EXACT AND VARIATIONAL CALCULATIONS OF THREE-DIMENSIONAL FREE ELECTRON LASER EIGENMODES WITH A WARM ELECTRON BEAM*

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I present an exact calculation of free electron laser (FEL) eigenmodes (fundamental as well as higher order modes) in the exponential-gain regime. These eigenmodes specify transverse profiles and exponential growth rates of the laser field, and they are self-consistent solutions of the coupled Maxwell-Vlasov equations describing the FEL interaction taking into account the effects due to energy spread, emittance and betatron oscillations of the electron beam, and diffraction and guiding of the laser field. The unperturbed electron distribution is assumed to be of Gaussian shape in four dimensional transverse phase space and in the energy variable, but uniform in longitudinal coordinate. A constant focusing of the electron beam similar to the matched natural wiggler focusing is assumed in both transverse planes. With these assumptions the eigenvalue problem can be reduced to a numerically manageable integral equation and solved exactly with a kernel iteration method. An approximate, but more efficient solution of the integral equation is also obtained for the fundamental mode by a variational technique, which is shown to agree well with the exact results. Furthermore, I present a handy formula, obtained from interpolating the numerical results, for quick calculations of FEL exponential growth rate of the fundamental mode. Comparisons with simulation code TDA will also be presented. Application of these solutions to the design and optimization in multi-dimensional parameter space for an X-ray FEL driven by SLAC linac will be demonstrated. In addition, a rigorous analysis of transverse mode degeneracy and hence the transverse coherence of SASE FEL will be presented based on the exact solutions of the higher order transverse modes.

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