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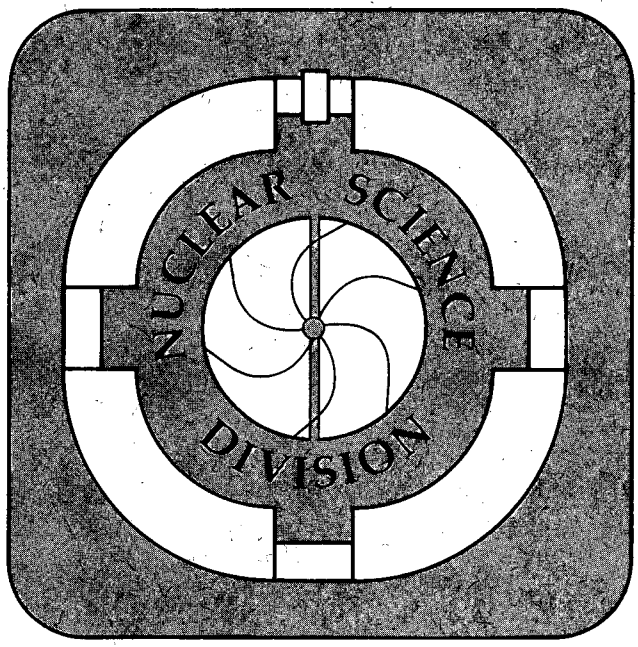

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Tests and Non-Tests of Time Reversal Invariance in Nuclear and Particle Physics

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ABSTRACT

There is a theorem that states that there can be no null test of time-reversal invariance (T symmetry) in exclusive reactions; that is, T symmetry does not require any *single* observable to vanish. This theorem is extended to inclusive reactions, and it is shown that purported null tests of T symmetry in electron scattering have been, in fact, non-tests of time-reversal invariance.

I. INTRODUCTION

Following the realization that there had been no experimental verification of parity *(P)* conservation in weak-interaction processes [1], there was an immediate investigation of possible experimental tests of the charge-conjugation *(C)* and time-reversal *(T)* symmetries because of the interrelation of the three symmetries that is provided by the *CPT* theorem [2]. Then, the subsequent experimental finding of *P* nonconservation in nuclear β-decay [3], and in pion and muon decays [4], was quickly followed by both theoretical and experimental activity concerning experimental tests of *T* symmetry.

Then, again, after the discovery of *CP* nonconservation in kaon decays [5], with the implied T noninvariance via the *CPT* theorem , there were increased efforts to devise and to carry out tests of *T* symmetry in other particle and nuclear systems. Null-test measurements of the proton and the deuteron analyzing powers [6], were proposed in $ep \rightarrow eX$ inclusive scattering [8] and in $ed \rightarrow ed$ elastic scattering [9]. These experiments were carried out with proton [10] and deuteron [11] targets, and the anticipated null results were found within the few to ten percent experimental uncertainties. In spite of this relatively modest precision, the very existence of this null-test observable could be important with respect to the level of experimental precision attainable in such a test of *T* symmetry. Since null tests of parity conservation exist, it has been possible to reach the remarkable precision of 2×10^{-8} in such tests [12,13], so a comparable null test of *T* would permit an improvement in experimental precision of *several orders of magnitude* over that achieved in past tests.

However, some fifteen years later a proof of the nonexistence of a *null* test of *T* in a two-particle in and two-particle out reaction was established. It states that in such a reaction there can be no *single* experimental observable that is required to vanish by *T* symmetry [14]. Thus, the often expressed view, that *T* symmetry requires a so-called *T*-odd observable to vanish, is in direct conflict with the theorem. It follows, then, that the finding

of a nonzero value of *any* observable cannot be taken as a proof of T violation, and the validity of the electron scattering tests is questionable.

The theorem, however, does not encompass total cross-section observables, and it has been shown that, there, T-odd spin-correlation observables provide null tests of T symmetry [15].

Since the importance of the "no null test" (of T) theorem has not been widely appreciated [16], it is developed in Section II from a more experimentally oriented perspective than that of the formal theoretical approach of reference 14, and the characterization of T-odd amplitudes and observables is discussed. The theorem is also extended to include the accessible observables of inclusive reactions $a + b \rightarrow c + X$. A parallel discussion of P-odd amplitudes and observables is presented, mainly for purposes of comparison and contrast, but the principal focus is on tests of T symmetry. Then, in Section III the purported null tests of T are examined, and the correct explanation of the results is provided. Section IV provides a summary of the discussion and conclusions.

II. NONEXISTENCE OF A NULL TEST OF TIME-REVERSAL INVARIANCE

For a reaction or scattering with two particles in and two out, the underlying reason for the lack of a null test of T symmetry can be clearly stated by comparison, for example, with a null test of parity conservation *(PC)*. In the latter case, one compares a transition amplitude or an experimental observable with the corresponding amplitude or observable for the *same*, but parity-transformed, process. Then, since PC requires that the corresponding amplitudes and observables be the same, any P-odd amplitude or observable vanishes [17]. The fundamental difference in a test of T is that one compares an observable in a reaction with a *different* observable in the inverse, i.e., time reversed, reaction, so that the difference (or sum) of the two observables is required by T symmetry to be zero. Thus, there is no single-observable null test of T in these reactions.

A. Reaction $a + b \rightarrow c + d$

For a more formal illustration of these remarks, consider a reaction with the simple spin-structure $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$. The matrix of amplitudes in the 2 X 2 spin-space can be expanded in terms of the Pauli spin matrices [18],

$$M(\theta) = \sum_j a_j(\theta)\, \sigma_j, \qquad j = o,x,y,z\,, \qquad \sigma_o = 1\,. \tag{1}$$

Choosing the center of mass helicity frame [19,20], in which the conditions imposed by T symmetry on the scattering/reaction amplitudes are most naturally expressed, unit vectors along the coordinate axes are

$$z_i\,(z_f) = k_i\,(k_f) \qquad y = k_i \times k_f \qquad x_i\,(x_f) = y \times z_i\,(z_f)\,, \tag{2}$$

where $k_i\,(k_f)$ is the c.m. momentum of particle $a\,(c)$. Then with the P and T transformations $k_{i,f} \rightarrow -k_{i,f}$, $\sigma \rightarrow \sigma$ and $k_i \leftrightarrow -k_f$, $\sigma \rightarrow -\sigma$, respectively, and noting that $\sigma_x \equiv \sigma \cdot x$ etc., one has the following transformations under the P, T symmetry operations:

$$\begin{aligned} P:&\quad \sigma_x, \sigma_y, \sigma_z \rightarrow -\sigma_x, \sigma_y, -\sigma_z \\ T:&\quad \sigma_x, \sigma_y, \sigma_z \rightarrow -\sigma_x, \sigma_y, \sigma_z\,. \end{aligned} \tag{3}$$

Then, the corresponding M-matrix amplitudes in (1) and in $M^t = \sum_j a^t_j\, \sigma_j$, the M matrix for the inverse reaction, can be classified according to their P and/or T symmetries as follows:

$$P: \qquad a_j = (-1)^{(n_x+n_z)}\, a_j \qquad (4a)$$

$$T: \qquad a_j = (-1)^{n_x}\, a^t_j \;; \qquad (4b)$$

and an amplitude a_j is P-odd (T-odd) if $n_x + n_z$ (n_x) is odd , where n_x (n_z) is the number of x (z) subscripts [21]. Thus, PC requires the P-odd (P-even) amplitudes to vanish when the product of the particles' intrinsic parities is even (odd), but T symmetry imposes no such condition on the T-odd amplitudes. It requires only that the T-odd amplitudes satisfy the condition (4b) that $a^t_x = -a_x$ [22]. Only in the case of elastic scattering, which is its own inverse reaction, does this condition force the amplitude to vanish.

Another form of this 2 X 2 matrix is that in which the elements, $M(if)$ are the actual transition amplitudes between initial and final helicity states,

$$M(\theta) = \begin{pmatrix} M(++) & M(-+) \\ M(+-) & M(--) \end{pmatrix} \qquad (5)$$

Comparison with (1), expressed in terms of the (invariant) amplitudes a_j,

$$M(\theta) = \sum_j a_j(\theta)\, \sigma_j = \begin{pmatrix} a_0+a_z & a_x-ia_y \\ a_x+ia_y & a_0-a_z \end{pmatrix}, \qquad (6)$$

shows the connection between the two sets of amplitudes. For the general case of all four particles with helicities $\alpha + \beta \rightarrow \gamma + \delta$, $M(if) = M(\alpha\beta,\gamma\delta)$, and the conditions imposed by the P and T symmetries directly on these helicity amplitudes $M(\alpha\beta,\gamma\delta)$ have been established [20], and these conditions are correspondingly satisfied in (6) by the conditions (4) imposed on the amplitudes a_j (a_{jk} in the general case).

Consider, now, the experimental observables for reactions with this particular spin-structure [7],

$$X(j,k) = Tr\, M\sigma_j M^\dagger \sigma_k \,/\, Tr\, MM^\dagger\,, \qquad j,k = o,x,y,z\,, \tag{7}$$

where j labels the polarization component of the initial-state particle, k labels the observed final-state polarization component, and $j\,(k) = o$ for unpolarized incident particles (unobserved final polarization). Since, by definition, the P transformation of the M-matrix is $M \rightarrow M$, the combination $M, M^\dagger$ contributes no change of sign in the P transformation of an observable, so its P symmetry is determined by the explicit spin-operators, σ_j and σ_k, in (7). Its T symmetry is determined in the same manner. Thus, with (3), it follows from (7) that these observables can be classified according to their P and T symmetries in exactly the same way as was found for the amplitudes in (4):

$$P: \qquad X(j,k) = (-1)^{(n_x+n_z)}\, X(j,k) \tag{8a}$$

$$T: \qquad X(j,k) = (-1)^{n_x}\; X^t(k,j) \tag{8b}$$

[23]. So, now, PC requires a P-odd *observable* to be zero, but the T symmetry condition is that an observable is equal to (+/-) a *different* observable in the inverse process (k,j), which proves the "no null test" theorem that there can be no single vanishing T-odd observable.

Even in the case of elastic scattering, where the T-odd amplitude a_x vanishes, the theorem applies. This circumstance can be understood from the specific expressions for appropriate pairs of observables in terms of the amplitudes; for example, analyzing powers and polarizing powers. From (1) and (7), with $I = \frac{1}{2} Tr\, MM^\dagger$, one obtains

$$IA_j \equiv IX(j,o) = 2(Re\; a_o a_j^* + Im\; a_k a_l^*), \tag{9a}$$

$$IP_j \equiv IX(o,j) = 2(Re\; a_o a_j^* - Im\; a_k a_l^*), \qquad j,k,l \text{ cyclic in } x,y,z\,. \tag{9b}$$

It is clear from (9), with the T-odd amplitude $a_x = 0$, that none of these observables goes to zero [24] and that

$$A_x = -P_x\,, \quad A_y = P_y\,, \quad A_z = P_z\,, \tag{10}$$

all in accord with (8b). Since there are *two* either P-odd or P-even amplitudes which vanish when parity is conserved, the P-odd observables vanish, now in accord with (8a).

In the more general case of a reaction, consider (9b) for the inverse reaction,

$$I^t P^t_j = 2(Re\ a^t_o a^{t*}_j - Im\ a^t_k a^{t*}_l). \tag{11}$$

Then, with the T-odd amplitudes $a^t_x = -a_x$, from (9a) and (11) one finds [25]

$$A_x = -P^t_x\,, \quad A_y = P^t_y\,, \quad A_z = P^t_z\,, \tag{12}$$

just as required by (8b).

The entire foregoing discussion is easily generalized to reactions/scattering of more complex spin-structures. Consider, for example, the case of particular interest in particle physics, $a + b \rightarrow c + d$, with four spin-$\frac{1}{2}$ particles. The required 4 X 4 M-matrix can now be expanded in terms of direct products of the 2 X 2 (a,c) and (b,d) matrices σ_j and σ_k, respectively [26],

$$M(\theta) = \sum_{j,k} a_{jk}(\theta)\ \sigma_j \otimes \sigma_k\ , \qquad j,k = o,x,y,z\ , \quad \sigma_o = 1\,. \tag{13}$$

In a more compact form, with the 4 X 4 matrix $\sigma_{jk} \equiv \sigma_j \otimes \sigma_k$,

$$M = \sum_{j,k} a_{jk}\ \sigma_{jk}\ , \tag{14}$$

and the 16 M-matrix amplitudes,

$$a_{oo},\ a_{ox},\ a_{oy},\ a_{oz},\ a_{xo},\ a_{xx},\ a_{xy},\ a_{xz}$$
$$a_{yo},\ a_{yx},\ a_{yy},\ a_{yz},\ a_{zo},\ a_{zx},\ a_{zy},\ a_{zz}, \tag{15}$$

can then be classified, as in (4), according to their P and/or T symmetries,

$$P: \quad a_{jk} = (-1)^{(n_x+n_z)}\ a_{jk} \tag{16a}$$
$$T: \quad a_{jk} = (-1)^{n_x}\ a^t_{jk} \tag{16b}$$

Also, again from (3), the experimental observables,

$$X(jk,lm) = Tr\ M\ \sigma_{jk}\ M^\dagger \sigma_{lm}\ /\ Tr\ MM^\dagger, \qquad j,k,l,m = o,x,y,z\ , \tag{17}$$

have, as in (8a) and (8b), the symmetries

$$P: \qquad X(jk,lm) = (-1)^{(n_x+n_z)}\ X(jk,lm) \tag{18a}$$
$$T: \qquad X(jk,lm) = (-1)^{n_x}\ X^t(lm,jk). \tag{18b}$$

Here, j,k designate the polarization components of particles a,b , and l,m the observed polarization components of c,d.

Finally, since the components, S_j , of the spin operator for any spin S transform just as the σ_j in (3), the symmetries (18a) and (18b) apply to reactions of particles with arbitrary spins. This includes the second (and higher) rank tensor observables, since the corresponding spin operators are constructed from combinations of the rank-one

operators S_j [23]. The equivalent symmetries imposed on the observables in their spherical-tensor form, rather than the cartesian form used here, have been established [27].

As is stated above in connection with (4) and (8) and now with (16) and (18), unlike the *PC* requirement that a *P*-odd amplitude or observable vanish, *T*-symmetry requires that a *pair* of amplitudes or observables satify the condition that

$$X - X^t = 0 \quad (T\text{-even}) \qquad or \qquad X + X^t = 0 \quad (T\text{-odd}). \tag{19}$$

Only in elastic scattering do the *T*-odd amplitudes vanish, but, in general, there is no single *T*-odd observable.

It is clear, then, that the valid argument, that the expectation value of a *P*-odd observable must vanish from *P* symmetry, has no counterpart with respect to *T*-odd observables and *T* symmetry. In fact, both (pairs of) *T*-even and *T*-odd observables, e.g. (12), are available for tests of *T* symmetry when *P* is not conserved, and it is only when the appropriate condition (19) is not satified that *T* symmetry violation is demonstrated. Since PC requires that $A_x = P_x = A_z = P_z = 0$, it is interesting to note that the rather standard nuclear physics test of $A_y - P^t_y = 0$ is a *T-even* test of *T* symmetry.

B. Inclusive reactions $a + b \rightarrow c + X$

In view of the fact that many inclusive experiments are pursued, especially in particle physics, it is of obvious interest to know whether or not there are *P* and/or *T* imposed symmetries on the available experimental observables in such reactions, $a + b \rightarrow c + X$, where only particle *c* is detected in the final state [28]. From energy and momentum conservation, *X* can be treated as a composite "particle" of known mass and momentum, with, however, unobservable spin. This latter fact has no effect on the observables involving

particles $a, b,$ and $c,$ and it will be seen that these observables retain the same symmetries as in the $2 \rightarrow 2$ exclusive reactions, namely (18a) and (18b).

Consider a reaction in which $a, b,$ and c are spin-$\frac{1}{2}$ particles, i. e., fermions. Then from baryon and lepton conservation, "particle" X is also a "fermion" and , for the purpose of illustration, is taken to be spin-$\frac{1}{2}$. Then the available observables are given as in (17) with $m = o,$ corresponding to the fact that the "polarization" of X is not observed [29],

$$X(jk,lo) = Tr\, M\, \sigma_{jk}\, M^{\dagger} \sigma_{lo} \,/\, Tr\, MM^{\dagger}. \tag{23}$$

Then, just as before, these observables have the symmetries given in (18). In order to better understand the specific details of these results, we consider again, for example, the expressions for the analyzing power A_{yo} and the inverse-reaction polarizing power $P^t{}_{yo}$, even though the latter cannot be determined experimentally. These are

$$IA_{yo} \equiv IX(yo,oo) = \frac{1}{4}\, Tr\, M\, \sigma_{yo}\, M^{\dagger}, \tag{24a}$$

$$I^tP^t{}_{yo} \equiv I^tX^t(oo,yo) = \frac{1}{4}\, Tr\, M^tM^{t\dagger}\, \sigma_{yo}\ , \tag{24b}$$

and with (14), Eq. (24a) becomes

$$IA_{yo} = \frac{1}{4}\, Tr\ \left[\left(\sum_{j,k} a_{jk}\, \sigma_{jk}\right) \sigma_{yo} \left(\sum_{j'k'} a_{j'k'}^{*}\, \sigma_{j'k'}\right)\right]. \tag{25}$$

Then, noting that

$$Tr\, \sigma_{jk}\, \sigma_{lm} = Tr\, [(\sigma_j\sigma_l) \otimes (\sigma_k\sigma_m)] = Tr\, \sigma_j\sigma_l\, Tr\, \sigma_k\sigma_m\ , \tag{26}$$

we have

$$IA_{yo} = \frac{1}{4} \sum_{j,k} \sum_{j',k'} a_{jk}\ a_{j'k'}^{*}\, Tr\, \sigma_j\sigma_y\sigma_{j'}\, Tr\, \sigma_k\sigma_{k'}\,. \tag{27}$$

Here one sees that the matrix operations factor, as they must, into operations in the separate (a,c) and (b,X) spin-spaces. Then using the properties $\sigma_j\sigma_k = i\sigma_l$, $\sigma_j^2 = \sigma_o$, $Tr\, \sigma_j\sigma_{j'} = 2\, \delta_{jj'}$, one finds

$$\begin{aligned} Tr\, \sigma_j\sigma_y\sigma_{j'} &= 2i\ (-2i) \quad \text{for} \quad (j,j') = (x,z)\ ((z,x)), \\ &= 2 \quad \text{for} \quad (j,j') = (o,y) \ \text{or}\ (y,o)\ , \\ &= 0 \quad \text{otherwise}\ , \end{aligned} \tag{28}$$

and (27) becomes

$$IA_{yo} = \sum_k 2(Re\ a_{ok}\ a_{yk}^* + Im\ a_{zk}\ a_{xk}^*), \tag{29}$$

and, similarly,

$$I^tP^t_{yo} = \sum_k 2(Re\ a^t_{ok}\ a^t_{yk}^* - Im\ a^t_{zk}\ a^t_{xk}^*) \tag{30}$$

for the inverse reaction. Comparing these two equations with (9a) and (11), one sees that they have identical forms, with the additional summation over k coming from taking the *trace* over the (b,X) part of the spin-space, which performs the sums over the spin projections of particles b and X. One then recovers the symmetries (12), and more generally (18b), among these inclusive observables, and these are independent of the "spin" of "particle" X. The parity-imposed symmetries on the (spherical-tensor) observables in a reaction with a three particle final-state have been discussed in a detailed treatment which uses the P-symmetries of the amplitudes to deduce those of the observables [30]. The corresponding inclusive observables are also included.

III. PURPORTED NULL TESTS OF TIME-REVERSAL INVARIANCE

A. Electron scattering

As is noted in section I, there have been inclusive experiments that searched for a nonvanishing analyzing power, A_{oy}, in the reaction $ep \rightarrow eX$ with a polarized proton target, at energies from 4 to 18 GeV [10]. This was purported to be a null test of T symmetry [8]. However, as has been detailed in Section II.B, even in this reaction there can be no null test of T. With that in mind it is straightforward to identify the error of reference 8. Their proposal was to measure the (presumably T-odd) correlation function

$$\boldsymbol{S} \cdot (\boldsymbol{k}_i \times \boldsymbol{k}_f), \tag{32}$$

where $\boldsymbol{S}$ is the polarization of the target proton. This, however, is my σ_y which from (3) is a T-even correlation, so there seems to have been a simple error of sign. The decay process $A \rightarrow B + e^+ + e^-$ was also discussed in [8], and the decay correlation-function

$$\sigma \cdot (\boldsymbol{k}_+ \times \boldsymbol{k}_-) \tag{33}$$

was examined. Here B is a spin-$\frac{1}{2}$ particle and $\boldsymbol{k}_+$ $(\boldsymbol{k}_-)$ is the e^+ (e^-) momentum. Although (33) has the same form as (32), it is, indeed, a T-odd correlation. The difference resides in the different T transformations: $\boldsymbol{k}_i \leftrightarrow -\boldsymbol{k}_f$, interchanging initial and final states, and $\boldsymbol{k}_+, \boldsymbol{k}_- \rightarrow -\boldsymbol{k}_+, -\boldsymbol{k}_-$, so that

$$(\boldsymbol{k}_i \times \boldsymbol{k}_f) \rightarrow (\boldsymbol{k}_f \times \boldsymbol{k}_i) = -(\boldsymbol{k}_i \times \boldsymbol{k}_f),$$

but

$$(\boldsymbol{k}_+ \times \boldsymbol{k}_-) \rightarrow (\boldsymbol{k}_+ \times \boldsymbol{k}_-), \tag{34}$$

and this difference may have gone unnoticed. So, even in the absence of the "no null test" theorem, the T-even observable A_{oy} would not have been required to vanish from T symmetry. It does vanish, however, from the one-photon exchange dynamics [7].

It now seems clear that the proposal to test T symmetry in the measurement of the same T-even observable, A_{oy}, or equivalently $P_{oy} \equiv X(oo,oy)$, in elastic ed scattering [9], which was done at 1 GeV [11], simply propagated this error; so, it is of obvious interest to know if this misconception has been corrected in the literature at some point during the past twenty-seven years. That it has not is indicated by continuing references to the reference 8 result. The same presumed T-odd correlation was investigated in pion photoproduction [31], and a connection was made to a test of C invariance [32]. Somewhat later, a review article [33] included the [8] result. Finally, references made in recent articles [34-37] show that the misconception still exists. Reference 35 does not quote the [8] result directly, but, in its treatment of A_{yo} in the deep-inelastic scattering of transversely polarized electrons from protons, it similarly arrives at the conclusion that a nonzero value of A_{yo} signals T violation. Reference 37, in its investigation of polarization observables in the inclusive electrodisintegration of the deuteron, $ed \rightarrow eX$, concludes that two form factors must vanish below the pion threshold because of T symmetry. These form factors correspond to the target analyzing power A_{oy} and the spin-correlation coefficient $X(z,yz;o,o)$, both of which are T-even observables. The latter involves the correlation between the electron polarization p_z and the deuteron tensor polarization p_{yz} [7,23].

The remaining point to be made , of course, is to show that a genuine T-odd, but P-even, observable does not vanish in (parity conserving) electron scattering. The appropriate observable is the spin-correlation coefficient $A_{zx} \equiv X(zx,oo)$, which in ep elastic scattering is proportional to G_M^2 [7], and the proton magnetic form factor G_M is certainly not zero.

IV. SUMMARY AND CONCLUSIONS

The theorem, that there can be no null test of T symmetry in a two-particle in and two-particle out reaction, has been extended to include inclusive reactions . As a result, the purported null tests in inclusive ep scattering and in elastic ed scattering are shown to have been non-tests of of T symmetry.

It follows, then, that there have been no direct tests of T symmetry in the electromagnetic interaction via electron scattering. It is somewhat discouraging to realize that the required comparison, between a reaction observable and its inverse-reaction counterpart, is very difficult in electron scattering. The possiblity in elastic ep scattering, for example, is to test the PC T-odd condition that

$$A_{zx} \equiv X(zx,oo) = - C_{zx} \equiv - X(oo,zx). \tag{42}$$

The most difficult part of this experimental comparison would seem to be the precise determination of the polarization p_z of the scattered electron.

Although T tests have been more readily available in strong-interaction processes, it is now important to improve the precision beyond the 10^{-2} to 10^{-3} level of experimental error that has been achieved. In view of the fact that a total cross-section observable is not included in the "no null test" theorem, and that such a T-test observable has been identified [15], it is important to pursue such tests because of the considerably higher precision that is inherently possible in null tests.

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22. The actual condition is $k_f a^t_X = -k_i a_X$ [20], but I drop the k factors since they cancel in the expressions for all of the observables except the differential cross-section.

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