LBL-36155
UC-414
Preprint

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

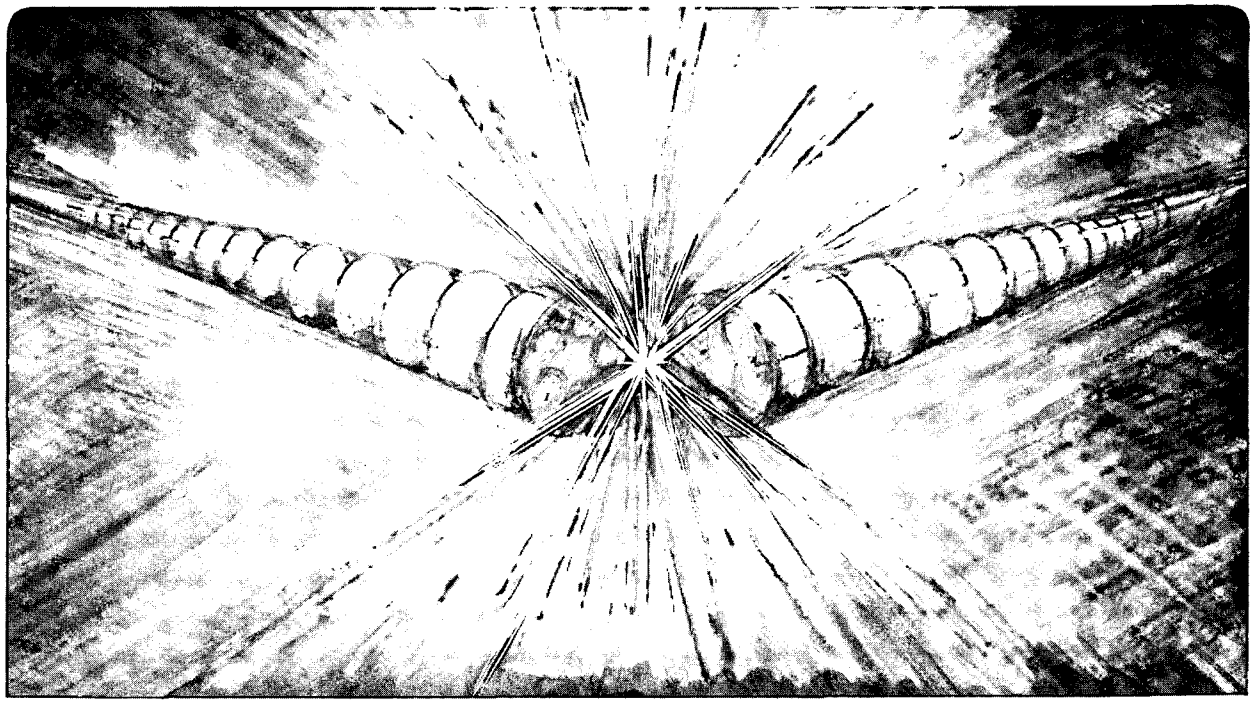
Accelerator & Fusion Research Division

Submitted to Nuclear Instruments and Methods in Physics Research A

Theory for Inductively Detuned Traveling Wave Structures

H. Li and S.S. Yu

December 1994



REFERENCE COPY
Does Not
Circulate
Bldg. 50 Library.
LBL-36155
Copy 1

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof, or The Regents of the University of California.

Lawrence Berkeley Laboratory is an equal opportunity employer.

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LBL-36155

Theory for Inductively Detuned Traveling Wave Structures*

Hai Li and Simon S. Yu

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

December 1994

* Work supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy, under Contract(s) No. DE-AC03-76SF00098 at Lawrence Berkeley Laboratory.

Theory of Inductively Detuned Traveling Wave Structures*

Hai Li and Simon S. Yu

Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

Abstract

As high power rf extraction cavities, traveling wave structures (TWS) have demonstrated significant advantages due to their inherent low field gradients. For applications involving long, multi-cavity devices such as a relativistic klystrons two-beam accelerator (RK-TBA), the extraction cavities must be inductively detuned to maintain longitudinal beam stability. In this paper, the theory of inductively detuned traveling wave cavities is developed within the framework of a coupled-cavity circuit description of TWS. We determine the output cell parameters (eigenfrequency ω_0 and quality factor Q) required for proper matching to avoid unwanted reflections. We then derive the power balance equation which quantifies the generation and the transmission of electromagnetic energy in each cell of the TWS. An analytic formula for predicting the power output from a TWS is obtained. Finally, using the analytic results derived we check the applicability of the computer code RKS for inductively detuned TWS's.

1. Introduction

In our recent work [1] on the design study of longitudinal dynamics of the drive beam in a 1 TeV relativistic klystron two-beam accelerator (RK-TBA) [2-3] we employed the so-called "inductive detuning" concept on the traveling wave (TW) extraction cavities to counter the debunching of the drive beam caused by space charge and rf-induced energy spread so that the level of output power can be maintained stably for many cavities. In this scheme the extraction cavities are inductively detuned so that the phase velocity of the operating wave mode is larger than the speed of light, c, and therefore, is off-synchronism with the drive beam. The particle

* Work supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy, under Contract No. DE-AC03-76SF00098 at the Lawrence Berkeley Laboratory.

bunches lag behind the decelerating crest of the wave (as illustrated in Figure 1), and the energy loss becomes phase dependent. This eventually leads to stable rf buckets [1].

The above design study was carried out via a computer code named "RKS" which was previously developed by Ryne and Yu [4], and is suitable for numerical investigations of beam-rf interactions in traveling wave structures (TWS's) that are used to extract power from RK's. To apply the RKS code to a TWS we need to know the eigenfrequency and the quality factor for the output cell (normally the last cell) of the TWS, so that only a forward wave is propagated and amplified, and there exists no reflected wave. This is the so-called "matching condition". Although such a matching condition was previously obtained in analytic work by Ryne and Yu [5], it applies only to the synchronism case when the wave and the beam are in phase. To use the RKS code to study the inductive detuning cases, a more general theoretical framework is needed and is derived in this paper.

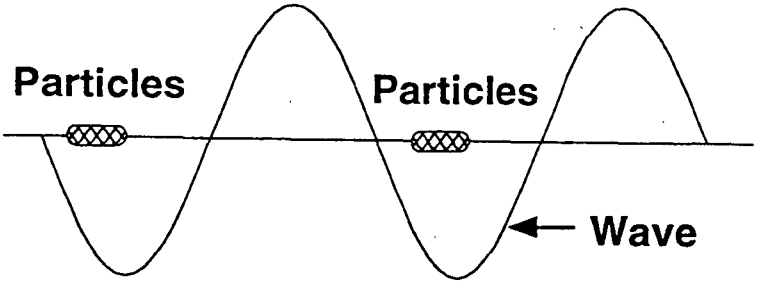


Fig. 1. Schematic of the "inductive detuning" concept

In this study, we start with the coupled circuit equations. We first derive the "matching conditions" under which a TWS propagates and amplifies only a forward traveling wave with the phase advance of the rf field being arbitrary with respect to that of the drive beam. We then obtain a power balance equation which quantifies the generation and the transmission of electromagnetic energy in each cell of the TWS. We also derive in this study an analytic formula that expresses the power extracted from the output cell of a TWS in terms of the induced current, the operating frequency and the cavity related parameters. This formula is useful for predicting the power output from a TWS. Finally, we derive the analytic expressions that characterize the amplitudes and the phases of a rf field in a detuned 3-cell TWS, and then use the obtained formula to check the applicability of the RKS code to the inductively detuned TWS's.

2. Theory of Inductively Detuned Traveling Wave Structures for Power Extraction

We adopt the analytical approach employed by Ryne and Yu in ref. [5]. Consider a TW structure consisting of N cells. Let the electric field in the nth cell of the structure be given by

$$\vec{E}_n(\vec{r},t) = a_n(t)\cdot \vec{\varepsilon}_n(\vec{r})\cdot e^{-i\omega t} \tag{1}$$

where $\vec{\varepsilon}_n$ denotes the eigenmode of the nth cell with eigenfrequency ω_n, and where we have assumed that $\omega_n \approx \omega$, with ω being the frequency of the RF field. It can be shown that, in the steady state, the excitation amplitudes a_n are governed by the following difference equations:

$$(\omega_n^2 - \omega^2 - i\frac{\omega\omega_n}{Q_n})\cdot a_n - (K_n^{n-1}\cdot a_{n-1} + K_n^{n+1}\cdot a_{n+1}) = \frac{i\omega}{\varepsilon_0}\int (d^3r)_n\cdot \vec{\varepsilon}_n^{\,*}\cdot \vec{J}_1, \tag{2}$$

where $n = 1, \ldots, N$ and $a_0 \equiv a_{N+1} \equiv 0$. In the above equations, K_n^{n-1} and K_n^{n+1} describe the coupling of cell n to cell $n-1$ and cell $n+1$, respectively. The quantity Q_n denotes the quality factor of the nth cell. $\vec{J}_1$ denotes the first harmonic of the RF current associated with the bunched beam. For the purpose of our analytic study, we assume that

$$K_n^{n-1} = K_n^{n+1} \equiv K. \tag{3}$$

Then the difference Eqs. (2) can be rewritten as

$$(\omega_n^2 - \omega^2 - i\frac{\omega\omega_n}{Q_n})\cdot a_n - K\cdot(a_{n-1} + a_{n+1}) = S_n, \tag{4}$$

where $S_n = \frac{i\omega}{\varepsilon_0}\int (d^3r)_n\cdot \vec{\varepsilon}_n^{\,*}\cdot \vec{J}_1$ is the drive term.

Most of the previous theoretical works on the TWS's assume an infinitely periodic structure, the treatment of which is straightforward. In this study, we too start by deriving the dispersion equation for an infinitely periodic structure. We then proceed to focus on the case of a structure with finite number of cells and develop a theory to show that if ω_N and Q_N for the last cell (the output cell) satisfy certain "matching conditions", the structure behaves just like a structure with infinite number of cells and there exists only a forward propagating wave. The properties of an infinitely periodic structure then applies also to this finite structure. The derivations are valid whether the wave is in synchronism with the particles or not.

2.1. Dispersion equation for an unloaded TWS with infinite number of cells

We will first review briefly the case with no source ($S_n = 0$) and no loss ($Q_n \to \infty$). We may assume a plane wave solution $a_n \sim e^{in\phi_p}$ and obtain from (4)

$$\omega_n - \omega \approx \frac{K}{\omega} cos(\phi_p), \tag{5}$$

where $\phi_p = kL_p$ is the phase advance of the wave across a single cell, k is the wave number and L_p is the longitudinal dimension of the cell. In obtaining (5) $\omega_n + \omega \approx 2\omega$ is used. The coupling constant K can be expressed in terms of the group velocity, V_g. Taking the derivative on both sides of Eq. (5) with respect to k, it is found that

$$K = \frac{\omega V_g}{L_p \cdot sin(\phi_p)}. \tag{6}$$

Substituting (6) into (5) we find that for an unloaded TWS with infinite number of cells (i.e., $N \to \infty$) the operating frequency ω and the eigenfrequency of cell n, ω_n, are related by the following dispersion equation

$$\omega_n = \omega + \frac{V_g}{L_p} ctg(kL_p). \tag{7}$$

2.2. Matching conditions for a TWS with finite number of cells

In reality, a TWS has only finite number of cells. We consider the case where, except for the first and last cells which may serve, respectively, as the input and the output cavities, the cells in between are identical, i.e.

$$\begin{aligned} Q_n &= Q_v \qquad (n = 2, \cdots, N\text{-}1), \\ \omega_n &= \omega_v \qquad (n = 2, \cdots, N\text{-}1). \end{aligned} \tag{8}$$

We will show that ω_1, Q_1, ω_N and Q_N can be determined in such a way that (i) for the other N-2 cells the TWS behaves just like an infinitely periodic structure, and (ii) there is no reflected wave in the structure but only a forward propagating wave. (i) is essentially the boundary condition(s), while (ii) is the so-called "matching condition(s)" in its original sense.

To obatin ω_1, Q_1, ω_N and Q_N that satisfy the above conditions, first, we solve difference equations (4) analytically. Since $a_0 = a_{N+1} = 0$, the equations for the 1st (n=1) and the last (n=N) cells are different in forms from those for the rest of the cells (n = 2, ..., N-1). By choosing

appropriate ω_1, Q_1, ω_N and Q_N, the equations for the first and the last cells may be written in the same form as those for the rest of the cells. Introducing parameters μ and η we define

$$\begin{aligned} \hat{a}_0 &= \mu a_1, \\ \hat{a}_{N+1} &= \eta a_N, \qquad (n = 1, ..., N) \\ \hat{a}_n &= a_n, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \mu &= \frac{1}{K}[(\omega_v^2 - i\frac{\omega\omega_v}{Q_v}) - (\omega_1^2 - i\frac{\omega\omega_1}{Q_1})] \\ \eta &= \frac{1}{K}[(\omega_v^2 - i\frac{\omega\omega_v}{Q_v}) - (\omega_N^2 - i\frac{\omega\omega_N}{Q_N})] \end{aligned} \tag{10}$$

In above, the first two equations of (9) combined with equations (10) form the so-called boundary conditions. In this way the system acts like a structure with infinite number of cells (for $n = 2, ..., N-1$), where ω_v can be determined by formula (7) ($Q_v >> 1$ is normally assumed). We may now rewrite Eq. (4) in the following form (with "hats" removed for simplicity of notation):

$$a_{n+1} - 2a_n \cdot cos(\alpha) + a_{n-1} = f_n \qquad (n = 1, ..., N), \tag{11}$$

where the phase advance of the field from cell to cell, α, and the drive terms, f_n, are defined, respectively, as follows

$$\cos\alpha = \frac{1}{2K}(\omega_v^2 - \omega^2 - i\frac{\omega\omega_v}{Q_v}), \tag{12}$$

and

$$f_n = -S_n/K. \tag{13}$$

The difference equations (11) with constant coefficients can be solved analytically [5]. The general solution is given by

$$a_n = e^{i\alpha n}[\frac{-i}{2sin(\alpha)}\sum_{r=1}^{n} f_r \cdot e^{-i\alpha r} + C_1] + e^{-i\alpha n}[\frac{i}{2sin(\alpha)}\sum_{r=1}^{n} f_r \cdot e^{i\alpha r} + C_2], \tag{14}$$

where C_1 and C_2 are constants that need to be determined by the boundary conditions and the matching condition. In obtaining (14), it is assumed that

$$\sum_{r=1}^{0} f_r \cdot e^{\pm i\alpha r} = 0 \tag{15}$$

and also that α is not equal to an integral multiple of π.

We now apply the above results to a TW output structure. Before proceeding further we assume a stiff beam, i.e., the drive terms, f_n, have constant amplitudes, but the phases of f_n increase from cell to cell by α' (to be distinguished from the corresponding phase advance of the field, α). When $\alpha = \alpha'$, the field and the beam travel in phase ($\omega/k = c$ for a relativistic beam), we call it the synchronism case, the theory of which has been developed in ref. [5]. When $\alpha \neq \alpha'$, the field and the beam are out of phase, we call it the non-synchronism case; in particular, for $\alpha < \alpha'$, the inductive detuning case, is the case we are most interested in [1]. By assuming a stiff beam we can write

$$f_n = f \cdot e^{i\alpha' n}. \tag{16}$$

Substituting (16) into (14) and after a few steps of algebra we have

$$\begin{aligned} a_n = e^{i\alpha n}[(\frac{-i}{2\sin\alpha})\cdot f \sum_{r=1}^{n} e^{ir\Delta^-} + (\frac{-i}{2\sin\alpha})\cdot f \cdot \frac{e^{i\Delta^+}}{1-e^{i\Delta^+}} e^{ir\Delta^-} + C_1] + \\ + e^{-i\alpha n}[(\frac{i}{2\sin\alpha})\cdot f \cdot \frac{e^{i\Delta^+}}{1-e^{i\Delta^+}} + C_2], \end{aligned} \tag{17}$$

where $\Delta^{\pm} \equiv \alpha' \pm \alpha$ with Δ^- being defined as the so called detuning angle. Then, $\Delta^- = 0$ corresponds to the synchronism case, while $\Delta^- \neq 0$ corresponds to the non-synchronism case(s). Since we have assumed that the fields vary as $e^{-i\omega t}$, so, in Eq. (17) the first term ($e^{i\alpha n}$) represents a wave traveling in the forward direction (from $n = 1$ to $n = N$) and the second term ($e^{-i\alpha n}$) represents a wave traveling in the backward direction. By appropriate choice of C_1 and C_2, the backward component can be eliminated.

We should now apply the boundary conditions given by (9) and (10). Substituting a_0 and a_1 into (14), respectively, and using (15) and (9) for a_0 we find that the condition relating a_0 and a_1 may be expressed as

$$\mu = \frac{C_1 + C_2}{C_1 \cdot e^{i\alpha} + C_2 \cdot e^{-i\alpha}}. \tag{18}$$

Since for the cases we are most interested in, there is no input cavity [1], therefore, in the following, we specify that the 1st cell in the TWS is the same as the N-2 cells that are behind it and only the last cell (n=N) is different from the rest. We then immediately find from (10) that

$$\mu = 0. \tag{19}$$

Equation (19) leads to $C_1 = -C_2$ from (18).

Our next task is to choose ω_N and Q_N such that the resulting solution would consist of just a forward traveling wave. Referring to (17), the backward traveling wave will vanish when

$$C_2 = -\frac{if}{2\sin\alpha}\,\frac{e^{i\Delta^+}}{1-e^{i\Delta^+}}. \tag{20}$$

Substituting a_N and a_{N+1}, respectively, into (17), assuming periodic boundary condition for α', i.e.,

$$\begin{aligned} \cos(N\alpha') &= 1, \\ \sin(N\alpha') &= 0 \end{aligned} \tag{21}$$

(e.g., in our present 1 TeV RK-TBA conceptual design [1], $\alpha'=2\pi/3$ and $N=3$) and then employing some mathematical manipulations, we find that

$$\begin{aligned} a_N &= f\cdot\frac{e^{i\alpha'}\cdot(1-e^{iN\alpha})}{(e^{i\Delta^-}-1)\cdot(e^{i\Delta^+}-1)}, \\ a_{N+1} &= f\cdot\frac{e^{i\alpha'}\cdot[e^{i\alpha'}-e^{i(N+1)\alpha}]}{(e^{i\Delta^-}-1)\cdot(e^{i\Delta^+}-1)}. \end{aligned} \tag{22}$$

In obtaining (22), $C_1 = -C_2$ and (20) are used. Plugging (22) in (9) for η, we get

$$\eta = e^{i\Delta^+/2}\left[\frac{\sin(N+1)\Delta^-/2}{\sin(N\Delta^-/2)}\right]. \tag{23}$$

It is noted that for the synchronism case when $\Delta^- = 0$, the above relation recovers (25) of ref. [5]. The "matching conditions" for the eigenfrequency and the external quality factor of the output cell of a detuned N-cell TWS can now be obtained by equating (23) to the 2nd equation of (10), and they are given as follows:

$$\begin{aligned} \omega_N &= \omega_V - \frac{V_g}{2L_p\cdot\sin(\alpha)}\left[\frac{\sin(N+1)\Delta^-/2}{\sin(N\Delta^-/2)}\cos\Delta^+/2)\right] \\ (Q_N)^{-1} &= \frac{V_g}{\omega_N L_p\cdot\sin(\alpha)}\left[\frac{\sin(N+1)\Delta^-/2}{\sin(N\Delta^-/2)}\sin(\Delta^+/2)\right] \end{aligned} \tag{24}$$

In deriving (24) relation (6) is used and $Q_V >> 1$ is assumed.

Now, we have obtained the formula for the conditions which, once satisfied, will guarantee a TWS to propagate a single forward traveling wave. The formula applies to the non-synchronism cases as well as the synchronism case, and therefore, once incorporated in the RKS code it will allow the code to simulate the physical processes of beam-rf interactions in detuned TWS's.

2.3. Power balance equation

Since this work is motivated by the need in our recent RK-TBA design study [1] to achieve stable power output for many TWS's with the inductive detuning scheme, it is helpful to obtain the power balance relation for a detuned TWS to quantify the generation and the transmission of *EM* energy inside the TWS so that we can have a better understanding of the physical process associated with it.

We start with equation (2). Applying Eq. (2) to the last cell of a N-cell TWS and taking the real portion of the equation after multiplying $i(a_N^*)$ on both sides of the equation, we have

$$\frac{\omega\omega_N}{Q_N}|a_N|^2 = Re[(\frac{-\omega}{\varepsilon_o}\int d^3r_N\cdot \vec{\varepsilon}_N^*\cdot \vec{J}_1)\cdot a_N^*] + Re(iKa_{N-1}a_N^*). \tag{25}$$

Rewriting $Re(iKa_{N-1}a_N^*)$ as $Re(-iKa_{N-1}^*a_N)$ and then repeating the above procedure sequentially from the $(N-1)^{th}$ cell to the 1st cell, we come to the following relation

$$\frac{\omega\omega_N}{Q_N}|a_N|^2 = \sum_{n=1}^{N} Re[(\frac{-\omega}{\varepsilon_o}\int d^3r_n\cdot \vec{\varepsilon}_n^*\cdot \vec{J}_1)\cdot a_n^*], \tag{26}$$

where $Q_n >> 1$ with $n = 1, 2, \ldots, N-1$ are assumed. Since we also know that the power extracted from the last cell of a N-cell TWS can be quantified by the following expression

$$\begin{aligned} P_{out} &\equiv \frac{\omega_N U_N}{Q_N} \\ &= \frac{\omega_N}{Q_N}[\frac{\varepsilon_o}{2}\int (d^3r)_N|E_N|^2] \\ &= \frac{\omega_N}{Q_N}[\frac{\varepsilon_o}{2}|a_N|^2\int (d^3r)_N|\vec{\varepsilon}_N(\vec{r})|^2], \end{aligned} \tag{27}$$

where $U_N \equiv \frac{\varepsilon_o}{2}\int |E_N|^2(d^3r)_N$ is defined as the stored energy in the cell N. Then, choosing the normalization convention $\int (d^3r)_n|\vec{\varepsilon}_n(\vec{r})|^2 = 1$ and substituting (26) into (27) we find the following power balance relation

$$P_{out} = \sum_{n=1}^{N} \frac{1}{2}Re[(-\int d^3r_n\cdot \vec{\varepsilon}_n^*\cdot \vec{J}_1)\cdot a_n^*]. \tag{28}$$

Now, if we further define

$$I_n^{id} \cdot e^{i\phi_n} \equiv (L_p)^{1/2} \int (d^3r)_n (\vec{\varepsilon}_n^* \cdot \vec{J}_1) \tag{29}$$

as the induced current and

$$V_n \cdot e^{i\psi_n} \equiv (L_p)^{-1/2} \cdot a_n \tag{30}$$

as the voltage across the cell n, respectively, we may rewrite the power balance equation (28) in the following form

$$P_{out} = \frac{1}{2} \sum_{n=1}^{N} (-I_n^{id} \cdot V_n) \cdot cos\,(\phi_n - \psi_n) \quad . \tag{31}$$

It is seen from (31) that the output power from a TWS is equal to an accumulation of the *EM* energy generated in each cell which is proportional to the induced current, I_n^{id} and the voltage across each cell, V_n. It is also seen that the maximum output power is achieved for the synchronism case when the beam and the rf field have the same phase advance across each cell (i.e., $\phi_n - \psi_n = 0$). For the non-synchronism cases the output power declines as the detuning angle increases.

2.4. Power extraction formula

For a stiff beam it is possible to derive an analytic expression for P_{out} in terms of the induced current and the cavity related parameters, e.g., the shunt impedance R/Q and the group velocity V_g. This formula is useful for zeroth order cavity design [1].

First, we treat the synchronism case when $\Delta^- = 0$. In this case, the matching conditions (24) reduce to

$$\begin{aligned} \omega_N &= \omega_V - \frac{V_g}{2L_p \cdot sin\,(\alpha)} \left(\frac{N+1}{N}\right) \cdot cos(\alpha) \\ (Q_N)^{-1} &= \frac{V_g}{\omega_N \cdot L_p \cdot sin\,(\alpha)} \left(\frac{N+1}{N}\right) \end{aligned} \quad , \tag{32}$$

the power balance equation (31) becomes

$$\begin{aligned} P(0) &\equiv P_{out}(\Delta^- = 0) \\ &= \frac{1}{2} \sum_{n=1}^{N} (-I_n^{id} \cdot V_n) \end{aligned} \tag{33}$$

and the field evolution (17) reduces to

$$a_n = e^{i\alpha n}[(\frac{-i}{2sin(\alpha)})\cdot nf] \,. \tag{34}$$

Also, from the definition of S_n ((4)), Eq. (11) and Eq. (16) we know that

$$\int (d^3r)_n \cdot (\vec{\varepsilon}_n)^* \cdot \vec{J}_1 = -(\frac{K\varepsilon_o}{i\omega})\cdot f\cdot e^{i\alpha n} \,. \tag{35}$$

Substituting (34) and (35) into (28) we then have

$$P(0) = [\frac{K\varepsilon_o}{4\omega\cdot sin(\alpha)} f^2]\cdot [\frac{N(N+1)}{2}]. \tag{36}$$

Introducing, respectively, the axial voltage across one cell

$$V_{axis} \equiv \int dz(E_n\cdot e^{ikz}) = a_n \int dz(\varepsilon_n\cdot e^{ikz}) \tag{37}$$

and the induced current

$$I_{ind} \equiv \int dS(J_1\cdot e^{ikz}), \tag{38}$$

we find that the corresponding shunt impedance may be expressed as

$$\frac{R}{Q} \equiv \frac{|V_{axis}|^2}{2\omega U_s} = \frac{\left|\int dz(\varepsilon\cdot e^{ikz})\right|^2}{\omega\varepsilon_o} \tag{39}$$

with $U_s \equiv \frac{1}{2}\varepsilon_o|a_n|^2$ being the stored energy in the cell, and also we find that Eq. (35) may be rewritten as

$$f\cdot e^{i\alpha n} = -\frac{i\omega}{K\varepsilon_o} I_{ind}\cdot \left[\int dz(\varepsilon_n\cdot e^{ikz})\right]^* . \tag{40}$$

From Eqs. (39) and (40) we have

$$\begin{aligned} f^2 &= \left| f\cdot e^{i\alpha n} \right|^2 \\ &= \frac{\omega^3}{K^2\varepsilon_o}(I_{ind})^2 \, (\frac{R}{Q}). \end{aligned} \tag{41}$$

Substituting (41) into (36) and after a few steps of algebra we obtain a formula that relates the extracted power from the TWS to the induced current (I_{ind}) of the beam, the frequency (ω) of the operating wave and several key cavity parameters – the shunt impedance R/Q, the group velocity V_g, the longitudinal dimension L_p and the cell number N. The formula is given as follows

$$P(0) = \frac{(I_{ind})^2}{4}\left(\frac{\omega L_p}{V_g}\right)\left[\left(\frac{R}{Q}\right)/2\right]\cdot N(N+1). \tag{42}$$

It is noted in (42) that $P(0)$ is proportional to $N(N+1)$.

Next, for the detuning cases when $\Delta^- \neq 0$, the extracted power may be expressed as

$$P(\Delta^-) = \Gamma(\Delta^-)\cdot P(0), \tag{43}$$

where the coefficient $\Gamma(\Delta^-)$ is a function of the detuning angle Δ^-. From Eq. (31) it is seen that $\Gamma(\Delta^-) = 1$ if $\Delta^- = 0$, and $\Gamma(\Delta^-) < 1$ if $\Delta^- \neq 0$. For any given Δ^-, $\Gamma(\Delta^-)$ may be determined from numerical simulation using the RKS code [1].

Eqs. (42) and (43) relate the output power requirement to the detuning angle and several other key cavity parameters. The cavity structure that meet the specific requirement(s) can then be designed with the *URMEL* [6] and the *MAFIA* codes [7], as has been done in ref. [1].

2.5. Field amplitudes and phases

In our present RK-TBA conceptual design, each TWS has 3 cells [1]. In the following we present two set of analytic formula to quantify the amplitudes and the phases of the rf field in a detuned 3-cell TWS:

(i) Between two adjacent cells in the TWS (with detuning angle Δ^-)

$$\begin{aligned} \frac{a_2}{a_1} &= e^{i(\alpha' - \Delta^-/2)}\cdot[2\cos(\Delta^-/2)], \\ \frac{a_3}{a_2} &= e^{i(\alpha' - \Delta^-/2)}\cdot\left[\frac{\sin(3\Delta^-/2)}{\sin(\Delta^-)}\right]. \end{aligned} \tag{44}$$

(44) describes how the rf field evolves across the TWS.

(ii) Between the cells in the detuned TWS (a_i) and the corresponding cells in the corresponding non-detuned TWS ($a_i(0)$) that is used as the base case

$$\frac{a_1}{a_1(0)} = e^{-i(\Delta^-/2)} \cdot \left[\frac{\sin(\alpha')}{\sin(\Delta^+/2)}\right],$$

$$\frac{a_2}{a_2(0)} = e^{-i(\Delta^-)} \cdot \left[\frac{\sin(\alpha')}{\sin(\Delta^+/2)} \frac{\sin(\Delta^-)}{2\sin(\Delta^-/2)}\right], \tag{45}$$

$$\frac{a_3}{a_3(0)} = e^{-i(3\Delta^-/2)} \cdot \left[\frac{\sin(\alpha')}{\sin(\Delta^+/2)} \frac{\sin(3\Delta^-/2)}{3\sin(\Delta^-/2)}\right].$$

(45) quantifies the effects of the cavity detuning on the amplitude and the phase of the rf field in each cell of the TWS.

3. RKS Code Checking For Detuned TWS's

RKS is a computer code developed by Ryne and Yu [4] for studying the interaction of a charged particle beam with an electromagnetic wave in a TWS or a standing wave structure (SWS) that are used to extract power from a RK. The code solves self-consistently the single particle equations of motion for the beam and the coupled circuit equations that govern the cavity excitation, and it includes the calculation of the space charge effect. It assumes a single dominant mode and cylindrical symmetry of its fields inside the cavity. The code has been checked against the relativistic klystron experiments conducted by the Microwave Source Facility group at LLNL [4] and has also been employed to assist in the design of the reacceleration experiment [8]. These studies have shown that results from the code are consistent with experimental results.

However, in all the previous RKS simulations of TWS's the beam bunches and the operating fields were in synchronism [4,8], while we are mostly interested in the nonsynchronism cases [1], therefore, in the following we use relations (45) to check the RKS code to ensure that the code can also be used for the inductive detuning cases. The ratio of the rf field (amplitude and phase) in each of the 3 cells of the detuned TWS to the corresponding one in the non-detuned TWS is evaluated both analytically (with (45)) and numerically (with RKS) for four different detuning cases, and the results are tabulated in Table 1 and Table 2, respectively. It is seen that in general the agreements are fairly good for the phases (ψ_i, i = 1,2,3) although there exists a 1-2° systematic discrepancy for almost all the cases. The agreements for the amplitudes ($|a_i|$, i = 1,2,3) are not as good as that of the phases (the discrepencies are 5~12%), but still, for any given detuning angle the ratio of the amplitudes does go down across the TWS in accord with the analytical calculation prediction.

Table 1. Analytical Results

$\alpha' - \alpha$	0°	10°	20°	30°	40°
$\lvert a_1/a_1(0)\rvert$	1.	0.956	0.922	0.896	0.879
$\lvert a_2/a_2(0)\rvert$	1.	0.952	0.908	0.865	0.826
$\lvert a_3/a_3(0)\rvert$	1.	0.946	0.884	0.816	0.742
$\psi_1 - \psi_1(0)$	0°	-5.0°	-10.0°	-15.0°	-20.0°
$\psi_2 - \psi_2(0)$	0°	-10.0°	-20.0°	-30.0°	-40.0°
$\psi_3 - \psi_3(0)$	0°	-15.0°	-30.0°	-45.0°	-60.0°

Table 2. Numerical Results (from *RKS* code)

$\alpha' - \alpha$	0°	10°	20°	30°	40°
$\lvert a_1/a_1(0)\rvert$	1.	1.04	1.07	1.07	1.02
$\lvert a_2/a_2(0)\rvert$	1.	1.02	1.03	1.00	0.94
$\lvert a_3/a_3(0)\rvert$	1.	1.02	1.01	0.948	0.84
$\psi_1 - \psi_1(0)$	0°	-3.1°	-8.3°	-13.0°	-18.2°
$\psi_2 - \psi_2(0)$	0°	-9.0°	-18.5°	-28.4°	-38.3°
$\psi_3 - \psi_3(0)$	0°	-14.2°	-29.0°	-44.0°	-59.0°

4. Summary

In this study we developed an analytic framework for using the inductively detuned TWS's to extract power from RK's. We obtained the "matching conditions", the power balance equation and also an analytic formula for predicting the power output from a TWS. We also checked the RKS code against a set of analytic formula for the cases of the detuned 3-cell TWS's, and found that the numerical results basically agree with the analytic ones although some small discrepancies do exist.

Acknowledgments

The authors would like to thank A. M. Sessler for his helpful discussions and careful review of the manuscript.

References

[1] H. Li, T. Houck, N. Goffeney, E. Henestroza, A. Sessler, G. Westenskow and S. Yu, "Design Study of Beam Dynamics Issues For 1 TeV Next Linear Collider Based Upon the Relativistic-Klystron Two-Beam Accelerator", Presented at the 1994 *Advanced Accelerator Concept Workshop*, to be Published in the Proceedings.

[2] A. M. Sessler and S. S. Yu, Phys. Rev. Lett. **58** (1987), 2439.

[3] S. Yu, F. Deadrick, N. Goffeney, E. Henestroza, T. Houck, H. Li, C. Peters, L. Reginato, A. Sessler, D. Vanecek and G. Westenskow, "Conceptual Design Report for Relativistic Klystron Two-Beam Accelerator", Presented at 17th Int'l LINAC Conf., Japan, August 22-26, 1994, to be published in the Proceedings.

[4] R. Ryne and S. S. Yu, Proc. 15th Int'l LINAC Conf., LA-12004-C (1990), 177.

[5] R. Ryne and S. S. Yu, Proc. 15th Int'l LINAC Conf., LA-12004-C (1990), 180.

[6] T. Weiland, Nucl. Instr. and Meth. A **216** (1983) 329.

[7] F. Ebeling, et al, Mafia User Guide, Los Alamos National Laboratory Report LA-UR-90-1307 (1989).

[8] G. M. Fiorentini, T. L. Houck and C. Wang, Proc. Intense Microwave Pulses, ed. by H. E. Brandt, SPIE 1872-17 (1993).

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL AND ELECTRONIC
INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720