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K.-J. Kim and A.V. Zege

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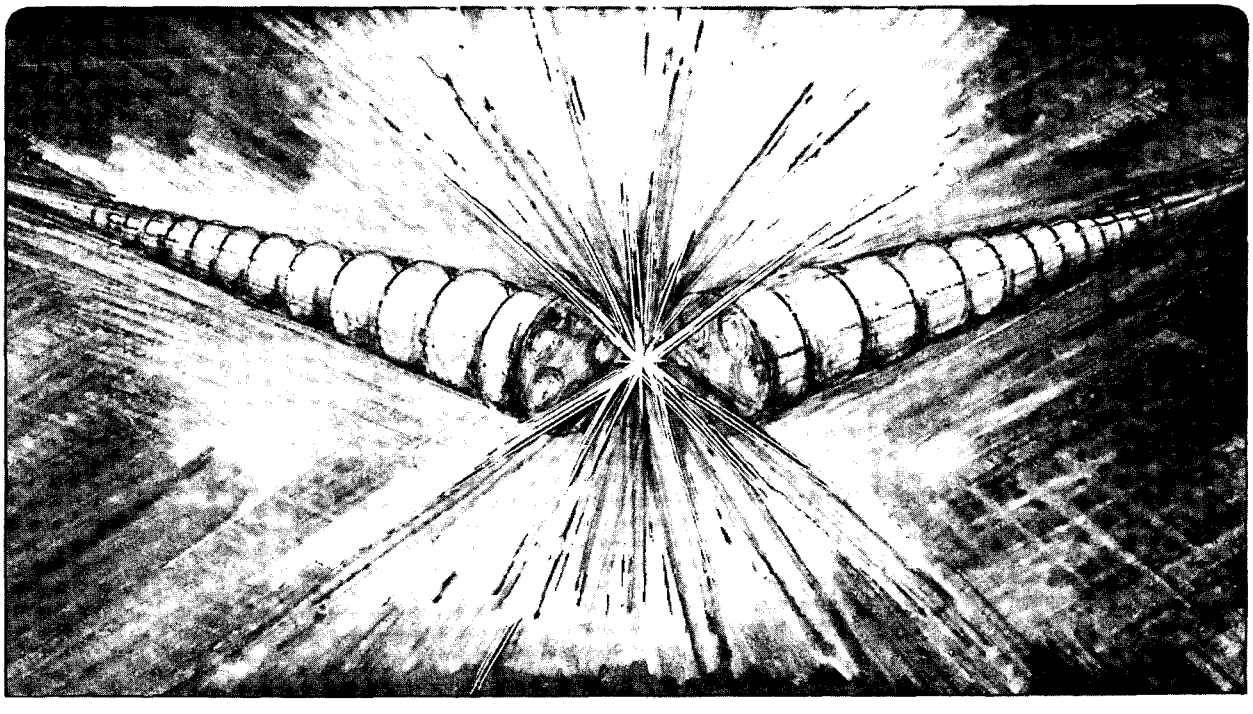

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K.-J. Kim and A.V. Zege

Accelerator and Fusion Research Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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ANALYSIS OF OPTICAL STOCHASTIC COOLING INCLUDING TRANSVERSE EFFECTS *

K.-J. Kim and A.V. Zege, Lawrence Berkeley Laboratory, Berkeley, CA, 94720 USA

Abstract

The phase space area plays an important role in determining the performance of optical stochastic cooling. Specifically, the number of samples in the beam consists of three factors corresponding to three dimensions, the factor in each dimension being given by the ratio of the total phase space area in that dimension to the radiation wavelength.

I. INTRODUCTION

In microwave stochastic cooling [1], the maximum cooling rate is limited by the number of particles n_s in a so-called "sample", defined to be the subset of particles that can interact with a given particle through the pick-up–amplifier–kicker system. The signal from the particles in the sample, other than that from the particle itself, contribute to the heating term. A smaller value of n_s therefore leads to a faster cooling. If the bandwidth of the pick-up–amplifier–kicker system is denoted by W, n_s is given by the well-known relationship

$$n_s = n\frac{1}{F}. \tag{1}$$

Here, n is the total number of the particles in the beam, $F = TW$, and T is the time duration of the beam. Equation (1) is the statement that the signal due to a given particle appearing at the kicker lasts about $1/W$, and hence only particles within that temporal distance can interact with the given particle. The quantity F is referred to as the number of the samples contained in the beam.

Recently, optical stochastic cooling was proposed to take advantage of the larger bandwidth possible with amplifiers in the optical wavelength region, which could be as high as 10^{13} –10^{14} Hz [2]. Thus the sample size n_s, and hence the cooling time, would be greatly reduced. A quadrupole undulator and a normal dipole undulator respectively play the role of the pick-up and the kicker in the original scheme in reference[2]. However, the requirement on the beam emittance in that scheme turns out to be very stringent, being about $\lambda\sigma_\delta$ or less, where λ is the wavelength of the radiation used for the cooling and σ_δ is the rms relative energy spread in the beam. The transit time scheme of optical stochastic cooling was proposed to avoid this limitation [3]. In this scheme, normal undulators are employed for both the pick-up and the kicker, and the information on particle coodinates is transmitted via the coordinate-dependent transit time of the particle trajectory between the pick-up and the kicker undulators.

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The purpose of this paper is to study the effect of the transverse structure of the radiation field on the cooling performance. To this end, we analyze the transit time scheme of the optical stochastic cooling in detail, using an approximate model for the undulator radiation to represent the space-time dependence of the undulator field [4]. Our main finding is that the number of samples in the beam F can be written as

$$F = F_L F_{Tx} F_{Ty}, \tag{2}$$

where F_L and F_{Tx} (F_{Ty}) are referred to respectively as the number of the longitudinal and the transverse samples in the x(y)-direction. We find that the number of the samples in each dimension is given by the ratio the total phase space area to the coherent phase space area λ.

The relevance of the transverse structure of the radiation field in optical stochastic cooling was first pointed out in reference[3] based on an intuitive argument. However, the expression for the number of the transverse samples conjectured there–the ratio of the beam size to the coherent mode size– is valid only in the limit of parallel beam.

The expression for F_L as the ratio of the total longitudinal phase space area to the coherent phase space area λ reduces the usual result TW in the long bunch limit. In the short bunch limit, however, F_L becomes the ratio of the beam energy spread to the coherent energy spread. This could have interesting consequences for the cooling of bunched beams.

II. PARTICLES AND FIELDS

Let x_j and x'_j be respectively the position and the angle of the betatron motion of the jth particle as it passes the center of the pick-up undulator, and δ_j its relative energy deviation. The total transverse coordinates are therefore

$$x_{Pj} = x_j + \eta\delta_j,\ x'_{Pj} = x'_j + \eta'\delta_j, \tag{3}$$

where η and η' are respectively the dispersion and its derivative at the center of the pick-up undulator. In the following, we will be mainly concerned with the behavior in the x-direction where the dispersion occurs. The behavior in the y-direction will become obvious.

Following the work in [3], we choose the betatron transfer matrix between the centers of the pick-up and the kicker undulators to be $-I$, and the dispersion function at the center of the kicker to satisfy the symmetry relation $(\eta_K, \eta'_K) = (\eta, -\eta')$. Thus, the transverse coordinates of the ith particle in the center of the kicker (x_{Ki}, x'_{Ki}) are related to those at the pick-up as follows:

$$x_{Ki} = -x_i + \eta\delta_i,\ x'_{Ki} = -x'_i - \eta'\delta_i. \tag{4}$$

The path length of the particle trajectory between the centers of the pick-up and the kicker in the linear approximation is given by

$$l_j = l_0 + \Delta l_j = l_0 + 2(\eta x'_{Pj} + Q\delta_j). \quad (5)$$

Here, $Q = \eta\eta' + \int_0^{l_0} ds D(s)/\rho(s)$, $D(s)$ being the contribution to the dispersion from the dipole magnets and $\rho(s)$ the radius of the curvature of the particle trajectory[3]. In this paper, the betatron focussing in the undulators is neglected so that particle trajectories are straight. The trajectory of an ith particle in the kicker undulator can therefore be written as

$$x_{Ki}(t) = x_{Ki} + x'_{Ki} z_{Ki}(t),\ x'_{Ki}(t) = x'_{Ki}. \quad (6)$$

The quantity $z_{Ki}(t)$ in Eq.(6) is the longitudinal coordinate given by:

$$z_{Ki}(t) = \beta_i(1 + x'^2_{Ki}/2)(ct - s_i - \Delta l_i), \quad (7)$$

where β_i is the speed of the ith electron divided by c, c the speed of light, and $s_i = ct_i$ the distance of the the ith particle behind the reference particle in the pick-up undulator. It is necessary in Eq.(7) to keep the second order term in the expansion of $\cos x'_{Ki}$ because the leading term is sometimes cancelled out. The constant term l_0 in the path length does not appear in Eq.(7) because the time $t = 0$ is defined to be the moment when the reference particle passes through the center of the kicker undulator.

To find the field produced by the jth electron $E_{Pj}(x; z_P) \exp i\omega(t - t_j)$, where $\omega = 2\pi c/\lambda = kc$, it is sufficient to note the relation at $z_P = 0$ (the center of the pick-up undulator) , $E_{Pj}(x; 0) = \exp[ikx'_{Pj}(x - x_{Pj})]E_0(x; 0)$, where E_0 is the field of the reference particle[4]. We will approximate $E_0(x; 0) = \exp(-x^2/4\sigma_r^2)$, where σ_r is the rms mode size at the waist. The field for $z_P \neq 0$ is then obtained by a Fresnel transformation. The resulting expression is further simplified by neglecting the z_P-dependence in the spotsize and the wavefront curvature. The result is then multipled by the factor $\exp[-(z - c(t - t_j))/4\sigma_C^2]$, where $\sigma_C \simeq N\lambda/\sqrt{2\pi}$, N being the number of the undulator periods. This factor is introduced to account for the fall-off the field strength due to the increase of the spotsize as well as the fact that the relative bandwidth of the undulator radiation is about $1/N$. In this way, we obtain

$$E_{Pj}(x; z_P, t) = E_0 \exp[ikz_P(1 - x_j'^2/2) - ik(ct - s_j) - \frac{(z_P - ct + s_j)^2}{4\sigma_C^2} + ikx'_j(x - x_j) - \frac{(x - x_j - z_P x'_j)^2}{4\sigma_r^2}] \quad (8)$$

In deriving Eq.(8), we have assumed that the undulator bandwidth, $1/N$, is much lager than both the relative energy spread of the particle beam σ_δ and the line width due to the angular spread, $\gamma^2 x'^2$, where γ is the relativistic factor and x' is the angle at the pick-up or kicker undulator.

We choose the optical ABCD matrix for the pick-up–amplifier–kicker system to be the same as that of the betatron transfer matrix, $-I$, with an an amplification factor g. The field at the kicker E_{Kj} is then related to that from the pick-up by $E_{Kj}(x, z, t) = gE_{Pj}(-x, z, t - l_0/c - \Delta s_j/c)$. Here, z is the longituninal coordinate measured from the center of the kicker undulator.

III. COOLING AND HEATING TERMS

In the kicker undulator, the energy of a particle becomes modified due to the coupling of its transverse motion in the undulator magnetic field and the transverse electric field of the amplified undulator radiation from the pick-up undulator. The change in the relative energy of the ith particle is

$$\Delta\delta_i = e\frac{gK_u}{2mc^2\gamma^2} Im \sum_j \int dt E_{Pj}(-x_{Ki}(t), z_{Ki}(t), t - \Delta t_i) \times \exp(ik_u z_{Ki}(t) - z_{Ki}^2/4\sigma_L^2). \quad (9)$$

The complex notation for the field is used here with Im implying the imaginary part, and e is the particle charge, m its mass, K_u the undulator deflection parameter, $k_u = 2\pi/\lambda_u$, λ_u the undulator period length, and the trajectory of the ith paricle is specified by Eq.(6) and Eq.(7). We are assuming that the undulator strength is tapering out as a Gaussian function with an rms length $\sigma_L \simeq L/\sqrt{2\pi}$. Also note the relation $\sigma_C = (1 - \beta)\sigma_L \simeq \sigma_L/2\gamma^2$. Performing the t–integral, we obtain

$$\Delta\delta_i = GIm\left(\Phi_{ii} + \sum_{j\neq i}\Phi_{ij}\right), \quad (10)$$

where $G = egK_u E_0 L/2mc^2\gamma^2$, and

$$\Phi_{ij} = \frac{\sigma_r}{\Sigma_{ij(2)}} \exp\left\{-\frac{k^2\sigma_L^2\sigma_r^2 x'^4_{ij}}{8\Sigma^2_{ij(2)}} - ik\Delta l_i - ik(s_i - s_j) - \frac{\left[\left(x_{ij} - x'_{ij}(s_i - s_j)\sigma_L/\sigma_C\right)^2 - x_{ij}^2 - (s_i - s_j)^2\sigma_r^2/\sigma_C^2\right]}{8\Sigma^2_{ij(2)}}\right\} \times \exp(i\varphi_{ij}) \quad (11)$$

Here

$$\Sigma_{ij(2)} = \sqrt{\sigma_r^2 + x'^2_{ij}\sigma_L^2/2},\ \Sigma_{ij(1)} = \sqrt{\sigma_r^2 + x'^2_{ij}\sigma_L^2}$$

$$l_i = \Delta t_i/c,\quad s_i = t_i/c,\quad x_{ij} = x_{Ki} + x_{Pj} = -x_i + x_j + \eta(\delta_i + \delta_j)$$

$$x'_{ij} = x'_{Ki} + x'_{Pj} = -x'_i + x'_j + \eta(-\delta_i + \delta_j)$$

The phase term φ_{ij} in the last factor in Eq.(11) is a real quantity with the property $\varphi_{ii} = 0$ and $|\varphi_{ij}| \ll |k(s_i - s_j)|$ for $i \neq j$. The explicite expression of such a phase term will not be important for the following calculations. The change in the average value of δ_i^2 is then

$$\langle\Delta\delta_1^2\rangle = 2GIm\langle\delta_1\Phi_{11}\rangle + n(G^2/2)\langle\Phi_{12}\Phi_{12}^*\rangle. \quad (12)$$

Here the angular brackets imply taking the average with respect to the variables s_i, δ_i, x_i, and x'_i. We assume that the

distribution function in these variables can be factorized into Gaussian functions in each variable, with rms widths $\sigma_z, \sigma_\delta, \sigma_x$, and $\sigma_{x'}$. Since all particles are equivalent, we have arbitrarily chosen two indices, $i = 1$ and $j = 2$. The first term in the above is the cooling term. In obtaining the heating term, the second term in Eq.(12), we have used the fact that the average of the quantity $\Phi_{1j}\Phi^*_{1j'}$ vanishes unless $j = j'$ because of the large, random phase $k(s_j - s_{j'})$.

The cooling term can be evaluated in a closed form as follows:

$$2GIm < \delta_1\Phi_{11} >= -\frac{4kGQ\sigma_\delta^2}{(1+2\eta^2\sigma_\delta^2/\sigma_r^2)^{3/2}}$$

$$\exp\left\{-2k^2\left[\sigma_{x'}^2\eta^2 + \frac{Q^2\sigma_\delta^2}{1+2\eta^2\sigma_\delta^2/\sigma_r^2}\right]\right\}. \qquad (13)$$

This reproduces the expression derived in reference[3] when $\eta\sigma_\delta \ll \sigma_r$.

The heating term can be written as $n_s G^2/2$, where $n_s = \langle\Phi_{12}\Phi^*_{12}\rangle$ is the number of particles in a sample for optical stochastic cooling. The averaging with respect to variables s_1 and s_2 involves Gaussian integrals. Assuming for simplicity that $\sigma_z \gg \sigma_C$, the result is

$$\langle\Phi_{12}\Phi^*_{12}\rangle = \sigma_C/\sigma_z \times$$

$$\langle\frac{\sigma_r^2}{\Sigma_{12(1)}\Sigma_{12(2)}}\exp\frac{-1}{4}\left\{\frac{x_{12}^2}{\Sigma_{12(1)}^2} + \frac{x_{12}^2}{\Sigma_{12(2)}^2} + \frac{k^2\sigma_r^2\sigma_L^2 x_{12}'^4}{\Sigma_{12(2)}^2}\right\}\rangle. \qquad (14)$$

The rest of the averaging will be carried out approximately by regarding the variables x_{12} and x'_{12} to be independent with Gaussian distributions of widths Σ_x and $\Sigma_{x'}$, respectively. These quantities will be roughly given by $\Sigma_x^2 \approx 2\sigma_x^2 + 2\eta^2\sigma_\delta^2$ and $\Sigma_{x'}^2 \approx 2\sigma_{x'}^2 + 2\eta^2\sigma_\delta^2$. Performing the x_{12}-average, we obtain

$$\langle\Phi_{12}\Phi^*_{12}\rangle = \frac{\sigma_C}{\sigma_z}\langle\frac{\sigma_r^2\exp[-(k\sigma_L\sigma_r x_{12}'^2/2\Sigma_{12(2)})^2]}{\Sigma_{12(2)}\sqrt{\Sigma_{12(1)}^2 + \Sigma_x^2}}\rangle. \qquad (15)$$

The final average in the variable x'_{12} can be estimated approximately as follows: First, we assume that the Rayleigh length of the undulator radiation is about σ_L so that

$$\sigma_r \approx \sqrt{\sigma_L/2k},\ \sigma_{r'} \approx 1/\sqrt{2k\sigma_L} \qquad (16)$$

The exponential factor in Eq.(15) can then be written as $\exp(-\xi^2/4(1+\xi))$, where $\xi = \sigma_L^2 x_{12}'^2/2\sigma_r^2$. Thus, the average is dominated by the region $\xi \leq 1$. For a rough estimate, therefore, we may set $\xi \approx 0$ everywhere except the exponential function, which we replace by $\exp(-\xi^2/4)$. The average of this term can be expressed in terms of the modified Bessel function $K_{1/4}$. By examining the result, we obtain the following estimate for the heating term:

$$\frac{G^2}{2}n\langle\Phi_{12}\Phi^*_{12}\rangle \equiv \frac{G^2}{2}n_s \approx \frac{\sigma_C}{\sigma_z}\frac{\lambda/2}{2\pi\sqrt{(\sigma_r^2+\Sigma_x^2)(\sigma_{r'}^2+\Sigma_{x'}^2)}}. \qquad (17)$$

Note here that the quantity $\lambda/2$ in the numerator is the coherent phase space area $2\pi\sigma_r\sigma_{r'}$. Equation (17) is the main result of our analysis in this paper. We consider the meaning of this result in the next section.

IV. THE ROLE OF PHASE SPACE AREA IN LONGITUDINAL AND TRANSVERSE SAMPLES

Comparing Eq.(17) and Eq.(1), we see that the number of samples in the beam F for the present case is indeed in the form of Eq.(2), with $F_L = \sigma_z/\sigma_C$, and

$$F_{Tx} \approx \frac{\sqrt{(\sigma_r^2+\Sigma_x^2)(\sigma_{r'}^2+\Sigma_{x'}^2)}}{\sigma_r\sigma_{r'}}. \qquad (18)$$

This reduces the conjecture of reference [3], Σ_x/σ_r, in the limit $\beta_x \gg \sigma_L$, where β_x is the beta function of the particle beam in the kicker undulator. However, note that Eq. (18) consists of two factors, the size and the angular factors. The angular factor arises from the fact that two particles with angular separation larger than the coherent mode angle do not interact, and therefore can not both belong to the same sample.

It is clear that n_s will be further reduced by the number of transverse samples in y-direction F_{Ty}, the expression of which will be similar to that of F_{Tx}.

The analysis of previous section gives $F_L \approx \sigma_z/\sigma_C$ because we assumed $\sigma_z \gg \sigma_C$. In analogy to the transverse samples, a more general expression is

$$F_L \approx \frac{2\pi\sqrt{(\sigma_C^2+\sigma_z^2)(\sigma_{C\delta}^2+\sigma_\delta^2)}}{\lambda/2}, \qquad (19)$$

where $\sigma_{C\delta} = 1/2k\sigma_C$ is the coherent bandwidth. In the limit $\sigma_z \gg \sigma_C$, $\sigma_{C\delta} \gg \sigma_\delta$, F_L becomes the usual expression σ_z/σ_C. In the opposite limit, $\sigma_z \ll \sigma_C$, $\sigma_{C\delta} \ll \sigma_\delta$, we obtain $F_L = \sigma_\delta/\sigma_{C\delta}$. Thus we find in this case that *a faster cooling would be achieved with a narrower bandwidth amplifier*. The implication of this conclusion for the cooling of bunched beams will be studied in a future paper.

V. ACKNOWLEDGEMENTS

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UNIVERSITY OF CALIFORNIA
TECHNICAL AND ELECTRONIC
INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720