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Flavor Symmetries and Fermion Masses

A. Rašin
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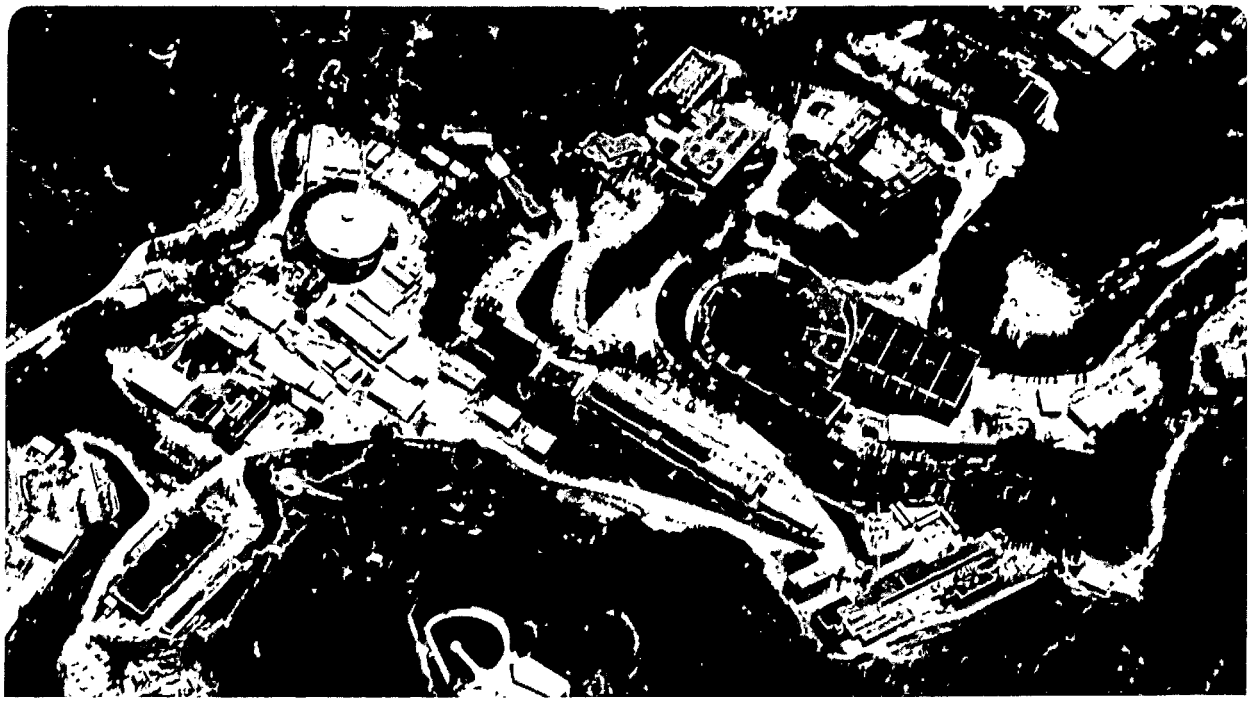

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Flavor symmetries and fermion masses [1]

Andrija Rašin [2]

Ph. D. Dissertation

Department of Physics, University of California, Berkeley, CA 94720

and Theoretical Physics Group, Lawrence Berkeley Laboratory

University of California, Berkeley, CA 94720

Abstract

We introduce several ways in which approximate flavor symmetries act on fermions and which are consistent with observed fermion masses and mixings. Flavor changing interactions mediated by new scalars appear as a consequence of approximate flavor symmetries. We discuss the experimental limits on masses of the new scalars, and show that the masses can easily be of the order of weak scale. Some implications for neutrino physics are also discussed.

Such flavor changing interactions would easily erase any primordial baryon asymmetry. We show that this situation can be saved by simply adding a new charged particle with its own asymmetry. The neutrality of the Universe, together with sphaleron processes, then ensures a survival of baryon asymmetry.

Several topics on flavor structure of the supersymmetric grand unified theories are discussed. First, we show that the successful predictions for the Kobayashi-Maskawa mixing matrix elements, $V_{ub}/V_{cb} =$

$\sqrt{m_u/m_c}$ and $V_{td}/V_{ts} = \sqrt{m_d/m_s}$, are a consequence of a large class of models, rather than specific properties of a few models. Second, we discuss how the recent observation of the decay $b \longrightarrow s\gamma$ constrains the parameter space when the ratio of the vacuum expectation values of the two Higgs doublets, $\tan\beta$, is large. Finally, we discuss the flavor structure of proton decay. We observe a surprising enhancement of the branching ratio for the muon mode in SO(10) models compared to the same mode in the SU(5) model.

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[2] On leave of absence from the Ruđer Bošković Institute, Zagreb, Croatia. Address after September 1, 1994: Department of Physics, University of Maryland, College Park 20742.

Contents

1. Introduction 1

2. Approximate flavor symmetries 5

2.1 Introduction 5

2.2 Approximate flavor symmetries 8

2.3 Many Higgs doublet model 12

2.4 Lepton sector 14

2.5 Conclusion 20

3. Preventing the erasure of baryon asymmetry in the early Universe 22

3.1 Introduction 22

3.2 General condition for survival of a baryon asymmetry 24

3.3 A simple model 30

3.4 Conclusion 35

4. Topics on supersymmetric grand unified theories 36

4.1 On the generality of certain predictions for quark mixing 36

4.2 The decay $b \to s\gamma$ and large $\tan\beta$ 46

4.3 The flavor structure of proton decay 52

Appendix A 64

Appendix B 68

Appendix C 69

References 72

List of Figures

Figure 1	A model of Yukawa couplings	82
Figure 2	The allowed parameter space for the mass and the lifetime of the X particle	83
Figure 3	The allowed region for CP asymmetries sin2α and sin2β for models in section 4.1.	84
Figure 4	One loop Standard Model contribution to the decay $b \rightarrow s\gamma$	85
Figure 5	One loop chargino contribution to the decay $b \rightarrow s\gamma$	86
Figure 6	Limits on the MSSM parameters for large tanβ from the chargino loop	87
Figure 7	One loop gluino contribution to the decay $b \rightarrow s\gamma$	88
Figure 8	Limits on the MSSM parameters for large tanβ from the gluino loop	89

List of Tables

Table 1 The ansatz for flavor symmetry breaking parameters 90

Table 2 Experimental lower limits on the exchanged scalar masses 91

Table 3 Neutrino mixing angle predictions in the three ansätze introduced 92

Table 4 Ratios of proton decay rates to neutrinos and muons in a specific class of models 93

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1 Introduction

In the Standard Model[1], Yukawa couplings λ are defined as couplings between fermions and Higgs scalars:

$$\mathcal{L}_Y = (\lambda_{ij}^{U^a} \bar{Q}_i U_j \frac{\tilde{H}_a}{\sqrt{2}} + \lambda_{ij}^{D^a} \bar{Q}_i D_j \frac{H_a}{\sqrt{2}} + \lambda_{ij}^{E^a} \bar{L}_i E_j \frac{H_a}{\sqrt{2}} + h.c.), \tag{1.1}$$

where Q_i and L_i are $SU(2)$ doublet quarks and leptons while U_i , D_i and E_i are $SU(2)$ singlets and $i = 1, 2, 3$ is a generation label. H_a are Higgs SU(2) doublets, and $a = 1, \ldots, n$, where n is the number of Higgs doublets.

The Standard Model can easily be extended to accommodate neutrino masses. We can introduce higher dimensional operators $\frac{1}{M} H H L_i L_j$ which give Majorana neutrino masses. We can also add SU(2) singlet neutrinos which can also have Majorana masses or Dirac masses similar to (1.1). Neutrino mass is then in general described by a 6×6 Yukawa matrix.

As opposed to, e.g. couplings of fermions and vector bosons, Yukawa couplings are not constrained nor related to each other by any symmetry principle; i.e., they enter the Lagrangian as arbitrary complex numbers. These numbers are then only fixed by experiment, namely by measuring fermion masses and mixing angles. However, a glance at the Review of Particle Properties[2] shows that some of the measurements are not easy at all. Although very good limits exist, neutrino masses and mixings have not yet been measured. Some quark masses and mixings have large uncertainties because of QCD. Also, the top quark has not been seen yet (although it might be just a matter of months now).

Therefore both theoretical and experimental motivations arise to go beyond

the Standard Model and to attempt to relate (or even set to zero) at least some of the couplings. Theoretically, this will reduce the arbitrariness of Yukawa couplings that plagues the Standard Model. Experimentally, predictions from the underlying theory might give clues as to where to look for new effects.

As of today, no theory has been generally accepted as the right way to go beyond the Standard Model, and this is even more true for an explanation of Yukawa couplings. However, many attempts have been made and, judging on the successes of their predictions, one has the feeling that some of these ideas will survive and will be a part of the correct underlying theory.

Several classes of models/ansätze for Yukawa couplings exist today, and we list them below:

- Approximate flavor symmetries[3]-[7], in which the entries in the Yukawa matrices are entered as small parameters by which the flavor symmetries are broken.
- Flavor democracy models[8], in which all the entries in the Yukawa matrices are equal (i.e., no flavor symmetry), and hierarchy comes from diagonalization and RGE running.
- Fritzsch and/or GUT inspired models[9]-[20], in which some entries in the Yukawa mass matrices are assumed to be zero (e.g. by discrete flavor symmetry), and others may be related by some GUT relation.
- String inspired models, composite models, ...

This is in no way a complete list, of course. Also, most of the work done actually falls into several categories above, showing that there are common ideas, which will hopefully lead us to a certain trail beyond the Standard Model.

Approximate flavor symmetries are described in section 2. We start with the basic observation that all fermion masses (except the top quark mass) are very small compared to the weak scale, suggesting that the Yukawa couplings are also small. This is easily understood in terms of the naturalness criterion[21], where the couplings are justified to be small if, in the limit that the couplings disappear, the theory gets a larger symmetry. And that is what actually happens; the Standard Model without Yukawa couplings has flavor symmetries. We introduce several ansätze for the way in which approximate flavor symmetries act on quarks and leptons and which are consistent with the observed masses and mixings. This leads us to interesting conclusions about the scale of flavor changing interactions and points us to which processes might be most promising for a discovery[4]-[6].

As soon as one allows flavor changing interactions, one might turn to cosmology and ask for consequences on the production of baryon asymmetry as seen today. It turns out that flavor changing interactions with approximate flavor symmetries would still be large enough to erase any primordial baryon asymmetry. Several solutions exist, such as the popular weak scale baryogenesis[22] in which there is no need for a primordial baryon asymmetry. In section 3, a different possibility is explained[23]. It is shown that the primordial baryon symmetry, in the presence of flavor changing interactions, can be saved by simply adding a new charged particle with its own asymmetry. Then, the requirement that the Universe is neutral, together with sphaleron processes, ensures survival of a baryon asymmetry.

A revival of interest in supersymmetric grand unified theories (SUSY GUTs) [24] was recently ignited by the success of the $\sin^2\theta_W$ prediction[25]. A number of models for Yukawa matrices were developed[15]-[19], and were shown to lead to

many successful predictions. Several aspects of these models are discussed in section 4. In particular, we will look at how general, regarding specific ansätze in these models, are the predictions for the relations among the Kobayashi-Maskawa mixing matrix[26] elements $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$ and $V_{td}/V_{ts} = \sqrt{m_d/m_s}$. Also, the problem of large $\tan\beta$ is discussed critically in light of recent $b \longrightarrow s\gamma$ measurements[27]. A large $\tan\beta$ can appear naturally in SO(10) models where the choice of flavor symmetries is such that the only nonzero renormalizable Yukawa coupling is that of the third generation. Finally, we discuss the flavor structure of several nucleon decay modes. We observe surprising enhancement of the decays in charged lepton modes[28], as opposed to more conventional SU(5) considerations.

2 Approximate flavor symmetries

2.1 Introduction

As more and more tests of the Standard Model confirm its predictions to ever higher accuracy, it becomes tempting to believe that new physics, especially if it involves flavor changing neutral currents, can only occur at energy scales very much larger than the weak scale. For example, $\Delta S = 2$ four fermion operators with coefficients $1/\Lambda^2$ give a $K_L - K_S$ mass difference $\Delta m_K/m_K \simeq (f_K/\Lambda)^2$ implying that $\Lambda \geq 1000$ TeV. The purpose of this section is to show that it is perfectly natural for physics involving new heavy scalars to occur at scales as low as the weak scale, 250 GeV, and to show that rare leptonic B meson decays will provide an excellent probe of this new physics.

In the Standard Model, the gauge interactions of the fermions

$$\mathcal{L}_0 = i\bar{Q}\not{D}Q + i\bar{U}\not{D}U + i\bar{D}\not{D}D + i\bar{L}\not{D}L + i\bar{E}\not{D}E \tag{2.1}$$

have a global symmetry $U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$, where Q_i and L_i are $SU(2)$ doublet quarks and leptons while U_i , D_i and E_i are $SU(2)$ singlets and $i = 1, 2, 3$ is a generation label. The Yukawa couplings

$$\mathcal{L}_Y = (\lambda_{ij}^{U^a}\bar{Q}_i U_j \frac{\tilde{H}_a}{\sqrt{2}} + \lambda_{ij}^{D^a}\bar{Q}_i D_j \frac{H_a}{\sqrt{2}} + \lambda_{ij}^{E^a}\bar{L}_i E_j \frac{H_a}{\sqrt{2}} + h.c.) \tag{2.2}$$

break the symmetry by varying degrees down to $U(1)_e \times U(1)_\mu \times U(1)_\tau \times U(1)_B$. The observed smallness of fermion masses (except maybe for the top quark) compared with the weak scale tells us that the flavor symmetries are approximately holding. We can think of Yukawa couplings as some higher order effects that break

the flavor symmetries in the underlying theory (which we do not know). For example, Froggatt and Nielsen[3] generate Yukawa couplings from higher dimensional operators which were generated by integrating out flavorless heavy fermions with mass M, as shown in Figure 1. When the heavy scalars ϕ_i, which carry the flavor of fermion f_i, get their vacuum expectation value, the flavor symmetry is broken, and we are left with effective Yukawa couplings suppressed by the ratios $\epsilon =< \phi_i > /M$.

However, here we are not interested in a specific model that generates Yukawa couplings. Rather, we will try to understand the general features of the idea that the flavor changing couplings between scalars and fermions are products of small parameters by which the flavor symmetries are broken.

In this section we introduce several forms for the way in which approximate flavor symmetries act on quarks and leptons. We then use this as a guide to infer the expected size of couplings between the known fermions and hypothetical, heavy scalar particles. The scalar mass M is then the only unknown parameter in the coefficient of the four fermion interactions induced by the exchange of this scalar. We derive the experimental limits on M from a variety of rare processes. If we, for the moment, assume the couplings to be real, the most powerful of these limits are of the order of the weak scale, giving hope to the possibility that we may discover physics at the weak scale much richer than in the minimal Standard Model.

There are two important advantages of our general approach. The scalar mass limits depend only on symmetry arguments and not on any specific model. Secondly, we can identify the most promising processes for discovering new physics in the next few years. In particular, we find that rare leptonic B decays are a very powerful probe of these new scalar interactions. For the case of leptoquarks, these

B decays will probe masses far above the present experimental limits. Also, the tree level exchange contribution to neutral K and B meson mass is at the level observed (or just below the current limit for ΔM_D[5]) by experiment.

An important application of our results is to flavor changing effects in models with many Higgs doublets[29]. We find that the approximate flavor symmetries, which we already know must be a part of any successful model of particle physics, are sufficient to make it natural to have any number of Higgs doublets coupling to up and down type quarks. In other words it is unnecessary to introduce discrete symmetries which act on Higgs doublets, as is so frequently done.

However, couplings need not be real in general and approximate flavor symmetries by themselves yield too much CP violation in the neutral kaon mass matrix. Hence in these theories CP must also be a good symmetry [5].

In the lepton sector, the masses are small indicating that the approximate flavor symmetries are preserved to a high degree of acuracy. Therefore lepton flavor changing interactions would be extremely hard to test [4]. Further, since the neutrino masses and mixing angles have not been measured yet, one cannot estimate the corresponding values of ϵs without additional assumptions. Nevertheless we find it important to address the question of approximate flavor symmetries in the lepton sector, because many experiments which aim to measure or set better limits on neutrino masses and mixings are under way or being planned for the near future. Indeed, we find that statements can be made about the lepton sector, regardless of additional assumptions about the ϵs[6]. For example, while the MSW solution to the solar neutrino problem can be easily fit, the predicted neutrino masses are unlikely to close the Universe. We study two cases of neutrino masses. In the

see-saw mechanism [30] case we show that the neutrino masses and mixings are independent of the right-handed neutrino flavor symmetry breaking mechanism. We also include for completness the case of Dirac neutrino masses only (as in the case of charged fermions). Then we provide several ansätze for the lepton flavor symmetry breaking parameters and list their predictions in terms of ratios of neutrino masses, and mixings. We study their relevance to the solar neutrino problem, atmospheric neutrino problem, closure of the Universe, etc. General features are noted which are independent of the particular ansatz used.

2.2 Approximate Flavor Symmetries

In the low energy theory flavor symmetries are broken by different amounts as is evident from the nonzero masses of fermions. We parametrize the lack of knowledge of the exact mechanism by which the symmetries were broken by a set of small parameters $\{\epsilon\}$, one for each of the chiral fermion fields, which describes the breaking of phase rotation invariance on each fermion. Thus λ^U_{ij} is suppressed by both ϵ_{Q_i} and ϵ_{U_j}. The idea is that the pattern of fermion masses and mixing angles can be described by the set $\{\epsilon\}$. However, this is not a precise numerical theory for fermion masses; equations of the form $\lambda^U_{ij} \approx \epsilon_{Q_i}\epsilon_{U_j}$ are only meant to be order of magnitude relations.

The lightness of the up quark tells us that flavor symmetries strongly suppress the $\bar{Q}_1 U_1$ operator. However, the mass eigenvalue does not allow us to infer whether this is because the approximate flavor symmetry is acting only on Q_1, only on U_1, or on both. However, we need to know whether the coefficient of a scalar coupling to $Q_1 X$ (where X is any fermion other than U_1) is strongly suppressed because

the up quark is very light.

We now argue that the approximate symmetries act both on left- and right-handed fields:

- The flavor symmetries do not act just on the right-handed fields because otherwise $U_3 \approx t_R$ couples to $(\alpha Q_1^u + \beta Q_2^u + \gamma Q_3^u) = t_L$ and $D_3 \approx b_R$ couples to $(\alpha' Q_1^u + \beta' Q_2^u + \gamma' Q_3^u) = b_L$ where $\alpha, \beta, \gamma, \alpha', \beta', \gamma'$ are arbitrary mixing angles of order unity, so that t_L and b_L would have no reason to be very nearly in the same $SU(2)_L$ doublet.

- The flavor symmetries do not act just on the left-handed fields because in this case the approximate flavor symmetries make no distinction between λ^U and λ^D. A large m_t/m_b ratio could be due to a large ratio of vevs v_2/v_1 in a two Higgs theory, but this would lead to an unacceptably large m_u/m_d ratio. In addition the KM angles are given by linear mass relations such as $\theta_c \approx m_d/m_s$ rather than the more successful square root form $\theta_c \approx \sqrt{m_d/m_s}$.

Therefore we conclude that *the underlying theory must have approximate flavor symmetries that act on both left- and right-handed fields* .

The approximate flavor symmetries and the associated set of small symmetry breaking parameters $\{\epsilon\}$ are defined on flavor eigenstates. In practice it is much more useful to know what suppression factors are induced on the mass eigenstates. Consider the up-type quarks. Assume that $\epsilon_{Q_i} \ll \epsilon_{Q_j}$ and $\epsilon_{U_i} \ll \epsilon_{U_j}$ for $i < j$, as suggested by $m_{U_i} \ll m_{U_j}$. Then the mass matrix is diagonalized by unitary rotations on the Q_i by a matrix with elements $|V_{ij}| \approx \epsilon_{Q_i}/\epsilon_{Q_j}$ $(i < j)$ and on the U_i by a matrix with elements $|V_{ij}^R| \approx \epsilon_{U_i}/\epsilon_{U_j}$ $(i < j)$. Relations between flavor and

mass eigenstates (Q_i') are of the form $Q_1' = Q_1 + O(\epsilon_{Q_1}/\epsilon_{Q_2})Q_2 + O(\epsilon_{Q_1}/\epsilon_{Q_3})Q_3$. This shows the important result that the flavor breaking parameters $\{\epsilon\}$ apply to mass eigenstates as well as to flavor eigenstates. For example, the three flavor eigenstate contributions to Q_1' all carry the same approximate flavor symmetry suppression factor of ϵ_{Q_1}.

The Yukawa matrices $\lambda^U, \lambda^D, \lambda^L$ contain a great deal of information about the form of the breaking of flavor symmetry. The quark mass matrices can be constructed approximately, under the assumption that they come from approximate flavor symmetries, from quark masses (i.e., the Yukawa matrices λ^U, λ^D eigenvalues) and the Kobayashi-Maskawa (KM) matrix[5]. Alternatively, we can give a simple ansatz for the flavor symmetry breaking parameters that fits known quark masses and mixings. Unfortunately, we cannot reconstruct the lepton matrices from the information which can be obtained from experiments, namely from the charged lepton masses and upper limits on neutrino masses and mixings. This information is insufficient to derive the form of the approximate lepton flavor symmetries which the underlying theory must have. Nevertheless it provides a strong guideline for giving simple predictive ansätze for the symmetry breaking paramaters.

A simple predictive ansatz for all the symmetry breaking parameters is shown in Table 1. It involves both left- and right-handed fermions and is predictive because it only involves quark and lepton masses. The rationale behind our choice is as follows. For the leptons, the flavor symmetries on L_i and E_i are equally responsible for suppressing the Yukawa couplings[31]. For quarks, we have again tried to have both left- and right- handed flavor symmetries equally responsible

for suppressing Yukawa couplings. However since Q_i appears in both up and down mass operators, we have taken the symmetry breaking parameter ϵ_{Q_i} to be the geometric mean of that expected from m_{U_i} and that expected from m_{D_i}. Note that we have allowed for a two Higgs doublet model. With only one Higgs boson $v_1 = v_2 = 250\,\mathrm{GeV}$. The ansatz gives reasonable values for the CKM mixing angles. $V_{ij} \approx \epsilon_{Q_i}/\epsilon_{Q_j} \approx (m_{U_i} m_{D_i}/m_{U_j} m_{D_j})^{\frac{1}{4}}, i < j$, which is correct at the factor of 2 level.

We use our ansatz to estimate the size of the Yukawa couplings and then the corresponding rates for various processes induced by the effective four fermion couplings. In Table 2 we list the limits on the exchanged scalar mass [32] obtained from a variety of experiments. For now we assume the scalar exchange does induce each process and that the flavor symmetry acts only on fermions. Once again, we obtained these numbers using our ansatz to estimate the Yukawa couplings, so we expect the values to be reliable up to a factor of perhaps 2 or 3.

The factor κ represents the ratio of the matrix element of the new four fermion operator relative to its vacuum insertion value. In the radiative μ decay, the τ-lepton contribution dominates in the loop because it has the largest Yukawa couplings.

First, we can see that these limits are nowhere near as strong as those for vector exchange [33]. Flavor nonconserving theories at the weak scale are not ruled out at all. Secondly, if the uncertainty factors of 2 or 3 go the right way, it is possible that the rare leptonic B_s decay will be the first place to discover this new physics, considering that branching ratios $10^{-7} - 10^{-8}$ will be obtained in the near future [34]. The branching ratio prediction for $B_s \to \mu^+\mu^-$ is about 10^{-9} in the Standard Model, and in two Higgs doublet models with discrete symmetries[35].

Also, we mention that the ΔM_D is expected to be close to its experimental limit, and that the exotic decays of the top quark $t \to bh^+$ and $t \to ch^0$ are expected to be large for small top quark masses and would probably prevent the t quark detection through the standard channel $t \to bW^+$[5].

There are cases where the scalar cannot induce all the processes considered, as in leptoquark models. The tree level exchange of leptoquarks generates four fermion operators which contain two quarks and two leptons. The limits from $K\bar{K}$ and $B\bar{B}$ mixing are therefore removed. In this case our results are particularly important: the rare leptonic B decay modes provide the most stringent test of models with scalar leptoquarks.

We discuss briefly the case in which approximate flavor symmetries act on the exchanged scalar too. Such is the case in R-parity violating supersymmetric models [36] where the exchanged scalar is a slepton or a squark which carries the same approximate flavor symmetry as its fermion partners. Then in Table 2 all mass limits are lowered by an additional symmetry breaking factor ϵ_a, where the approximate symmetry of type "a" is carried by the scalar. It is even less likely that such theories could have been excluded.

2.3 Many Higgs doublet model

In this subsection we apply our results to the case of the minimal Standard Model extended only by the addition of an arbitrary number of Higgs doublets. In this case it is already known that, for the special case of Fritzsch-like Yukawa matrices, the additional scalars need not be heavier than a TeV [11]. However, our results are independent of the particular texture and depend only on the approximate

flavor symmetry.

Let us look at a two Higgs doublet model (the generalization to many Higgs doublets is trivial). For example, the up quark Yukawa couplings are

$$\begin{pmatrix} \bar{Q}_1 & \bar{Q}_2 & \bar{Q}_3 \end{pmatrix} \begin{pmatrix} \sim \epsilon_{Q_1}\epsilon_{U_1} & \sim \epsilon_{Q_1}\epsilon_{U_2} & \sim \epsilon_{Q_1}\epsilon_{U_3} \\ \sim \epsilon_{Q_2}\epsilon_{U_1} & \sim \epsilon_{Q_2}\epsilon_{U_2} & \sim \epsilon_{Q_2}\epsilon_{U_3} \\ \sim \epsilon_{Q_3}\epsilon_{U_1} & \sim \epsilon_{Q_3}\epsilon_{U_2} & \sim \epsilon_{Q_3}\epsilon_{U_3} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} \frac{\tilde{H}_1}{\sqrt{2}}$$

$$+ \begin{pmatrix} \bar{Q}_1 & \bar{Q}_2 & \bar{Q}_3 \end{pmatrix} \begin{pmatrix} \sim \epsilon_{Q_1}\epsilon_{U_1} & \sim \epsilon_{Q_1}\epsilon_{U_2} & \sim \epsilon_{Q_1}\epsilon_{U_3} \\ \sim \epsilon_{Q_2}\epsilon_{U_1} & \sim \epsilon_{Q_2}\epsilon_{U_2} & \sim \epsilon_{Q_2}\epsilon_{U_3} \\ \sim \epsilon_{Q_3}\epsilon_{U_1} & \sim \epsilon_{Q_3}\epsilon_{U_2} & \sim \epsilon_{Q_3}\epsilon_{U_3} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} \frac{\tilde{H}_2}{\sqrt{2}}$$

and similarly for down type quark matrices, keeping in mind that each entry in the matrices is uncertain by a factor of 2 or 3 (denoted by $\sim$).

Notice that because of the numerical factors in front of the ϵs, the matrices for H_1 and H_2 are not equal in general. That means that if we diagonalize the matrix of H_1, the matrix of H_2 will not be diagonalized and will keep the same general form as above. In particular, we can always choose H_1 to be the only doublet that acquires a vev (rotating the doublets will not change the form of matrices). Therefore we see that in the quark mass eigenstate basis, the new Higgs H_2 couplings are not diagonal: we have flavor changing couplings. The nice thing now is that since the flavor changing couplings are small the stringent experimental limits on flavor changing neutral currents (FCNC) actually translate only into lower limits on the mass of new scalars, about 1 TeV, as we saw above for the plausible ansatz.

To avoid problems with large flavor-changing neutral currents, Glashow and Weinberg [29] argued that only one Higgs doublet could couple to up-type quarks

and only one Higgs to down-type quarks. However, this naturality constraint, known as the Glashow-Weinberg criterion (or natural flavor conservation (NFC)), was based on an unusual definition of what is "natural." For them the avoidance of flavor-changing neutral currents was natural in a model only if it occured for all values of the coupling constants of that model. For us a model will be natural provided the smallness of any coupling is guaranteed by approximate symmetries [21], and we find that this implies the Glashow-Weinberg criterion is not necessary.

Notice that if the matrix for H_2 is nearly equal to the matrix for H_1, then diagonalization of the H_1 matrix will almost diagonalize the second matrix. In this case the flavor changing couplings become even smaller. This is of course no surprise, because if the two matrices were exactly equal then only one linear combination of Higgses $(H_1 + H_2)$ couples to the quarks: we have NFC! This is actually the starting point of Leurer, Nir and Seiberg [7]; i.e., use broken flavor symmetries in combination with weakly broken NFC.

One potential problem arises from the smallness of the observed CP violation, as noted by Hall and Weinberg[5]. The CP violating parameter $\epsilon_{CP} = Im(\Delta M_K)/\sqrt{2}|\Delta M_K|$ would naively be expected to be of order unity in our case, contrary to the observed value of 10^{-3}. To avoid this problem one might go back to NFC, or, more in the philosophy of naturalness of small couplings, just say that CP is another approximately conserved quantity broken by ϵ_{CP}.

2.4 Lepton sector

By adding the right-handed neutrinos N_i, $i = 1, 2, 3$, to the particle content of the Standard Model we can allow for Dirac type masses. Under the action of

approximate flavor symmetries, whenever an N_i enters a Yukawa interaction, the corresponding coupling must contain the symmetry breaking parameter ϵ_{N_i}.

A natural way to justify the smallness of neutrino masses is to use the *see-saw mechanism* in which the smallness of the left-handed neutrino masses is explained by the new scale of heavy right-handed neutrinos. The mass matrices will have the structure

$$m_{N_{Dij}} \approx \epsilon_{L_i}\epsilon_{N_j} v_{SM} \,, \tag{2.3}$$

$$m_{N_{Mij}} \approx \epsilon_{N_i}\epsilon_{N_j} v_{Big} \,, \tag{2.4}$$

$$m_{E_{ij}} \approx \epsilon_{L_i}\epsilon_{E_j} v_{SM} \,, \tag{2.5}$$

where m_{N_D} and m_E are the neutrino and charged lepton Dirac mass matrices, m_{N_M} is the right-handed neutrino Majorana mass matrix, $v_{SM} = 174\,\mathrm{GeV}$ and v_{Big} is the new large mass scale. The generation indices i and j run from 1 to 3. In the following we assume a hierarchy in the ϵs (i.e. $\epsilon_{L_1} << \epsilon_{L_2} << \epsilon_{L_3}$, etc.) as suggested by the hierarchy of quark and charged lepton masses. Then the diagonalization of the neutrino mass matrix will give a heavy sector with masses $m_{N_{Hi}} \approx \epsilon_{N_i}^2 v_{Big}$ and a very light sector with mass matrix

$$m_{N_{Lij}} \approx (m_{N_D} m_{N_M}^{-1} m_{N_D}^T)_{ij} \approx \epsilon_{L_i}\epsilon_{L_j}\frac{v_{SM}^2}{v_{Big}} \,, \tag{2.6}$$

where the number $Tr(\epsilon_N(\epsilon_N\epsilon_N)^{-1}\epsilon_N)$ is assumed to be of order unity. We have the expected result: the heavy right-handed neutrino decouples from the theory leaving behind a very light left-handed neutrino. The masses and mixing angles are independent of the right-handed symmetry breaking parameters ϵ_{N_i}:

$$m_i^N \approx \epsilon_{L_i}^2 \frac{v_{SM}^2}{v_{Big}} \,,$$

$$m_i^E \approx \epsilon_{L_i}\epsilon_{E_i}v_{SM} \quad (no\ sum\ on\ i)\ ,$$

$$V_{ij} \approx \frac{\epsilon_{L_i}}{\epsilon_{L_j}} \quad (i<j)\ . \tag{2.7}$$

Therefore, besides the unknown scale v_{Big}, only two sets of ϵs are needed: ϵ_{L_i} and ϵ_{E_i}. In fact, the neutrino masses and mixings depend only on ϵ_{L_i} and they are approximately related through

$$V_{ij} \approx \sqrt{\frac{m_i^N}{m_j^N}}\ . \tag{2.8}$$

Equation (2.8) reduces the number of parameters needed to describe neutrino masses and mixings by three; for example, given two mixing angles and one neutrino mass, we can predict the third mixing angle and the other two neutrino masses. These results are extremely general. They follow simply from the factorization of the Dirac masses, regardless of the specific form of $m_{N_M}^{-1}$, which only contributes to set the scale.

For completeness, we note that in the case of *Dirac masses only*, the neutrino and the charged lepton mass matrices become

$$m_{N_{Dij}} \approx \epsilon_{L_i}\epsilon_{N_j}v_{SM}\ , \tag{2.9}$$

$$m_{E_{ij}} \approx \epsilon_{L_i}\epsilon_{E_j}v_{SM}\ . \tag{2.10}$$

Their diagonalization yields

$$m_i^N \approx \epsilon_{L_i}\epsilon_{N_i}v_{SM} \quad (no\ sum\ on\ i)\ ,$$

$$m_i^E \approx \epsilon_{L_i}\epsilon_{E_i}v_{SM} \quad (no\ sum\ on\ i)\ , \tag{2.11}$$

whereas the lepton mixing matrix is approximately diagonal with off diagonal elements of the order of

$$V_{ij} \approx \frac{\epsilon_{L_i}}{\epsilon_{L_j}} \quad (i<j)\ . \tag{2.12}$$

Therefore in this case we need three sets of ϵs to explain masses and mixings (cf. Eqns.(2.11)-(2.12)): ϵ_{L_i}, ϵ_{N_i} and ϵ_{E_i}.

In addition to having one more set of (unknown) ϵs than the *see-saw* case, the *Dirac only* case has no natural explanation for the very low scale associated with the neutrino masses, i.e. that the flavor symmetries of right-handed neutrinos are much better preserved than for other fermions. It turns out that one typically gets the ϵ_{N_i} of order 10^{-12} or so, compared to quark and charged lepton ϵs which are typically 10^{-3} and larger. Therefore, in the following we will mostly concentrate on the *see-saw* case.

While it was possible to fix quantitatively the flavor symmetry breaking parameters in the quark sector from quark masses and mixings, in the lepton sector the situation is much more difficult. At this time direct laboratory experiments provide only upper limits on neutrino masses and mixings, although some indirect sources like solar neutrinos or cosmology point to some specific allowed ranges. Therefore, to estimate the sizes of lepton flavor symmetry breaking parameters ϵs, additional assumptions are needed.

Our strategy is as follows: we list several plausible or GUT motivated ansätze which relate ϵs of different fields. This will enable us to estimate ratios of neutrino masses and mixings (listed in Table 3). If the mixings are consistent with the allowed range for the MSW solution[37] of the solar neutrino problem, we take this as a hint to fix the mass scale and predict all neutrino masses. We then look at further predictions. As is the case with any calculation based on these approximate flavor symmetries ([3]-[7]), the factors of two or three may contribute coherently

factors of an order of magnitude or so. In addition, the numerical results depend on the specific ansatz. What we seek are the general features of those results rather than the detailed numerical results themselves.

I. As our first ansatz we assume that $\epsilon_{L_i} = \epsilon_{E_i}$ [4]. This ansatz can be justified as follows [38]. Assume that in the lepton sector the only combination of symmetry which is broken is the vector flavor symmetry. This means that, instead of breaking separately left(L) and right(R) flavor symmetries, only the combination L-R is broken. Therefore we need only one set of ϵs, which are then determined from $\epsilon_{L_i} \approx \sqrt{\frac{m_i^E}{v_{SM}}}$. The $\nu_e - \nu_\mu$ mixing found in this way is consistent with the small mixing angle region[39] of the MSW explanation for the Solar Neutrino Problem (SNP). The mixing angles for this and the other ansätze can be found in table 3. We checked that for these mixing angles the third flavor does in fact decouple (see [40]). If this is indeed the solution for that problem, the mass of the muon neutrino must be around 3×10^{-3} eV. This then sets the scale for the new physics and the other neutrino masses at

$$v_{Big} \approx \frac{\epsilon_{L_2}^2 v_{SM}^2}{m_{\nu_\mu}} \approx 10^{13}\,\text{GeV} , \tag{2.13}$$

$$m_{\nu_e} \approx \frac{\epsilon_{L_1}^2}{\epsilon_{L_2}^2} m_{\nu_\mu} \approx 10^{-5}\,\text{eV} ,$$

$$m_{\nu_\tau} \approx \frac{\epsilon_{L_3}^2}{\epsilon_{L_2}^2} m_{\nu_\mu} \approx 10^{-2}\,\text{eV} . \tag{2.14}$$

Taking into account the excluded regions due to the IMB experiment [41], the predicted muon to tau neutrino mixing ($\Delta m^2 \approx 10^{-3}\,\text{eV}^2$, $\sin^2(2\theta_{\mu\tau}) \approx 0.2$) is around a factor of five away from the parameters consistent with a $\nu_\mu - \nu_\tau$ oscillation explanation of the Atmospheric Neutrino Problem (ANP) [42], and still two orders of magnitude away from the laboratory limits for this mixing. Finally, the small-

ness of all three neutrino masses in this ansatz suggests that neutrinos cannot be responsible for closing the Universe.

II. Another interesting ansatz is suggested by the fact that in the quark sector $\epsilon_{Q_i} \approx \epsilon_{U_i} \quad i = 1,2,3$, as found by Hall and Weinberg [5]. Inspired by an $SU(5)$ type of unification we are led to look at an ansatz in which

$$\epsilon_{L_i} \propto \epsilon_{D_i} \ , \qquad \epsilon_{E_i} \propto \ \epsilon_{Q_i} \approx \epsilon_{U_i} \ . \tag{2.15}$$

In this ansatz we *predict* (using the numerical values of ϵ_Q, ϵ_U and ϵ_D from [5]) the ratios of charged lepton masses to be within factors of three of the measured values. We consider this an interesting result. Further, the mixing angles are consistent with the three flavour mixing explanations of the SNP [40] for squared masses of ν_μ and ν_τ of order 10^{-4} eV^2. Therefore, the $\nu_\mu - \nu_\tau$ oscillation explanation of the ANP is unlikely. In addition this mass scale cannot be tested in the laboratory nor provide an explanation for Dark Matter.

III. Finally, one might look for inspiration in the breaking of $SO(10)$ into $SU(4) \otimes SU(2)_L \otimes SU(2)_R$. We know that at the renormalization scale of 1 GeV we have been considering, the $SU(2)_R$ symmetry must be badly broken since $m_t >> m_b$ and $m_e >> m_{\nu_e}$. Further, assuming the ansatz $\epsilon_{L_i} \propto \epsilon_{Q_i}$, $\epsilon_{E_i} \propto \epsilon_{D_i}$, and $\epsilon_{N_i} \propto \epsilon_{U_i}$ would lead to $m_e/m_\mu \approx 4.8 \times 10^{-2}$, which is wrong by an order of magnitude. Assuming that the $SU(4)$ might still provide some useful information for the $SU(2)_L$ singlets we look at the ansatz

$$\epsilon_{E_i} \propto \epsilon_{D_i} \ , \qquad \epsilon_{N_i} \propto \epsilon_{U_i} \ . \tag{2.16}$$

In this ansatz we again predict a $\nu_e - \nu_\mu$ mixing angle consistent with the small angle MSW solution to the SNP. Again, assuming that this is indeed the solution

for that problem fixes the see-saw neutrino masses at

$$
\begin{aligned}
m_{\nu_\mu} &\approx 10^{-3}\,\mathrm{eV}\ , \\
m_{\nu_e} &\approx (\frac{\epsilon_{L_1}}{\epsilon_{L_2}})^2 m_{\nu_\mu} \approx 10^{-6}\,\mathrm{eV}\ , \\
m_{\nu_\tau} &\approx (\frac{\epsilon_{L_3}}{\epsilon_{L_2}})^2 m_{\nu_\mu} \approx 0.5\,\mathrm{eV}\ .
\end{aligned} \tag{2.17}
$$

We again find it unlikely that the values obtained can close the Universe or solve the ANP.

2.5 Conclusion

In this section we have introduced a simple ansatz for how the approximate flavor symmetries act on quarks and leptons, as shown in Table 1. It reproduces the KM matrix elements at the factor of 2 level. If the interactions of additional scalars respect these approximate symmetries, then the mass limits on the scalars from various experiments are shown in Table 2. From this viewpoint, new flavor changing physics at the weak scale is not excluded, and is natural. In particular, extra Higgs doublets can couple to both up and down type quarks; there is no need to impose additional discrete symmetries on the scalars. We find that rare leptonic B decay modes, such as $B_s^0 \to \mu^+\mu^-$, could uncover this new scalar-mediated physics in the coming years.

We also extended the concept of approximate flavor symmetries to the lepton sector. In particular we considered the see-saw mechanism as a source of the neutrino masses and showed that the predictions do not depend on the neutrino flavor symmetry breaking parameters. This yelds a simple relation (cf. eq. (2.8)) between neutrino masses and mixing angles which reduces the number of parame-

ters needed to describe the lepton sector. The lack of information on the neutrino masses and mixing angles led us to propose several ansätze. These exhibit the following common features. They are consistent with the MSW solution of the SNP. The ANP is unlikely to be explained through $\nu_\mu - \nu_\tau$ oscillations and the scale of neutrino masses is too small to close the Universe.

3 Preventing the erasure of baryon asymmetry in the early Universe

3.1 Introduction

Various authors[43, 44] have placed cosmological bounds on the size of baryon and lepton number violating interactions in theories where baryogenesis occurs before the electroweak phase transition. The baryon asymmetry of the Universe is threatened by a combination of these interactions and a large electroweak instanton rate[45, 46]. Electroweak instanton interactions are expected to be in equilibrium for temperatures above T_{min}, approximately the weak breaking scale, up to some very high temperature $T_{max} \simeq 10^{12}$ GeV. Such reactions create $SU(2)_L$ transforming fermions out of the vacuum[45]. Lepton and baryon violating interactions such as R-parity breaking terms in supersymmetry[47] or Majorana neutrino masses, when in equilibrium simultaneously with instanton reactions, can break all linear combinations of conserved quantum numbers which involve baryon number.

In the previous section the many Higgs doublet model had interactions which break all flavors. Let us check whether these interactions erase the primordial flavor asymmetry. A 4-dimensional interaction (like the interactions of scalars and fermions in the previous section) with coupling λ have a thermal equilibrium rate $\Gamma_4 \sim \lambda^2 T$. This interaction falls out of equilibrium when its rate is less than the rate of the expansion of the Universe $\Gamma_H \sim 17T^2/M_P$. In order to prevent a baryon asymmetry washout, such a flavor changing interaction must fall out of equilibrium before the sphaleron interactions do. The strongest limit on the coupling λ then arises at the lowest temperature where the sphalerons are in equilibrium, namely

about 100 GeV or so. This gives a limit of about $\lambda < 10^{-8}$. In the previous section the couplings which are smallest are the electron flavor changing couplings. However, the largest among them are still of the order $10^{-3} - 10^{-4}$.

Naively, one is led to believe that the baryon asymmetry of the Universe is therefore washed away. In this section we examine the general circumstances in which this outcome is avoided. We find that in many models there will be additional symmetries and, even though these symmetries apparently have nothing to do with baryon number, they automatically lead to a protection of it.

It is well known that a symmetry which involves baryon number itself, such as $B - 3L_i$, can preserve the baryon asymmetry [44]. Approximate symmetries involving B have been found in the minimal supersymmetric model which can be used to help prevent erasure of the baryon asymmetry[48]. We have found that the protection of the baryon asymmetry is extremely common and is a typical feature of theories with extra symmetries, even when those symmetries do not transform quarks. We illustrate this by a very simple example: assume that there exists a particle, X, which carries hypercharge but not SU(2) or SU(3) gauge interactions[49]. Assume that reactions occurring at temperatures well above T_{min} generate an asymmetry in the X species, and that at lower temperatures the reactions which change X number are sufficiently weak that this X asymmetry persists. A crucial role is played by the requirement that the early Universe is hypercharge neutral. Because X particles carry hypercharge, the asymmetry in their number contributes to the hypercharge density of the Universe. The remaining particles in the theory must carry an opposite hypercharge density to cancel this. Chemical equilibrium equations specify how this hypercharge density is shared. A baryon

asymmetry can develop either through added B violating interactions or once the weak instanton becomes effective. In general any X asymmetry together with chemical equilibrium requires a baryon asymmetry[51]. This illustrates just how easy it is to preserve the baryon asymmetry and, to our way of thinking, puts the issue of direct detection of baryon and lepton number violation back where it belongs: with the experimentalists.

3.2 General condition for survival of a baryon asymmetry

In this section we discuss, in a very general way, the conditions under which an extra $U(1)$ symmetry preserves the cosmological baryon asymmetry.

In thermodynamic equilibrium the number density of particle species i is determined by its chemical potential, μ_i. If a given reaction, say $p_1 + p_2 \rightleftharpoons p_3 + p_4$, is in equilibrium then $\mu_1 + \mu_2 = \mu_3 + \mu_4$. It is straightforward, yet tedious, to solve all chemical equilibrium equations. One can simplify the process by noticing that these equations are the same equations one would write down to determine the $U(1)$ symmetries of the equilibrium theory. One need only replace μ_i with q_i, the charge of particle i. Solving for q_i determines the possible assignments of $U(1)$ charge to each particle so that all equilibrium reactions conserve that charge. In general such $U(1)$ symmetries need not be exact symmetries of the Lagrangian. They are symmetries of those interactions in thermal equilibrium at temperature T, and we refer to them as effective $U(1)$ symmetries at this temperature.

Thus, a solution to the chemical equilibrium equations corresponds to an assignment of effective $U(1)$ charges to each particle, and the possible effective $U(1)s$ in a given theory are usually easy to identify. Suppose that at a certain

temperature there are N such effective U(1)s: $U(1)_A, A = 1, ...N$; then the most general solution is

$$\mu_i = \sum_A C^A q_i^A \,, \tag{3.1}$$

where q_i^A is the charge of particle i under $U(1)_A$. The constant C^A we refer to as the asymmetry constant for $U(1)_A$. As soon as some interaction which violates $U(1)_A$ comes into thermal equilibrium, C^A rapidly tends to zero: $U(1)_A$ is no longer able to support particle asymmetries.

This general solution is restricted, however. We assume that the Universe is homogeneous and that no charge asymmetry has developed for the unbroken gauged $U(1)s$ of the theory[52]. This forces the charge density for these $U(1)s$ to zero. We can write the charge density for $U(1)_A$ as

$$Q^A = \sum_i q_i^A \, n_i \,, \tag{3.2}$$

where n_i is the particle asymmetry density of species i. If particle asymmetry densities are small then they can be written, for $T \gg m_i$, as

$$n_i \simeq \frac{T^2}{6} \tilde{g}_i \mu_i \,, \tag{3.3}$$

where $\tilde{g}_i$ is the number of internal degrees of freedom of particle i, g_i, multiplied by a factor of two for bosons. (However, see reference [53] for an interesting look at small mass effects.) Under these conditions the charge density constraint is a simple linear equation in the μ_is. Q^A can be written using n_i from (3.3) and μ_i from (3.1):

$$Q^A \simeq \frac{T^2}{6} \sum_B C^B \sum_i \tilde{g}_i q_i^B q_i^A = \frac{T^2}{6} \sum_B C^B \, \overline{B} \cdot \overline{A} \,, \tag{3.4}$$

where we define $\overline{B} \cdot \overline{A}$ by

$$\overline{B} \cdot \overline{A} = \sum_i \tilde{g}_i q_i^B q_i^A \,. \tag{3.5}$$

Should the diagonal generators of non-Abelian gauge groups, such as T_{3L}, be included in the list of effective $U(1)s$? The answer is no, as can be seen easily from the above equations. Call such a generator α, then neutrality of the Universe with respect to this charge requires

$$\sum_A C^A \overline{A} \cdot \overline{\alpha} = 0 \,, \tag{3.6}$$

where

$$\overline{A} \cdot \overline{\alpha} = \sum_i \tilde{g}_i q_i^A q_i^\alpha \,. \tag{3.7}$$

When A refers to a $U(1)$ generator (not embedded in a non-Abelian gauge group) then $\overline{A} \cdot \overline{\alpha} = 0$. This is because the $\tilde{g}_i$ and q_i^A are the same for all components of an irreducible representation of α, and hence the sum in (3.7) can be written as a sum of zero terms, one for each irreducible multiplet of α. When $A = \beta$ is a diagonal generator of a non-Abelian group the orthogonality property of the generators within each multiplet ensures that $\sum_i q_i^\alpha q_i^\beta$ vanishes for $\beta \neq \alpha$. Hence the sum in (3.6) just has one term: $C^\alpha \overline{\alpha} \cdot \overline{\alpha} = 0$. Since $\overline{\alpha} \cdot \overline{\alpha} \neq 0$, we have proved that $C^\alpha = 0$ follows from $Q^\alpha = 0$. This implies that such $U(1)s$ need not be included in the list of effective $U(1)s$.

Now let us apply this formalism. We are interested in the situation in which additional particles and interactions have been added to the Standard Model such that at temperatures T, $T_c < T < T_{max}$, where T_c is the weak breaking temperature, there are just two effective $U(1)s$: Y and X, where $Y = 2(Q - T_3)$ denotes

hypercharge and X is an ungauged effective symmetry. The charge neutrality condition (3.6) when applied to hypercharge gives

$$C^Y = -\frac{\overline{X}\cdot\overline{Y}}{\overline{Y}^2}C^X\,. \tag{3.8}$$

Using (3.8) in equation (3.4) the asymmetry in baryon number is just

$$n_B \simeq \frac{T^2}{6}C^X\left(\overline{X} - \frac{\overline{X}\cdot\overline{Y}}{\overline{Y}^2}\overline{Y}\right)\cdot\overline{B}\,, \tag{3.9}$$

where we have rewritten Q^B, the baryon density, as n_B. This is the general result of this paper. Any effective $U(1)_X$, whether or not it contains a piece of baryon number, will in general contribute to n_B if $C^X \neq 0$. The extension of (3.9) to many extra X symmetries is straightforward. Providing such a $U(1)_X$ exists, there is no limit to how large the B and L violating interactions can be.

We will examine the case in which X particles carry no baryon number themselves. Then

$$n_B \simeq \frac{T^2}{6}C^X\left(-\frac{\overline{Y}\cdot\overline{B}}{\overline{Y}^2}\right)\overline{X}\cdot\overline{Y}\,. \tag{3.10}$$

In the Standard Model $\overline{Y}\cdot\overline{B}/\overline{Y}^2 = \frac{1}{11}$. Additional particles will change this, but would generally give some non-zero value which we call α. Then $n_B \simeq -\frac{T^2}{6}\alpha C^X\left(\overline{X}\cdot\overline{Y}\right)$. Thus to obtain $n_B \neq 0$ we require that some particles with $X_i \neq 0$ have $Y_i \neq 0$. Hypercharge neutrality then forces other particles to have an asymmetry, some of which carry baryon number, thus providing a baryon asymmetry.

Cline *et al.* [50] point out that in the Standard Model right handed electron number is conserved down to a temperature of about 10 TeV, and thus can insure that baryon/lepton violating interactions do not wash away the baryon asymmetry

above this temperature. Thus the Standard Model already contains X particles in the form of right handed electrons. In section 3.3, we discuss another possibility, an X symmetry which does not transform any Standard Model particles. In this case (3.8) can be rewritten in terms of the hypercharge density carried by the Standard Model sector, $Q^Y(SM)$, and by the X sector, $Q^Y(X) \equiv \sum_i q_i^Y n_{X_i}$:

$$Q^Y(SM) = Q^Y(X)\,. \tag{3.11}$$

In terms of $Q^Y(X)$ equation (3.10) becomes

$$n_B \simeq -\frac{1}{11} Q^Y(X)\,. \tag{3.12}$$

(We have assumed that $T < 10\,\mathrm{TeV}$ so that right handed electrons are in equilibrium.)

Equation (3.12) does not assume that X number density is small or proportional to its chemical potential. Thus it is valid even when the temperature drops below the mass of certain X particles. When this happens the heavier species carrying X might decay into lighter ones. Nevertheless, providing the particles with $X \neq 0$ possess a hypercharge asymmetry the baryon asymmetry will survive. In particular the $X \neq 0$ particles must continue to carry such an asymmetry until a temperature T_0, beneath which B and L violating reactions are sufficiently weak that a symmetry having a baryon number component has become an effective $U(1)$. The resulting baryon asymmetry after X decay depends on the specifics of the model. In the least complicated scenario, in which baryon number is a good symmetry below T_0, today's baryon asymmetry is simply derived from (3.12) and entropy considerations.

We note that it is not necessary for our X sector to be neutral under $SU(2)$. Adding additional $SU(2)$ transforming fermions to the Standard Model will mean that these particles also take part in instanton mediated reactions. Nevertheless, in a consistent theory, instanton reactions will conserve the hypercharge asymmetry carried by the X sector of the theory. This is true because instantons neither violate hypercharge in the Standard Model sector nor in the theory overall, and thus must conserve hypercharge in the X-sector.

In this section we have tacitly assumed that some component of baryon number is a good symmetry below T_c, the weak breaking temperature. If this is not the case, then, for temperatures T, $T_0 < T < T_c$, the role of hypercharge is played by electric charge. In this case the X sector must carry an electric charge asymmetry.

An intriguing possibility exists if the lightest X particle is stable and electrically neutral. If this is the case, the particle is a candidate for the dark matter in the Universe [54, 55]. To realize such a scenario, the X sector would still have to maintain a hypercharge asymmetry for temperatures above T_0. (For convenience,we have assumed $T_0 \geq T_c$.) However, at a lower temperature, charged X particles would decay to Standard Model particles plus these electrically neutral X particles. If Ω_X is the fraction of the critical density contributed by the electrically neutral X particles then their mass is given by

$$\frac{m_X}{m_{proton}} \simeq \frac{\langle q_X \rangle}{11} 10^2 \Omega_X \,, \tag{3.13}$$

where $\langle q_X \rangle$ is the appropriate average of X-particle hypercharges. Low-background Ge detector experiments [56, 57] indicate that an electrically neutral dark matter particle with nonzero hypercharge must have a mass greater than $\sim 1000\,\mathrm{GeV}$. Thus, we can effectively rule out a dark matter X particle with nonzero hyper-

charge. One possible candidate is the neutral component of a new hyperchargeless $SU(2)$ multiplet. Such a particle is expected to interact via loop diagrams with nuclei and thus its cross section with Ge is approximately $10^{-35}\,\mathrm{cm}^2$ or smaller [55], effectively evading relevant experimental limits [56]. Another candidate is a new particle with no gauge interactions whatsoever [54].

3.3 A simple model

In this section we illustrate the general ideas discussed above with a very simple model. We add to the Standard Model a single fermion X, of mass m_X, which is SU(2) neutral but has three units of electric charge. It is unstable, decaying to three charged leptons via the effective interaction

$$\frac{1}{2}\frac{1}{M^2}\sum_{ijk} f_{ijk}\left(\overline{X}\, e_R^i\right)\left((e_R^j)^T C e_R^k\right) \;+\; h.c.\,, \tag{3.14}$$

where e_R^i is the right handed lepton field of flavor i, X is the X particle field, C is the charge conjugation matrix, M is a constant with units of energy, and f_{ijk} $(= f_{ikj})$ is a flavor dependent constant of order 1. In addition we let our model include unspecified lepton and/or baryon violating terms which together with the instanton reaction break all linear combinations of B and L numbers.

Both the mass of the X particle, m_X, and the constant M are constrained by the various requirements of our theory. First we must insure that the X asymmetry develops before all baryon violating interactions fall out of equilibrium. Otherwise the X asymmetry has no effect on baryon number. Let T_X be the temperature at which X violating reactions drop out of equilibrium. Without specifying the exact scenario, we assume that an X asymmetry develops at some temperature

lower than T_X but above the temperature at which instantons freeze out (see [58] and references therein for numerous methods by which number asymmetries can develop). In this way the instanton reaction provides the baryon violation required for our mechanism to work. This is a convenient choice, but not a necessary one if other baryon violation exists in the theory.

It is interesting to note that the only baryon violation required in this model is instantons. If an X asymmetry exists or develops during the epoch in which instantons are in equilibrium, then it will necessarily generate a proportional baryon asymmetry.

In our example X particles will eventually decay into Standard Model particles. Various constraints must be imposed on this decay. To make things simple we require X particles to survive past the temperature at which instantons freeze out. We assume that after this temperature baryon number is a good symmetry. Thus, the only possible effect on the produced baryon density comes from the change in entropy of the Universe upon X decay.

The standard nucleosynthesis scenario places limits on this decay [58]. If X particles decay after nucleosynthesis, they must not dump more than a factor of ~ 15 times the entropy density present at the time of nucleosynthesis. If they did then the observed baryon to photon density would be incompatible with standard nucleosynthesis. Also, if the mass of the X particle is larger than a few MeV, which it must be to avoid strict limits on the width of the Z boson, then energetic photons from X decay can destroy too much deuterium. Further, depending on the era of decay, photons from X decay can destroy the uniformity of the cosmic microwave background radiation or contribute too much to the diffuse photon

background. If X particles decay before nucleosynthesis, their mass and density prior to decay must be compatible with the known baryon to photon ratio, η, during nucleosynthesis.

Let us examine our first constraint. The rate for X violating 4-fermion interactions is given by

$$\Gamma_X \simeq \frac{49 f^2 \pi^5}{12960\zeta(3)} \frac{T^5}{M^4}, \tag{3.15}$$

where f^2 is an average of terms like $f^*_{ijk} f_{lmn}$, and we have dropped terms of order $\frac{m_X}{T}$.

The Hubble constant, H, is $17\frac{T^2}{M_p}$. The 4-fermion interaction drops out of equilibrium when its rate falls below the Hubble expansion rate[59]. Calling the temperature at which this occurs T_X, we have

$$M^4 \simeq \frac{49 f^2 \pi^5}{220320\zeta(3)} M_p T_X^3 . \tag{3.16}$$

Although X number changing interactions freeze out at T_X, X particles stay in thermodynamic equilibrium below this temperature through their gauge interactions. These gauge interactions freeze out at a much lower temperature given by the standard cold relic freeze out criteria.

Now we examine the decay of the X particles. The decay rate for these particles is given by

$$\Gamma \simeq \frac{f^2}{256\pi^3} \frac{m_X^5}{M^4}, \tag{3.17}$$

where we have ignored terms of order the temperature over m_X since they will be seen to be negligible. The X particles decay when this rate is approximately equal to the Hubble expansion rate. Calling the temperature at which these rates

become equal T_D, we have

$$M^4 \simeq \frac{f^2}{4352\pi^3} \frac{m_X^5}{T_D^2} M_p \,. \tag{3.18}$$

If significant entropy is generated by X decay then T_D is the "reheat" temperature after decay.

Equations (3.16) and (3.18) can be combined to give

$$m_X^5 \simeq 7.6 \times 10^3 \, T_X^3 T_D^2 \,. \tag{3.19}$$

In Figure 2 we plot the allowed parameter space by considering the constraints discussed above. (We have assumed $T_D \leq T_{min} \simeq 10^2\,\mathrm{GeV}$ and required $T_X > 10T_{min}$.)

The diagonal dotted lines in this figure are lines of constant T_X and are labeled in GeV. The allowed region is divided up into three regimes. The first, corresponding to $T_D > 10^{-3}\,\mathrm{GeV}$, covers the case in which X particles decay before the onset of nucleosynthesis. In this case the X density just before decay may be quite large, leading to an early matter dominated era and a significant increase in entropy density upon X decay. This is because for large m_X, X particle gauge interactions freeze out when there is still a large anti-X particle density. In this situation, the X number asymmetry is a small fraction of the symmetric relic freeze out density. A large symmetric relic density leads to large entropy dumping when X particles decay. Let us call the factor by which entropy is increased R. Since, in our model, today's observed baryon asymmetry is proportional to the X asymmetry divided by R, a large X asymmetry is required when R is large. We have plotted a dot-dashed line which corresponds to the onset of significant entropy generation when X particles decay. At this line entropy is increased by 10% upon X decay. As we

rise above this line the amount of entropy generated when the X particles decay increases. At the top boundary of our allowed region the X asymmetry required to generate today's observed baryon asymmetry becomes infinite. Above this line there is no way to generate enough baryon asymmetry.

In the second regime 10^{-4} GeV $< T_D < 10^{-3}$ GeV, during which nucleosynthesis is taking place, we impose the conservative requirement that X decay increases the Universe's entropy by less than 10%. This is shown as a dip in the top boundary of the allowed region.

The last regime, $T_D < 10^{-4}$ GeV, in which X particles decay after nucleosynthesis, is bounded on the left by the requirement that decay products don't destroy too much deuterium [60]. The curved line marked with an arrow takes account of this limit (We have used Lindley's rough calculation for heavy dark matter particles [60]). This constraint is more severe than those arising from cosmic microwave background and diffuse photon background observations. The top limit of this region is determined by entropy dumping considerations. Since in this case X particles are still present during nucleosynthesis, we know that the required X asymmetry is equal to $\frac{-g_X}{11}$ times the baryon asymmetry at the time of nucleosynthesis. When X particles decay they can increase the entropy and thus decrease the value of η today relative to its value during nucleosynthesis. We allow at most a decrease by a factor of 15, and this gives us our top limit. Figure 2 illustrates how general our mechanism is. The X particle's mass can range over 12 orders of magnitude, from 45 GeV to 10^{12} GeV.

3.4 Conclusion

We have shown that in order to avoid the strict cosmological limits placed on lepton and baryon number violating interactions it is not necessary to resort to low temperature baryon generation or to the addition of new symmetries which affect baryons. Any symmetry which allows one sector of the theory to acquire a net hypercharge density will suffice. This includes a symmetry under which Standard Model particles are neutral, as our example shows. The key observation is that, although this new symmetry seems decoupled from the rest of the theory, the gauged $U(1)$ symmetries can connect it. Thus an asymmetry in X particles, because they are charged, is enough to ensure a proportional asymmetry in all charged particles independent of whether or not their particle number is conserved.

If a scenario similar to the one proposed here was realized in the early Universe, than experimental searches for lepton and baryon violating interactions may prove successful. Such a success would not only directly signal exciting new L and/or B number violating physics, but would also indirectly signal the existence of a baryon number protection mechanism.

4 Topics on supersymmetric grand unified theories

4.1 On the generality of certain predictions for quark mixing

A theory of fermion masses should explain both the values of the quark and lepton masses and the sizes of the four independent parameters of the Kobayashi-Maskawa (KM) mixing matrix[26]. In the Standard Model these quantities appear as free Yukawa coupling parameters and must be determined from experiment. While we are far from a fundamental understanding of fermion masses, theories which go beyond the Standard Model can possess symmetries which reduce the number of free parameters of these Yukawa coupling matrices, giving relationships between the KM matrix elements and the quark masses. The first relationship so obtained in a gauge theory was the very successful prediction for the Cabibbo angle: $|V_{us}| = \sqrt{\frac{m_d}{m_s}}$[61], where $|V_{us}| = 0.221 \pm 0.002$ and $\sqrt{\frac{m_d}{m_s}} = 0.226 \pm 0.009$[62]. Much interest has also centered around the relation $|V_{cb}| = \sqrt{\frac{m_c}{m_t}}$ obtained by Harvey, Ramond and Reiss[13] working with the form for the Yukawa matrices written down by Georgi and Jarlskog[12]. If this relation were valid at the weak scale the top quark would be predicted to be too heavy[63]. However, inclusion of renormalization group (RG) corrections show that such a relation in a supersymmetric grand unified theory leads to a prediction of $130 < m_t < 195$ GeV([15]-[17]).

We choose $|V_{us}|$, $|V_{cb}|$, $\frac{|V_{ub}|}{|V_{cb}|}$ and $\frac{|V_{td}|}{|V_{ts}|}$ as the the four independent parameters of the KM matrix. Of these $|V_{us}|$ and $|V_{cb}|$ are the two which are best measured. In this section we concentrate on predictions for $\frac{|V_{ub}|}{|V_{cb}|}$ and $\frac{|V_{td}|}{|V_{ts}|}$. These are predicted in several schemes for fermion masses in terms of ratios of quark masses([9],[64],

[63],[15]):

$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{m_u}{m_c}} \simeq 0.061 \pm 0.009\,, \tag{4.1}$$

and

$$\frac{|V_{td}|}{|V_{ts}|} = \sqrt{\frac{m_d}{m_s}} \simeq 0.226 \pm 0.009\,, \tag{4.2}$$

where mass values from reference[62] have been used, keeping in mind that the values in the ratios must be taken at the same renormalization scale μ. In this section we make two comments about these relations: they are very successful, and they are quite generic, following from a simple pattern for the Yukawa matrices.

The success of these relations has been magnified by the 1992 announcement by the CLEO collaboration[65] of lower values for $\frac{|V_{ub}|}{|V_{cb}|}$. They find central values of $\frac{|V_{ub}|}{|V_{cb}|}$ of 0.053, 0.062, 0.065 and 0.095 in four phenomenological models used to analyze the data. The experimental uncertainty is about ± 0.020. Also the value of the top quark mass obtained from precision electroweak data from LEP[66] $m_t = 145 \pm 25$ GeV, is relevant because $|V_{td}|$ is probed experimentally via the $B^0 - \bar{B}^0$ mixing parameter x_d which is strongly dependent on m_t:

$$\begin{aligned} x_d &= \tau_b \frac{G_F^2}{6\pi^2}(\sqrt{B_B} f_B)^2 m_B \eta_B m_t^2 S(y_t) Re(V_{td}^* V_{tb})^2 \\ &= 0.69(\frac{\sqrt{B} f_B}{0.17GeV})^2(\frac{\eta_B}{0.85})(\frac{m_t}{145GeV})^2(\frac{S(y_t)}{0.59})(\frac{\frac{|V_{td}|}{|V_{ts}|}}{0.226})^2(\frac{|V_{cb}|}{0.043})^2\,, \end{aligned} \tag{4.3}$$

where $y_t = m_t^2/M_W^2$, $S(y_t) = 1 - \frac{3}{4}\frac{y_t(1+y_t)}{(1-y_t)^2}[1 + \frac{2y_t}{1-y_t^2} ln(y_t)]$ and η_B is the QCD correction factor. From this it can be seen that by using central values for m_t and other parameters, together with the experimental result that $x_d = 0.70 \pm 0.10$, the prediction of equation (4.2) is highly successful.

Given the success of these two predictions, it is interesting to ask whether they result from just a few specific models, or whether they are generic features

of a wide class of theories[67]. In the rest of this section we show that predictions (4.1) and (4.2) occur whenever two conditions on the elements of the Yukawa matrices are satisfied. We also show that CP violation measurements with neutral B mesons will provide a test of whether the relations (4.1) and (4.2) provide a correct understanding of $|V_{ub}|$ and $|V_{td}|$.

General constraint on the Yukawa matrices

What conditions must the Yukawa matrices $\mathbf{Y}$ ($\mathbf{Y}$=$\mathbf{U}$ or $\mathbf{D}$) satisfy in order to get relations (4.1) and (4.2)? The observed hierarchy of quark masses and mixing angles leads us to the assumption that the entries in the Yukawa matrices have a hierarchical structure, with Y_{33} being the largest. We first take Y_{ij} to be real and later consider how the analysis is modified by CP violating phases. The matrices $\mathbf{Y}$ can be diagonalized by three successive rotations in the (2,3), (1,3) and (1,2) sectors (denoted by s_{23}, s_{13} and s_{12}):

$$
\begin{pmatrix} \tilde{\tilde{Y}}_{11} & 0 & 0 \\ 0 & \tilde{Y}_{22} & 0 \\ 0 & 0 & Y_{33} \end{pmatrix} =
$$

$$
\begin{pmatrix} 1 & -s^Y_{12} & 0 \\ s^Y_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -s^Y_{13} \\ 0 & 1 & 0 \\ s^Y_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -s^Y_{23} \\ 0 & s^Y_{23} & 1 \end{pmatrix} \times
$$

$$
\times \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \times
$$

$$
\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & s'^Y_{23} \\ 0 & -s'^Y_{23} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & s'^Y_{13} \\ 0 & 1 & 0 \\ -s'^Y_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & s'^Y_{12} & 0 \\ -s'^Y_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{4.4}
$$

The small rotation angles are given to leading order by

$$
s^Y_{23} \simeq \frac{Y_{23}}{Y_{33}} + \frac{Y_{32}Y_{22}}{Y^2_{33}} , \; s'^Y_{23} \simeq \frac{Y_{32}}{Y_{33}} + \frac{Y_{23}Y_{22}}{Y^2_{33}} , \tag{4.5}
$$

$$
s^Y_{13} \simeq \frac{\tilde{Y}_{13}}{Y_{33}} + \frac{\tilde{Y}_{31}Y_{11}}{Y^2_{33}} , \; s'^Y_{13} \simeq \frac{\tilde{Y}_{31}}{Y_{33}} + \frac{\tilde{Y}_{13}Y_{11}}{Y^2_{33}} , \tag{4.6}
$$

$$
s^Y_{12} \simeq \frac{\tilde{Y}_{12}}{\tilde{Y}_{22}} + \frac{\tilde{Y}_{21}\tilde{Y}_{11}}{\tilde{Y}^2_{22}} , \; s'^Y_{12} \simeq \frac{\tilde{Y}_{21}}{\tilde{Y}_{22}} + \frac{\tilde{Y}_{12}\tilde{Y}_{11}}{\tilde{Y}^2_{22}} . \tag{4.7}
$$

The successive rotations produce elements

$$
\tilde{\tilde{Y}}_{11} \simeq \tilde{Y}_{11} - \frac{\tilde{Y}_{12}\tilde{Y}_{21}}{\tilde{Y}_{22}} , \; \tilde{Y}_{11} \simeq Y_{11} - \frac{\tilde{Y}_{13}\tilde{Y}_{31}}{Y_{33}} , \; \tilde{Y}_{22} \simeq Y_{22} - \frac{Y_{23}Y_{32}}{Y_{33}} , \tag{4.8}
$$

and

$$
\tilde{Y}_{12} = Y_{12} - Y_{13}s'^Y_{23} , \; \tilde{Y}_{21} = Y_{21} - Y_{31}s^Y_{23} , \tag{4.9}
$$

$$\widetilde{Y}_{13} = Y_{13} + Y_{12}s_{23}^{Y}\,,\ \widetilde{Y}_{31} = Y_{31} + Y_{21}s_{23}^{Y}\,. \tag{4.10}$$

The KM matrix which results from these rotations is

$$\mathbf{V} = \begin{pmatrix} 1 & s_{12} + s_{13}^{U}s_{23} & s_{13} - s_{12}^{U}s_{23} \\ -s_{12} - s_{13}^{D}s_{23} & 1 & s_{23} + s_{12}^{U}s_{13} \\ -s_{13} + s_{12}^{D}s_{23} & -s_{23} - s_{12}^{D}s_{13} & 1 \end{pmatrix}, \tag{4.11}$$

where $s_{23} = s_{23}^{D} - s_{23}^{U}$, $s_{13} = s_{13}^{D} - s_{13}^{U}$ and $s_{12} = s_{12}^{D} - s_{12}^{U}$.

To get relations (4.1) and (4.2) it is sufficient to have

- $\frac{|V_{ub}|}{|V_{cb}|} = |s_{12}^{U}|$ and $\frac{|V_{td}|}{|V_{ts}|} = |s_{12}^{D}|$ which is obtained by

$$|s_{13}| << |s_{12}^{U}s_{23}| \text{ and } |s_{13}| << |s_{12}^{D}s_{23}|\,, \tag{4.12}$$

- $|s_{12}^{U}| = \sqrt{\frac{m_u}{m_c}}$ and $|s_{12}^{D}| = \sqrt{\frac{m_d}{m_s}}$ which is obtained by

$$|\widetilde{Y}_{11}| << |\frac{\widetilde{Y}_{12}\widetilde{Y}_{21}}{\widetilde{Y}_{22}}| \text{ and } |\widetilde{Y}_{12}| = |\widetilde{Y}_{21}|\,. \tag{4.13}$$

The conditions (4.12) and (4.13) on the Yukawa matrices **U** and **D** allow for a wide class of mass ansätze. It is possible that there are some other cases which will lead to (4.1) and (4.2) but we believe that they would be quite special, involving for example nontrivial cancellations. All proposed ansätze that we know of which lead to (4.1) and (4.2) satisfy the conditions (4.12) and (4.13).

Now consider redoing the analysis with Y_{ij} complex. The sequence of rotations in (4.4) will now be interspersed with various diagonal rephasing matrices. This will change the above equations in several ways. For example, in (4.5), (4.6), (4.7) and (4.8) the right hand sides of the equations must be replaced by their absolute values. In equations (4.9) and (4.10) there will be relative phases between the

terms on the right-hand sides. Finally the phase rotations will affect the KM matrix. While the phase transformations cannot induce any new terms in V_{ij}, they can multiply any of the existing ones by phases. However, it is clear that even in this case equations (4.12) and (4.13) *are* the correct conditions for yielding the predictions (4.1) and (4.2).

The conditions (4.12) and (4.13) are very simple; however, when expressed in terms of U_{ij} and D_{ij} via equations (4.5)-(4.10), they appear quite cumbersome. Nevertheless, a simple heuristic way of stating the conditions on U_{ij} and D_{ij} is as follows:

- Y_{11}, Y_{13} and Y_{31} must be small.

  While conditions (4.12) and (4.13) are the precise statement on the smallness of these elements, a feel for their meaning can be grasped as follows. The smallness and hierarchy of fermion masses and mixings can be restated in terms of approximate chiral and flavor symmetries which act on each fermion type([3]-[5]). From these approximate symmetries alone one finds that the inequalities of (4.12) and (4.13) become approximate equalities, thus $|s_{13}| \approx |s_{12}^U s_{23}|$, etc. Hence these approximate chiral and flavor symmetries are not sufficient to guarantee results (4.1) and (4.2). These results follow only if the 11, 13 and 31 entries of the Yukawa matrices are constrained by some more powerful means, for example by some new exact symmetry. In many specific ansätze, family symmetries force these to vanish([9],[64],[63],[15]).

- $|\widetilde{Y}_{12}| = |\widetilde{Y}_{21}|$ usually results, to sufficient accuracy, whenever $|Y_{12}| = |Y_{21}|$.

  While the 12 entries must be symmetric, other entries need not have any

symmetry. None of the models discussed in reference[19] has symmetric Yukawa matrices, but they all have predictions (4.1) and (4.2).

The general conditions (4.12) and (4.13) are satisfied by many special forms for the Yukawa matrices, so that it is not possible to use them to derive a definite hierarchical structure for **U** and **D**. However, this can be done for the subclass of theories in which the Yukawa matrices are also symmetric, $Y_{ij} = Y_{ji}$, and the resulting hierarchical patterns are given in the Appendix A.

The KM matrix

We now study the KM matrix which results from Yukawa couplings which satisfy the conditions (4.12) and (4.13). In particular, the 11, 13 and 31 entries are found to be sufficiently small that they give only negligable corrections to the diagonalization of **U** and **D**,

$$\mathbf{L_U^\dagger U R_U = U_d}\ ,\ \mathbf{L_D^\dagger D R_D = D_d}\,, \tag{4.14}$$

by the unitary matrices

$$\mathbf{L_U^\dagger} = \begin{pmatrix} 1 & -s_2 & 0 \\ s_2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi_U} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -s_U \\ 0 & s_U & 1 \end{pmatrix}, \tag{4.15}$$

$$\mathbf{L_D^\dagger} = \begin{pmatrix} 1 & -s_1 & 0 \\ s_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi_D} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -s_D \\ 0 & s_D & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.16}$$

where s_U and s_D are s_{23} rotations in the $\mathbf{U}$ and $\mathbf{D}$, respectively, and ϕ_U, ϕ_D and ϕ are the necessary phase redefinitions. This gives the KM matrix

$$\mathbf{V} = \mathbf{L_U^\dagger L_D} = \begin{pmatrix} e^{i\psi} & s_1 e^{i\psi} - s_2 e^{i\phi} & -s_2(s_D e^{i\phi} - s_U) \\ s_2 e^{i\psi} - s_1 e^{i\phi} & e^{i\phi} & (s_D e^{i\phi} - s_U) \\ -s_1(s_U e^{i\phi} - s_D) & (s_U e^{i\phi} - s_D) & 1 \end{pmatrix}, \quad (4.17)$$

where $\psi = \phi_U - \phi_D$. For this matrix to yield the predictions (4.1) and (4.2) we impose constraint (4.13), which implies $|s_1| = \sqrt{\frac{m_d}{m_s}}$ and $|s_2| = \sqrt{\frac{m_u}{m_c}}$. After an additional phase redefinition, the KM matrix can be brought into the form introduced in reference[15]:

$$\mathbf{V} = \begin{pmatrix} 1 & s_1 + s_2 e^{-i\phi} & s_2 s_3 \\ -s_2 - s_1 e^{-i\phi} & e^{-i\phi} & s_3 \\ s_1 s_3 & -s_3 & e^{i\phi} \end{pmatrix}, \quad (4.18)$$

where $\phi \to \psi - \phi$ and $s_2 \to -s_2$. In (4.18) we do not lose any generality by choosing the corresponding angles θ_1, θ_2 and θ_3 to lie in the first quadrant.

This form is now seen to result directly from the straightforward diagonalization of a large class of Yukawa textures. It has the appealing feature that to a very good approximation s_1, s_2 and ϕ are renormalization group invariants, while s_3 obeys a simple scaling law. Note that while $s_1 = \sqrt{\frac{m_d}{m_s}}$ and $s_2 = \sqrt{\frac{m_u}{m_c}}$ are given by quark masses and directly yield the predictions (4.1) and (4.2), s_3 and ϕ are determined from $|V_{cb}|$ and $|V_{us}|$, giving $s_3 = 0.043 \pm 0.007$ and, using the numbers quoted in (4.1) and (4.2), $\sin\phi = 0.98^{+0.02}_{-0.07}$[68]. Hence we find that if the Yukawa matrices satisfy (4.12) and (4.13), the entire KM matrix can be determined quite accurately.

Since all four independent parameters of this KM matrix have now been spec-

ified by CP conserving magnitudes $|V_{ij}|$, a crucial question is whether the resulting prediction for CP violation agrees with data. Since V_{us} in (4.18) is not real, we use a rephase invariant result for the CP violating kaon parameter ϵ[69]:

$$|\epsilon| = \frac{G_F^2}{12\pi^2}(\sqrt{B_K}f_K)^2\frac{m_K}{\sqrt{2}\Delta m_K}m_t^2\frac{1}{|\xi_u|^2}\sum_{ij}S(y_i,y_j)Im(\xi_i\xi_j(\xi_u^*)^2)\eta_{ij}\,, \tag{4.19}$$

where $y_i = m_i^2/m_W^2$, $\xi_i = V_{is}V_{id}^*$,

$$\begin{aligned} S(y_i,y_j) &= \frac{y_iy_j}{y_t}\times\{[\frac{1}{4}+\frac{3}{2}(1-y_j)^{-1}-\frac{3}{4}(1-y_j)^{-2}]\frac{lny_j}{y_j-y_i}+ \\ &+ (y_j\to y_i)-\frac{3}{4}[(1-y_i)(1-y_j)]^{-1}\}\,, \end{aligned} \tag{4.20}$$

and $\eta_{tt} = 0.63$, $\eta_{ct} = 0.34$ are the QCD correction factors. Note that $S(y_t) = S(y_t,y_t)$. Using central values we find

$$\begin{aligned} |\epsilon| &= 2.26\cdot 10^{-3}\sin\phi(\frac{\sqrt{B_K}f_K}{0.16\,\mathrm{GeV}})^2(\frac{m_t}{145\,\mathrm{GeV}})^2(\frac{0.221}{|\xi_u|})^2\times \\ &[\ \frac{S(y_t)}{0.59}(\frac{s_1}{0.226})^3\frac{s_2}{0.061}(\frac{s_3}{0.043})^4\frac{\eta_{tt}}{0.63}+ \\ &+\ 0.12\frac{S(y_c,y_t)}{0.24\cdot 10^{-3}}(\frac{s_1}{0.226})^3\frac{s_2}{0.061}(\frac{s_3}{0.043})^2\frac{\eta_{ct}}{0.34}]\,. \end{aligned} \tag{4.21}$$

Here we used $Im(\xi_t^2(\xi_u^*)^2)\approx 2s_1^3s_2s_3^4\sin\phi$ and $Im(\xi_c\xi_t(\xi_u^*)^2)\approx s_1^3s_2s_3^2\sin\phi$. We see that the experimental results indicate a large (CP violating) phase ϕ, consistent with its determination from $|V_{us}|$. An alternative way of stating the prediction for CP violation is via the quantity J [70]:

$$\begin{aligned} J &= Im(V_{ud}V_{ub}^*V_{td}^*V_{tb}) &&= s_1s_2s_3^2\sin\phi \\ &= \sqrt{\tfrac{m_d}{m_s}}\sqrt{\tfrac{m_u}{m_c}}V_{cb}^2\sin\phi &&= (2.6\pm 0.9)10^{-5}\sin\phi. \end{aligned} \tag{4.22}$$

CP asymmetries in B decays

A good test of this KM matrix comes from looking at the allowed values for the CP asymmetries in B decays[71]. The asymmetries, given by sin2α (coming from $B_d \to \pi^+\pi^-$) and sin2β (coming from $B_d \to \psi K_S$), can be expressed in terms of the Cabibbo angle $s_c \equiv |V_{us}|$, s_1 and s_2[72]:

$$\sin 2\alpha = -2\cos\phi\sin\phi, \tag{4.23}$$

$$\sin 2\beta = \frac{2s_1 s_2 \sin\phi}{s_c^2}(1 + \frac{s_2\cos\phi}{s_1}). \tag{4.24}$$

In Figure 3 we plot the allowed region for sin2α and sin2β. The dotted region is the region allowed by the Standard Model[73]. The sin2α variation comes mainly from the uncertainty in s_1 (i.e., from the uncertainties in d and s masses), while the sin2β variation comes mainly from the uncertainty in s_2 (i.e., from the uncertainties in u and c masses)[62]. Precise measurements of sin2α and sin2β will reduce the experimental uncertainties on s_1 and s_2, thereby providing a stringent test of (4.1) and (4.2).

In conclusion, we have shown that $\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{m_u}{m_c}}$ and $\frac{|V_{td}|}{|V_{ts}|} = \sqrt{\frac{m_d}{m_s}}$ are highly successful relations which result from a wide range of models: the Yukawa matrices **U** and **D** need only satisfy the constraints (4.12) and (4.13). This typically means that these matrices have small 11, 13, and 31 entries, and symmetric 12 entries. Given the generality of these results, one might question whether $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$ can be used as a probe of specific mass matrix ansätze in future B-physics experiments. The answer is that they can, but only if these schemes are able to predict m_d/m_s and m_u/m_c more accurately than they are currently extracted from experiment.

4.2 The decay $b \to s\gamma$ and large $\tan\beta$

Last year CLEO[27] announced the discovery of the decay $B \to K^*(892)\gamma$ with the average branching ratio

$$BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9)10^{-5}. \tag{4.25}$$

This flavor changing decay is a stringent test of both the Standard Model and many theories beyond the Standard Model. In this section, we will study how this measurement constrains the parameters of the minimal supersymmetric standard model (MSSM) with large value of $\tan\beta$, the ratio of vacuum expectation values of the two Higgs doublets.

Considerations of long distance effects show that the origin of this decay is most probably the quark level decay $b \to s\gamma$[74]. In the Standard Model $b \to s\gamma$ happens at the one loop level via the exchange of a top quark and a W boson (Figure 4). Notice that this diagram also changes helicity so that the amplitude is proportional to m_b (in the limit $m_s = 0$).

The QCD corrections to this process can be very large. This is computed by running down the coefficients of all relevant operators for this process from the weak scale $\sim M_W$ down to scale of m_b. Many calculations exist([74],[75]), with the general result that the rate for $b \to s\gamma$ is enhanced 3 to 5 times for top quark masses between 160 and 120 GeV. A consensus is that the Standard Model value is about[76]

$$BR_{SM}(b \to s\gamma) = (2-4)10^{-4}\,. \tag{4.26}$$

However, the largest uncertainties come from the calculation of hadronization, i.e. the translation of the total decay rate for $b \to s\gamma$ to the rate for $B \to K^*(892)\gamma$.

Here the calculations for the fraction of $b \to s\gamma$ which end up in $B \to K^*(892)\gamma$ range from about 4% to 40%[77]. We think it is safe to conclude that this implies that CLEO's result translates into observed radiative b quark decay rate of

$$BR_{obs}(b \to s\gamma) = 10^{-3} - 10^{-4}\,. \tag{4.27}$$

This agrees with the Standard Model estimate (4.26) and it is a stringent test for many theories beyond the Standard Model [78].

In addition to the Standard Model contribution, in supersymmetric theories the decay $b \to s\gamma$ receives additional contributions from loops which involve supersymmetric particles. In this section, we will study how CLEO's measurement constrains the parameter space in supersymmetric theories. In particular, there might be substantial enhancements in comparison to the Standard Model for large values of $\tan\beta$ as is the case in some supersymmetric SO(10) grand unified theories. This in turn constrains the parameter space of the MSSM. A large $\tan\beta$ can appear naturally in SO(10) models with one Higgs multiplet where the choice of flavor symmetries is such that the only nonzero renormalizable Yukawa coupling is that of the third generation.

Let us now ask how the constraint (4.27) affects the MSSM parameters with the additional condition of large $\tan\beta$.

MSSM has, in addition to Standard Model parameters, five more parameters: μ, the coupling of H_1H_2 in the superpotential; $\tilde{m}$, the (SUSY breaking) squark mass; M, the (SUSY breaking) gaugino mass; A, the (SUSY breaking) trilinear scalar term; and B, the (SUSY breaking) coupling of H_1H_2[79]. These parameters

are then run down to low scales and then compared to experimental limits in order to constrain the available phase space. Further reduction of parameters is possible by more constraints; for example, flat Kähler metric requires $B = A - \tilde{m}$, or we might require that the low energy behavior must reproduce the correct scale of electroweak breaking. However, here we will not impose any further constraints on the five parameters and will keep them free.

Let us see now how $\tan\beta$ is related to the MSSM parameters. The neutral direction of the Higgs potential at low energies is

$$V = \mu_1^2|H_1^0|^2 + \mu_2^2|H_2^0|^2 - \mu_3^2(H_1^0 H_2^0 + h.c.) + \frac{1}{8}(g^2 + g'^2)(|H_1^0|^2 - |H_2^0|^2)\,, \quad (4.28)$$

where $\mu_i, i = 1,2,3$ are the parameter μ scaled (differently) down from the GUT scale. The minimum of this potential is found for

$$\sin 2\beta \equiv \frac{2v_2 v_1}{v_2^2 + v_1^2} = \frac{2\mu_3^2}{\mu_1^2 + \mu_2^2} = \frac{2B\mu}{\mu_1^2 + \mu_2^2} \quad (4.29)$$

where B and μ are the MSSM parameters at the scale considered.

Large $\tan\beta$ arises naturally in the simplest SO(10) theories where the Yukawa couplings of the heaviest generation come from a term $16_3 10 16_3$ at a GUT scale, where the 16_3 denotes the matter multiplet of the heaviest generation, and 10 is the scalar multiplet which contains the two SU(2) doublets responsible for giving the masses to fermions. Such coupling for the heaviest generation, but not for the lighter ones, was introduced in several SUSY GUT theories (see, for example, [15],[19]) by an ad hoc choice of discrete flavor symmetries. This unification of Yukawa couplings combined with the requirement that we obtain correct tau lepton and bottom quark masses, gives as a prediction a large value of the top quark mass (typically 160-180 GeV), and therefore a large value for $\tan\beta \approx \frac{m_t}{m_b}$. From (4.29)

we have for large $\tan\beta$

$$\frac{1}{\tan\beta} \approx \frac{B\mu}{\mu_1^2 + \mu_2^2}. \tag{4.30}$$

We can explain large $\tan\beta$ by having small B or μ[80]. However, this is not totally warranted by approximate symmetries and still requires fine tuning[81]. Therefore we still keep all MSSM parameters free even for large $\tan\beta$, and look to experiment instead.

Let us see now how the observed rate (4.27) for the radiative b decay constrains the MSSM parameters. First, it is obvious that, for $\tan\beta$ of order unity, no significant enhancement will occur in MSSM over the Standard Model. This is because we have, in the additional loops: 1) supersymmetric particles with masses of order M_W or larger, from experimental limits, therefore suppressing the loops; and 2) the couplings at vertices are certainly not larger than order unity, which the Standard Model couplings (g_2) are. Therefore, we will only concentrate on loops which can have large $\tan\beta$ in any of the couplings at the vertices.

We did a computation of the ratio of the amplitudes of supersymmetric loops to the Standard Model contribution

$$r \equiv \frac{|A_{SUSY}|}{|A_{SM}|} \tag{4.31}$$

for various values of the MSSM parameters. The value of $\tan\beta$ was held large and fixed ($\tan\beta = 60$). Since the Standard Model prediction for the decay $b \to s\gamma$ is within an order of magnitude of the measured value, we have chosen to call a part of parameter space "forbidden" when the ratio of the amplitudes was larger than 3 ($r > 3$). On the other hand, if $r < 3$, the supersymmetric amplitude contribution is within the experimental and theoretical uncertainties of the observed rate, and

we call such a solution "allowed."

There are two diagrams that grow with tanβ. The first one is the chargino exchange (Figure 5), where in the loop the fermion partner of charged Higgs doublets and the stop particle are exchanged. The vertices with quarks have the Yukawa coupling elements λ^d_{33} and λ^u_{23}. The change of the helicity of the stop goes through a term proportional to $A\lambda^u_{33} < H_2 >$. In order to compare this to the Standard Model contribution we must factor out the b quark mass. However, in the loop we have λ^d_{33} and $< H_2 >$. This then gives a factor of tanβ after $m_b = \lambda^d_{33} < H_1 >$ is pulled out. In Figure 6 the allowed and forbidden regions for A and $\tilde{m}$ are shown for fixed plausible values of the other parameters ($M = 200\,\text{GeV}$, $m_t = 180\,\text{GeV}$ and $\mu = 10\,\text{GeV}$(dashed line), $\mu = 50\,\text{GeV}$(solid line), $\mu = 350\,\text{GeV}$(dotted line)). The region to the right of the line has $r < 3$ (allowed), and the region to the left has $r > 3$ (forbidden). It is clear that, for large values of tanβ, the CLEO measurement starts to substantially enter the parameter space of the MSSM. The SUSY scalar mass parameter $\tilde{m}$ appears to be pushed to higher values for a plausible choice of parameters. Numerical calculation show that there is not much dependence on M and m_t but, as we can see from Figure 6, there is some dependence on μ.

It is well known that the down type squarks can receive substantial flavor changing contributions from renormalization group running[82]. This can change a $\tilde{s}$ into $\tilde{b}$ with a factor

$$\delta m^2_{Q_{23}} \approx \frac{1}{16\pi^2} V_{cb} \tilde{m}^2 ln(\frac{M_G}{M_W}) \approx \frac{1}{5} V_{cb}. \tag{4.32}$$

Therefore, a potentially large (with large tanβ) contribution can come from the diagram (Figure 7) with the gluinos and the down type squarks in the loop. The

scalar coupling $\mu\lambda^d_{33}$ in the diagram comes from the cross term in $|\partial W/\partial H_1|^2$ where

$$W = \mu H_1 H_2 + d^c \lambda^d Q H_1 \tag{4.33}$$

is the relevant part of the superpotential. This effectively couples the "wrong" Higgs H_2 to the down Yukawa couplings[83]. Hence in Figure 7, in order to pull out a factor of m_b, we are again left with a factor of $\tan\beta$. An example of our numerical results for this contribution is shown in Figure 8 where the allowed and forbidden regions for M and $\tilde{m}$ are shown. In this figure, we have again fixed a plausible set of the other parameters ($m_t = 180\,\mathrm{GeV}$ and $\mu = 10\,\mathrm{GeV}$(dotted line), $\mu = 50\,\mathrm{GeV}$(solid line), $\mu = 130\,\mathrm{GeV}$(dashed line)). The region to the right of the line has $r < 3$ (allowed), and the region to the left has $r > 3$ (forbidden). This diagram is not as powerful as the chargino contribution mainly because of the suppression by small $\delta m^2_{Q_{23}}$. However, some excluded parts of the parameter space are already comparable to best direct measurements.

In conclusion, we showed that the CLEO measurement of $B \to K^*(892)\gamma$ already excludes substantial portions of the parameter space of MSSM, despite theoretical and experimental uncertainites that plague the extraction of the rate for $b \to s\gamma$. Therefore, $b \to s\gamma$ proves to be a powerful tool in constraining flavor changing models and it is important to further refine theoretical estimates as well as do more accurate measurements in the near future.

4.3 The flavor structure of proton decay

In this section we want to study the flavor structure of couplings of effective four fermion operators leading to proton decay in supersymmetric grand unified theories (SUSY GUTs). As is well known, in these theories an effective 5-dimensional operator is generated by the exchange of heavy color triplets between two pairs of fermions and scalar partners of fermions (sfermions). Sfermions are converted into fermions by an exchange of a wino[84] (so called "wino dressing"), giving the effective four fermion operator responsible for the proton decay[85].

The couplings of the color triplets to fermions and sfermions define the flavor structure of proton decay because they come from Yukawa couplings of matter multiplets with Higgs multipets in SUSY GUTs.

For example, in S0(10), the simplest Yukawa coupling is a coupling between a Higgs of 10 and matter multiplets 16_i

$$h^{ij}\, 16_i\, 10\, 16_j\,, \tag{4.34}$$

which is decomposed under $SU(2)$ fields as

$$\begin{aligned} h^{ij}\{\ &(u_i^c Q_j + u_j^c Q_i + \nu_i^c L_j + \nu_j^c L_i) H_f \ + \ (Q_i Q_j + e_i^c u_j^c + e_j^c u_i^c - \nu_i^c d_j^c - \nu_j^c d_i^c) H_c \\ &+ (d_i^c Q_j + d_j^c Q_i + e_i^c L_j + e_j^c L_i) \bar{H}_f \ + \ (Q_i L_j + Q_j L_i + u_i^c d_j^c - u_j^c d_i^c) \bar{H}_c\ \}, \end{aligned} \tag{4.35}$$

where H_f, $\bar{H}_f$, H_c, $\bar{H}_c$ are the SU(2) doublets and color triplets of 10, Q_i, L_i, u_i^c, d_i^c, e_i^c, ν_i^c are the fermion SU(2) doublets and singlets of 16_i, and i=1,2,3 is the generation label.

In SU(5), the simplest Yukawa couplings come from

$$f^{ij} T_i T_j H + g^{ij} T_i \bar{F}_j \bar{H}\,, \tag{4.36}$$

where T_i and $\bar{F}_i$ are the SU(5) 10 and $\bar{5}$ matter mulltiplets, and H and $\bar{H}$ are the SU(5) 5 and $\bar{5}$ Higgs multiplets. Under SU(2), the multiplets decompose as

$$f^{ij}\{\ (u_i^c Q_j + u_j^c Q_i)H_f \ \ + \ \ (Q_i Q_j + e_i^c u_j^c + e_j^c u_i^c)H_c\ \}$$

$$g^{ij}\{\ (Q_i d_j^c + e_i^c L_j)\bar{H}_f \ \ + \ \ (Q_i L_j + u_i^c d_j^c)\bar{H}_c\ \}\,. \tag{4.37}$$

The forms (4.35) and (4.37) are also useful in the case when the Yukawa couplings in the theory get generated by higher dimensional operators which reduce to effective forms (4.34) or (4.36). Suppose an SO(10) theory has a higher dimensional operator $16_i\ 45_{B-L}\ 10\ 16_j$. After 45_{B-L} takes a vev in the direction of $B-L$, the higher dimensional operator assumes form (4.34) where the 16_i has all the fermion fields multiplied with their $B-L$ value.

Therefore we can write down the most general form of Yukawa couplings of matter fields to Higgs doublets and color triplets coming from effective couplings (4.34) (or (4.36) in SU(5)) (generation indices suppressed):

$$(u^{cT}h^u Q + \nu^{cT}h^\nu L)H_f \ \ + \ \ (Q^T h^Q Q + e^{cT}h^{eu}u^c - \nu^{cT}h^{\nu d}d^c)H_c$$

$$+(d^{cT}h^d Q + e^{cT}h^e L)\bar{H}_f \ \ + \ \ (Q^T h^L L + u^{cT}h^{ud}d^c)\bar{H}_c\,, \tag{4.38}$$

where the elements of 3×3 matrices $h^u, h^d, \ldots$ are simply obtained from the matrix h and relevant Clebsches of the higher dimensional operator.

We can simplify the couplings in (4.38) by going to the basis where up quarks and charged leptons are in their mass eigenstates. Matrices h^u, h^d, h^e and h^ν are diagonalized by biunitary transformations $U_{aR}h^a U_{aL}^\dagger = h_d^a,\ \ a = u, d, e, \nu$, and we choose to go to a basis where

$$Q \to U_{uL}^\dagger Q,\ \ L \to U_{eL}^\dagger L,\ \ a^{cT} \to a^{cT}U_{aR},\ a = u, d, e, \nu. \tag{4.39}$$

This also implies

$$h^Q \to U^*_{uL} h^Q U^\dagger_{uL} \equiv U^Q, \;\; h^L \to U^*_{uL} h^L U^\dagger_{eL} \equiv U^L, \tag{4.40}$$

and similarly for h^{eu}, $h^{\nu d}$ and h^{ud}.

By integrating out the heavy triplets H_c and $\bar{H}_c$ from (4.38), we generate five dimensional operators in the superpotential

$$\frac{1}{M_c}(Q^T\, U^Q\, Q)(Q^T\, U^L\, L) + \frac{1}{M_c}(e^{cT} U^{eu} u^c - \nu^{cT} U^{\nu d} d^c)(u^{cT} U^{ud} d^c)\,, \tag{4.41}$$

where M_c is the mass of the exchanged triplet.

Dressing of the left handed part of (4.41) with winos produces the effective six dimensional operators which contribute to nucleon decays[84]:

$$\begin{aligned} L^{uude} &= \frac{1}{M_c}\frac{\alpha}{2\pi}[U^Q_{ik}U^L_{jl} - U^Q_{ij}U^L_{kl}](u_i d'_j)(u_k e_l)[f(\tilde{d}'_k, \tilde{\nu}_l) + f(\tilde{u}_j, \tilde{d}'_i)] \\ L^{udd\nu} &= \frac{1}{M_c}\frac{\alpha}{2\pi}[U^Q_{ik}U^L_{jl} - U^Q_{ij}U^L_{kl}](d'_i u_j)(d'_k \nu_l)[f(\tilde{u}_k, \tilde{e}_l) + f(\tilde{u}_i, \tilde{d}'_j)], \end{aligned} \tag{4.42}$$

where $d' = Vd$, and V is the Kobayashi-Maskawa matrix (at the weak scale) and u and d are the mass eigenstates of left handed up and down quarks, respectively. The f's are the triangle diagram factors, which are functions of scalar masses running through the triangle. For simplicity we assume all the scalars to be of the same mass, and we set from now on all the f's equal. Hence,

$$\begin{aligned} L^{uude} &= \frac{1}{M_c}\frac{\alpha}{2\pi}2f[U^Q_{ik}(V^T U^L)_{ml} - (U^Q V)_{im} U^L_{kl}](u_i d_m)(u_k e_l) \\ L^{udd\nu} &= \frac{1}{M_c}\frac{\alpha}{2\pi}2f[(V^T U^Q V)_{mn} U^L_{jl} - (V^T U^Q)_{mj}(V^T U^L)_{nl}](d_m u_j)(d_n \nu_l), \end{aligned} \tag{4.43}$$

and we see that the proton decay will crucially depend on the flavor matrices U^Q and U^L, defined in (4.40).

The Decay $p \to K^+\nu_i$

Following the notation of [86]-[87], the rate for the decay $p \to K^+\nu_i$ is given by

$$\Gamma(p \to K^+\nu_i) = \frac{(m_p^2 - m_K^2)^2}{32\pi m_p^3 f_\pi^2}|C(sud\nu_i)\frac{2m_p}{3m_B}D + C(dus\nu_i)(1 + \frac{m_p}{3m_B}(D + 3F))|^2, \tag{4.44}$$

where the chiral lagrangian factors are estimated to be $m_B = 1150\,\text{MeV}$, $D = 0.81$ and $F = 0.44$. The factors C are the coefficients of corresponding operators in the Lagrangian and from (4.43) we read off

$$\begin{aligned} C(sud\nu_i) &= \frac{1}{M_c}\frac{\alpha}{2\pi}2f[(V^TU^QV)_{21}U^L_{1i} - (V^TU^Q)_{21}(V^TU^L)_{1i}] \\ C(dus\nu_i) &= \frac{1}{M_c}\frac{\alpha}{2\pi}2f[(V^TU^QV)_{12}U^L_{1i} - (V^TU^Q)_{11}(V^TU^L)_{2i}]. \end{aligned} \tag{4.45}$$

Evaluating the constants, we can rewrite (4.44) as

$$\begin{aligned} \Gamma(p \to K^+\nu_i) &= \frac{1}{2.81 \times 10^{18}\,\text{yr}}(\frac{\beta}{0.003\,\text{GeV}^3})^2\frac{1}{(\frac{f_\pi}{130\,\text{MeV}})^2}\frac{1}{(\frac{M_c}{10^{17}\,\text{GeV}})^2}(\frac{2f}{1\,\text{TeV}})^2 \times \\ &\times \Big| \left[(V^TU^QV)_{21}U^L_{1i} - (V^TU^Q)_{21}(V^TU^L)_{1i}\right] 0.44 + \\ &+ \left[(V^TU^QV)_{12}U^L_{1i} - (V^TU^Q)_{11}(V^TU^L)_{2i}\right] 1.58 \Big|^2 . \end{aligned} \tag{4.46}$$

The decay $p \to K^0e_i$

The decay $p \to K^0e_i$

$$\Gamma(p \to K^0e_i) = \frac{(m_p^2 - m_K^2)^2}{32\pi m_p^3 f_\pi^2}|C(suue_i)(1 - \frac{m_p}{3m_B}(D - F))|^2 \tag{4.47}$$

gets a contribution from L^{uude} in (4.43) and we write a formula similar to (4.46)

$$\begin{aligned} \Gamma(p \to K^0e_i) &= \frac{1}{2.81 \times 10^{18}yr}(\frac{\beta}{0.003GeV^3})^2\frac{1}{(\frac{f_\pi}{130MeV})^2}\frac{1}{(\frac{M_c}{10^{17}GeV})^2}(\frac{2f}{1TeV})^2 \times \\ &\times \Big| \left[U^Q_{11}(V^TU^L)_{2i} - (U^QV)_{12}U^L_{1i}\right] 0.70 \Big|^2 . \end{aligned} \tag{4.48}$$

Flavor structure

To completely determine the decay rates we need to know the matrix elements inside the square brackets in (4.46) and (4.48), and those are given, via U^Q and U^L, by the flavor structure of the underlying SUSY GUT.

For example, in minimal SU(5) we have from (4.37)

$$h^u = 2\,h^Q = symmetric, \;\; h^d = h^{L^T} = h^{e^T}. \tag{4.49}$$

If we go to the basis of mass eigenstates for up quarks ($h^u = diag(\lambda_u, \lambda_c, \lambda_t)$) and charged leptons ($h^e = diag(\lambda_e, \lambda_\mu, \lambda_\tau)$) , h^Q also becomes diagonal with the same values as h^u, up to two phases (phases cannot be absorbed by redefinition of Q fields: their phases are already fixed by requiring the diagonal h^u to be real), while $h^d = h^{L^T} = h^e V_G^\dagger = diag(\lambda_e, \lambda_\mu, \lambda_\tau) V_G^\dagger = diag(\lambda_d, \lambda_s, \lambda_b) V_G^\dagger$. The index G on the Kobayahi-Maskawa matrix indicates that it is to be evaluated at the scale M_{GUT}, rather than the weak scale. Notice that, by (4.40), in the new basis $U^Q = h^Q$ and $U^L = h^L$. To summarize, in SU(5)

$$U^Q = diag(\lambda_u e^{i\phi_u}, \lambda_c e^{i\phi_c}, \lambda_t e^{i\phi_t}), \;\; U^L = V_G^* diag(\lambda_d, \lambda_s, \lambda_b)\,. \tag{4.50}$$

The square brackets of (4.46) are largest for $i = 2$ ($p \to K^+\nu_\mu$) and are about $\lambda_c \lambda_s \Theta^2_{Cabbibo} \approx 1.5 \times 10^{-7}$, giving for the partial lifetime about 3.1×10^{31} yr. The square bracket of (4.48) is largest for $i = 2$ ($p \to K^+\mu$), and is about $\lambda_u \lambda_s \approx 8.3 \times 10^{-9}$, giving for the partial lifetime about 8.4×10^{34} yr, or a factor of about 2.7×10^3 longer than $p \to K^+\nu_\mu$ decay [86]. Notice that the charged lepton mode in SU(5) is suppressed because U^Q is diagonal, and therefore makes the square bracket in (4.48) proportional to the small parameter λ_u. This is a special property of SU(5), tracing it back to h^Q being proportional to h^u.

However, the SU(5) group does not predict realistic fermion masses[12]; thus, it is hard to believe that such a theory can reliably predict proton decay which, as we showed, depends crucially on flavor structure.

A higher unified group which has realistic fermion masses, such as SO(10), will have general Yukawa couplings of the form (4.38). In general, different Clebches of triplet and doublet Yukawa couplings will cause h^Q to *not* be proportional to h^u, and we expect that the charged lepton mode may not to be so severely suppressed.

Let us see what we can say in general about the decay rates before going to a specific example ([19]) in the next section. First, let us assume that all Yukawa matrices in (4.38) are of the hierarchical form $(A >> B, E >> C)$

$$h^i = \begin{pmatrix} 0 & z_i' C & 0 \\ z_i C & y_i E e^{i\phi} & x_i' B \\ 0 & x_i B & A \end{pmatrix}, \tag{4.51}$$

where $i = u, d, e, \nu$ are the ordinary couplings Higgs doublet couplings, and $i = Q, L$ are the Higgs triplet couplings. Here, for h^Q it is understood that a factor 1/2 multiplies the left hand side.

For simplicity we assumed the (1,1), (1,3) and (3,1) entries to be zero (most of the successful S0(10) models have this property). The coefficients $x_i, x_i', y_i, z_i, z_i'$ are Clebsches coming from vevs of fields entering the higher dimensional operators. In the SU(5) limit (4.49), $x_Q = x_u = x_u'$, $y_Q = y_u$, $z^Q = z_u = z_u'$, $x_L = x_d' = x_e$, $x_L' = x_d = x_e'$, $y_L = y_d = y_e$, $z_L = z_d' = z_e$, $z_L' = z_d = z_e'$.

By requiring the theory to predict realistic fermion masses and mixings we can fix combinations of A, B, C, E and Clebsches from h^u and h^d. We require, for example, $V_{cb} = (x_d - x_u)B/A$. The observed fact that $\lambda_c/\lambda_t << \lambda_s/\lambda_b$ can easily be

accommodated if y_u is much smaller than y_d and $E >> B^2/A$. Then $\lambda_s = y_d E_d$ and $\lambda_c = -x_u x'_u B^2/A$. It also follows that because of the observed $\lambda_u/\lambda_c << \lambda_d/\lambda_s$, $|s_2|\sqrt{\frac{z_d}{z'_d}} < |s_1|\sqrt{\frac{z_d}{z'_d}}$ for zs of the same order, where $s_2 = -\frac{z_u C}{\lambda_c} = -\sqrt{\frac{\lambda_u}{\lambda_c}}\sqrt{\frac{z_u}{z'_u}}$ and $s_1 = \frac{z_d C}{\lambda_s} = \sqrt{\frac{\lambda_d}{\lambda_s}}\sqrt{\frac{z_d}{z'_d}}$. We can therefore in principle compute matrices U^Q and U^L responsible for the proton decay.

The left handed rotations on h^u and h^e are (following the notation of [19])

$$U_{uL} = \begin{pmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi_u} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{c}_3 & \bar{s}_3 \\ 0 & -\bar{s}_3 & \bar{c}_3 \end{pmatrix} \tag{4.52}$$

$$U_{eL} = \begin{pmatrix} c_{2e} & s_{2e} & 0 \\ -s_{2e} & c_{2e} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi_e} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{c}_{3e} & \bar{s}_{3e} \\ 0 & -\bar{s}_{3e} & \bar{c}_{3e} \end{pmatrix}, \tag{4.53}$$

where $\bar{\Theta}_3 = -x_u B/A$, $\bar{\Theta}_{3e} = -x_e B/A$, $\Theta_2 = -z_u C/y_u E_u$ and $\Theta_{2e} = -z_e C/y_e E_e$.

This gives for the proton decay flavor matrices U^Q and U^L in (4.40)

$$U^Q = \frac{1}{2} \begin{pmatrix} 2z_Q C s_2 e^{i\phi_u} + s_2^2 y_Q E'_Q & z_Q C e^{i\phi_u} + s_2 y_Q E'_Q & (x_Q - x_u) B s_2 \\ & y_Q E'_Q & (x_Q - x_u) B \\ & & A \end{pmatrix} \tag{4.54}$$

$$U^L = \begin{pmatrix} z'_L C s_{2e} e^{i\phi_u} + s_2(z_L C e^{i\phi_e} + s_{2e} y_L E'_L) & z'_L C e^{i\phi_u} + s_2 y_L E'_L & (x'_L - x_u) B s_2 \\ z_L C e^{i\phi_e} + s_{2e} y_L E'_L & y_L E'_L & (x'_L - x_u) B \\ (x_L - x_e) B s_{2e} & (x_L - x_e) B & A \end{pmatrix}, \tag{4.55}$$

where $y_Q E'_Q = y_Q E e^{i\phi} - x_Q x_u \frac{B^2}{A}$ and $y_L E'_L = y_L E e^{i\phi} - x'_L x_e \frac{B^2}{A}$, and we remember that U^Q is symmetric by definition. Notice that for reasonably small values of xs ($x < 10$), we can write $E'_Q = E'_L = E e^{i\phi}$.

The equations (4.54) and (4.55) are the central result of this section. Given a theory we would know the values of different Clebsches and therefore we would be able to compute the matrices U^Q and U^L from equations (4.54) and (4.55). This then leads to proton decay rates (4.46) and (4.48).

The exact result of course depends on the Clebsches from the underlying theory that generates the Yukawa couplings. Before we go to a specific example in the next section with exact numerical results, let us try to get a feeling for how large the rates can be.

Let us now look at proton decay rates to kaons and muon leptons ($i = 2$). Before estimating the matrix elements note that the neutrino mode (4.46) has chiral factors 0.44 and 1.58 inside the square brackets as opposed to 0.70 in muon mode (4.48). The flavor structure matrices inside the square brackets wear the same indices (2112) both in the neutrino and the muon mode, and are expected to be of the same order for all Clebsches of order unity (barring some accidental cancellations) So, we expect the neutrino mode to be generally larger than the muon mode by at least the chiral factor $(2.02/0.70)^2 = 8.4$.

First, let us estimate the matrix elements when all the Clebsches are of order unity. Then our matrices U^Q in (4.54) and U^L in (4.55) are of the general form (4.51), up to the nonzero (1,1), (1,3) and (3,1) elements. Various multiplications of U^Q or U^L by the KM matrix V, as needed for computation of the decay reates (4.46) and (4.48), will not affect seriously sizes of their matrix elements. Hence, in the case of Clebsches of order unity we expect for the neutrino mode, from (4.46),

$$\Gamma(p \to K^+\nu_\mu) \sim |U^Q_{21} U^L_{12}\, 2.02|^2 = |C^2\, 2.02|^2 = |\lambda_s^2 s_1^2\, 2.02|^2\,, \qquad (4.56)$$

and for the muon mode, from (4.48),

$$\Gamma(p \to K^0\mu) \sim |U^Q_{21}U^L_{12}\,0.70|^2 = |C^2\,2.02|^2 = |\lambda_s^2 s_1^2\,0.70|^2\,. \tag{4.57}$$

Thus, in the case of Clebsches of order unity the neutrino mode is larger than the muon mode only by the chiral factor

$$\frac{\Gamma(p \to K^+\nu_\mu)}{\Gamma(p \to K^0\mu)} = \frac{|2.02|^2}{|0.70|^2} = 8.4 \tag{4.58}$$

instead of the minimal SU(5) factor of several thousand!

On the other hand, a large ratio is expected for example if z_Q, the (1,2) element of h^Q, were very small and other Clebsches of the same order. Whereas in this case the proton decay rate to neutrino will stay of the same order as above ($\sim |\lambda_s^2 s_1^2|^2$) due to the multiplications with the KM matrix, notice that the muon mode has always a U^Q element with the row index equal to 1. Looking at the first row of U^Q in (4.54) we see that, for small z_Q, the terms are proportional to s_2. This means that

$$\frac{\Gamma(p \to K^+\nu_\mu)}{\Gamma(p \to K^0\mu)} = \frac{|\lambda_s^2 s_1^2 2.02|^2}{|\lambda_s^2 s_1 s_2 0.70|^2} = 94.5\,, \tag{4.59}$$

i.e. the charged lepton mode would be considerably smaller but still larger than the SU(5) case.

However, the experiments will not be able to tag the flavor of the neutrino, of course, so what we should do is compute the proton decay rate to all neutrinos, and then compare to the decay rate for $p \to K^0\mu$. We expect $p \to K^+\nu_e$ to be small because it involves a lighter generation neutrino, but $p \to K^+\nu_\tau$ need not be small, as we will show now[88]. Similar to the case of ν_μ, we expect for the ν_τ mode

$$\Gamma(p \to K^+\nu_\tau) \sim |U^Q_{21}U^L_{13}\,2.02|^2 = |CBs_1\,2.02|^2 = |\lambda_s\lambda_b V_{cb} s_1^2\,2.02|^2, \tag{4.60}$$

which is of the same size or possibly even larger than the ν_μ mode (depending on the relative signs of the Clebsches). We therefore expect the total decay rate to be a factor of $\sim 15-20$ larger than the μ mode, rather than the 8.4 from only the ν_μ mode, for Clebsches of order unity.

It is interesting to note that if z_Q is small (which we saw is unfavorable for the μ mode), it makes the ν_τ mode smaller by $|s_2/s_1|^2 \approx 10$ as well. The reason that the ν_τ mode is proportional to $s_2 s_1$ rather than s_1^2 is as follows. If we look at the expression (4.46) for $i = 3$ we have terms with $(V^T U^L)_{13}$, $(V^T U^L)_{13}$ and U^L_{13} . Now, U^L_{13} is already proportional to s_2, and this is multiplied by $(V^T U^Q V)_{21}$ which has a s_1. $(V^T U^L)_{13}$ (or $(V^T U^L)_{23}$) is proportional to s_1, but these terms are multiplied by U^Q from the first column (or first row, because U^Q is symmetric) which is proportional to s_2 (for small z_Q).

Previous discussion shows that the muon decay mode in S0(10) is not expected to be hopelessly small as in the minimal SU(5) mode. Depending on the Clebsches this mode might be detectable in the near future if the total proton decay rate is close to the experimental limit today. Namely, the proposed new proton decay facilities [89] should have an improvement of about a factor of ~ 10 in rate detection. This should be added to the fact that the experiments are by a factor of ~ 3 more sensitive to the charged lepton mode than the neutrino mode. Again, however, the exact rate depends on the Clebsches, and it must be checked for each individual model.

Depending on the model the muon mode may either completely dominate the total decay rate, or lead to about 10% for equal Clebsches, or be suppressed for

specific choice of Clebsches (small z_Q). This means that, by measuring the charged lepton mode, or even by not seeing it, the experiments will be able to discriminate between different theories.

Note at this point that the overall proton decay mode goes as $|\lambda_s^2|^2$ rather than the naive SU(5) $|\lambda_s\lambda_c|^2$ behaviour, which could mean an overall enhancement in the decay rate. However, the overall rate depends on the sizes of the Clebsches as well as on how the colored triplet mass was generated[28].

Finally, we check in Appendix B the SU(5) limit (4.50) of the general expressions for U^Q and U^L in (4.54 - 4.55). U^Q assumes the diagonal form proportional to up quark masses, and U^L receives the form as in (4.50), up to first order in relevant couplings.

In conclusion, *in a realistic SO(10) model the branching ratio of the proton decay to charged lepton mode might be significantly enhanced in contrast to the minimal SU(5) model.* In particular, the matrix element U^Q_{12} now enters the charged lepton decay mode (4.48) and can be expected to be substantially larger than U^Q_{11}, on grounds of the underlying flavor structure.

Let us now look at the class of predictive models discussed in [19].

Proton decay branching ratios in SO(10) theories of Anderson *et al.* [19]

As an example of the discussion in the previous section let us look at the "22" texture models of [19], i.e. textures coming from four operators $O_{33} + O_{23} + O_{22} + O_{12}$, where the indices denote generations. It is the minimal set of operators

consistent with fermion masses and mixings.

There are 9 possible O_{23} and 6 possible O_{22} operators. Although all 6 0_{22} operators give the same Clebsch for up, down and charged lepton matrices, they give different Clebsches to h^Q and h^L. This means that in principle proton decays could distinguish between those six models. For completeness we list in Appendix C the Yukawas for all 54 models.

We immediately note that $z_Q = -\frac{1}{27}$, so we expect the charged lepton decay mode to be suppressed as in (4.59). Also, it turns out that the ν_τ mode was not too much suppressed. Although it is true that individual terms in the decay rate of proton to ν_τ are somewhat smaller than the ν_μ mode term, addition of several terms of the same order make ν_τ close to ν_μ, and in some cases even slightly bigger.

We did a full numerical computation of the matrix elements for all neutrino modes and the muon mode. Results for the most realistic models are shown in Table 4. The ratio of proton decay rate to all neutrinos over the proton decay rate to muon is typically between 100 and 200 for most of the models, with the lowest number being 96, for the model $O_{22} = 5, O_{23} = 7$.

Although rates to the charged lepton mode are substantially higher in models of Anderson *et al.*[19] than in the SU(5) case, they are still small and will probably escape detection in the future proton decay experiments. The muon mode in this class of models is suppressed because of the two reasons mentioned above, namely small z_Q and the rates to ν_μ and ν_τ being of the same order. But, in the general case as shown in the previous section, there is no reason to believe that the charged lepton mode is suppressed. Therefore, we strongly support the searches for charged lepton mode as they are a powerful method to distinguish between different models.

Appendix A. Forms of U and D with the additional constraint $|Y_{ij}| = |Y_{ji}|$

Let us try to find the most general forms of *symmetric* U and D which lead to (4.1) and (4.2),

$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{m_u}{m_c}} \simeq 0.061 \pm 0.009\,, \tag{A.1}$$

and

$$\frac{|V_{td}|}{|V_{ts}|} = \sqrt{\frac{m_d}{m_s}} \simeq 0.226 \pm 0.009\,. \tag{A.2}$$

We neglect phases for simplicity. As mentioned before, we assume no accidental cancellations, so if a sum of two elements is small it is because they are both small. First, because of symmetry, expressions (4.5)-(4.10) simplify: $s^Y_{23} = s'^Y_{23} = \frac{Y_{23}}{Y_{33}}$ and therefore

$$\tilde{Y}_{12} = \tilde{Y}_{21} = Y_{12} - Y_{13}\frac{Y_{23}}{Y_{33}}\,, \tag{A.3}$$

$$\tilde{Y}_{13} = \tilde{Y}_{31} = Y_{13} + Y_{12}\frac{Y_{23}}{Y_{33}}\,, \tag{A.4}$$

$$s^Y_{13} = s'^Y_{13} = \frac{\tilde{Y}_{13}}{Y_{33}}\,, \tag{A.5}$$

$$s^Y_{12} = s'^Y_{12} = \frac{\tilde{Y}_{12}}{Y_{33}}\,, \tag{A.6}$$

and

$$\tilde{\tilde{Y}}_{11} \simeq \tilde{Y}_{11} - \frac{\tilde{Y}^2_{12}}{\tilde{Y}_{22}},\ \tilde{Y}_{11} \simeq Y_{11} - \frac{\tilde{Y}^2_{13}}{Y_{33}},\ \tilde{Y}_{22} \simeq Y_{22} - \frac{Y^2_{23}}{Y_{33}}\,. \tag{A.7}$$

We will express all mass matrix elements in terms of their eigenvalues (recall that $m_1 = \tilde{\tilde{Y}}_{11}$, $m_2 = \tilde{Y}_{22}$ and $m_3 = Y_{33}$). Because of our assumption of no accidental cancellations we divide possible forms into two categories: either $Y_{22} << m_2$ and $Y_{23} = \sqrt{m_2 m_3}$, or $Y_{22} = m_2$ and $Y_{23} << \sqrt{m_2 m_3}$.

Now we use conditions (4.12) and (4.13). Let us first use condition (4.13) because it does not depend on whether $\mathbf{Y}$ is $\mathbf{U}$ or $\mathbf{D}$. It tells us that

$$\tilde{Y}_{12} = \sqrt{m_1 m_2}, \tag{A.8}$$

$$Y_{11} << m_1, \tag{A.9}$$

$$\tilde{Y}_{13} << \sqrt{m_1 m_3}. \tag{A.10}$$

Since $Y_{23} \leq \sqrt{m_2 m_3}$ (from $m_2 = \tilde{Y}_{22}$), it follows from equations (A.8) and (A.10) that $Y_{12} = \sqrt{m_1 m_2}$ and $Y_{13} << \sqrt{m_1 m_3}$. Therefore the symmetric $\mathbf{U}$ and $\mathbf{D}$ that obey (4.13) must take one of the following forms:

$$\begin{pmatrix} (Y_{11} << m_1) & \sqrt{m_1 m_2} & (Y_{13} << \sqrt{m_1 m_3}) \\ \sqrt{m_1 m_2} & (Y_{22} << m_2) & \sqrt{m_2 m_3} \\ (Y_{13} << \sqrt{m_1 m_3}) & \sqrt{m_2 m_3} & m_3 \end{pmatrix}, \tag{A.11}$$

where $Y_{22} << \frac{Y_{23}^2}{Y_{33}}$, or

$$\begin{pmatrix} (Y_{11} << m_1) & \sqrt{m_1 m_2} & (Y_{13} << \sqrt{m_1 m_3}) \\ \sqrt{m_1 m_2} & m_2 & (Y_{23} << \sqrt{m_2 m_3}) \\ (Y_{13} << \sqrt{m_1 m_3}) & (Y_{23} << \sqrt{m_2 m_3}) & m_3 \end{pmatrix}, \tag{A.12}$$

where $Y_{22} >> \frac{Y_{23}^2}{Y_{33}}$.

Let us now use the constraint (4.12) which will further constrain some of the bracketed elements in (A.11) or (A.12). Using reasonable values for quark masses we see that in (4.12) the more stringent constraint is

$$s_{13} << \sqrt{\frac{u_1}{u_2}} s_{23}, \tag{A.13}$$

where $s_{13} = s_{13}^D - s_{13}^U$ and $s_{23} = s_{23}^D - s_{23}^U$. We use $u_1 = m_u$, $u_2 = m_c$, etc. In particular, both s_{13}^U and s_{13}^D must be less than $\sqrt{\frac{u_1}{u_2}}s_{23}$:

$$|\frac{U_{13}}{u_3} + \frac{\sqrt{u_1u_2}}{u_3}\frac{U_{23}}{u_3}| << \sqrt{\frac{u_1}{u_2}}|\frac{D_{23}}{d_3} - \frac{U_{23}}{u_3}|, \tag{A.14}$$

$$|\frac{D_{13}}{d_3} + \frac{\sqrt{d_1d_2}}{d_3}\frac{D_{23}}{d_3}| << \sqrt{\frac{u_1}{u_2}}|\frac{D_{23}}{d_3} - \frac{U_{23}}{u_3}|. \tag{A.15}$$

Notice that consistency of solutions is automatically obeyed since $\frac{\sqrt{d_1d_2}}{d_3} << \sqrt{\frac{u_1}{u_2}}$ and $\frac{\sqrt{u_1u_2}}{u_3} << \sqrt{\frac{u_1}{u_2}}$ for reasonable quark masses. Therefore, we conclude that limits on U_{13} and D_{13} may be somewhat stringent:

$$U_{13} << min\{\sqrt{u_1u_3}, \sqrt{\frac{u_1}{u_2}}u_3s_{23}\} \equiv a\,, \tag{A.16}$$

$$D_{13} << min\{\sqrt{d_1d_3}, \sqrt{\frac{u_1}{u_2}}d_3s_{23}\} \equiv b\,. \tag{A.17}$$

If $\mathbf{U}$ or $\mathbf{D}$ is of type (A.12) somewhat stringent limits on U_{23} and D_{23} are also possible:

$$U_{23} << min\{\sqrt{u_2u_3}, \frac{u_3^2}{u_2}s_{23}\} \equiv c\,, \tag{A.18}$$

$$D_{23} << min\{\sqrt{d_2d_3}, \sqrt{\frac{u_1}{u_2}}\frac{d_3^2}{\sqrt{d_1d_2}}s_{23}\} \equiv d\,. \tag{A.19}$$

We can now write possible forms of symmetric $\mathbf{U}$ and $\mathbf{D}$ which lead to successful predictions (A.1) and (A.2). There are four possibilities depending on whether $\mathbf{U}$ or $\mathbf{D}$ take on the form (A.11) or (A.12)[90]

1)

$$\mathbf{U} = \begin{pmatrix} (U_{11} << u_1) & \sqrt{u_1u_2} & (U_{13} << \sqrt{u_1u_3}) \\ \sqrt{u_1u_2} & (U_{22} << u_2) & \sqrt{u_2u_3} \\ (U_{13} << \sqrt{u_1u_3}) & \sqrt{u_2u_3} & u_3 \end{pmatrix}, \tag{A.20}$$

$$\mathbf{D} = \begin{pmatrix} (D_{11} << d_1) & \sqrt{d_1 d_2} & (D_{13} << \sqrt{d_1 d_3}\sqrt{\frac{u_1 d_2}{u_2 d_1}}) \\ \sqrt{d_1 d_2} & (D_{22} << d_2) & \sqrt{d_2 d_3} \\ (D_{13} << \sqrt{d_1 d_3}\sqrt{\frac{u_1 d_2}{u_2 d_1}}) & \sqrt{d_2 d_3} & d_3 \end{pmatrix}. \quad \text{(A.21)}$$

2)

$$\mathbf{U} = \begin{pmatrix} (U_{11} << u_1) & \sqrt{u_1 u_2} & (U_{13} << \sqrt{u_1 u_3}) \\ \sqrt{u_1 u_2} & u_2 & (U_{23} << \sqrt{u_2 u_3}) \\ (U_{13} << \sqrt{u_1 u_3}) & (U_{23} << \sqrt{u_2 u_3}) & u_3 \end{pmatrix}, \quad \text{(A.22)}$$

$$\mathbf{D} = \begin{pmatrix} (D_{11} << d_1) & \sqrt{d_1 d_2} & (D_{13} << \sqrt{d_1 d_3}\sqrt{\frac{u_1 d_2}{u_2 d_1}}) \\ \sqrt{d_1 d_2} & (D_{22} << d_2) & \sqrt{d_2 d_3} \\ (D_{13} << \sqrt{d_1 d_3}\sqrt{\frac{u_1 d_2}{u_2 d_1}}) & \sqrt{d_2 d_3} & d_3 \end{pmatrix}. \quad \text{(A.23)}$$

3)

$$\mathbf{U} = \begin{pmatrix} (U_{11} << u_1) & \sqrt{u_1 u_2} & (U_{13} << a) \\ \sqrt{u_1 u_2} & (U_{22} << u_2) & \sqrt{u_2 u_3} \\ (U_{13} << a) & \sqrt{u_2 u_3} & u_3 \end{pmatrix}, \quad \text{(A.24)}$$

$$\mathbf{D} = \begin{pmatrix} (D_{11} << d_1) & \sqrt{d_1 d_2} & (D_{13} << b) \\ \sqrt{d_1 d_2} & d_2 & (D_{23} << d) \\ (D_{13} << b) & (D_{23} << d) & d_3 \end{pmatrix}. \quad \text{(A.25)}$$

4)

$$\mathbf{U} = \begin{pmatrix} (U_{11} << u_1) & \sqrt{u_1 u_2} & (U_{13} << a) \\ \sqrt{u_1 u_2} & u_2 & (U_{23} << c) \\ (U_{13} << a) & (U_{23} << c) & u_3 \end{pmatrix}, \quad \text{(A.26)}$$

$$\mathbf{D} = \begin{pmatrix} (D_{11} << d_1) & \sqrt{d_1 d_2} & (D_{13} << b) \\ \sqrt{d_1 d_2} & d_2 & (D_{23} << d) \\ (D_{13} << b) & (D_{23} << d) & d_3 \end{pmatrix}, \quad \text{(A.27)}$$

where a,b,c and d are given in equations (A.16)-(A.19).

In the above it is understood that U_{23} and D_{23} cannot be simultaneously equal to zero since they are constrained by the condition $V_{cb} = s_{23}$.

Some specific mass matrix ansätze can be recovered by setting the bracketed elements to zero. For example, 1) contains the Fritzsch scheme[9], while 2) is the generalization of the Harvey, Reiss and Ramond form[13]. Nevertheless, it is important to notice that although the bracketed elements can in many cases be set to zero, they need not to be. As long as they obey the limits, relations (A.1) and (A.2) will follow. For example, Y_{13} can be as big as Y_{12}!

Appendix B. SU(5) limit of matrices U^Q and U^L

Let us look at the SU(5) limit of (4.54-4.55). We can rewrite U^Q in terms of quantities that vanish in the SU(5) limit (4.49): $\Delta x_Q = x_Q - x'_u$,$\Delta x_u = x'_u - x_u$, $\Delta y_Q = y_Q - y_u$, $\Delta z_Q = z_Q - z'_u$, $\Delta z_u = z'_u - z_u$,

$$U^Q = \frac{1}{2}\begin{pmatrix} -\lambda_u e^{i\phi_u}(1+\frac{\Delta z_Q}{z'_u}) + s_2\Delta U^Q_{12} & \Delta U^Q_{12} & (\Delta x_Q + \Delta x_u)Bs_2 \\ & \lambda_c e^{i\phi_c} + \Delta U^Q_{22} & (\Delta x_Q + \Delta x_u)B \\ & & \lambda_t \end{pmatrix}, \tag{B.1}$$

where $\lambda_u = \frac{z_u z'_u C^2}{\lambda_c}$, $\lambda_c = y_u E_u$, $\lambda_t = A$, $s_2 = -\frac{z_u C}{\lambda_c} = \sqrt{\frac{\lambda_u}{\lambda_c}}\sqrt{\frac{z_u}{z'_u}}$, and $\Delta U^Q_{12} = -s_2\lambda_c\frac{\Delta z_Q + \Delta z_u}{z_u}e^{i\phi_u} + s_2\Delta U^Q_{22}$, and $\Delta U^Q_{22} = \Delta y_Q E e^{i\phi} - \Delta x_Q \frac{x_u B^2}{A}$.

Similarly, we rewrite U^L in terms of $\Delta x_{Ld} = x'_L - x_d$, $\Delta y_{Ld} = y_L - y_d$, $\Delta x_{Le} = x_L - x_e$, $\Delta x'_{Le} = x'_L - x'_e$ $\Delta x_{ed} = x_e - x'_d$, $\Delta z_{ed} = z_e - z'_d$, $\Delta z_{Ld} = z'_L - z_d$

and $\Delta z_{Le} = z_L - z_e$,

$$U^L = \begin{pmatrix} [\lambda_d e^{i\phi_u} \frac{(1+\frac{\Delta z_{ed}}{z'_d})}{1+(\Delta U^L_{22d}-\Delta U^L_{22e})e^{-i\phi_d}} + & [s_1\lambda_s(1+\frac{\Delta z_{Ld}}{z_d})e^{i\phi_u} + & \\ s_2(\Delta z_{Le}Ce^{i\phi_e} + s_{2e}\Delta U^L_{22e})] & +s_2(\lambda_s e^{i\phi_d} + \Delta U^L_{22d})] & [\Delta x_{Ld}B + V_{cb}\lambda_b]s_2 \\ & & \\ \Delta z_{Le}Ce^{i\phi_e} + s_{2e}\Delta U^L_{22e} & \lambda_s e^{i\phi_d} + \Delta U^L_{22d} & \Delta x_{Ld}B + V_{cb}\lambda_b \\ \Delta x_{Le}Bs_{2e} & \Delta x_{Le}B & \lambda_b \end{pmatrix}, \tag{B.2}$$

where $s_1 = \frac{z_d C}{\lambda_s} = \sqrt{\frac{\lambda_d}{\lambda_s}}\sqrt{\frac{z_d}{z'_d}}$, $s_{2e} = -\frac{z_e C}{\lambda_\mu} = (-s_1(1+\frac{\Delta z_{ed}}{z'_d}))\frac{\lambda_s}{\lambda_\mu}$, $U^L_{22d} = \Delta y_{Ld}Ee^{i\phi} - (\Delta x_{Ld}x'_d + \Delta x_{ed}x_d + \Delta x_{Ld}\Delta x_{ed})\frac{B^2}{A}$ and $U^L_{22e} = \Delta y_{Le}Ee^{i\phi} - \Delta x_{Le}x_e\frac{B^2}{A}$.

We can easily check the SU(5) limit (4.50) (all Δs going to zero); U^Q assumes the diagonal form proportional to up quark masses, and U^L receives the form as in (4.50), namely $U^L \longrightarrow V_G^* diag(\lambda_d, \lambda_s, \lambda_b)$, up to the leading order in relevant couplings.

Appendix C. Yukawa matrices for the "22" textures of [19]

Here we list the mass matrices as well as h^Q and h^L of the 6 O_{22} and 9 O_{23} models in the "22" texture of [19]. It is interesting to note that although mass matrices h^u, h^d and h^e do not disinguish between the 6 O_{22} models, h^Q, h^L and h^ν do. Also note that z_Q is small.

$$h^{\nu} = \begin{pmatrix} 0 & 125C & 0 \\ 125C & (-\frac{6}{5},\frac{6}{5},6,\frac{6}{5},6,-\frac{6}{25})Ee^{i\phi} & (0,0,0,0,0,0,\frac{1}{9},-\frac{1}{3},1)B \\ 0 & (-\frac{3}{5},-\frac{9}{5},\frac{1}{5},\frac{3}{5},0,0,\frac{1}{25},\frac{3}{25},\frac{9}{25})B & A \end{pmatrix} \quad \text{(C.1)}$$

$$h^{u} = \begin{pmatrix} 0 & -\frac{C}{27} & 0 \\ -\frac{C}{27} & 0 & (-4,-4,-4,-4,-4,-4,1,1,1)B \\ 0 & (1,-1,1,-1,-4,-4,1,-1,1)B & A \end{pmatrix} \quad \text{(C.2)}$$

$$h^{e} = \begin{pmatrix} 0 & C & 0 \\ C & 3Ee^{i\phi} & (-2,6,-2,6,6,6,\frac{1}{9},-\frac{1}{3},1)B \\ 0 & (-3,-9,1,3,-18,6,1,3,9)B & A \end{pmatrix} \quad \text{(C.3)}$$

$$h^{d} = \begin{pmatrix} 0 & C & 0 \\ C & Ee^{i\phi} & (2,2,-\frac{2}{3},-\frac{2}{3},2,-\frac{2}{3},1,1,1)B \\ 0 & (-\frac{1}{3},\frac{1}{3},-\frac{1}{3},\frac{1}{3},-\frac{2}{3},-\frac{2}{3},\frac{1}{9},-\frac{1}{9},\frac{1}{9})B & A \end{pmatrix} \quad \text{(C.4)}$$

$$h^Q = \frac{1}{2}\begin{pmatrix} 0 & -\frac{C}{27} & 0 \\ -\frac{C}{27} & (-\frac{3}{4},-\frac{3}{2},-\frac{1}{2},\frac{3}{2},\frac{1}{2},\frac{9}{4})Ee^{i\phi} & (1,1,1,1,1,1,1,1,1)B \\ 0 & (1,1,1,1,1,1,1,1,1)B & A \end{pmatrix} \tag{C.5}$$

$$h^L = \begin{pmatrix} 0 & C & 0 \\ C & (\frac{3}{4},\frac{5}{2},\frac{3}{2},\frac{3}{2},\frac{5}{2},\frac{3}{4})Ee^{i\phi} & (-\frac{1}{3},1,-\frac{1}{3},1,1,1,\frac{1}{9},-\frac{1}{3},1)B \\ 0 & (-3,-3,1,1,-3,1,1,1,1)B & A \end{pmatrix} \tag{C.6}$$

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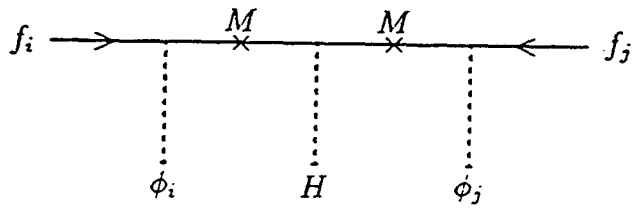

Figure 1: A model for Yukawa couplings. The f_i are the matter fields of flavor i, H is the Higgs doublet and ϕ_i are some scalars which carry flavor i. When the heavy fermions of mass M are integrated out we are left with an effective interaction $\frac{1}{M^2}(f_i\phi_i)H(f_j\phi_j)$ (we assumed that the coupling at the vertices in the figure are of order unity). When the scalars ϕ_i assume their vacuum expectation values, the flavor symmetries get broken, and we are left with an effective Yukawa coupling $\epsilon_i\epsilon_j$, where $\epsilon_i =< \phi_i > /M$.

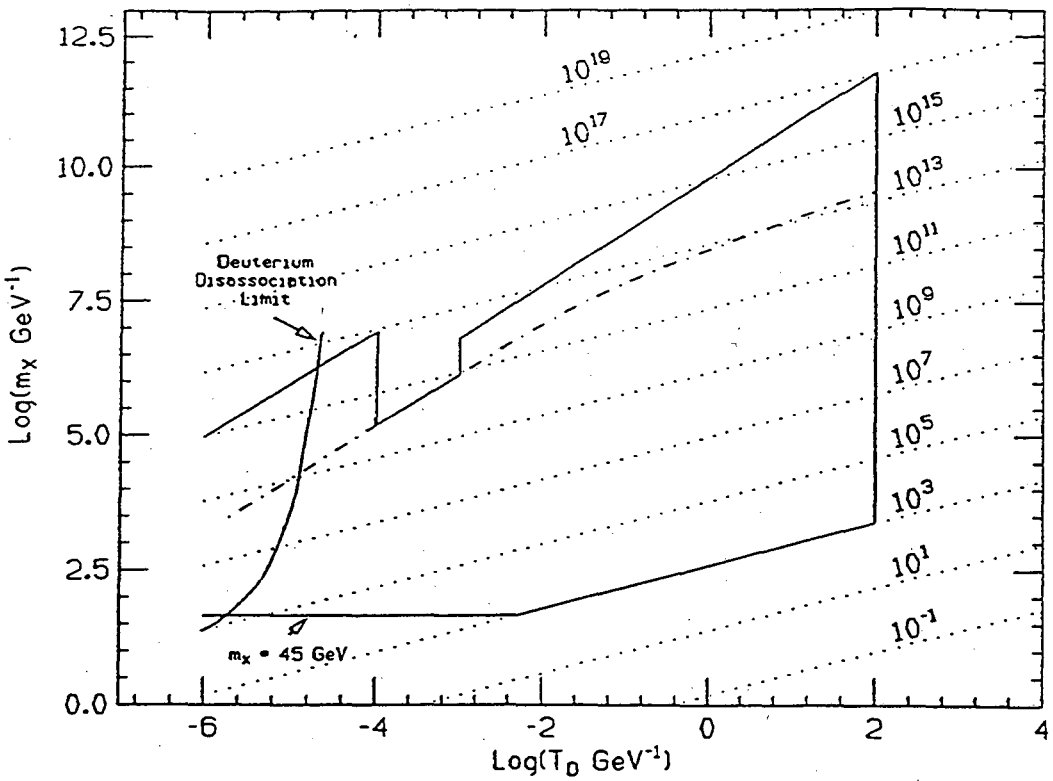

Figure 2: The allowed parameter space in our example is shown, bounded by solid lines. We have assumed $T_D \leq T_{min} \simeq 10^2\,\mathrm{GeV}$ and required $T_X > 10T_{min}$ and $m_X > 45\,\mathrm{GeV}$. The diagonal dotted lines are lines of constant T_D, and are labeled in GeV. Our parameter M is also constant on these dotted lines, $M = 2.9 \times 10^4 \left(\frac{T_X}{\mathrm{GeV}}\right)^{3/4}$ GeV. On the dot-dashed line the entropy of the universe is increased by 10% when X particles decay. In determining this line as well as the top boundary line we have assumed that X particle gauge interactions freeze out according to the standard cold relic freeze out criteria. We have made conservative assumptions in determining the relative increase in entropy upon X decay, allowing the cosmic scale factor to scale as t^n where n ranges from 1/2 to 2/3. We have used a value for η at the time of nucleosynthesis equal to $\left(\frac{11}{4}\right) 3 \times 10^{-10}$.

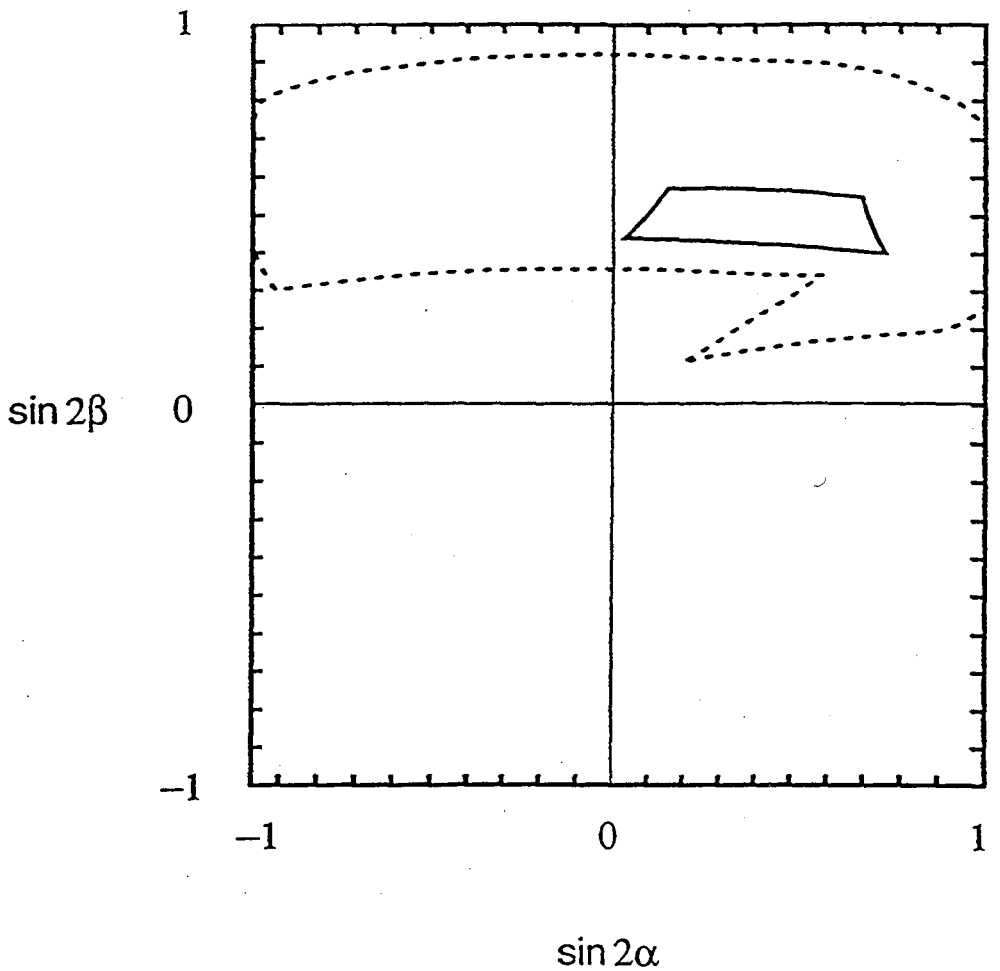

Figure 3: The allowed region (solid line) for CP asymmetries sin2α and sin2β using relations $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$ and $V_{td}/V_{ts} = \sqrt{m_d/m_s}$. The dotted region is allowed by the Standard Model.

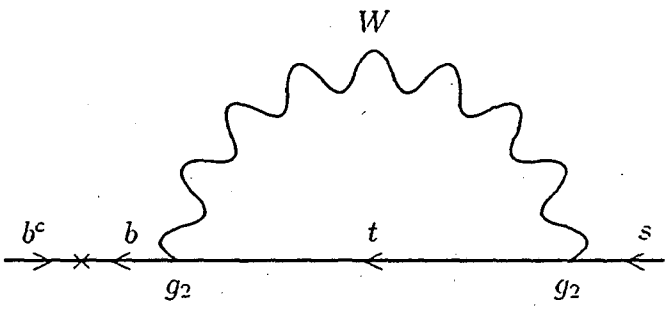

Figure 4: Standard Model contribution to the decay $b \to s\gamma$. A photon is understood to be attached to any of the lines.

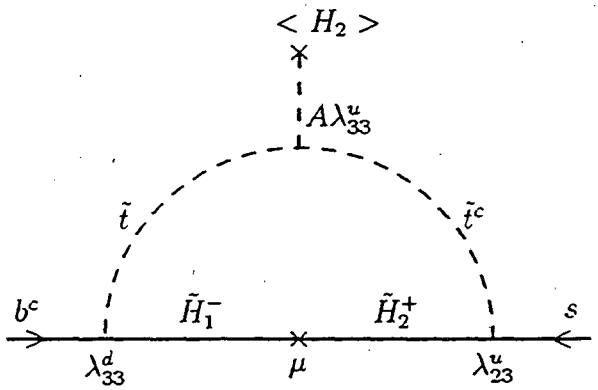

Figure 5: Leading chargino contribution to the decay $b \to s\gamma$ when $\tan\beta$ is large. A photon is understood to be attached to any of the lines of the charged particles.

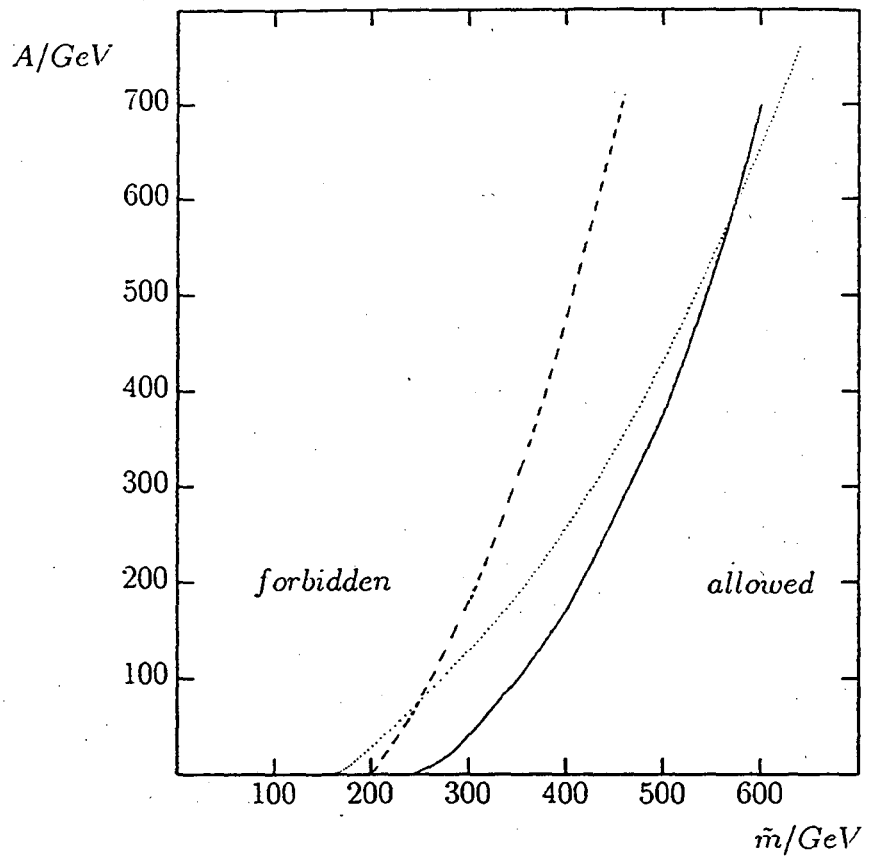

Figure 6: Limits on the MSSM parameters $\tilde{m}$ and A from the chargino loop contribution to radiative b decay when $\tan\beta$ is large. The lines represent $r = 3$ for different values of μ: $\mu = 10\,\mathrm{GeV}$ (dashed line), $\mu = 50\,\mathrm{GeV}$ (solid line), $\mu = 350\,\mathrm{GeV}$ (dotted line), $\tan\beta = 60$, $m_t = 180\,\mathrm{GeV}$ and $M = 200\,\mathrm{GeV}$. There is not much dependence on m_t (when varied between 150 GeV and 180 GeV) and M (when varied between 100 GeV and 400 GeV).

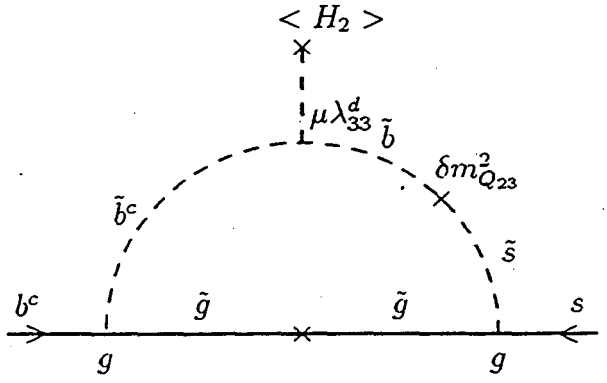

Figure 7: Gluino contribution to the decay $b \to s\gamma$. The flavor changing scalar vertex $\delta m^2_{Q_{23}}$ comes from renormalization group running. A photon is understood to be attached to any of the lines of the charged particles.

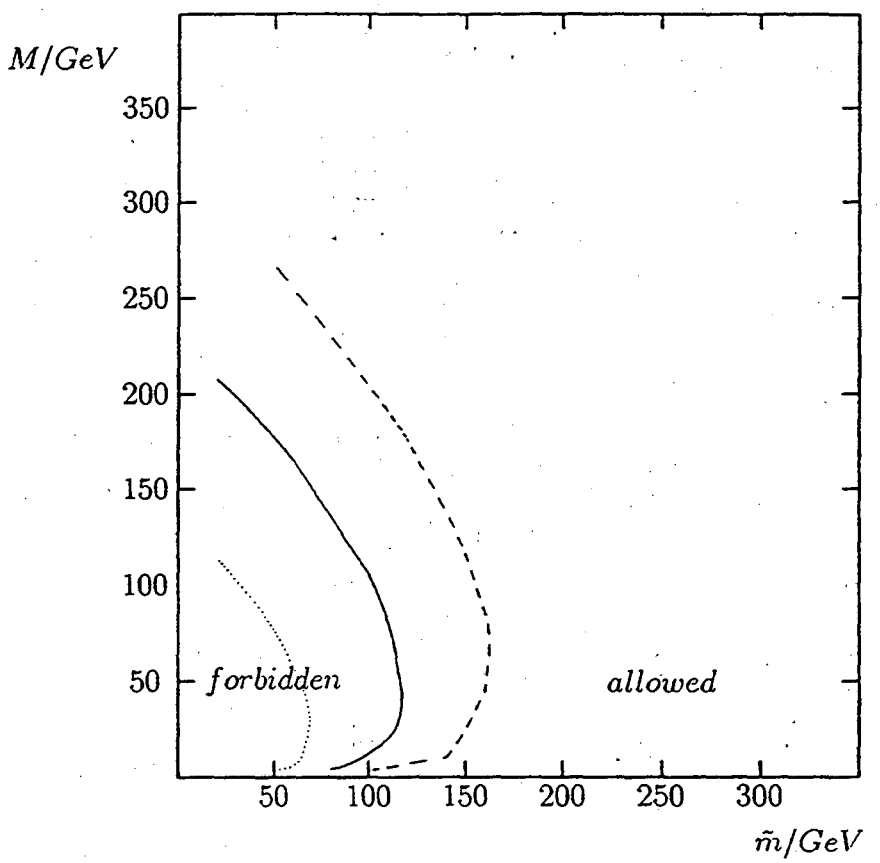

Figure 8: Limits on the MSSM parameters $\tilde{m}$ and M from the gluino loop contribution to radiative b decay when $\tan\beta$ is large. The lines represent $r = 3$ for different values of μ: $\mu = 10\,\mathrm{GeV}$ (dotted line), $\mu = 50\,\mathrm{GeV}$ (solid line), $\mu = 130\,\mathrm{GeV}$ (dashed line), $\tan\beta = 60$ and $m_t = 180\,\mathrm{GeV}$. There is not much dependence on m_t (when varied between 150 GeV and 180 GeV).

FIELD	FLAVOR SYMMETRY BREAKING PARAMETER
Q_i	$\sqrt{\eta_i \xi_i}$
U_i	$\sqrt{\frac{\eta_i}{\xi_i}}\eta_i$
D_i	$\sqrt{\frac{\xi_i}{\eta_i}}\xi_i$
L_i, E_i	$\sqrt{\frac{m_{E_i}}{v_1}}$

Table 1: The ansatz for flavor symmetry breaking paramaters associated with the chiral fermion fields. $\eta_i = \sqrt{\frac{m_{U_i}}{v_2}}$ and $\xi_i = \sqrt{\frac{m_{D_i}}{v_1}}$.

process	$M/GeV(250GeV/v_1)$
$\mu \to 3e$	1
$\mu \to e\gamma$	4
$\mu N \to eN$	10
$K_L^0 \to \mu^\pm e^\mp$	20
$B_d \to \tau^+\tau^-$	20 $(\frac{10^{-4}}{B.R.})^{\frac{1}{4}}$
$B_s \to \mu^+\mu^-$	70 $(\frac{10^{-6}}{B.R.})^{\frac{1}{4}}$
$\Delta m(B_d^0 - \bar{B}_d^0)$	400 $\sqrt{\kappa}$
$\Delta m(K^0 - \bar{K}^0)$	500 $\sqrt{\kappa}$

Table 2: Experimental lower limits on the exchanged scalar masses.

ansatz	$\sin^2(2\theta_{e\mu})$	$\sin^2(2\theta_{e\tau})$	$\sin^2(2\theta_{\mu\tau})$
I	2×10^{-2}	10^{-3}	0.2
II	0.2	0.1	0.8
III	2×10^{-3}	8×10^{-6}	2×10^{-2}

Table 3: Neutrino mixing angle predictions in the three ansätze introduced. As noted in the text, these results are meant as estimates rather than precise calculations.

22model	23model	$\frac{\Gamma(p\to K^+\nu_e)}{\Gamma(p\to K^0\mu)}$	$\frac{\Gamma(p\to K^+\nu_\mu)}{\Gamma(p\to K^0\mu)}$	$\frac{\Gamma(p\to K^+\nu_\tau)}{\Gamma(p\to K^0\mu)}$	$\sum\frac{\Gamma(p\to K^+\nu_i)}{\Gamma(p\to K^0\mu)}$
1	1	2.26	67.6	43.4	113.3
1	3	2.41	72.6	57.9	132.9
1	4	3.47	95.9	73.0	172.4
1	6	3.35	92.3	102.3	198.0
1	7	4.45	106.3	104.1	214.9
1	8	3.36	83.7	86.3	173.3
2	1	2.55	69.4	48.3	120.2
2	3	2.69	73.3	63.1	139.1
2	4	3.85	100.6	80.0	184.4
2	6	3.89	101.8	118.2	223.9
2	7	4.44	107.8	104.2	216.4
2	8	3.01	74.6	77.2	154.8
3	1	1.51	53.1	47.9	102.5
3	3	1.63	58.1	62.6	122.2
3	4	3.65	108.3	96.2	208.1
3	6	3.63	107.4	139.6	250.6
3	7	5.92	139.6	128.1	273.6
3	8	2.81	70.3	73.2	146.2
4	1	4.01	92.0	40.2	136.2
4	3	4.10	93.8	53.4	151.3
4	4	3.57	83.7	54.2	141.5
4	6	3.41	79.9	75.0	158.3
4	7	2.86	72.0	77.1	151.9
4	8	3.79	93.1	95.3	192.2
5	1	5.52	115.6	28.2	149.3
5	3	5.52	114.6	39.2	159.3
5	4	3.12	67.6	33.1	103.8
5	6	2.75	59.4	41.7	103.9
5	7	1.58	41.8	53.2	96.6
5	8	6.08	148.1	149.1	303.3
6	1	3.72	87.4	42.4	133.5
6	3	3.83	89.6	56.0	149.4
6	4	3.66	87.3	59.4	150.4
6	6	3.55	84.8	83.7	172.1
6	7	3.15	78.8	82.3	164.2
6	8	3.54	87.1	89.4	180.0

Table 4: Ratios of proton decay rates to neutrinos over the rate to muons in models of [19].

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720