function of density) of neutron star matter decisively links neutron stars with nuclear and particle physics (plus various other branches of physics). It is the basic input quantity whose knowledge over a broad range of densities, ranging from the density of iron at the star's surface up to ~ 15 times the density of normal nuclear matter reached in the cores of massive stars, is necessary when solving the Einstein equations for the properties of neutron stars.

2.1. Non-Relativistic Models for the Equation of state

The starting point when determining non-relativistic models of the equation of state is a phenomenological nucleon-nucleon interaction. In the case of the equations of state reported here, different two-nucleon potentials (denoted V_{ij}) which fit nucleon-nucleon scattering data and deuteron properties have been employed. Most of these two-nucleon potentials are supplemented with three-nucleon interactions (denoted V_{ijk}). Hence the Hamiltonian is of the form

$$H = \sum_i \left(\frac{-\hbar^2}{2m}\right) \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} \ . \tag{1}$$

The many-body method adopted to solve the Schroedinger equation is based on the variational approach[1,2,3].

1.2 Relativistic Models for the Equation of state

Relativistic models for the equation of state of neutron star matter are derived from a lagrangian of the following type[4,5],

$$\begin{aligned} \mathcal{L}(x) &= \sum_{B=p,n,\Sigma^{\pm,0},\Lambda,\Xi^{0,-},\Delta^{++,+,0,-}} \mathcal{L}_B^0(x) \\ &+ \sum_{M=\sigma,\omega,\pi,\varrho,\eta,\delta,\phi} \left\{ \mathcal{L}_M^0(x) + \sum_{B=p,n,\ldots,\Delta^{++,+,0,-}} \mathcal{L}_{B,M}^{\mathrm{Int}}(x) \right\} \\ &+ \sum_{\lambda=e^-,\mu^-} \mathcal{L}_\lambda(x) \ . \end{aligned} \tag{2}$$

The quantities $\mathcal{L}_B^0$, $\mathcal{L}_M^0$, $\mathcal{L}_{B,M}^{\mathrm{Int}}$ and $\mathcal{L}_\lambda$ refer to the lagrangians of free baryons, free mesons, interacting baryons, and leptons respectively. Their explicit expressions are given in Refs. [4,5,6]. The summation index B extends over all charged baryon states whose thresholds are reached in dense neutron star matter[7]. The nuclear forces are mediated by scalar, vector, and isovector mesons.

The equations of motion of the various baryon and meson fields, which follow from

$$\frac{\delta \mathcal{L}}{\delta \chi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \chi)} = 0 \ , \qquad \text{where} \quad \chi = \psi_B(x);\ \sigma(x), \omega_\mu(x), \ldots \ , \tag{3}$$

are to be solved subject to the constraints of electric charge neutrality and generalized β-equilibrium of neutron star matter [4,5]. Solving the equations of motion constitutes an extremely complicated problem. As demonstrated elsewhere, it can be accomplished on the footing of three different levels of complexity, i.e. the relativistic Hartree[4,5], relativistic Hartree-Fock[5] and relativistic ladder (Brueckner-Hartree-Fock type)[13,17] approximation.

Table 1: Nuclear equations of state (EOS) applied for the construction of models of general relativistic rotating neutron star models. Their tabulated representations, i.e. pressure versus energy and baryon density $P(\epsilon, \varrho)$, are given in Ref. [6].

Label	EOS	Description (see text)	Reference
	Relativistic field theoretical equations of state		
1	G_{300}	H, K=300	8
2	HV	H, K=285	7,5
3	G^{DCM2}_{B180}	Q, K=265, $B^{1/4}$ = 180	9,10
4	G^{DCM2}_{265}	H, K=265	11
5	G^{π}_{300}	H, π, K=300	8
6	G^{π}_{200}	H, π, K=200	12
7	Λ^{00}_{Bonn} + HV	H, K=186	13
8	G^{DCM1}_{225}	H, K=225	11
9	G^{DCM1}_{B180}	Q, K=225, $B^{1/4}$ = 180	9,10
10	HFV	H, Δ, K=376	5
11	Λ^{00}_{HEA} + HFV	H, Δ, K=115	13
	Non-relativistic potential model equations of state		
12	BJ(I)	H, Δ	14
13	WFF(UV_{14}+TNI)	NP, K=261	15
14	FP(V_{14}+TNI)	N, K=240	16
15	WFF(UV_{14}+UVII)	NP, K=202	15
16	WFF(AV_{14}+UVII)	NP, K=209	15

1.3 Collection of Nuclear Equations of state

A representative collection of nuclear equations of state, which are determined in the above described frameworks, i.e non-relativistic Schroedinger theory and relativistic nuclear field theory, is listed in Table 1. It is this collection that will be applied for the construction of models of general relativistic rotating neutron star models. The specific properties of these equations of state are described in the third column of Table 1, where the following abbreviations are used: N = pure neutron; NP = n, p, leptons; π = pion condensation; H = composed of n, p, hyperons ($\Sigma^{\pm,0}$, Λ, $\Xi^{0,-}$), leptons; Δ = Δ_{1232}-resonance; Q = quark-hybrid composition, i.e. n, p, hyperons in equilibrium with u, d, s quarks, leptons; K = incompressibility (in MeV); $B^{1/4}$ = bag constant (in MeV). Not all equations of state of this collection account for neutron matter in β-equilibrium (i.e. entries 13-16). These models treat neutron star matter as being composed of only neutrons (entry 14), or neutrons and protons in equilibrium with leptons (entries 13, 15, 16), which is however not the ground-state of neutron star matter predicted by theory[7]. In contrast, the relativistic equations of state *account for all baryon states* that become populated in dense star models constructed from them. An inherent feature of the relativistic equations of state is that they do not violate *causality*, i.e. the velocity of sound

is smaller than the velocity of light at all densities, which is not the case for the non-relativistic equations of state. Among the latter only the WFF(UV_{14} + TNI) equation of state does not violate causality up to densities relevant for the construction of models of neutron stars from it[18].

3. Rotating Massive Stars in General Relativity

As outlined in the Introduction, neutron stars are objects of highly compressed matter so that the geometry of space-time is changed considerably from flat space-time. Thus for the construction of realistic models of rapidly rotating pulsars one has to resort to Einstein's theory of general relativity. In the case of a star rotating at its *absolute* limiting rotational periods, i.e. the Kepler (or mass-shedding) frequency, Einstein's equations,

$$\mathcal{R}^{\kappa\lambda} - \frac{1}{2} g^{\kappa\lambda} \mathcal{R} = 8\pi \mathcal{T}^{\kappa\lambda}(\epsilon, P(\epsilon)) \ , \tag{4}$$

are to be solved in combination with the general relativistic expression describing the onset of mass-shedding at its equator:[19,20,21]

$$\Omega_{\rm K} = \omega + \frac{\omega'}{2\psi'} + e^{\nu-\psi}\sqrt{\frac{\nu'}{\psi'} + \left(\frac{\omega'}{2\psi'}e^{\psi-\nu}\right)^2} \ . \tag{5}$$

The line element has the form[19,20]

$$ds^2 = -e^{2\nu(r,\theta;\Omega)}dt^2 + e^{2\psi(r,\theta;\Omega)}(d\phi - \omega(r,\theta;\Omega)dt)^2 + e^{2\mu(r,\theta;\Omega)}d\theta^2 + e^{2\lambda(r,\theta;\Omega)}dr^2 \ .$$

The quantities $\mathcal{R}^{\kappa\lambda}$, $g^{\kappa\lambda}$, and $\mathcal{R}$ denote respectively the Ricci tensor, metric tensor, and Ricci scalar (scalar curvature). The dependence of the energy-momentum tensor $\mathcal{T}^{\kappa\lambda}$ on pressure and energy density, P and ϵ respectively, is indicated in Eq. (4). The quantities ω, ν, and ψ in Eq. (5) denote the frame dragging frequency of local inertial frames and time- and space-like metric functions, respectively. The primes denote derivatives with respect to Schwarzschild radial coordinate, and all functions on the right are evaluated at the star's equator. All the quantities on the right hand side of Eq. (5) depend also on $\Omega_{\rm K}$, so that it is not an equation for $\Omega_{\rm K}$, but a transcendental relationship which the solution of the equations of stellar structure, resulting from Eq. (4), must satisfy if the star is rotating at its Kepler frequency. (For more details, we refer to Ref. [20].)

4. Neutron Stars Rotating at the Kepler Frequency

The computed general relativistic Kepler periods $P_{\rm K}$ ($\equiv 2\pi/\Omega_{\rm K}$), defined in Eq. 5, are graphically depicted in Fig. 1 for a representative sample of equations of state listed in Table 1.[†] The smallest rotational periods are obtained for WFF(AV_{14}+

[†]An investigation of the limiting rotational Kepler period of neutron stars that is performed *without*

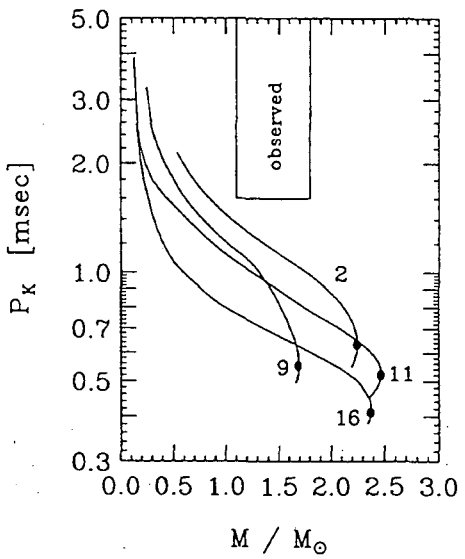

Figure 1: Kepler period versus rotational neutron star mass. The labeling of the curves is explained in Table 1. Only pulsar periods $P > P_K$ are possible, which is consistent with the pulsar periods known to date.

UVII) (label 16). We find that the relativistic equations of state lead in general to larger rotational periods than the non-relativistic ones due to the somewhat larger radii of the associated star models[20]. The upper limit on the Kepler period is set by the relativistic HV (label 2) equation of state. The periods obtained from all other equations of state listed in Table 1 lie between curves "2" and "16". The rectangle in Fig. 1 covers both the approximate range of neutron star masses as determined from observations[23], as well as the measured rotational periods ($P \geq 1.56$ msec)[24]. One sees that all pulsar periods so far observed are larger than the absolute limiting Kepler values.

Our investigation predicts limiting rotational Kepler periods for a $M = 1.4\,M_\odot$ pulsar in the range $0.7 < P_K/\text{msec} < 1.2$, depending on equation of state. In as much as our collection of equations of state contains representatives from theories of non-relativistic sort as well as relativistic approaches at several levels of development it appears that the observation of a pulsar with period below about 0.7 msec would be hard to reconcile with theories of dense hadronic matter. As shown elsewhere[25,20,6], this conclusion is strengthened by investigating the onset of emission of gravity waves from rotating neutron stars which sets an even more stringent limit on rapid rotation than mass shedding. In short it is found that the latter

taking recourse to any particular models of dense matter but derives the limit only on the general principles that (a) Einstein's equations describe stellar structure, (b) matter is microscopically stable, and (c) causality is not violated has only recently been performed by Glendenning[22]. He establishes a lower bound for the minimum Kepler period for a $M = 1.44\,M_\odot$ neutron star of $P_K = 0.33$ ms. This value sets an absolute limit on rotation on any star bound by gravity. Of course, the equation of state that nature has chosen need not be the one that allows stars to rotate most rapidly, so the above is a strict model independent limit.

instability confines stable rotational neutron star periods to be *larger than about 1 msec*. Half-millisecond periods, for example, are completely excluded for pulsars made of baryon matter. Therefore, the possible future discovery of a single sub-millisecond pulsar, rotating with a period of say ~ 0.5 msec, would give a strong hint that such an object is not a neutron star, but a rotating, self-bound *strange star*, and that 3-flavor strange quark matter is the true ground-state of the strong interaction, as pointed out by Glendenning[22,26] (see Sect. 5).

The fact that any successful model for the nuclear equation of state must accommodate pulsars with rotational periods of (at least) 1.56 msec and masses larger than typically $\sim 1.5\ M_\odot$ leads to an overall constraint on its density dependence (double constraint of fast rotation and a large enough neutron star mass): it must behave soft in the vicinity of the density of normal nuclear matter and intermediate nuclear densities in order to lead to small enough rotational pulsar periods, but rather stiff at high nuclear densities to account for large enough masses![13,27,28]

5. Strange Quark Matter as the true Ground State of Matter

The hypothesis that strange quark matter may be the absolute ground state of the strong interaction (not ^{56}Fe) has been raised by Witten in 1984 (cf. Fig. 2)[29]. If the hypothesis is true, then a *separate* class of compact stars could exist, which are called strange stars. They form a distinct and disconnected branch of compact stars and are not part of the continuum of equilibrium configurations that include white dwarfs and neutron stars. In principle both strange and neutron stars could exist. However if strange stars exist, the galaxy is likely to be contaminated by strange quark nuggets which would convert all neutron stars that they come into contact with to strange stars[26,30,31]. This in turn means that the objects known to astronomers as pulsars are probably rotating strange matter stars, *not* neutron matter stars, as is usually assumed.

6. Neutron Stars versus Strange Stars

6.1. Mass-Radius Relationship

The mass-radius relationship of strange stars with crust, whose inner density is equal to neutron drip, is shown in Fig. 3[32,33]. The solid dots denote the maximum-mass stars of the neutron (NS) and strange quark star (SS) sequence. The arrows indicate the minimum-mass star of each sequence ('a': strange star, 'b': neutron star). White-dwarf-like strange star configurations ('sd': strange dwarfs) terminate at the crossed point labeled 'd'. The symbol 'wd' indicates the region of ordinary white dwarfs. A value for the bag constant of $B^{1/4} = 145$ MeV has been chosen. This choice represents strongly bound strange matter with an energy per baryon ~ 830 MeV, and thus corresponds to strange quark matter being absolutely bound with respect to ^{56}Fe.

Since the crust is bound by the gravitational interaction (and not by confinement, which is the case for the strange matter core), the mass-radius relationship of

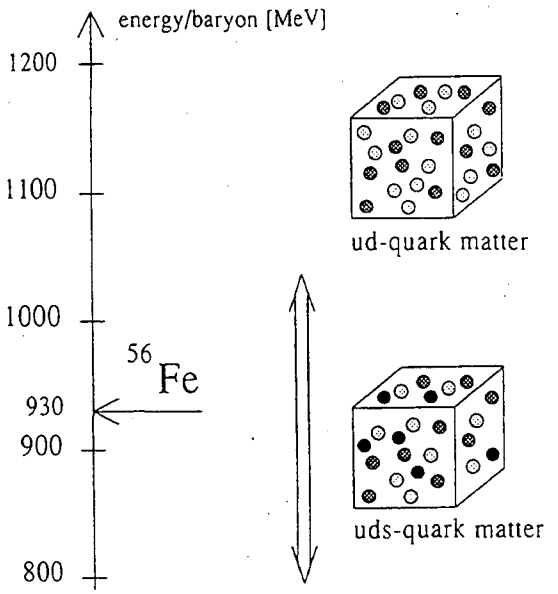

Figure 2: Comparison of the energy per baryon of ^{56}Fe (i.e. nuclear matter) with those of 2-flavor (u,d quarks) and 3-flavor (u,d,s quarks) quark matter. The latter system, also referred to as strange matter, is always lower in energy than 2-flavor quark matter due to the extra Fermi well, accessible to strange quarks. The vertical arrow roughly comprises the possible range of the energy per baryon of strange matter as predicted by theory. It should be noticed that it can be even smaller than the energy per baryon of ^{56}Fe, in which case strange matter would be more stable than ordinary matter.

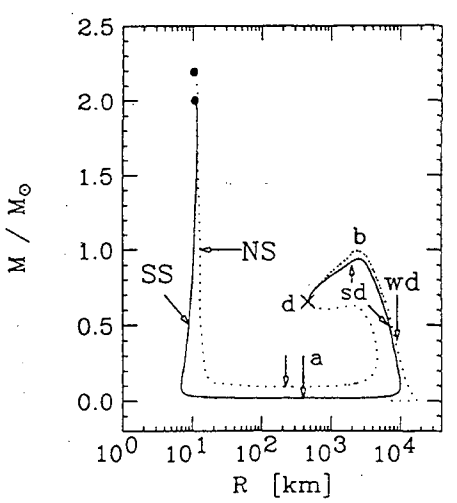

Figure 3: Mass versus radius of strange star configurations with nuclear crust (solid curve) and gravitationally bound stars (dotted curve). The following abbreviations are used: NS=neutron star, SS=strange star, wd=white dwarf, sd=strange dwarf.

strange stars is qualitatively similar to the one for neutron stars. The radius being largest for the lightest and smallest for the heaviest stars in the sequence. Just as for neutron stars the relationship is not necessarily monotonic at intermediate masses. The radius of the strange quark core is proportional to $M^{1/3}$ which is typical for self-bound objects. This proportionality is only appreciably modified near the mass where gravity terminates the stable sequence.

Unfortunately the bulk properties of models of neutron and strange quark stars of masses that are typical for neutron stars, $1.1 \lesssim M/M_\odot \lesssim 1.8$, are relatively similar, as can be seen from Fig. 3, and therefore do not allow the distiction between the two possible pictures. The situation changes however as regards the possibility of *fast rotation* of neutron stars and strange stars. This has its origin in the different mass-radius relationships of neutron stars and strange quark stars[34]. As a consequence of this the entire familiy of strange stars can rotate rapidly, not just those near the limit of gravitational collapse to a black hole, as is the case for neutron stars. For example, in Sect. 4 it has been pointed out that the minimum possible rotational period of a neutron star possessing a typical pulsar mass of $M \sim 1.4\, M_\odot$ is larger than ~ 1 msec. The minimum possible Kepler periods of strange stars of the same mass, carrying a nuclear crust whose inner density is equal to neutron drip, was found to lie in the range $\sim (0.6 - 0.8)$ msec, depending on bag constant[32]. Strange stars without nuclear cursts can rotate even faster, possessing Kepler periods of $0.6 - 0.7$ msec[35].

The sequence of strange stars has a minimum mass of $\sim 0.015\, M_\odot$ (radius of ~ 400 km) or about 15 Jupiter masses (label 'a'), which is smaller than that of the neutron star sequence, about $0.1\, M_\odot$[36]. These low-mass strange stars may be of considerable importance since they may be difficult to detect and therefore may effectively *hide baryonic matter.*[‡] Furthermore, of interest to the subject of cooling of strange stars is the crust thickness of strange stars[43]. It ranges from $\sim$ 400 km for stars at the lower mass limit to a fraction of a kilometer for the star at the maximum mass[32]. Those strange stars which result from solving the Oppenheimer-Volkoff equations for central star densities that are smaller than the corresponding central density of the minimum-mass star, but larger than the smallest possible one (determined by $\epsilon = 3\, P_{\rm drip} + 4\, B$, $P_{\rm drip}$ denotes the drip pressure) are shown too in Fig. 3. The cross refers to that particular star whose strange-matter-core radius has shrunken to zero, thus possessing mass and radius values of an ordinary white dwarf star[44].

[‡]Promising in this respect are microlensing searches which can give definitive results on dark objects in the mass range 10^{-5} to $10^2\, M_\odot$ [37,38]. In fact the presently performed searches may eventually be sensitive to the entire theoretically possible range of baryonic dark matter, 10^{-9} to $10^6\, M_\odot$ [39]. If the dark matter consists of these objects, these experiments should find it! [40]. Only recently the observation of the effects of an invisible object at least as massive as the planet Jupiter has been reported independently by the MACHO and French collaboration (for status reports, see Refs. [41] and [42], respectively).

6.2. Pulsar Glitches

A crucial astrophysical test of the strange matter hypothesis is whether or not strange quark stars can give rise to the observed phenomena of pulsar glitches. In the crust quake model an oblate solid nuclear crust in its present shape slowly comes out of equilibrium with the forces acting on it as the rotational period changes, and fractures when the built up stress exceeds the sheer strength of the crust material. The only existing investigation which deals with the calculation of the thickness, mass and moment of inertia of the nuclear crust that can exist on the surface of a rotating, general relativistic strange quark star has been performed by Glendenning and Weber[32]. It was found that the data on relative frequency changes $\Delta\Omega/\Omega$ of observed glitches (Ω denotes the star's rotational frequency) and the measured values of the ratio $(\Delta\Omega/\Omega)/(\Delta\dot{\Omega}/\dot{\Omega})$ for the Crab and Vela pulsars *can* be understood from the computed crustal moment of inertia of strange stars.

6.3. Stability of Quark Stars against Radial Oscillations

Below we give the equations that are to be solved to obtain the eigenfrequencies and eigenfunctions of radial normal modes of a star. The analysis is carried out on the basis of Einstein's field equations for a metric of the form

$$ds^2 \;=\; e^{2\nu(r)}\,dt^2 \;-\; e^{2\lambda(r)}\,dr^2 \;-\; r^2\left(d\theta^2 \;+\; \sin^2\theta\,d\phi^2\right) \;. \tag{6}$$

The adiabatic motion of the star in its nth normal mode ($n = 0$ is the fundamental mode) is expressed in terms of an amplitude $u_n(r)$ by

$$\delta r(r,t) \;=\; e^{\nu}\,u_n(r)\,e^{i\omega_n t}\,/\,r^2 \;, \tag{7}$$

where $\delta r(r,t)$ denotes small perturbations in r. The quantity $\omega_n(t)$ is the star's oscillation frequency, which we want to compute. The eigenequation for $u_n(r)$, which governs the normal modes, has the Sturm-Liouville form

$$\frac{d}{dr}\left(\Pi(r)\,\frac{du_n(r)}{dr}\right) \;+\; \left(Q(r) \;+\; \omega_n^2\,W(r)\right)\,u_n(r) \;=\; 0 \;. \tag{8}$$

The functions $\Pi(r)$, $Q(r)$, and $W(r)$ are expressed in terms of the equilibrium configurations of the star by

$$\Pi \;=\; e^{(\lambda+3\nu)}\,r^{-2}\;\Gamma\,P \;, \tag{9}$$

$$\begin{aligned} Q \;=\;& -4\,e^{(\lambda+3\nu)}\,r^{-3}\,\frac{dP}{dr} \;-\; 8\,\pi\,e^{3(\lambda+\nu)}\,r^{-2}\,P\,(\epsilon \;+\; P) \\ & +\,e^{(\lambda+3\nu)}\,r^{-2}\;(\epsilon \;+\; P)^{-1}\left(\frac{dP}{dr}\right)^2 \;, \end{aligned} \tag{10}$$

$$W \;=\; e^{(3\lambda+\nu)}\,r^{-2}\;(\epsilon \;+\; P) \;. \tag{11}$$

The quantities ϵ and P in Eqs. (9) - (11) denote the energy density (total mass-energy) and the pressure of the stellar equilibrium configuation as measured by a

local observer. The pressure gradient, dP/dr, is obtained from the Oppenheimer-Volkoff equations. The symbol Γ denotes the varying adiabatic index at constant entropy, given by

$$\Gamma = \frac{(\epsilon + P)}{P} \frac{\partial P}{\partial \epsilon} . \tag{12}$$

The boundary conditions for Eq. (8) are

$$u_n \sim r^3 \quad \text{at star's origin,} \quad r = 0 , \tag{13}$$

$$\frac{du_n}{dr} = 0 \quad \text{at star's surface,} \quad r = R . \tag{14}$$

Solving Eq. (8) subject to the boundary conditions (13) and (14) leads to the frequency spectrum ω_n^2 $(n = 0, 1, 2, \ldots)$ of the normal radial modes of a given stellar model. As a characteristic feature, the eigenfrequencies ω_n^2 form an infinite discrete sequence, i.e. $\omega_0^2 < \omega_1^2 < \omega_2^2 < \ldots$. We compute them for the sequence of strange stars exhibited in Fig. 3. As a side issue, the stability of charm stars (hydrostatic equilibrium configurations composed of u, d, c, s quarks) against radial oscillations is treated too.

The four lowest-lying eigenfrequencies of massive strange stars and strange dwarfs, whose inner crust density is equal to neutron drip, are shown in Fig. 4. A comparison with the mass-central density relationships of strange stars[44,45] shows that these equilibrium configurations possess a characteristic mode of vibration of zero frequency ($\omega_n^2 = 0$) when and only when $\partial M/\partial \epsilon_c = 0$, that is, only when the star's mass attains an extremum (critical point associated with an inflection point of mass). What is not known from the theorem however is *which* mode is possessing a zero point. (Of course, it must be the lowest-lying one, which was previously stable, i.e. $0 < \omega_n^2$.) We find that it is the $n = 0$ mode which becomes zero first at two different density values, which correspond to the maximum- and minimum-mass strange star configurations labeled 'a' and 'b' in Fig. 3 (for details, we refer to Ref. [44]). The $n = 0$ mode passes through zero at 'b' and remains negative for all densities down to the central density of the strange dwarf at the termination point (cross). This is of decisive importance for the question of stability of these strange dwarfs against radial oscillations: Since $\omega_0^2 < 0$ is associated with an exponentially growing mode of oscillation, all these strange dwarfs are *unstable* against radial oscillations. This finding now enables us to make the very important statement that no stable strange dwarfs located between 'b' and the sequence's crossed termination point can exist in nature! The higher modes shown in Fig. 4, $n = 1, 2$, and 3, form a discrete sequence, i.e. these obey $\omega_1^2 < \omega_2^2 < \omega_3^2$, as enforced by the mathematical structure of the eigenequation (8).

The eigenfrequencies and the locations of the zero frequency vibrations of massive strange stars possessing central densities up to those at which even *charm* quark states become populated in their dense interiors (charm stars), are shown in Fig. 4

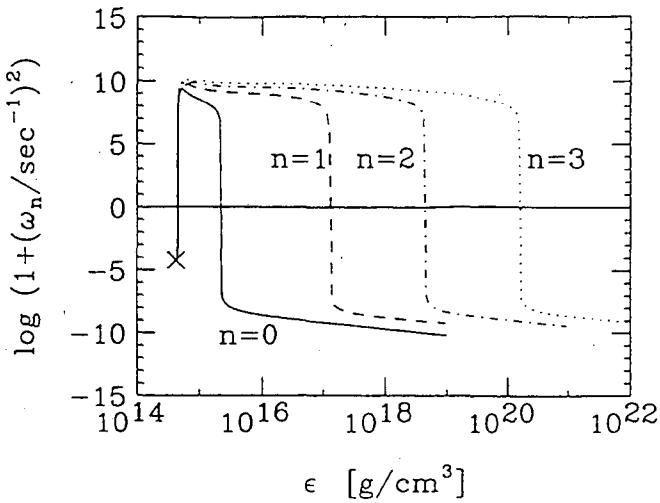

Figure 4: Pulsation frequencies of the lowest four ($n = 0, 1, 2$, and 3) normal radial modes of strange matter stars as a function of central star density. The labeling of the y-axis is an abbreviation for sign(a) log $(1 + \text{abs}(a))$, where $a \equiv (\omega_n/\text{sec}^{-1})^2$. The cross refers to the termination point of the strange dwarf sequence (cf. Fig. 3).

too. Each density for which $\omega_n^2 = 0$ corresponds to an extremum of mass. The second zero point of the $n = 0$ mode is located at that density at which the strange star sequence attains its maximum mass (solid dot in Fig. 3). Since ω_0^2 remains negative at all densities larger than this one, it follows that no quark matter stars can exist in nature that are more compact than the hypothetical strange stars. Specifically this *rules out the possible existence of charm stars*. In fact, as one sees from Fig. 4, going to higher and higher central star densities leads to the successive excitation of more and more unstable modes ($\omega_n^2 < 0$, $n = 1, 2, 3, \ldots$), and thus no quark stars more massive than strange stars fulfill the condition $\omega_n^2 > 0$ for all $n \geq 0$, which is necessary for stability. This situation is analogous to that of hydrostatic equilibrium configurations in the neutron star sequence with central densities above that of the maximum-mass neutron star.

7. Summary

The main issues of this contribution can be stated briefly under two categories, the one concerning the determination of the minimum possible rotational period of gravitationally bound neutron stars, and the other concerning the properties of any new pulsar whose rotational period lies considerably below that limit.

1. The minimum stable rotational period of (gravitationally bound) neutron stars of mass $M \sim 1.4\, M_\odot$, constructed for a collection of realistic nuclear equations of state is about 1 msec. From this it follows that the nature of any pulsar that is found to have a shorter period, say 0.5 msec, must be different from the

one of a neutron star. According to our present understanding of superdense matter, the hypothesis that most comfortably fits such objects are rapidly rotating strange stars (composed of u, d, s strange quark matter).

2. Concerning the structure and stability of strange stars, it is found that:

   (a) Strange stars can carry nuclear crusts of thickness ranging from $1 \sim$ km to $\sim 10^3$ km, depending on mass. Thick crusts on strange stars might be of great relevance to their cooling behavior.

   (b) Strange stars possess masses in the range $\sim 2\,M_\odot$ down to $\sim 10^{-4}\,M_\odot$. The latter value, which refers to the minimum-mass star of the sequence, depends on the value of inner crust density of the nuclear crust. Such a small mass is even smaller than Jupiter's mass (which amounts $10^{-3}\,M_\odot$). The important astrophysical implication of the existence of such light objects would be that they occur as natural stellar candidates which effectively hide baryonic matter, linking light (as well as massive) strange stars to the fundamental dark matter problem.

   (c) Since mass and radius values of $\sim 10^{-4}\,M_\odot$ and $\sim 10^3$ km are completely excluded for both neutron stars as well as white dwarfs, they may serve as additional signatures for hypothetical strange stars (besides small rotational periods).

   (d) The computed crustal moment of inertia of strange stars can account for the observed relative frequency changes of pulsars (glitches).

   (e) From the Oppenheimer-Volkoff equations one obtains strange matter configurations, carrying nuclear crusts, whose masses and radii are similar to those of white dwarf stars. We thus call such objects strange dwarfs.

   (f) Strange dwarfs carrying a nuclear crust whose inner crust density is equal to neutron drip are unstable against radial oscillations.

   (g) Charm stars, composed of u, d, s and c quarks, are unstable against radial oscillations.

There remain many fascinating aspects of fast pulsars that need to be worked out in detail, a few of them are alluded to above. By means of marrying nuclear physics (and various other branches of physics) with astrophysics, the rapid increase of the body of data on newly observed millisecond pulsars opens up the unique possibility to learn about the behavior of super-dense nuclear matter as well as the true ground state of the strong interaction. As noticed by M. A. Ruderman and W. A. Fowler, this intimacy is still fairly new. What will come out of it, while not forseeable, is likely to be lively, entertaining, and perhaps even beautiful [46].

One of us (F. W.) thanks the Deutsche Forschungsgemeinschaft for supporting his participation in the conference by a grant. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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