LBL-36633
UC-413
Preprint

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Submitted to Physical Review C

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December 1994

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Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

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This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy, under contract number DE-AC03-76SF00098.

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Abstract

We discuss the appearance of $\Delta I = 2$ staggering effects in the gamma-ray energies of some superdeformed bands as due to the mixing of a series of K-bands. These bands are described by the rotational Hamiltonian of a symmetric top having a four-fold symmetry axis. Using the known properties of SD bands we have found limits to the possible values of the model parameters by the analysis of effective moments of inertia, deviations from the $I(I+1)$ rigid-rotor behavior and B(E2) values. Our results indicate that the value of $\mathcal{J}_3$, the moment of inertia due to the Y_{44} deformation, is close to that expected for a rigid ellipsoid.

PACS numbers: 21.10Re,21.60Ev

Very recently, a $\Delta I = 2$ staggering was observed in the gamma-ray energies of the yrast superdeformed (SD) band[1] in ^{149}Gd and in the SD bands[2] in ^{194}Hg. It has been suggested that such perturbations in the level energies could arise from Y_{44} deformations of the nuclear shape[3,4]. In fact, in Ref. 3, Hamamoto and Mottelson have shown that staggering can occur as a result of tunneling between the four minima in angular momentum space generated by a potential related to the Y_{44} deformation. In this work we will present a somewhat different approach to describe this phenomenon, namely one that involves the mixing of multiple K-bands. We will also attempt to limit the model parameters using the known properties of SD bands. While implicit in the tunneling picture, our derivation explicitly shows the appearance of the K-bands, thus providing a familiar framework to introduce effective coupling terms in the Hamiltonian. As will be shown our Hamiltonian is indeed equivalent to that in Ref. 3) and therefore our results are relevant to the two descriptions.

We start with the rotational Hamiltonian:

$$H = A_1 I_1^2 + A_2 I_2^2 + A_3 I_3^2, \tag{1}$$

where the coefficients A_i are related to the moments of inertia, $A_i = \hbar^2/2\mathcal{J}_i$. For a nucleus which has a Y_{44} deformation around the 3-axis (an axis of four-fold symmetry C_4) then $A_1 = A_2 = A$ and (1) becomes:

$$H = AI^2 + (A_3 - A)K^2, \tag{2}$$

which is the general Hamiltonian describing the motion of a symmetric top. From our current understanding of the rotational properties of superdeformed nuclei we expect that the motion will be strongly determined by the large value of the quadrupole deformation (Y_{20}). A priori this implies[5] $A_3 >> 1$, resulting in a small component of rotational angular momentum along the 4-fold symmetry axis.

The Hamiltonian (2) describes a series of different rotational bands labeled by the quantum number K. The C_4 symmetry restricts[6] the possible values of K due to the invariance of the total wave function for a rotation $\mathcal{R}_3(\pi/2)$ around the 3-axis. If ϕ

represents an intrinsic state with eigenvalue Ω for the J_3 operator, then

$$e^{-iJ_3\pi/2}\phi e^{iI_3\pi/2}D^I_{KM} = e^{2\pi i(K-\Omega)/4}\phi D^I_{KM}$$

implies $K = \Omega, \Omega \pm 4, \Omega \pm 8, ...$ etc.

Therefore, for an intrinsic configuration relevant to the ground state of an even-even nucleus ($\Omega = 0$) the possible values of K are $K = 0, 4, 8, ...$ etc., giving rise to a sequence of rotational bands that differ by ΔK=4. Without mixing, the yrast states will be those of the K=0 band, and as a consequence of eq. (2) there will be no staggering. Mixing between these bands has to involve to lowest order I_+^4 and I_-^4 to provide a ΔK=4 change. We thus introduce the simplest effective coupling of fourth order in the angular momentum operators that includes both a non-diagonal (h_4...) and a diagonal term (h_0...):

$$\begin{aligned} H_c &= h_4(I_+^4 + I_-^4) + h_0(I_+^2 I_-^2 + I_-^2 I_+^2 + I_+ I_- I_- I_+ + I_+ I_- I_+ I_- + ...) \\ &= h_4(I_+^4 + I_-^4) + h_0(I_+^2 I_-^2 + I_-^2 I_+^2 + 4(I^2 - K^2)^2), \end{aligned} \tag{3}$$

The total Hamiltonian $H + H_c$ was diagonalized in the basis:

$$|IKM> = \begin{cases} (\frac{2I+1}{16\pi^2})^{1/2}(D^I_{KM} + D^I_{\bar{K}M}) & K > 0 \\ (\frac{2I+1}{16\pi^2})^{1/2}\sqrt{2}D^I_{0M} & K = 0. \end{cases} \qquad I : even$$

The relevant matrix elements are given by

$$\begin{aligned} &< IK+4|H_c|IK >=< IK|H_c|IK+4 >= \\ &= h_4\sqrt{(I-K-3)(I-K-2)(I-K-1)(I-K)} \\ &\times\sqrt{(I+K+1)(I+K+2)(I+K+3)(I+K+4)} \times \begin{cases} \sqrt{2} & K = 0 \\ 1 & K > 0 \end{cases} \end{aligned}$$

and

$$< IK|H_c|IK > \; = h_0[(I-K-1)(I-K)(I+K+1)(I+K+2)$$

$$+(I-K+1)(I-K+2)(I+K-1)(I+K)$$
$$+4(I(I+1)-K^2)^2]$$
$$\approx 6h_0(I^2-K^2)^2,$$

with the approximation valid for $I >> 1$. In the basis introduced above the rotational wave function takes the form,

$$\Psi_{IM} = \sum_K C_{IK}|IKM> .$$

It is interesting to note that the Hamiltonian matrix increases its dimension every four units of spin due to the existence of bands differing by $\Delta K = 4$. The inclusion of an aditional state for $I = 4, 8, 12, ...$ etc. provides an important ingredient for the appearance of staggering effects.

It can be shown by rearranging the terms in Eq. (3) that H_c can be written as in Ref. 3, that is:

$$B_1(I_1^2 - I_2^2)^2 + B_2(I_1^2 + I_2^2)^2,$$

with $B_1 = 4h_4$ and $B_2 \approx 2(3h_0 - h_4)$. To make our discussion consistent with Ref. 3, the results of our calculations will be presented using the parameters B_1 and B_2 rather than h_4 and h_0. Let us take $A_3 = 90$ and $B_1 = 1$ (used in Ref. 3)) and let B_2 vary. In Fig. 1 we show the level energies of the yrast states divided by $I(I+1)$ as a function of B_2, with the different curves corresponding to different spins. Only for values of $B_2 \approx 0$ does the system behave like a rigid rotor but with a renormalized moment of inertia which, for this particular case, is ≈ 20 times smaller than the initial $\mathcal{J}$.

We conclude from this result that these parameters need to be interpreted in relation to the value of $A = \hbar^2/2\mathcal{J}$, which gives the (dominant) I^2 dependence in the rotational energy. The authors of Ref. 3) have not considered this renormalization in their analysis.

From the known properties of SD bands we will try to limit the parameter space. We know that in SD bands deviations from $I(I+1)$ are not large; for example in the Hg region an expansion $E(I) = \alpha I(I+1) + \beta[I(I+1)]^2$ gives $\beta/\alpha \approx 10^{-4}$. As seen in Fig. 1 one needs $B_2 \approx 0$ ($h_0 \approx h_4/3$), as otherwise the I^4 term gives rise to considerable

deviations from $I(I+1)$. This conclusion applies only if we restrict ourselves to the effective coupling (3). It can be argued that it is possible to add a $K-$independent I^4 term that compensates the effects introduced by the B_2 term; however that scenario will then require a very specific cancellation of the two I^4 coefficients.

We also know that for SD bands both the kinematic moment of inertia $\mathcal{J}^{(1)}$ and the quadrupole moment Q_0 are reasonably well described by rigid-body values. Assuming only a contribution from Q_{20} we have calculated reduced transition probabilities, B(E2), for the mixed bands using the expression,

$$B(E2)/B(E2)_0 = (\sum_K C_{IK} C_{(I-2)K} < IK20|I-2K >)^2 / < I020|I-20 >^2,$$

referred to the value $B(E2)_0$ when the coupling term, H_c, is zero. We found that for $A_3 \approx 90$ and $B_1 = 1$ the B(E2) values show a reduction of 20% while $\mathcal{J}^{(1)}$, as mentioned earlier, changes by a factor of ≈ 20, not consistent with the experimental findings. In order to overcome this inconsistency we need to reduce the value of A_3, which then reduces the effect on the renormalized moment of inertia. The set of parameters $A_3 \approx 2$ and $B_1 = 0.01$ results in a reduction of 20% both in B(E2) and $\mathcal{J}^{(1)}$. These are not entirely consistent with rigid-body values, but we think that the discrepancy is probably acceptable if we remember that effects like pairing and alignments are left out in the present description. Again, for these parameters we require that $B_2 \approx 0$ in order to avoid deviations from the $I(I+1)$ dependence.

This value of A_3 is not as large as we expected (from a presumably small Y_{44} deformation) and corresponds to a ratio $\mathcal{J}_3/\mathcal{J} \approx 1/2$, which can be compared with that calculated for a rigid ellipsoid of semi-axes c and a; $\mathcal{J}_3/\mathcal{J} = 2/(1+(c/a)^2) = 2/5$ for a 2:1 axes ratio. The rather small value of A_3 raises an important question related to the behavior of the excited bands. In fig. 2 the energies of the *yrast* and first excited band (*yrare*), obtained after diagonalization, are given as a function of $I(I+1)$. The "rigid-rotor"-like behavior is immediately seen, with effective inertia parameters ($A_{eff} = E(I)/I(I+1)$) of 1.2 and 1.7, respectively, for these two bands. Using these results we can estimate that, for example, in the Hg region the first excited band will lie around 3.5 MeV above the *yrast* line

at spins around 40, probably too high in excitation energy to be populated with enough intensity to be observed experimentally.

The K-components of the wave functions, C^2_{IK}, of these two bands (Fig. 3) give us some insight into a classical picture of the nuclear motion. On the one hand the *yrast* band has a K-distribution peaked at $K = 4$ and extending to high-K values for high angular momentum; the ratio $< K > /I \approx 1/4$ implies a constant tilting angle of the rotation axis of only 15^0 with respect to the perpendicular to the symmetry axis. On the other hand, in the *yrare* band the K-distribution peaks at $K \approx I$ and $< K > /I \approx 2/3$, thus corresponding more to a rotation around the C_4 symmetry axis. For such a motion the staggering effects are extremely pronounced due to the $\Delta K = 4$ "jumps". and are "propagated" to the *yrast* band through the K mixing (Fig. 4). The important renormalization factor in the moment of inertia, described earlier, can be explained by the broad distribution in K-values of the wave functions Ψ_{IM}. Since the average rotational energy can be written as $< H >= I(I+1) + (A_3 - 1) < K^2 > + < H_c >$, the larger the value of A_3 the more important the contribution of the $A_3 < K^2 >$ term (which is proportional to I^2) to the rotational energy and therefore the larger the renormalization.

Calculated values of $B(E2)$ for the two bands are presented in Fig. 5. The *yrast* band shows a rather smooth behavior with a value around 80% of the uncoupled case; the perturbations seen at low spins are perhaps too small to be detected experimentally. The pronounced staggering in E_γ and $B(E2)$ in the *yrare* band makes the finding of these peculiar bands an attractive experimental challenge. Following the argument about the relatively high excitation energy given earlier this may not be possible in the SD regime; however tentative evidence[7] for $\Delta I = 2$ staggering in the ground state bands in ^{238}Pu and $^{236,238}U$ at around spin 14 suggests that these systems could be ideal cases to look for those bands, expected to be at an excitation energy of ≈ 600 keV.

In conclusion, we have studied the effect of a Y_{44} deformation on the rotational motion of nuclei, with emphasis on SD bands. Complementary to Ref. 3 we discuss the appearance of staggering effects through the mixing of K-bands described by the Hamiltonian (2). The existence of bands differing by $\Delta K = 4$, imposed by the C_4 symmetry,

is a necessary ingredient for the staggering since the Hamiltonian matrix increases its dimension every four units of spin. Although it is still too early to attempt a "fit" of the parameters with experimental data, we have shown that special attention has to be given to the renormalization effects caused by the coupling terms. In fact, without resorting to an extra term in the Hamiltonian that will cancel the renormalization of the moment of inertia we believe that the experimental values of B(E2) and $\mathcal{J}^{(1)}$ rule out $A_3 >> 1$, contrary to our initial premise and that of Ref. 3)

Therefore, the result $\mathcal{J}_3 \approx \mathcal{J}_{rigid}$ would seem to imply a rather large value of the Y_{44} deformation if this is a collective motion. However, perhaps we can interpret A_3 as an effective inertia generated by the alignment of single particle angular momentum[8]. It appears to be difficult to obtain values of $B_1 \approx 0.01$ from microscopic models; current calculations based on the Tilted Axis Cranking model[8] give values about 10 times smaller.

Aknowledgments

We would like to thank Profs. B.R. Mottelson and S. Frauendorf for a number of illuminating discussions. This work was supported by the U.S. DOE under contract DE-AC03-76SF00098

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[2] B.Cederwall et al., Phys.Rev.Lett. **72** 3150 (1994)

[3] I.Hamamoto and B.Mottelson, Phys.Lett. **B333** 294 (1994)

[4] I.M. Pavlichenkov and S.Flibotte , Proceedings of the "Conference on Physics from Large γ-ray Detector Arrays", Berkeley, California, USA, August 2-6 1994.

[5] In what follows we will define $A \equiv 1$. The results should then be interpreted as "in units of A"

[6] A.Bohr and B.Mottelson, Nuclear Structure Vol. II, Benjamin, New York (1975).

[7] L.K.Peker, J.H.Hamilton and J.O.Rasmussen, submitted to Phys.Rev.Lett.

[8] S.Frauendorf, in Ref. 4.

Figure Captions

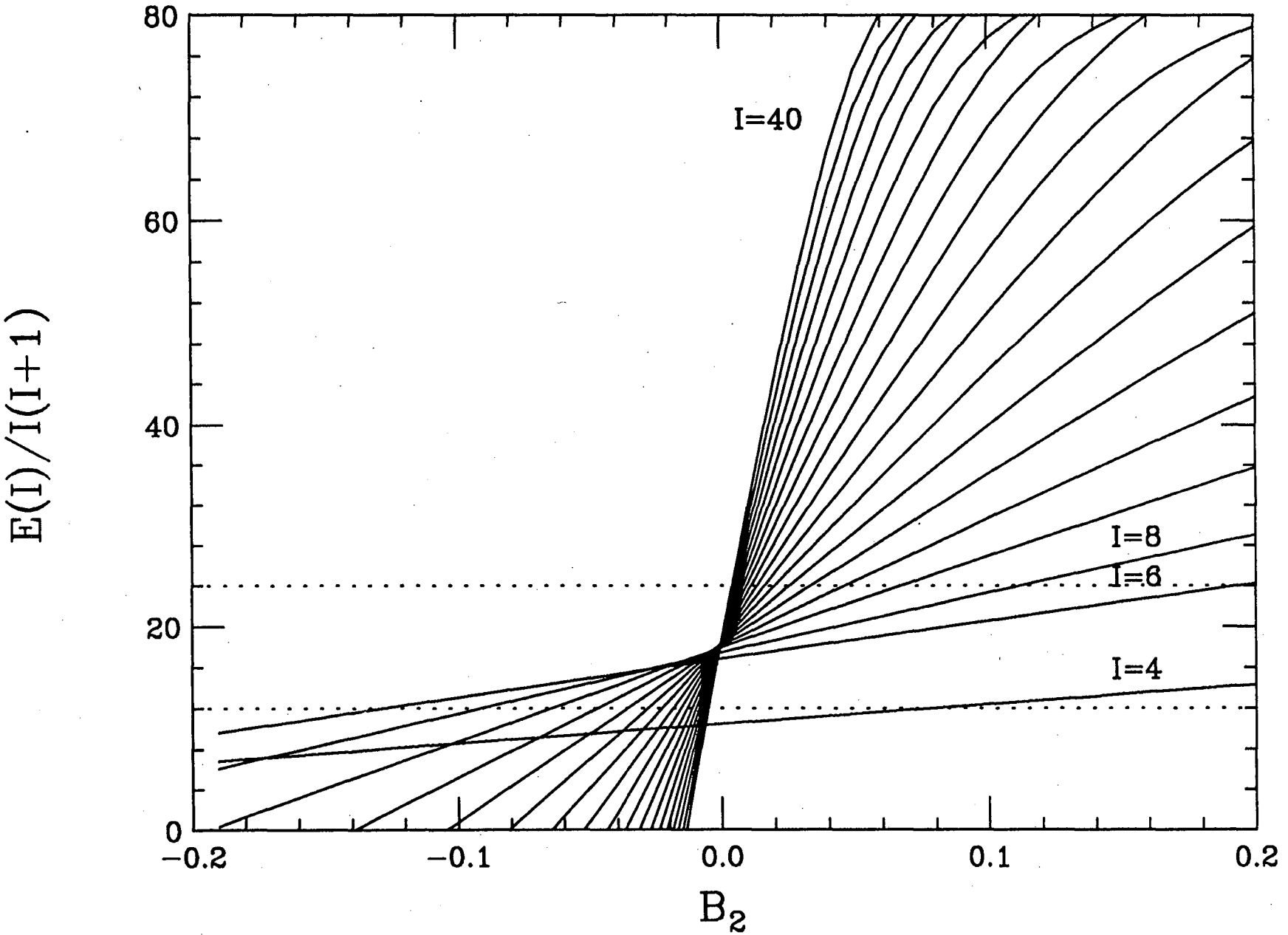
Fig.1 Energy levels, normalized to $I(I+1)$, obtained by diagonalization of the total Hamiltonian $H + H_c$ as a function of the parameter B_2 for increasing values of I. The other parameters are $A_3 = 90$ and $B_1 = 1$. The dotted lines represent the deviation expected at spin 40 for a value of $\beta/\alpha \approx 10^{-4}$ (see text). Note that quantities in both axes are "in units of A".

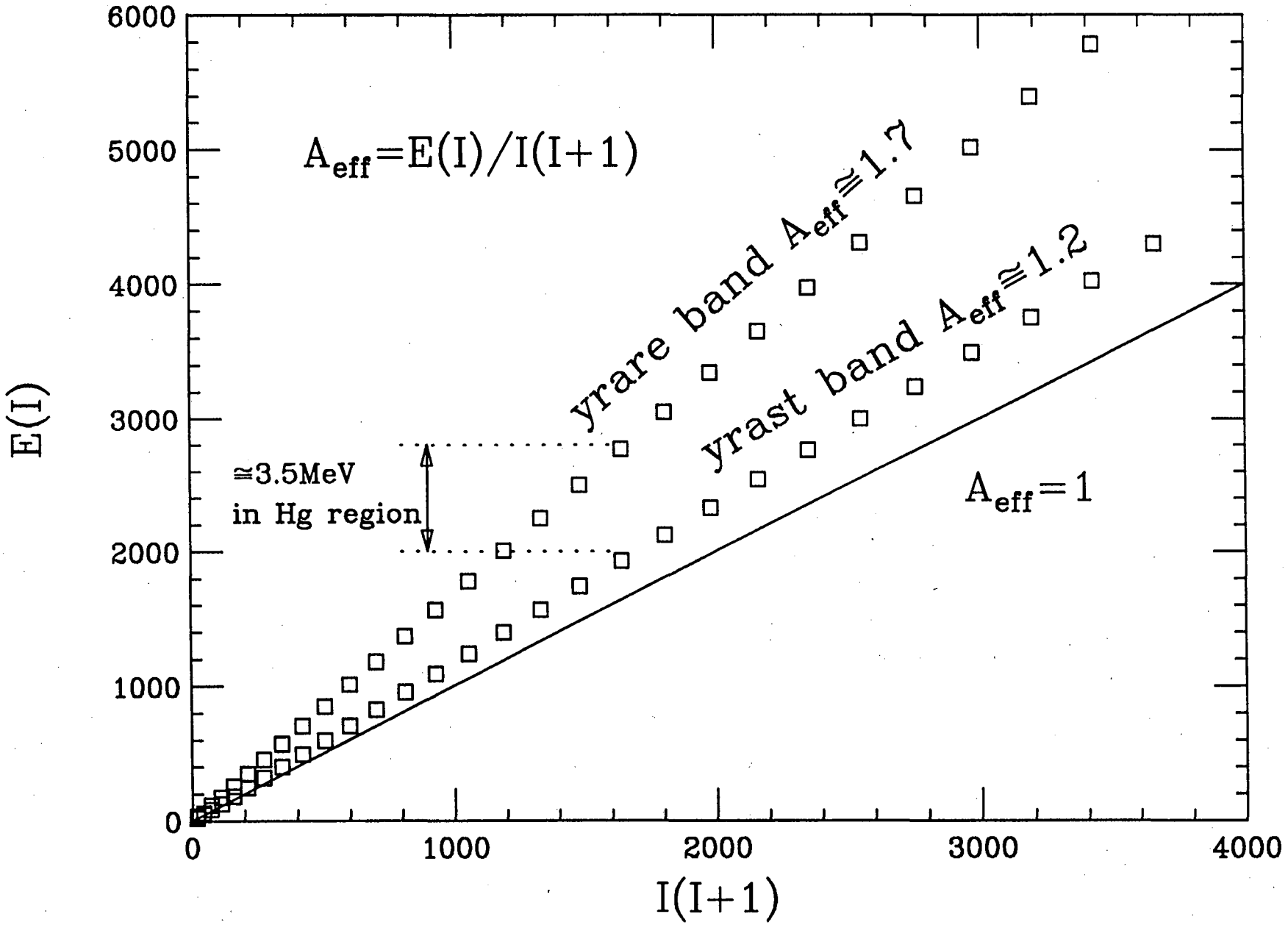
Fig.2 Energy ("in units of A") of the *yrast* and *yrare* bands as a function of $I(I+1)$ for $A_3 = 1.9$ and $B_1 = 0.01$. The slopes of these lines define A_{eff} (see text).

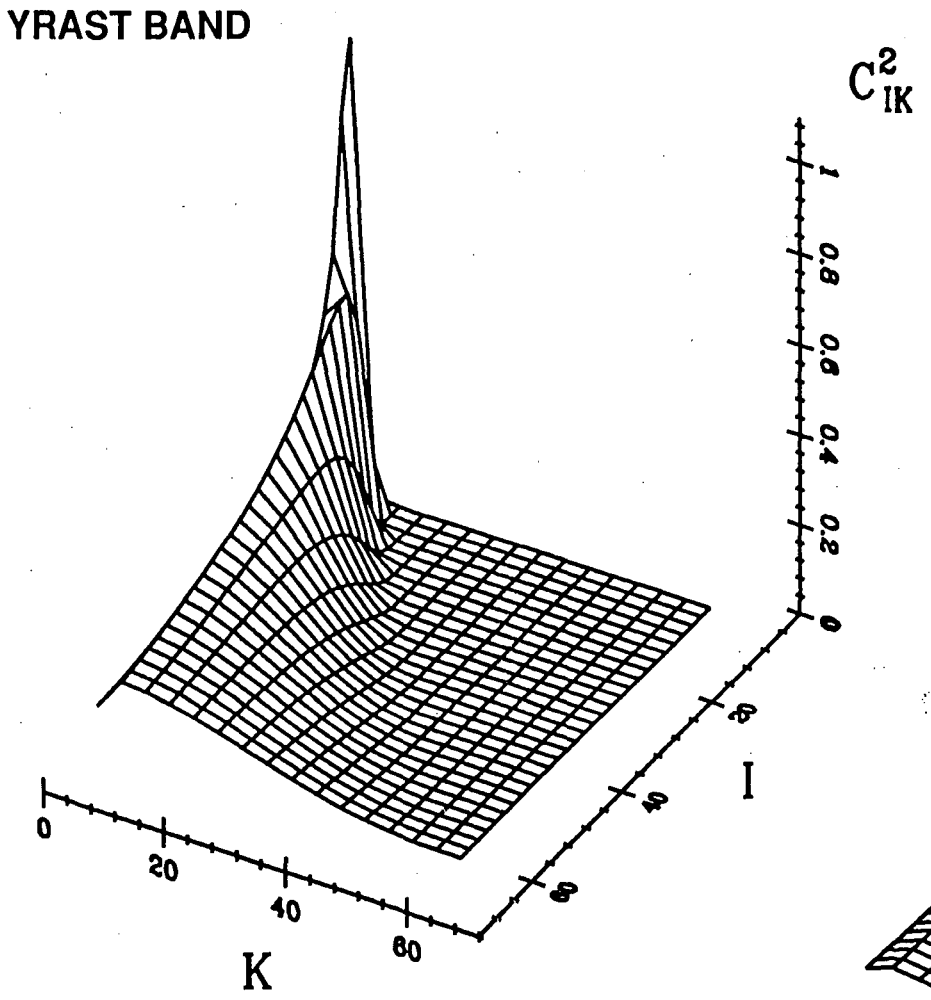
Fig.3 Three-dimensional plot of the amplitudes squared, C^2_{IK}, of the wave functions for the *yrast* and *yrare* bands given in Fig. 2

Fig.4 Staggering plot for the *yrast* and *yrare* bands discussed in fig.2. This is the second derivative of E_γ with respect to I. Note that gamma ray energies are also given in "units of A".

Fig.5 Reduced transition probabilities $B(E2)$ for the *yrast* and *yrare* bands relative to the value $B(E2)_0$ when the coupling term, H_c, is zero. This case corresponds to the example of Fig. 2.


80
60
40
20
0
−0.2
−0.1
0.0
0.1
0.2
E(I)/I(I+1)
B2
I=40
I=8
I=6
I=4


$A_{eff}=E(I)/I(I+1)$
yrare band $A_{eff}\cong 1.7$
yrast band $A_{eff}\cong 1.2$
$A_{eff}=1$
≅3.5MeV
in Hg region
E(I)
I(I+1)
0
1000
2000
3000
4000
5000
6000


YRAST BAND
C^2_{IK}
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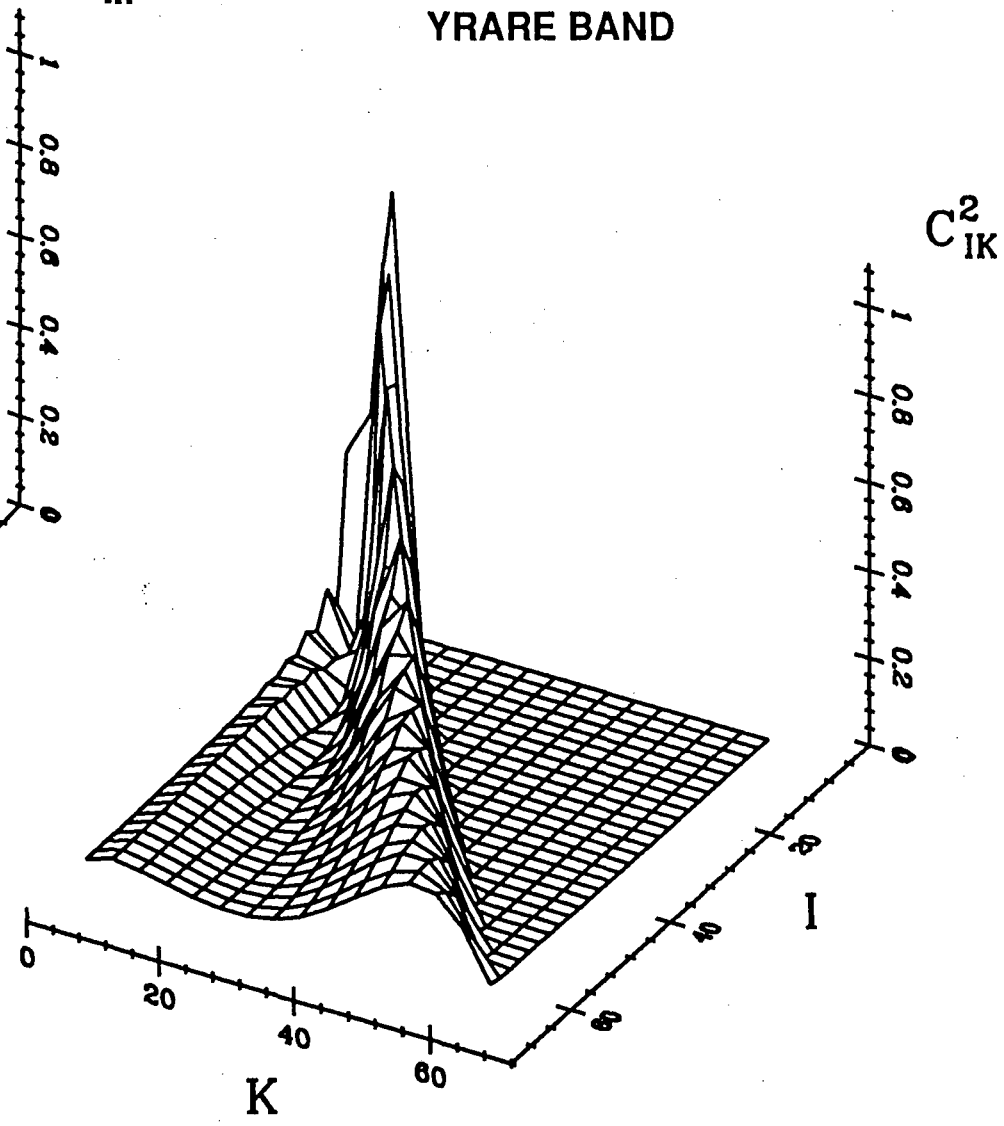
0.6
0.4
0.2
0
0
20
40
60
K
0
20
40
60
I


YRARE BAND
C^2_{IK}
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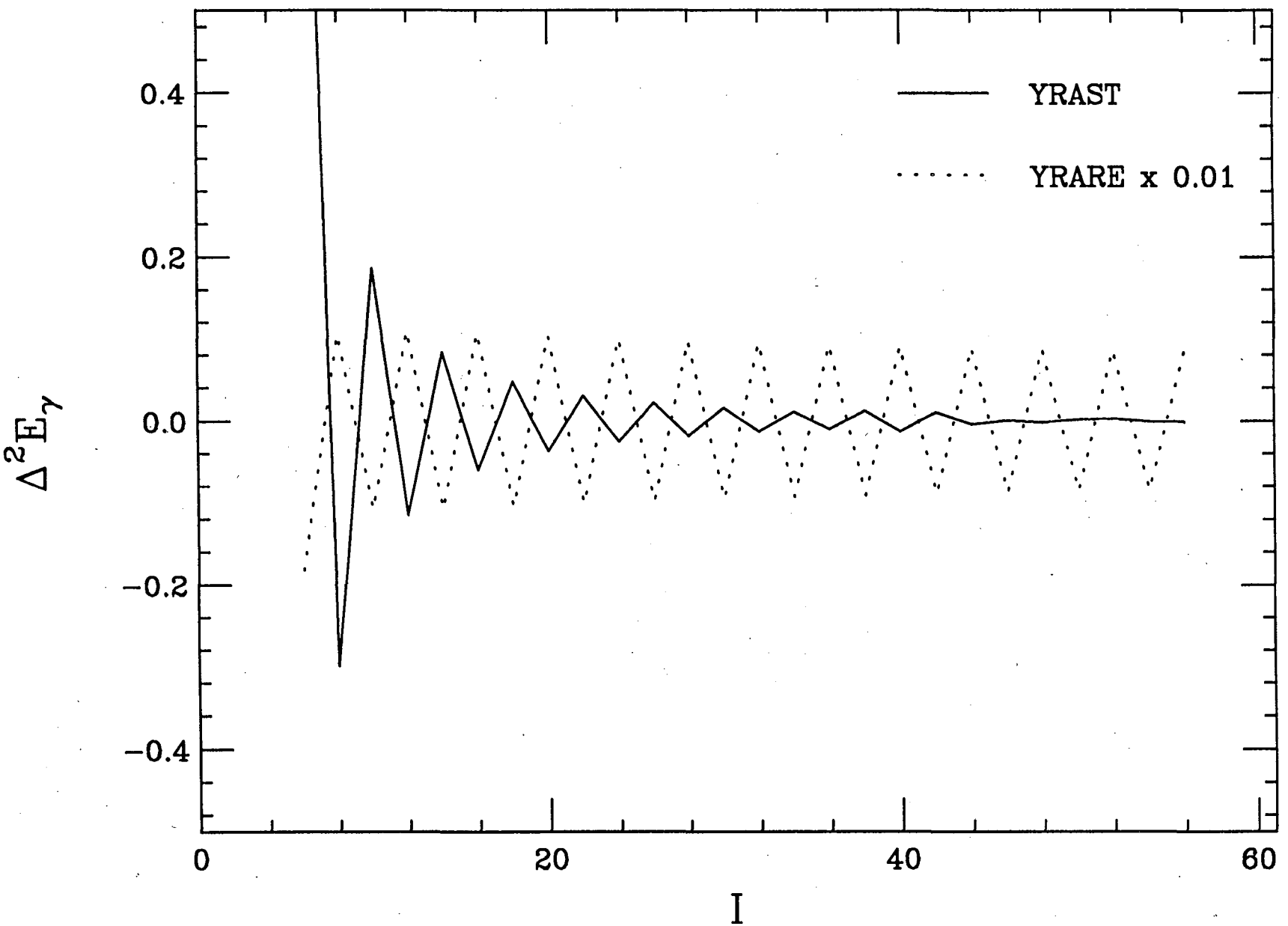
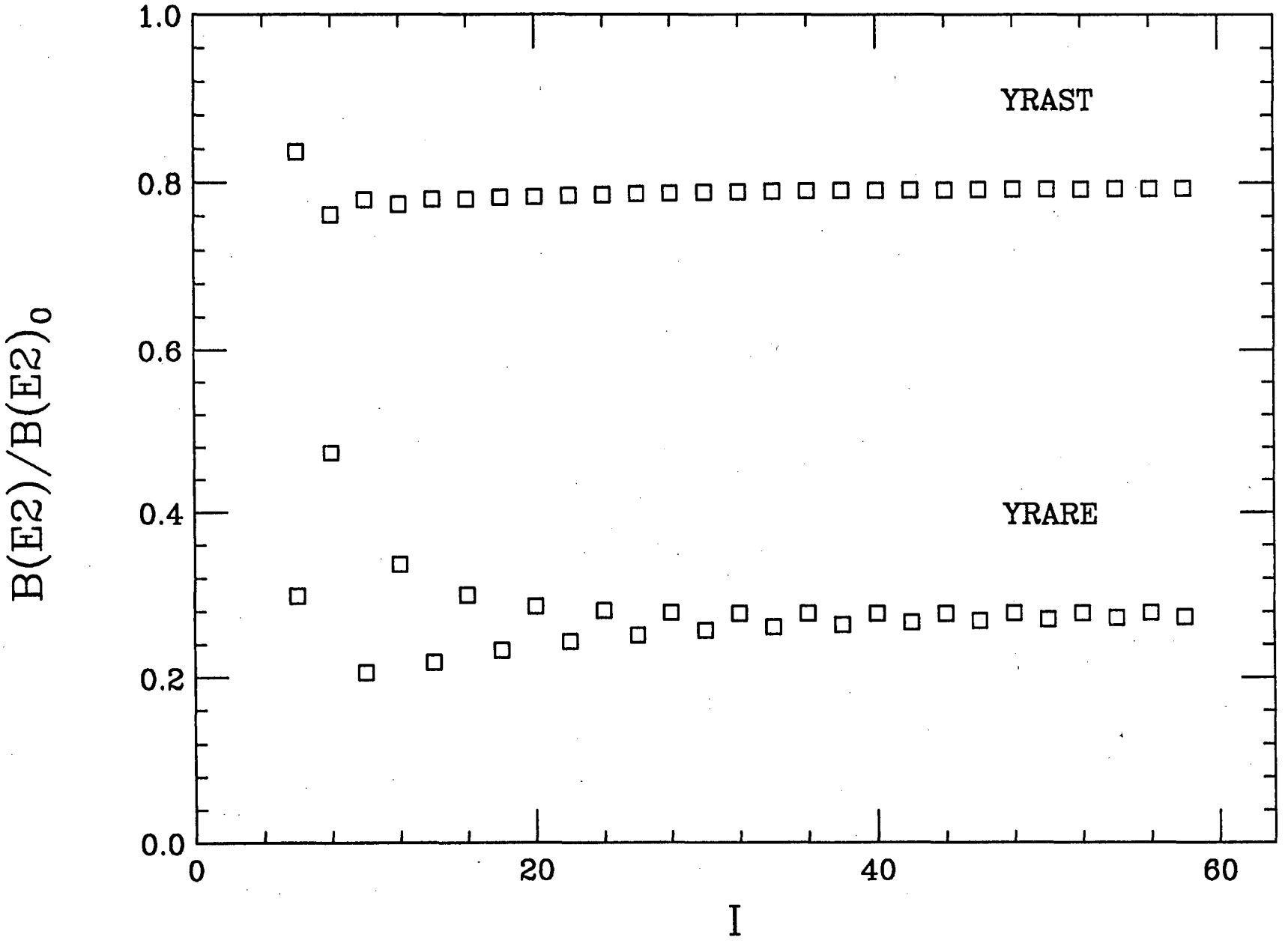
0.6
0.4
0.2
0
0
20
40
60
K
0
20
40
60
I


YRAST
YRARE x 0.01
0.4
0.2
0.0
−0.2
−0.4
$\Delta^2 E_\gamma$
0
20
40
60
I


1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
I
B(E2)/B(E2)$_0$
YRAST
YRARE

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