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INITIAL-STATE RADIATIVE AND RESOLUTION CORRECTIONS AND RESONANCE PARAMETERS IN e^+e^- ANNIHILATION

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J. D. Jackson and D. L. Scharre

March 5, 1975

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INITIAL-STATE RADIATIVE AND RESOLUTION CORRECTIONS AND RESONANCE PARAMETERS IN e^+e^- ANNIHILATION*

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March 5, 1975

ABSTRACT

The effects in e^+e^- annihilation of soft photon emission by the electrons and a finite energy spread on the shapes of resonant cross sections and determination of resonance parameters are treated in a simple coherent manner. The modification of the well-known area method for finding partial widths in the presence of appreciable radiative corrections is discussed in detail and illustrated. Experimental line shapes are expressed in terms of a radiatively corrected energy resolution function $G_R(W - W')$. Numerical values and an explicit functional approximation for G_R are given for a Gaussian distribution of beam energies. The emphasis is on narrow resonances such as the recently discovered states at 3.1 and 3.7 GeV.

1. Introduction

The problem of radiative corrections is familiar to workers at electron accelerators or e^+e^- storage rings, but the recent discovery of resonant states with widths small compared to the beam energy resolution introduces aspects not normally considered. The purpose of this paper is to collect in one place formulas relevant for the analysis of

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resonant line shapes and the determination of resonance parameters in e^+e^- annihilation and to illustrate their use on the narrow resonances at 3.1 and 3.7 GeV. Most of the material is known to, and perhaps derived by, physicists at SLAC and elsewhere. Our purpose is frankly didactic, although the results on the radiatively corrected Gaussian are, to our knowledge, new.

Only radiative corrections for the initial e^+e^- state are treated. These are all that need be considered in discussing the total cross sections or integrated partial cross sections. Furthermore, the large masses of other particles relative to electrons mean that the radiative corrections in the final state are less important, even for detailed differential angular and energy distributions, except for the elastic channel.

In e^+e^- annihilation the lowest order radiative corrections are described by the six diagrams in fig. 1(b). The first two diagrams correspond to real photon emission and their sum contributes incoherently to the cross section. The other four renormalization diagrams are higher order in α and contribute in lowest order only by interference with the nonradiative amplitude of fig. 1(a). The calculation of the lowest order radiative corrections is done in several places. The most immediately applicable reference is Bonneau and Martin[1]. Their eq. (16) is given below as eq. (1).

A more complete treatment of the problem of the infrared divergences associated with the vanishing of the photon mass involves the consideration of emission of arbitrary numbers of very soft photons. A basic understanding of the soft photon problem was achieved by Bloch and Nordsieck in 1937. A comprehensive modern

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treatment is given by Yennie, Frautschi, and Suura[2]. A pedagogical version of this work appears in the Brandeis lectures of Yennie[3]. The consequence of including the multiple emission of soft photons is an "exponentiation" of the lowest order logarithmic corrections into power law corrections. A nice discussion of this exponentiation for soft photon emission by a classical current source can be found in Bjorken and Drell[4].

Radiative corrections for high-energy electron scattering by nucleons and by nuclei are treated authoritatively by Mo and Tsai[5] and Tsai[6].

A brief account of some of the present considerations, as applied to the analysis of the resonance at 3.1 GeV, has been given by Yennie[7].

2. Basic Formulas

(a) Notation We consider ultrarelativistic electrons and use the following notation:

W = total energy in the center of mass

E = $W/2$ = energy of each of the electrons in the initial state

k = energy of an emitted photon

$$t = \frac{2\alpha}{\pi}\left[\ln\left(\frac{W^2}{m_e^2}\right) - 1\right]$$

$\sigma_o(W)$ = a cross section for e^+e^- annihilation at energy W in the absence of radiative corrections

$\sigma(W)$ = cross section with radiative corrections

$\tilde{\sigma}(W)$ = cross section with radiative corrections and folded with a resolution function

M = mass of narrow resonant state

Γ = total width (FWHM) of a resonance

Γ_a = partial width of a resonance into channel a

$(\text{Area})_o$ = energy integral of an isolated resonance cross section, in the absence of radiation corrections

$G(W - W')$ = normalized resolution function

σ = standard deviation parameter of a (Gaussian) resolution function

ΔW = $2.3548\,\sigma$ = FWHM of a Gaussian resolution function

$(\Delta W)_{obs}$ = FWHM of $\tilde{\sigma}(W)$ for an isolated resonance .

The quantity t is the (classical) energy radiated per unit frequency interval at low frequencies when electron and positron in head-on collision disappear.

(b) Bonneau-Martin first order formula

Equation (16) of Bonneau and Martin for the cross section including photon emission and renormalization corrections can be written in our notation as

$$\sigma(W) = \sigma_o(W)\left\{1 + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{6} - \frac{17}{36}\right) + \frac{13}{12}t + t\int_0^E \frac{dk}{k}\left[\left(1 - \frac{k}{E} + \frac{k^2}{2E^2}\right)\frac{\sigma_o(W-k)}{\sigma_o(W)} - 1\right]\right\} . \quad (1)$$

In the integral over dk the argument $(W - k)$ in σ_o should more correctly be $(W^2 - 2Wk)^{\frac{1}{2}}$, but for narrow resonances our approximation is perfectly adequate. Bonneau and Martin's upper limit of integration q_{max} has been put equal to E, corresponding to the fact that an electron can lose all its energy in radiation. The soft-photon emission is contained in the dk/k term and is just the classical result, corrected for energy conservation by the cross section $\sigma_o(W - k)$. It is convenient to rewrite eq. (1) with the soft-photon part displayed separately from the "hard" photon rerms:

$$\sigma(W) = \sigma_o(W)\left[1 + \varepsilon\right] + t\int_0^E \frac{dk}{k}\left[\sigma_o(W - k) - \sigma_o(W)\right] - \frac{t}{E}\int_0^E dk\left(1 - \frac{k}{2E}\right)\sigma_o(W - k) \qquad (2)$$

where

$$\varepsilon = \frac{2\alpha}{\pi}\left(\frac{\pi^2}{6} - \frac{17}{36}\right) + \frac{13}{12}t \qquad (3)$$

is a small number that changes slowly with energy. (For the $\psi(3100)$, where $t = 0.076$, $\varepsilon = 0.085$.) The last term in eq. (2) is small compared to the first two unless the energy W is far off resonance. From now on we omit this "hard" photon piece, although at an appropriate point below we will come back and pick it up.

With the omission of the "hard" photon terms, our slightly simplified version of the Bonneau-Martin formula reads

$$\sigma(W) = \sigma_o(W)\left[1 + \varepsilon\right] + t\int_0^E \frac{dk}{k}\left[\sigma_o(W - k) - \sigma_o(W)\right] . \qquad (4)$$

(c) Exponentiated form of the radiatively-corrected cross section

The emission of arbitrarily large numbers of soft photons with energies less than k leads to the introduction of a factor[2,3]

$$\exp\left[- t\,\ln(E/k)\right] = \left(\frac{k}{E}\right)^t \qquad (5)$$

in the integrand of the integral in eq. (4). Then we find that the radiatively corrected cross section becomes

$$\sigma(W) = t\int_0^E \frac{dk}{k}\left(\frac{k}{E}\right)^t \cdot \sigma_o(W - k) + \varepsilon\,\sigma_o(W) . \qquad (6)$$

The justification for keeping the $\varepsilon\,\sigma_o(W)$ term after exponentiation is not clear. The presence of the factor $(k/E)^t$ makes the integral convergent at the lower limit. In fact,

$$t\int_0^E \frac{dk}{k}\left(\frac{k}{E}\right)^t = 1 ,$$

showing that the radiative processes redistribute the effective energy of annihilation but do not alter the normalization. This argues for the omission of at least part of the term $\varepsilon\,\sigma_o(W)$ in (6). The first part of ε is very small (0.0027) and can be viewed as some sort of "inner" correction to the width Γ_e in the entrance channel. The second, energy-dependent, part of ε is larger, but is not greater than 0.1 even at PEP or EPIC energies of $W = 30$ GeV. For simplicity, we omit the $\varepsilon\,\sigma_o(W)$ term from (6) from now on. The reader who wishes to add its contribution may do so.

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(d) Folding with the energy resolution function

The incident beams in a storage ring have inherent spreads in energy coming mostly from the quantum fluctuations in the emission of synchrotron radiation. Each beam is approximately Gaussian in energy and so the total energy W is distributed approximately in a Gaussian fashion. If the normalized resolution function for a mean beam energy W is $G(W - W')$, the observed cross section is

$$\tilde{\sigma}(W) = \int_{-\infty}^{\infty} dW' \, \sigma(W') \, G(W - W') \quad . \tag{7}$$

The resolution function G is assumed to fall off sufficiently rapidly that the limits of integration can be taken formally as $\pm\infty$ without damage to the physics. Using the radiative correction formula (6) for $\sigma(W')$ this becomes

$$\tilde{\sigma}(W) = t \int_{-\infty}^{\infty} dW' \, G(W - W') \cdot \int_0^{E'} \frac{dk}{k} \left(\frac{k}{E'}\right)^t \cdot \sigma_o(W' - k) \quad . \tag{8}$$

In using eq.(8) below, we assume that the cross sections $\sigma_o(W)$ and $\tilde{\sigma}(W)$ are resonant cross sections, that is, each is the part of the cross section in a particular channel that shows relatively rapid energy variation above (or below) a smooth, essentially constant, background. The background cross section has its own radiative corrections, but that is a separate and comparatively simple problem. For relatively narrow resonances it will be convenient to make certain quite justifiable approximations. For example, with $\sigma_o(W)$ as a resonance whose width Γ is small compared to its mass M (the ρ^o meson with $M = 0.77$ GeV, $\Gamma = 0.15$ GeV qualifies), it is

justified to replace the variable upper limit on the k integration by $E = \frac{W}{2}$ and to approximate

$$\left(\frac{k}{E'}\right)^t = \left(\frac{2k}{W}\right)^t \cdot \left(\frac{W}{2E'}\right)^t \approx \left(\frac{2k}{W}\right)^t \quad .$$

(Note in this connection that t ranges from $t = 0.06$ at $W = 0.5$ GeV to $t = 0.10$ at $W = 40$ GeV.) Then eq. (8) reads

$$\tilde{\sigma}(W) = t \int_{-\infty}^{\infty} dW' \int_0^{W/2} \frac{dk}{k} \left(\frac{2k}{W}\right)^t \sigma_o(W' - k) \, G(W - W') \quad . \tag{9}$$

A change of variable from W' to $W'' = W' - k$ yields

$$\tilde{\sigma}(W) = \int_{-\infty}^{\infty} dW'' \, \sigma_o(W'') \, G_R(W - W'') \tag{10}$$

where

$$G_R(w) = t \int_0^{W/2} \frac{dk}{k} \left(\frac{2k}{W}\right)^t G(w - k) \tag{11}$$

is the radiatively corrected resolution function.

The resolution function is assumed to be relatively narrow (tens of MeV at worst). Since we are interested in w values in $G_R(w)$ of the order of the width of the resonance, or at least not comparable to $W/2$, the integrand in (11) vanishes rapidly for large k and the upper limit on the integral can be replaced by infinity. Suppose now that the normalized energy resolution function (not necessarily a Gaussian) has a standard deviation σ that measures its width in energy. If we use k/σ as variable in the integral in (4) then $G_R(w)$ can be written

$$G_R(w) = \left(\frac{2\sigma}{W}\right)^t \frac{1}{\sigma} F(z,t) \quad , \qquad z = w/\sigma \quad , \qquad (12)$$

where $F(z,t)$ is a dimensionless function,

$$F(z,t) = t\int_0^\infty \frac{dx}{x}\, x^t \cdot \sigma\, G\big((z - x)\sigma\big) \quad . \qquad (13)$$

Note that t varies with energy only logarithmically from resonance to resonance $\big(t = 0.063$ at the ρ^o, $t = 0.076$ at the $\psi(3100)\big)$. The function $F(z,t)$ therefore almost depends only on the shape of $G(w)$ with w in units of σ. The dependence of G_R on the beam energy and the energy spread is essentially all contained in the factor $(2\sigma/W)^t$ in eq. (12).

Equation (10) shows that, given $G_R(w)$, the folding of the cross section is the same as always. The radiative corrections have been transferred to the beam energy resolution function. For a specific function $G(w)$, the integral (13) can be performed to obtain $F(z,t)$ and hence $G_R(w)$. The example of a Gaussian resolution function $G(w)$ is treated in Section 9 below.

3. High-energy Radiative Tail

A characteristic feature of the cross sections is a radiative tail on the high-energy side of a resonance, as shown in fig. 2. This corresponds physically to the emission of a photon by one or the other of the incident electrons causing the energy of annihilation to be in the neighborhood of the resonance mass M, even though the incident energy W is considerably higher. For energies sufficiently far above the resonance that $(W - M)$ is large compared to both the natural line width Γ and the width ΔW of the resolution function, an asymptotic

expansion for $\tilde{\sigma}(W)$ can be obtained. From eq. (10) we see that in this limit, the important arguments of G_R are large compared to ΔW. This means that we can neglect the width of the resolution function and approximate $G(w - k) \simeq \delta(w - k)$ in eq. (11). Then we find

$$G_R(w) \simeq \frac{t}{w}\left(\frac{w}{E}\right)^k \cdot \left[\theta(w) - \theta\left(w - \frac{W}{2}\right)\right] \qquad (14)$$

for $w \gg \sigma$. Not surprisingly, eq. (10) then becomes the same as the integral in eq. (6). Since $\sigma_o(W')$ peaks at $W' = M$, the relatively slowly varying function G_R, eq. (14), can be expanded around $w = W - M$ in eq. (10). Provided $(W - M) \gg \Gamma$ and σ_o is symmetric around $W = M$, we obtain the asymptotic expansion,

$$\tilde{\sigma}(W) = t(\text{Area})_o \frac{1}{W - M}\left(\frac{W - M}{E}\right)^t$$

$$\times \left[1 + \frac{(1 - t)(2 - t)}{2(W - M)^2}\left\langle (W - M)^2 \right\rangle_o + \cdots\right] \qquad (15)$$

where

$$(\text{Area})_o = \int \sigma_o(W')\, dW' \qquad (16)$$

is the area of the cross section without radiative corrections and $\left\langle (W - M)^2 \right\rangle_o$ is the mean square width of the resonant cross section $\sigma_o(W)$. There are three necessary comments here:

(1) The leading term, proportional to the area, is independent of the detailed shape or symmetry of $\sigma_o(W)$.

(2) If we had kept the lowest order terms in the width σ of $G(w)$, the mean square width would be replaced by $\left\langle (W - M)^2 \right\rangle_o + \sigma^2$.

(3) For a Breit-Wigner resonance, decreasing as $(W - M)^{-2}$ asymptotically, the mean square width is infinite. This is a signal

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that (15) is incorrect beyond the first term for such a line shape. The equivalent expansion is not just in inverse powers of $(W - M)$. If accuracy demands more than the first term in (15), it is best to perform a numerical evaluation of (6) or (10).

For narrow resonances, with widths smaller than or comparable to the resolution, only the first term, basically the $1/\Delta E$ bremsstrahlung spectrum, in (15) is important once $(W - M) \gg \underset{\sim}{\sigma}$. For quite large values of $(W - M)$ it is possible that the neglected "hard" photon terms are not negligible. If we keep only the leading term in eq. (15), but go back and pick up the "hard" photon contributions from eq. (2), the high-energy tail becomes

$$\tilde{\sigma}(W) = t(\text{Area})_o \cdot \left[\left(\frac{W - M}{E}\right)^t \frac{1}{W - M} - \frac{1}{E} + \frac{W - M}{2E^2}\right] \quad . \qquad (17)$$

The lowest order (Bonneau-Martin) cross section gives (17) with the factor $\left[(W - M)/E\right]^t$ omitted. As observed by Yennie[7], an accurate measurement of the radiative tail, (17), allows a determination of the area of the cross section independent of the beam energy spread.

4. Cross Section For a Resonance Whose Width is Large Compared to the Energy Resolution

For a resonance like the ρ^o and even the ω^o, its width Γ is large compared to the beam resolution. Then the resolution function $G(W - W')$ can be taken as a delta function and the observed cross section $\tilde{\sigma}(W)$ is essentially equal to (6). An integration by parts gives

$$\sigma(W) = - \int_0^E dk \left(\frac{k}{E}\right)^t \frac{d}{dk} \sigma_o(W - k) \quad . \qquad (18)$$

In writing (18) we have dropped a term $\sigma_o(W/2)$, the assumption being that, if $\sigma_o(W)$ is a resonant cross section peaking at $W = M$ and W is not too far from resonance, such a term is negligible. Suppose that $\sigma_o(W)$ is a Breit-Wigner resonance with widths whose energy variation can be neglected. Then

$$\sigma_o(W) = (\sigma_o)_{max} \frac{\Gamma^2/4}{(M - W)^2 + \frac{\Gamma^2}{4}} \quad . \qquad (19)$$

The cross section $(\sigma_o)_{max}$ is the peak cross section; its value depends on the particular channel or channels being considered. With (19) inserted, eq. (18) becomes

$$\sigma(W) = (\sigma_o)_{max} \frac{\Gamma^2}{4} \int_0^E \left(\frac{k}{E}\right)^t \frac{2(M - W + k)}{\left[(M - W + k)^2 + \frac{\Gamma^2}{4}\right]^2} dk \quad . \qquad (20)$$

It is probably simplest at this stage to integrate numerically in order to see what (20) gives.

For reference and orientation we evaluate (20) at $W = M$. This is essentially, but not quite, the peak cross section, the maximum being infinitesimally higher in magnitude and in position. We find

$$\sigma(M) = \frac{(\pi t/2)}{\sin(\pi t/2)} \left(\frac{\Gamma}{M}\right)^t (\sigma_o)_{max} \quad . \qquad (21)$$

The first factor can be approximated as $(\pi t/2)/\sin(\pi t/2) \approx (1 + \pi^2 t^2/24)$. It is equal to unity within 0.004 or less up to $W = 30$ GeV. Thus $\sigma(M) \approx (\Gamma/M)^t (\sigma_o)_{max}$. Numerically for the ρ^o meson, with $M \simeq 770$ MeV, $t = 0.063$, $\Gamma = 150$ MeV, we find $(\Gamma/M)^t \simeq 0.90$. For the ω^o meson, with $\Gamma = 10$ MeV, $(\Gamma/M)^t \simeq 0.76$. The reduction in peak cross section because of

radiative processes is thus not negligible and is larger the smaller the width (provided the energy resolution is good--see the next section).

For completeness we note that the lowest order radiative correction gives a factor $\left[1 - t\ \ell n(M/\Gamma)\right]$ instead of $(\Gamma/M)^t$, corresponding to the first terms in an expansion of $(\Gamma/M)^t = \exp\left[-t\ \ell n(M/\Gamma)\right]$ in powers of t. For the ω^o the linear radiative correction factor is 0.723 instead of 0.758. Inclusion of $\varepsilon\ \sigma_o'(M)$ from eq. (2) or (6) adds 0.072 to the 0.723, giving 0.795.

The high-energy radiative tail is given by eq. (17) with

$$(\text{Area})_o = \frac{\pi}{2}\Gamma(\sigma_o)_{max} \quad . \tag{22}$$

5. Cross Section For a Resonance Whose Width is Very Narrow Compared to the Energy Resolution

Although the general circumstance of a resonance whose width is comparable to the energy resolution can be dealt with effectively only by numerical integration, the limit of a very narrow resonance (or very poor energy resolution) can be discussed simply if the resolution function $G(w)$ is known. For a resonant cross section with a total width Γ that is very small compared to the width $\underset{\sim}{\sigma}$ of $G(w)$, we can approximate the cross section $\sigma_o(W'')$ in (10) by $\sigma_o(W'') = (\text{Area})_o\ \delta(W'' - M)$. Then we obtain

$$\tilde{\sigma}(W) = G_R(W - M) \times (\text{Area})_o \tag{23}$$

where $G_R(W - M)$ is the radiatively corrected resolution function (11).

We discuss the detailed shape of $G_R(w)$ for a Gaussian G below. Here we content ourselves with considering the value of the cross section at $W = M$. From (12) and (13) we find

$$\tilde{\sigma}(M) = \left(\frac{2\underset{\sim}{\sigma}}{M}\right)^t G(0)\cdot N(t) \tag{24}$$

where

$$N(t) = \frac{F(0,t)}{\underset{\sim}{\sigma}\ G(0)} = t\int_0^\infty \frac{dx}{x}\ x^t\ \frac{G(\underset{\sim}{\sigma}x)}{G(0)} \tag{25}$$

is a number depending upon t and on the <u>shape</u> of $G(w)$. It is a number close to unity, as can be seen qualitatively as follows. Suppose that the resolution function is of rectangular shape, $G(w) = (2\sqrt{3}\,\underset{\sim}{\sigma})^{-1}$, $-\sqrt{3}\,\underset{\sim}{\sigma} < w < \sqrt{3}\,\underset{\sim}{\sigma}$. Then $N(t) = (3)^{t/2} \simeq 1 + 0.5493\ t$.

For the more realistic Breit-Wigner resolution function,

$$G(w) = \frac{(\Delta W/2\pi)}{w^2 + (\Delta W/2)^2} \quad ,$$

with ΔW playing the role of $2\underset{\sim}{\sigma}$, the value of $N(t)$ can be read off from (21) to be $N(t) = (\pi t/2)\ \text{cosec}(\pi t/2) \simeq 1 + \pi^2 t^2/24$. For the still more realistic Gaussian shape, we find

$$N(t) = 2^{t/2}\ \Gamma(1 + \tfrac{t}{2}) \simeq 1 + 0.058\ t. \tag{26}$$

For an inverted, truncated parabola, $N(t) = 2(5)^{t/2}/(2 + t) \simeq 1 + 0.3047\ t$. For reasonable resolution functions, then, we find $N(t)$ to be a number within a few percent of unity for $t < 0.1$, and for the Breit-Wigner or Gaussian much closer. The observed peak cross section for a resonance whose width is negligible compared to the energy resolution is therefore given closely by (24) with $N(t) = 1$.

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The radiative processes decrease the peak cross section by a factor $(2\sigma/M)^t$. Typical values for the $\psi(3100)$ are $M = 3095$ MeV, $\sigma = 0.78$ MeV, $t = 0.076$. These give $(2\sigma/M)^t = 0.561$, a very significant reduction.

6. Area Method of Determining Resonance Widths in the Presence of Radiative Corrections

The area method is one of the most reliable for determining resonance parameters because the details of the energy resolution are minimized to a great degree (In principle, they are eliminated entirely.). The method is well known in nuclear physics. The only new aspect here is the presence of soft-photon processes. Firstly, consider only the energy resolution. If the resonant cross section is $\sigma_o(W)$ then the folded cross section is

$$\tilde{\sigma}(W) = \int G(W - W')\, \sigma_o(W')\, dW' \quad .$$

We now integrate the cross section, smeared by the resolution function, from W_{min} to a variable upper limit W. The lower limit W_{min} is chosen in practice to be where the resonant part of the cross section first begins to be visible above the background. The integral is

$$A(W,W_{min}) = \int_{W_{min}}^{W} \tilde{\sigma}(W')\, dW' = \int_{W_{min}}^{W} dW' \int dW''\, G(W' - W'')\, \sigma_o(W'') .$$

The behavior of $A(W,W_{min})$ as a function of W is shown schematically in fig. 3(b). In fig. 3(a) the cross section and the folded cross section are sketched. At positive values of $(W - M)$ large compared to the observed width Γ_{obs} of $\tilde{\sigma}(W)$ the integral becomes constant. Its values is found by interchanging orders of integration above.

Since G is normalized we find

$$\lim_{(W-M)\gg\Gamma_{obs}} A(W,W_{min}) = \int dW''\, \sigma_o(W'') \equiv (\text{Area})_o \quad . \tag{27}$$

The plateau value of $A(W,W_{min})$ is thus equal to the area of the original cross section, independent of the form or details of $G(W - W')$.

The method needs only slight modification because of the radiative corrections. We begin with the smeared cross section with radiative corrections, eq. (8), and integrate it from W_{min} to W:

$$A(W,W_{min}) = t \int_{W_{min}}^{W} dW' \int_{-\infty}^{\infty} dW''\, G(W' - W'') \int_0^{E''} \frac{dk}{k} \left(\frac{k}{E''}\right)^t \times \sigma_o(W'' - k) \quad .$$

Since we are concerned with resonances whose observed widths are small compared with W it is permissible to neglect the energy variation of a factor $(M/2E'')^t$--even for the ρ^o it causes an error of less than 1%--and also put the upper limit on the dk integration as $M/2$. Then an integation by parts in the dk integral gives

$$A(W,W_{min}) = \int_{W_{min}}^{W} dW' \int_{-\infty}^{\infty} dW''\, G(W' - W'') \int_0^{M/2} dk \left(\frac{2k}{M}\right)^t \times \frac{d\sigma_o(W'' - k)}{dW''} \quad .$$

If we now perform the dW'' integration and integrate by parts, we obtain

$$A(W,W_{min}) = \int_{W_{min}}^{W} dW' \int_0^{M/2} dk \left(\frac{2k}{M}\right)^t \int_{-\infty}^{\infty} dW'' \, \sigma_o(W'' - k)$$

$$\times \frac{dG(W' - W'')}{dW'} .$$

Now we can do the W' integration:

$$A(W,W_{min}) = \int_0^{M/2} dk \left(\frac{2k}{M}\right)^t \cdot \int_{-\infty}^{\infty} dW'' \, \sigma_o(W'' - k)$$

$$\times \left[G(W - W'') - G(W_{min} - W'')\right] .$$

Since $W = W_{min}$ is the energy below which there is no resonant cross section, the last term $G(W_{min} - W'')$ can be dropped. Then a change of variables in the dW'' integral gives

$$A(W,W_{min}) = \int_0^{M/2} dk \left(\frac{2k}{M}\right)^t \int_{-\infty}^{\infty} dW' \, \sigma_o(W') \, G(W - k - W') . \quad (28)$$

To see the behavior of $A(W,W_{min})$ we note that the integral over dW' is confined by the resonant cross section $\sigma_o(W')$ to a range of the order of $\pm\Gamma$ around $W' = M$. The resolution function, on the other hand, is nonvanishing only for $|W - k - W'| \lesssim \Delta W$. If $(W - M)$ is large compared with the larger of Γ and ΔW, the range of the dk integration is confined to $k \simeq (W - M) \pm \Gamma \pm \Delta W$. Since the factor $(2k/M)^t$ is slowly varying provided k is not too small, it can be evaluated at $k = (W - M)$ in this limit and taken outside the integral. The remaining integrals are just as in the radiationless situation, provided $(W - M) \gg \Delta W$. We thus obtain

$$A(W,W_{min}) \rightarrow \left[\frac{2(W - M)}{M}\right]^t \cdot (\text{Area})_o$$

or

$$(\text{Area})_o = \lim_{(W-M)\gg\Gamma,\Delta W} \left[\frac{M}{2(W - M)}\right]^t \cdot A(W,W_{min}) . \quad (29)$$

This is the generalization of eq. (27) to include the effects of the radiative corrections. The integral $A(W,W_{min})$ continues to increase slowly with increasing W because of the radiative tail, (15), rather than levelling off to a plateau as in fig. 3(b). The factor $\left[M/2(W - M)\right]^t$, which is larger than unity but decreases with increasing energy, corrects for the rise in $A(W,W_{min})$. The product is larger than $A(W,W_{min})$, compensating for the reduced cross section near the resonance, but levels off to a well-defined plateau that is the true area of $\sigma_o(W)$ in the absence of soft-photon processes. In practice, one calculates $A(W,W_{min})$ as a function of W and then begins multiplying by the correction factor for values of W slightly above the resonance.

7. Example of determination of Γ_e for $\psi(3100)$

To illustrate the use of eq. (29) we consider the integration of the cross section $\sigma(e^+e^- \rightarrow \text{hadrons})$ to determine Γ_e for the $\psi(3100)$. We use the data from Augustin et al.[8] These are not the latest or best data, but they are accessible in the literature and serve for purposes of illustration.

The data on $e^+e^- \rightarrow$ hadrons near 3.1 GeV are plotted on a linear ordinate scale in fig. 4. A background cross section of 25 nb has been subtracted. Also shown are the radiative correction factor $\left[M/2(W - M)\right]^t$, the integral $A(W,W_{min})$ and the product of the two.

The plateau value for the area according to eq. (29) is

$$(\text{Area})_o = 11.5 \times 10^3 \text{ nb} \times \text{MeV} \quad .$$

For a narrow resonant line (19) the area is given by (22). If a $J = 1^-$ assignement is assumed, the peak cross section for a reaction $a \to b$ is

$$(\sigma_o)_{max} = \frac{12\pi}{M^2} \frac{\Gamma_a \Gamma_b}{\Gamma^2} \tag{30}$$

where Γ_a, Γ_b are the partial widths for the two channels and Γ is the total width. The product $\Gamma_a \Gamma_b / \Gamma$ is thus determined to be

$$\frac{\Gamma_a \Gamma_b}{\Gamma} = \frac{M^2}{6\pi^2} \left[(\text{Area})_o\right]_{ab}$$

$$= 4.34 \times 10^{-14} \left[M(\text{MeV})\right]^2 \cdot \left[\text{Area}(\text{nb} \times \text{MeV})\right] \text{ MeV} \quad . \tag{31}$$

The numerical factor converts nb to $(\text{MeV})^{-2}$, as well as including the factor $(6\pi^2)^{-1}$. For $M = 3105$ MeV and $(\text{Area})_o = 11.5 \times 10^3$ nb × MeV, this gives

$$\frac{\Gamma_e \Gamma_{had}}{\Gamma} = 4.81 \times 10^{-3} \text{ MeV} \quad .$$

A rough estimate of the error in determining the area and the resonant mass indicates a generous uncertainty of 5 to 10%, apart from systematic errors.

Integration according to eq. (29) of the cross section $\sigma(e^+e^- \to \mu^+\mu^-)$ of Augustin et al.[8] (called by them the cross section for 2-body, collinear events that are not pairs of electrons) gives

$$(\text{Area})_o^{\mu\mu} \simeq 7.8 \times 10^2 \text{ nb} \times \text{MeV}$$

with an error of 20% at most, apart from systematics. Assuming this cross section is truly $e^+e^- \to \mu^+\mu^-$ we have

$$\frac{\Gamma_e \Gamma_\mu}{\Gamma} = 3.3 \times 10^{-4} \text{ MeV} \quad .$$

We thus have $\Gamma_\mu / \Gamma_{had} = 0.068$. Assuming that $\Gamma = \Gamma_e + \Gamma_\mu + \Gamma_{had}$ and that $\Gamma_\mu = \Gamma_e$ (This is consistent with the observed resonant contribution in the $e^+e^- \to e^+e^-$ channel and the results of Ford et al.[9]), we can solve for Γ_e, Γ_{had}, Γ:

$$\Gamma_e = 5.5 \pm 0.5 \text{ keV}$$

$$\Gamma_{had} = 81 \text{ keV}$$

$$\Gamma = 92 \text{ keV} \quad .$$

The error on Γ_{had} and Γ is probably less than 20%, apart from systematics.

With a larger sample of data and better estimates of efficiencies the LBL-SLAC collaboration have presented more recent results.[10] Including estimates of systematic errors, these are $(\text{Area})_o = (10.8 \pm 2.7) \times 10^3$ nb - MeV and

$$\Gamma_e = 5.2 \pm 1.3 \text{ keV}$$

$$\Gamma = 77 \pm 19 \text{ kev}$$

$$M = 3095 \pm 5 \text{ MeV} \quad .$$

8. Quick and Dirty Method of Estimating $(\text{Area})_o$

While the area method is most reliable in determining $(\text{Area})_o$, a fair estimate can be made quickly using the radiative correction factor for the peak cross section. For a resonance seen in good resolution (Sect. 4) we found a factor $(\Gamma/M)^t$ and for one seen in poor resolution (Sect. 5), $(2\sigma/M)^t$ where σ is the standard deviation of a Gaussian resolution function. Since the FWHM of a Gaussian is $\Delta W = 2.3548\sigma$, a general radiative reduction factor for whatever conditions of narrowness is

$$(\sigma_{max})_{\substack{\text{with radiative}\\ \text{processes}}} = \left(\frac{\Delta W_{obs}}{M}\right)^t \cdot (\sigma_{max})_{\substack{\text{no radiative}\\ \text{processes}}} \qquad (32)$$

where ΔW_{obs} is the observed FWHM of the line. This interpolates between the two limits smoothly and is in error by less than 1% in comparison with $(2\sigma/M)^t$.

If the observed line is assumed to be Gaussian in shape, its area, without radiative tail, is

$$(\text{Area})_{obs} = 1.0645(\sigma_{max})_{obs} \cdot \Delta W_{obs} \quad . \qquad (33)$$

If the line shape is a Breit-Wigner, the coefficient 1.0645 becomes $\pi/2 = 1.57$. We assume here that the resonance is narrow and the resolution function is Gaussian, so that the observed line shape is approximately Gaussian.

The true area can thus be estimated from eqs. (32) and (33) to be

$$(\text{Area})_o \simeq 1.0645\left(\frac{M}{\Delta W_{obs}}\right)^t (\sigma_{max})_{obs} \cdot \Delta W_{obs} \quad . \qquad (34)$$

The radiative tail produces a skewing of the symmetrical line shape, but makes an effect of only a few percent on ΔW_{obs}.

As an example of the use of (34) we consider the data of Augustin et al., used previously in fig. 4. The observed peak cross section and FWHM are $(\sigma_{max})_{obs} = 2300$ nb, $\Delta W_{obs} = 2.5$ MeV. With $M = 3105$ MeV and $t = 0.076$, eq. (34) gives

$$(\text{Area})_o = 1.0645 \times 1.72 \times 2300 \times 2.5 = 10.5 \times 10^3 \text{ nb} \times \text{MeV} \; .$$

This is to be compared with the value of 11.5×10^3 nb × MeV found by the area method.

9. Radiatively Corrected Gaussian Resolution Function

Since the beams in e^+e^- storage rings are expected to give a roughly Gaussian spread in energy, we consider

$$G(w) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp(-w^2/2\sigma^2) \quad . \qquad (35)$$

The function $F(z,t)$ of eq. (13) is then

$$F(z,t) = \frac{t}{\sqrt{2\pi}} \int_0^\infty dx\, x^{t-1}\, e^{-(z-x)^2/2} \quad .$$

This can be recognized[11] as an integral representation of a Weber's parabolic cylinder function, $D_\nu(x)$. Thus we have

$$F(z,t) = \frac{\Gamma(1+t)}{\sqrt{2\pi}} e^{-z^2/4} D_{-t}(-z) \quad . \qquad (36)$$

0 0 0 0 4 3 0 1 9 7 3

Some mathematical properties are

(a) In the limit $t \to 0$, $D_o(-z) = e^{-z^2/4}$ and we recover the Gaussian (35).

(b) For large <u>positive</u> z, the asymptotic expansion of $F(z,t)$ is

$$F(z,t) \simeq t\, z^{t-1}\left[1 + \frac{(1-t)(2-t)}{2z^2} + \cdots\right]. \quad (37)$$

This is just the soft-photon part of the radiative tail discussed in Sect. 3.

(c) For large <u>negative</u> z, the asymptotic expansion is

$$F(z,t) \simeq \frac{\Gamma(1+t)}{\sqrt{2\pi}}\, e^{-z^2/2}\left(\frac{1}{|z|}\right)^t\left[1 - \frac{t(1+t)}{2z^2} + \cdots\right]. \quad (38)$$

Note that on the negative side the radiatively corrected Gaussian has its exponential decrease augmented by a power law factor, $|z|^{-t}$.

(d) The parabolic cylinder functions can be expressed in terms of confluent hypergeometric or Kummer functions. One such representation is

$$F(z,t) = \frac{2^{t/2}\,\Gamma\left(1+\frac{t}{2}\right)}{\sqrt{2\pi}}\, M\left(\frac{1-t}{2}, \frac{1}{2}, -\frac{1}{2}z^2\right)$$

$$+ \frac{t}{2}\,\frac{\Gamma(1+t)}{2^{t/2}\,\Gamma\left(1+\frac{t}{2}\right)}\cdot z\, M\left(1-\frac{t}{2}, \frac{3}{2}, -\frac{1}{2}z^2\right). \quad (39)$$

In eq. (39) the function $M(a,b,x)$ is the standard confluent hypergeometric function defined by the series,

$$M(a,b,x) = \sum_{n=0}^{\infty} \frac{a(a+1)(a+2)\cdots(a+n-1)}{b(b+1)(b+2)\cdots(b+n-1)}\,\frac{x^n}{n!}$$

and sometimes written as ${}_1F_1(a;\, b;\, x)$.

(e) The values of $F(0,t)$ and $F'(0,t)$ can be read off from eq. (39).

The function $F(z,t)$ has been evaluated for the interesting range of t values, 0.06 to 0.10 ($W = 0.5$ GeV to $W = 40$ GeV). The evaluation was done with the original integral representation, after an integration by parts, and also with the representation (39), using a library routine for confluent hypergeometric functions in the LBL computer. Except for very large $|z|$, where the convergence of the hypergeometric series routine is poor, the two sets of values agree to four or more significant figures. The table contains values of $F(z,t)$ for $t = 0.070, 0.075, 0.080, 0.085$, and z values of $-5(0.5) - 2$, $-2(0.2)2$, $2(0.5)8$. The t range corresponds to $W \simeq 1.6$ GeV to 8.0 GeV. Figure 5 shows a typical curve of $F(z,t)$ for $t = 0.08$, together with the Gaussian, eq. (35). The soft-photon emission changes the position and height of the peak only slightly, but causes a skewing of the curve and adds the radiative tail. The full width at half maximum is only a few percent greater than for the uncorrected Gaussian. Remember, however, that $G_R(w)$ has the factor $(2\sigma/M)^t$ multiplying $F(z,t)$. The height of the peak is therefore reduced, often considerably, as discussed in Sect. 5.

10. Approximate Explicit Representation of $G_R(w)$ for a Gaussian $G(w)$

To facilitate the folding of cross sections according to eq. (10) without the necessity of computing the function $F(z,t)$ for a large array of z values, a relatively simple explicit approximation to $F(z,t)$ has been evolved. The functional form is

$$F_A(z,t) = \frac{\Gamma(1+t)}{\sqrt{2\pi}} e^{-z^2/2} \left[0.31 - \frac{0.73\, z}{\sqrt{1 + \left(\frac{z}{1+1.37t}\right)^2}} + z^2 \right]^{-t/2}$$

$$+ \theta(z) t\, z^t \left(\frac{z^{2.18}}{1 + z^{3.18}} \right) \left\{ 1 + \frac{(1-t)(2-t)/2}{\left[\left(z - \frac{46}{z^2+21} \right)^2 + 2.44 + 1.5t \right]} \right\} . \tag{40}$$

This is a somewhat ungainly expression, but it does incorporate the proper asymptotic behaviors (37) and (38) including $O(z^{-2})$, has approximately the correct values for $F(0,t)$ and $F'(0,t)$, and is found to differ from the true $F(z,t)$ by less than 0.5% in absolute value for an infinite strip in the z-t plane, $-\infty < z < \infty$, $0 \leq t \leq 0.10$. Over most of the important range of z it represents $F(z,t)$ to within one or two parts in 10^3.

If one percent accuracy is adequate, eq. (40), together with (12) and (10), can be employed to compute the energy resolution-folded, radiatively-corrected cross section in the same manner as without soft-photon processes. It is important to remember in such integrations that $F(z,t)$ has a radiative tail that falls off slowly with energy. Unless the cross section itself cuts off the integrand in (10), the

integral may appear to diverge. This is one reason for defining $\sigma_0(W)$ to be the (resonant) cross section above background.

11. A Few Examples of Cross Sections, With and Without Interference

Figures 6, 7, and 8 are examples of the energy dependence of cross sections for $e^+e^- \to$ hadrons and $e^+e^- \to \mu^+\mu^-$, with the effects of finite beam energy spread and radiative corrections included. It has been assumed that the resonant state has $J^P = 1^-$ and that the coupling to leptons is parity-conserving and the same for electrons and for muons. For $e^+e^- \to \mu^+\mu^-$, the ordinary one-photon annihilation amplitude interferes with the resonant amplitude. The cross section for $e^+e^- \to$ hadrons is actually the total cross section, minus the resonant contribution from the leptons (it involves a factor $\Gamma_h/\Gamma = 1 - 2\Gamma_e/\Gamma$), plus an energy independent background cross section σ_b. For the $e^+e^- \to \mu^+\mu^-$ channel, the background cross section is $(\sigma_{\mu\mu})_{QED} = 4\pi\alpha^2/3M^2$.

Figure 6 is the cross section for $e^+e^- \to$ hadrons with parameters appropriate for the $\psi(3100)$ seen with an energy resolution typical of the beams in SPEAR II. The dotted curve here, and in figs. 7 and 8, is the cross section $\sigma_0(W)$ without either radiative corrections or smearing with the energy resolution function. The solid curve is the cross section with both these effects included. The reduction in peak cross section and the radiative tail on the high side of the resonance are the significant features.

Figures 7 and 8 represent cross sections for the channel $e^+e^- \to \mu^+\mu^-$ for the $\psi(3100)$ and $\psi(3700)$ respectively, again with typical SPEAR II energy resolution. The one-photon and resonant

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amplitudes interfere destructively below the resonance, giving a dramatic dip in the "bare" cross section. The radiative corrections and, more importantly, the finite energy resolution reduce the effect to 10-30%, but at the same time move the minimum a few MeV away from the peak. The interference for the $\psi(3100)$ has been observed at the level of a three standard deviation effect,[10] leading to the not unexpected spin-parity assignment of $J^P = 1^-$. At present, the cross section for $e^+e^- \to \mu^+\mu^-$ near 3700 MeV is not accurately enough known to check the $J^P = 1^-$ assignment assumed here. It should be noted, too, that the total width $\Gamma = 300$ keV assumed for fig. 8 is merely exemplary. The total width of the $\psi(3700)$ is presently only known to be somewhere between 200 keV and 800 keV.[10]

12. Acknowledgments

We wish to acknowledge our debt to J. D. Bjorken, G. Feldman, H. Lynch, Y.-S. Tsai, and D. R. Yennie for introducing us to the world of explicit radiative corrections. R. Giles and R. Pearson of SLAC performed similar calculations to those of figs. 6, 7, and 8 during the SLAC Theory Workshop, November, 1974. In that same Workshop,[12] Y.-S. Tsai discussed in briefer form much of the material presented here.

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0 0 0 0 4 3 0 1 9 7 5

Table of the radiatively corrected Gaussian energy resolution function F(z,t), given by eq. (36), for four t values

z	t = 0.070	t = 0.075	t = 0.080	t = 0.085
-5.0	1.280 (-6)	1.267 (-6)	1.254 (-6)	1.241 (-6)
-4.5	1.386 (-5)	1.372 (-5)	1.358 (-5)	1.345 (-5)
-4.0	1.169 (-4)	1.158 (-4)	1.148 (-4)	1.137 (-4)
-3.5	7.691 (-4)	7.624 (-4)	7.557 (-4)	7.492 (-4)
-3.0	3.944 (-3)	3.913 (-3)	3.881 (-3)	3.850 (-3)
-2.5	1.578 (-2)	1.566 (-2)	1.555 (-2)	1.544 (-2)
-2.0	4.925 (-2)	4.894 (-2)	4.863 (-2)	4.833 (-2)
-1.8	7.245 (-2)	7.203 (-2)	7.161 (-2)	7.119 (-2)
-1.6	1.025 (-1)	1.019 (-1)	1.014 (-1)	1.008 (-1)
-1.4	1.393 (-1)	1.386 (-1)	1.379 (-1)	1.373 (-1)
-1.2	1.821 (-1)	1.813 (-1)	1.805 (-1)	1.797 (-1)
-1.0	2.288 (-1)	2.279 (-1)	2.271 (-1)	2.262 (-1)
-0.8	2.765 (-1)	2.757 (-1)	2.748 (-1)	2.740 (-1)
-0.6	3.215 (-1)	3.207 (-1)	3.199 (-1)	3.192 (-1)
-0.4	3.595 (-1)	3.590 (-1)	3.584 (-1)	3.579 (-1)
-0.2	3.870 (-1)	3.867 (-1)	3.865 (-1)	3.862 (-1)
0.0	4.010 (-1)	4.011 (-1)	4.013 (-1)	4.015 (-1)
0.2	4.002 (-1)	4.009 (-1)	4.015 (-1)	4.022 (-1)
0.4	3.850 (-1)	3.862 (-1)	3.874 (-1)	3.886 (-1)
0.6	3.573 (-1)	3.591 (-1)	3.608 (-1)	3.625 (-1)
0.8	3.204 (-1)	3.226 (-1)	3.247 (-1)	3.269 (-1)
1.0	2.780 (-1)	2.805 (-1)	2.831 (-1)	2.856 (-1)
1.2	2.340 (-1)	2.368 (-1)	2.396 (-1)	2.424 (-1)

Table continued next page

Table continued.

Table of the radiatively corrected Gaussian energy resolution function $F(z,t)$, given by eq.(36), for four t values.

z	t = 0.070	t = 0.075	t = 0.080	t = 0.085
1.4	1.916 (-1)	1.946 (-1)	1.976 (-1)	2.006 (-1)
1.6	1.535 (-1)	1.565 (-1)	1.596 (-1)	1.627 (-1)
1.8	1.209 (-1)	1.239 (-1)	1.269 (-1)	1.300 (-1)
2.0	9.436 (-2)	9.731 (-2)	1.003 (-1)	1.032 (-1)
2.5	5.205 (-2)	5.463 (-2)	5.721 (-2)	5.982 (-2)
3.0	3.304 (-2)	3.521 (-2)	3.741 (-2)	3.962 (-2)
3.5	2.487 (-2)	2.673 (-2)	2.860 (-2)	3.049 (-2)
4.0	2.079 (-2)	2.241 (-2)	2.404 (-2)	2.570 (-2)
4.5	1.822 (-2)	1.965 (-2)	2.111 (-2)	2.259 (-2)
5.0	1.632 (-2)	1.762 (-2)	1.894 (-2)	2.028 (-2)
5.5	1.482 (-2)	1.600 (-2)	1.721 (-2)	1.844 (-2)
6.0	1.359 (-2)	1.468 (-2)	1.580 (-2)	1.694 (-2)
6.5	1.256 (-2)	1.358 (-2)	1.462 (-2)	1.567 (-2)
7.0	1.168 (-2)	1.264 (-2)	1.361 (-2)	1.460 (-2)
7.5	1.093 (-2)	1.183 (-2)	1.274 (-2)	1.367 (-2)
8.0	1.027 (-2)	1.112 (-2)	1.198 (-2)	1.286 (-2)

FIGURE CAPTIONS

Fig. 1. e^+e^- annihilation via one-photon exchange. (a) Lowest order diagram; (b) Higher order diagrams, the top two involving real (soft) photon emission and the next four each involving one additional virtual photon.

Fig. 2. Resonant cross section, without radiative corrections (dashed line) and with (solid line).

Fig. 3. Area method of determining $\Gamma_a\Gamma_b/\Gamma$ for a resonant cross section. (a) Resonant line with and without smearing from finite energy resolution; (b) The integral $A(W,W_{min})$ as a function of W.

Fig. 4. Illustration of the use of the area method for determination of $\Gamma_e\Gamma_h/\Gamma$ for the $\psi(3100)$. The data of reference 8 for $e^+e^- \to$ hadrons are plotted with a background cross section of 25 nb subtracted. The dashed curve is one tenth of the area $A(W,W_{min})$, while the solid curve above it is one tenth of the radiatively corrected area (right-hand side of eq. (29)). Also shown is the radiative correction factor $\left[M/2(W - M)\right]^t$ (scale at the right).

Fig. 5. Gaussian energy resolution function $G(w)$, eq. (35) (dotted curve) and radiatively corrected Gaussian $F(z,t)$, eq. (36), for $t = 0.08$ as functions of $z = w/\sigma_m$.

Fig. 6. Cross section for $e^+e^- \to$ hadrons at the $\psi(3100)$ as a function of total cm energy W. Resonance parameters are $M = 3095$ MeV, $\Gamma_e = 5.2$ keV, $\Gamma = 77$ keV. The standard deviation of the Gaussian energy resolution function is $\sigma_m = 1.0$ MeV. Background cross section is $\sigma_b = 22$ nb.

Fig. 7. Cross section for $e^+e^- \to \mu^+\mu^-$ at the $\psi(3100)$ as a function of total cm energy W. The same resonance parameters as for fig. 6.

Fig. 8. Cross section for $e^+e^- \to \mu^+\mu^-$ at the $\psi(3700)$ as a function of total cm energy W. The resonance parameters are chosen to be $M = 3684$ MeV, $\Gamma_e = 2.2$ keV, $\Gamma = 300$ keV. The energy resolution is $\sigma = 1.5$ MeV.

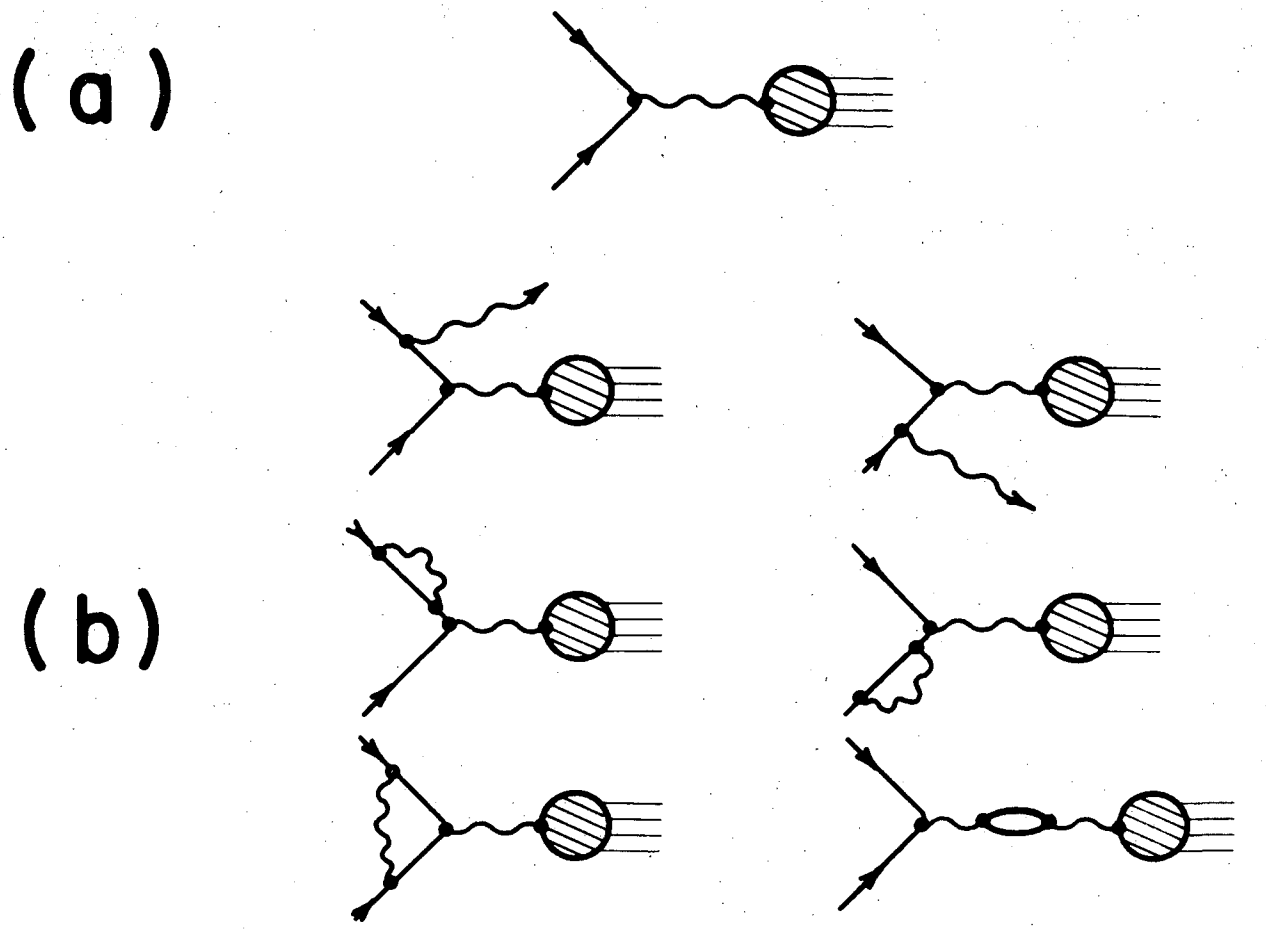

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Fig. 1

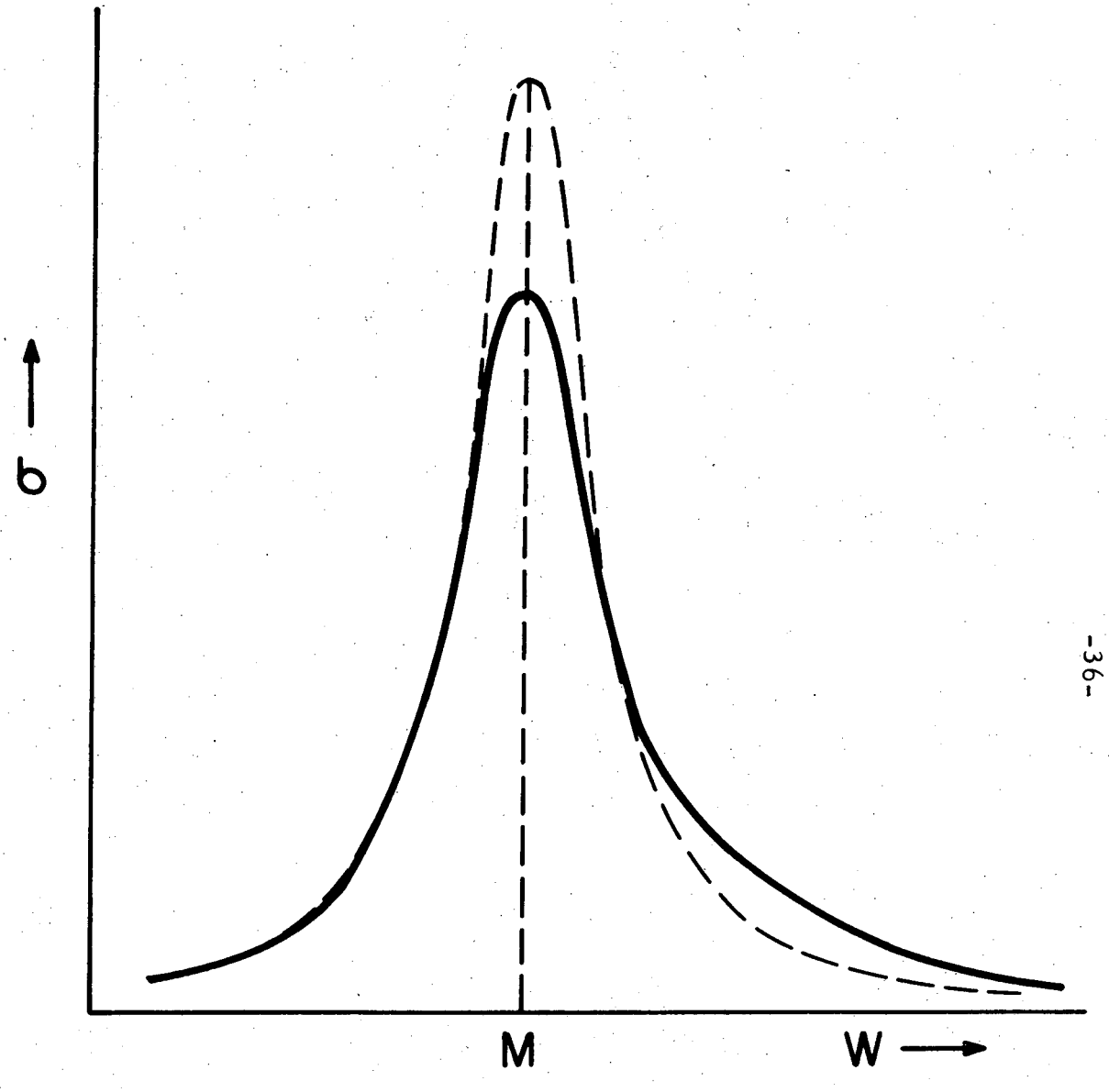

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Fig. 2

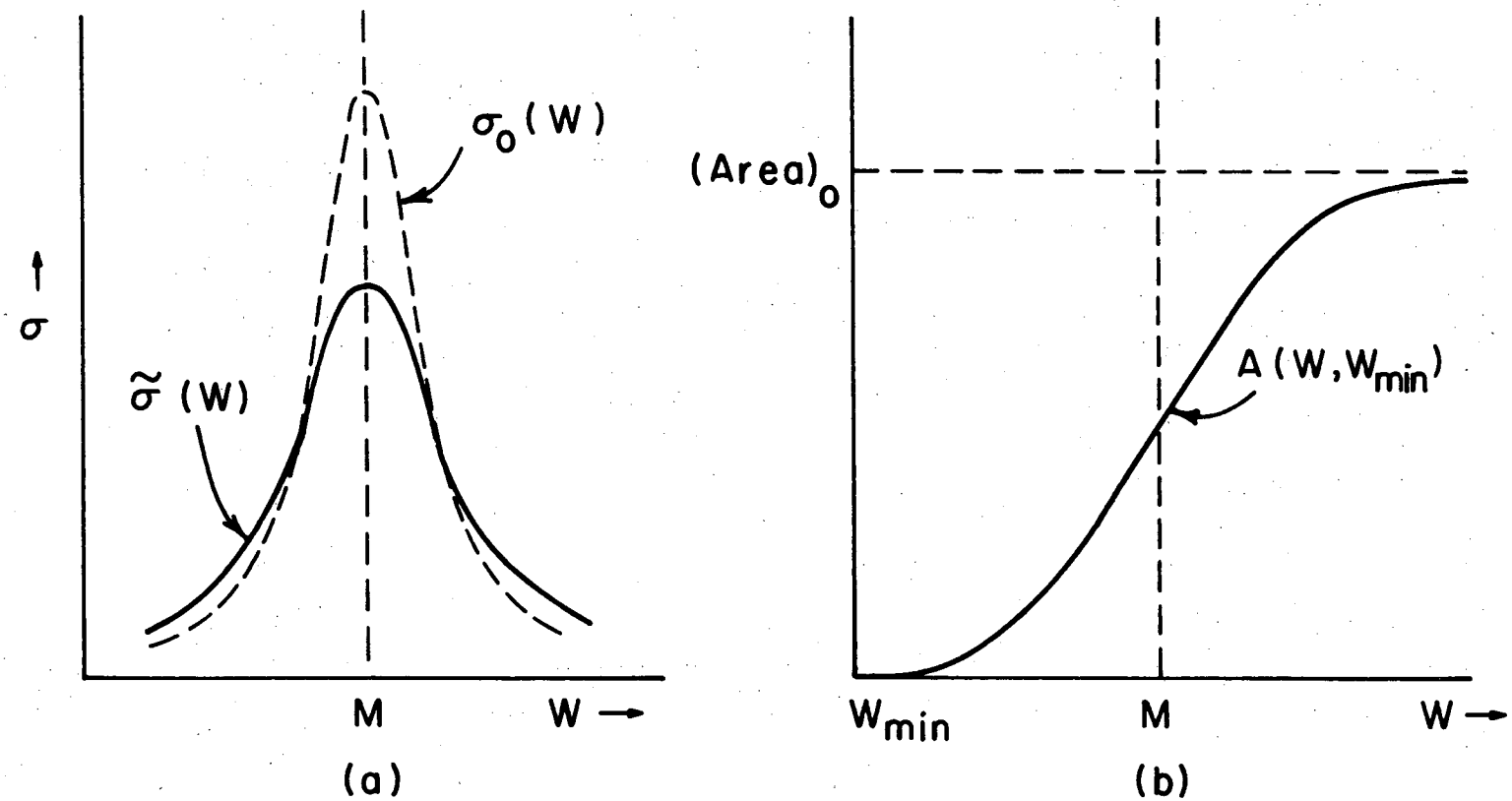

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Fig. 3

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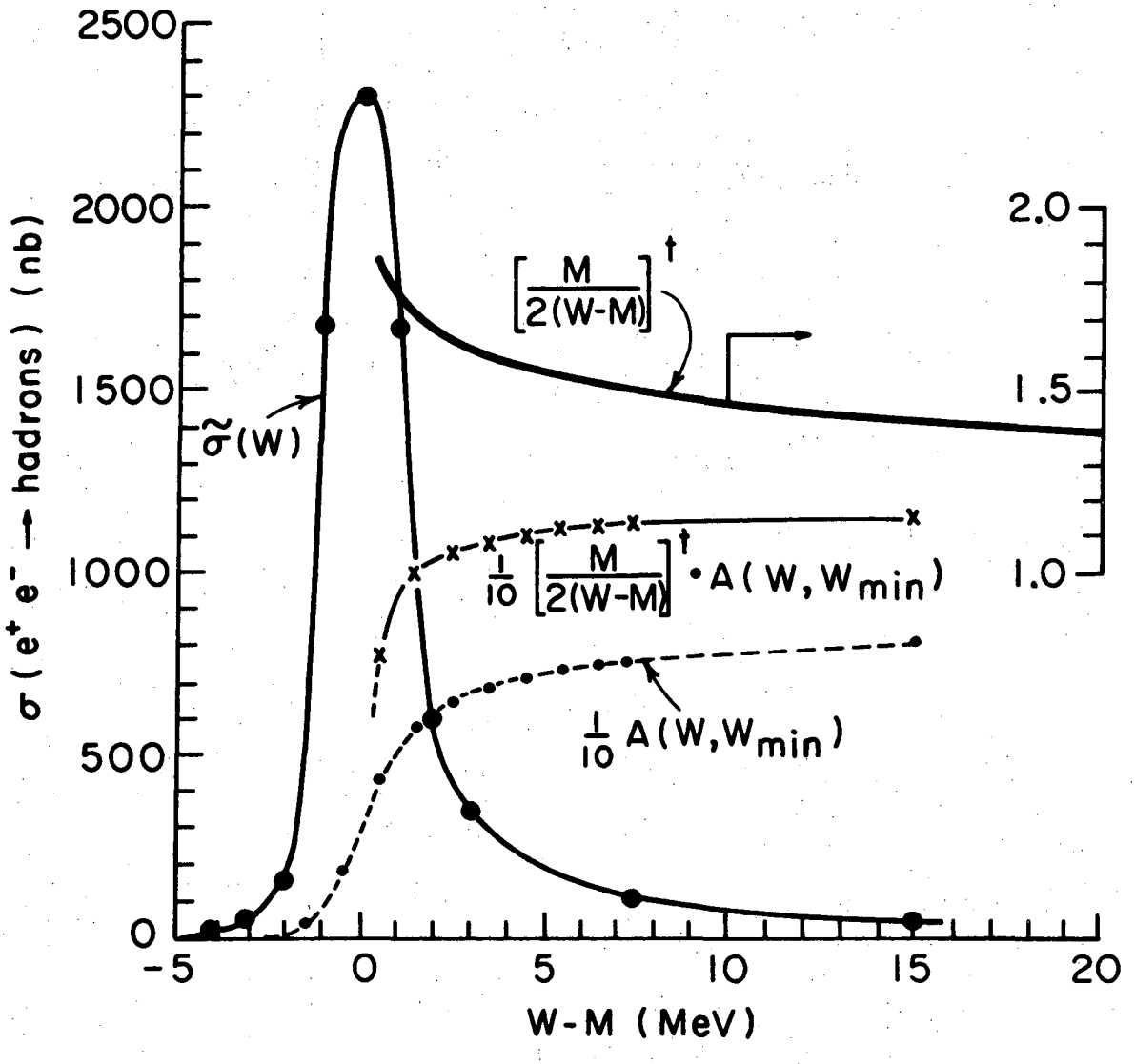

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Fig. 4

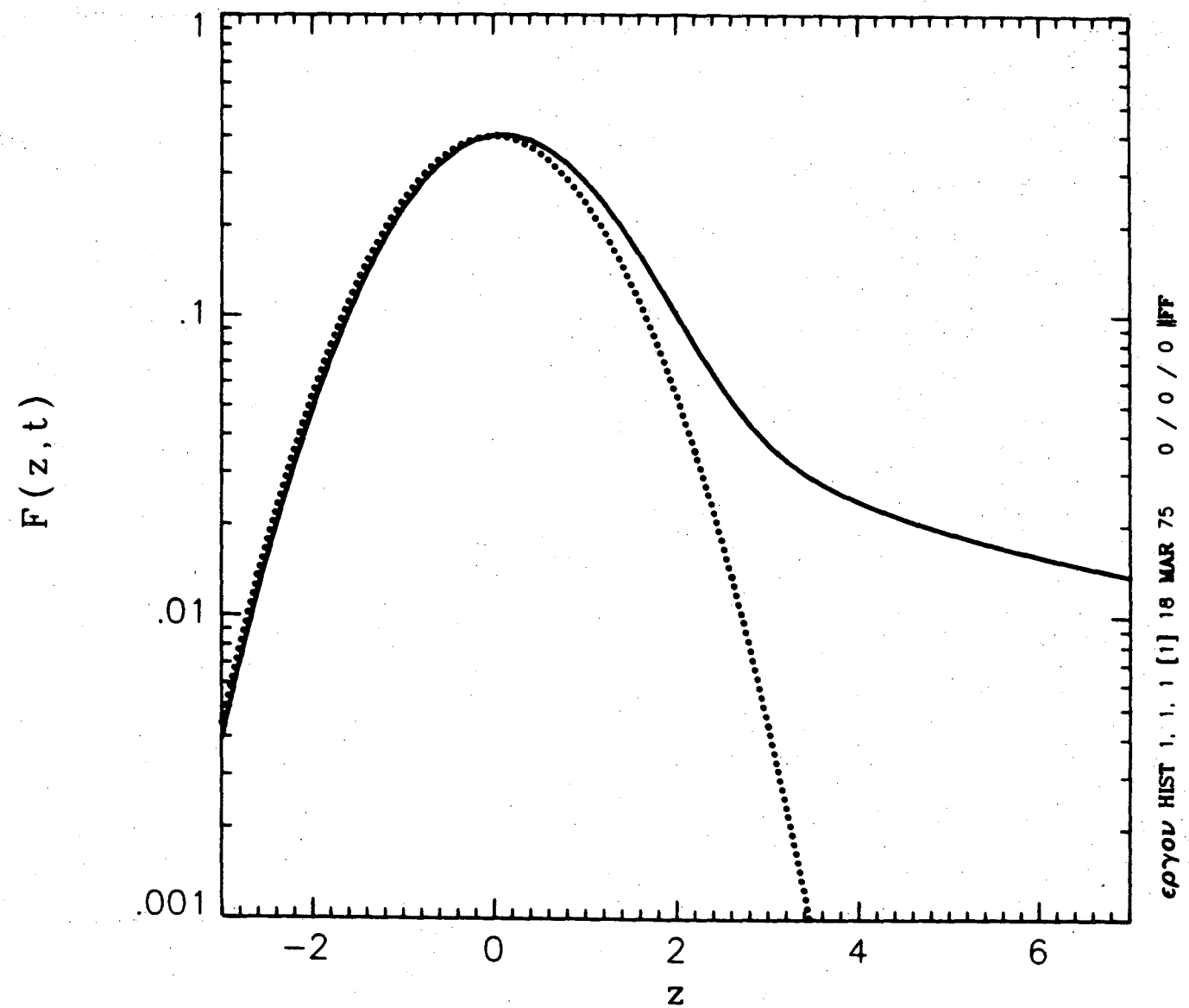

Fig. 5

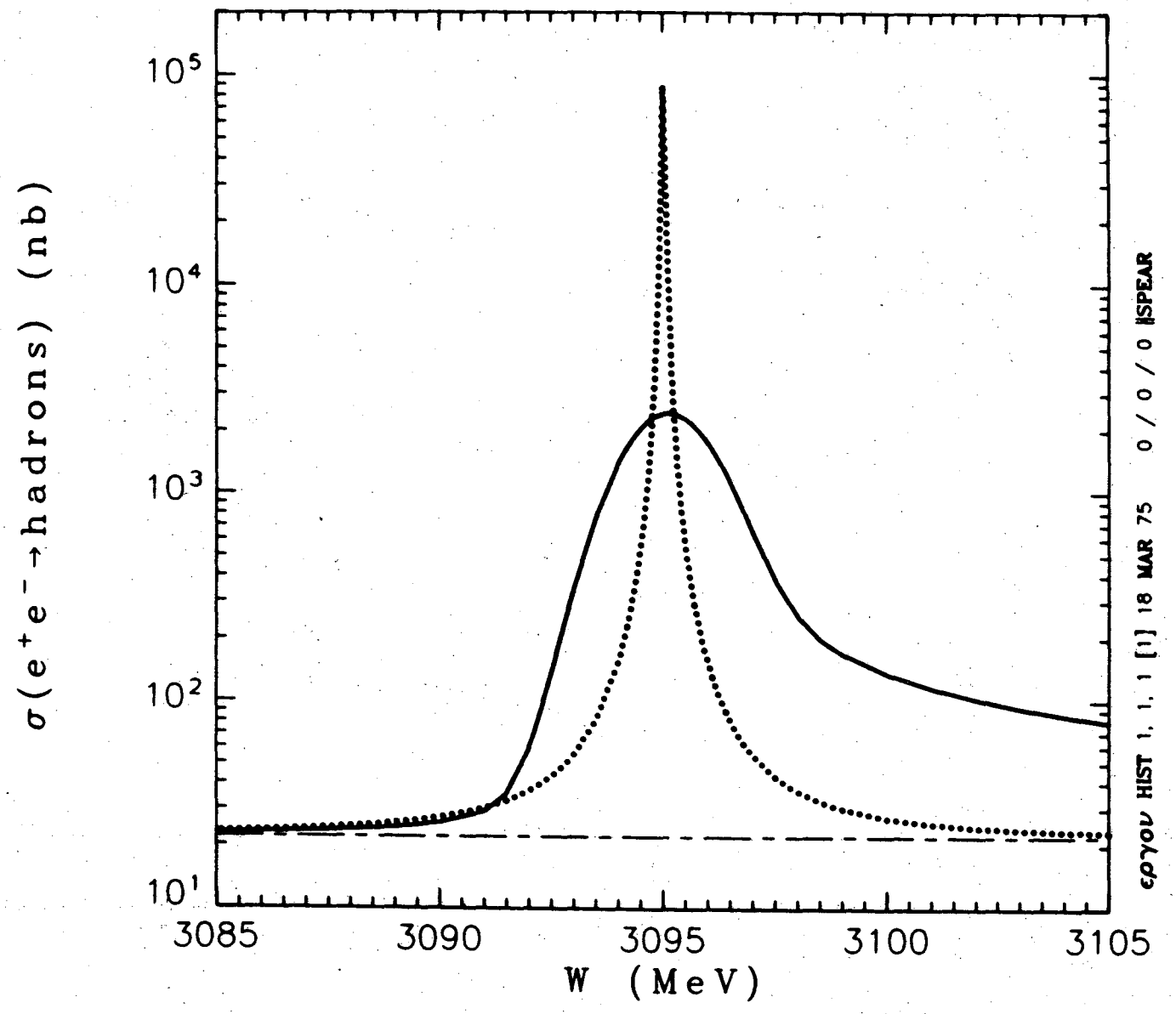
10^5
10^4
10^3
10^2
10^1
$\sigma(e^+e^- \to hadrons)$ (nb)
3085
3090
3095
3100
3105
W (MeV)
εργον HIST 1, 1, 1 [1] 18 MAR 75 0 / 0 / 0 ISPEAR

Fig. 6

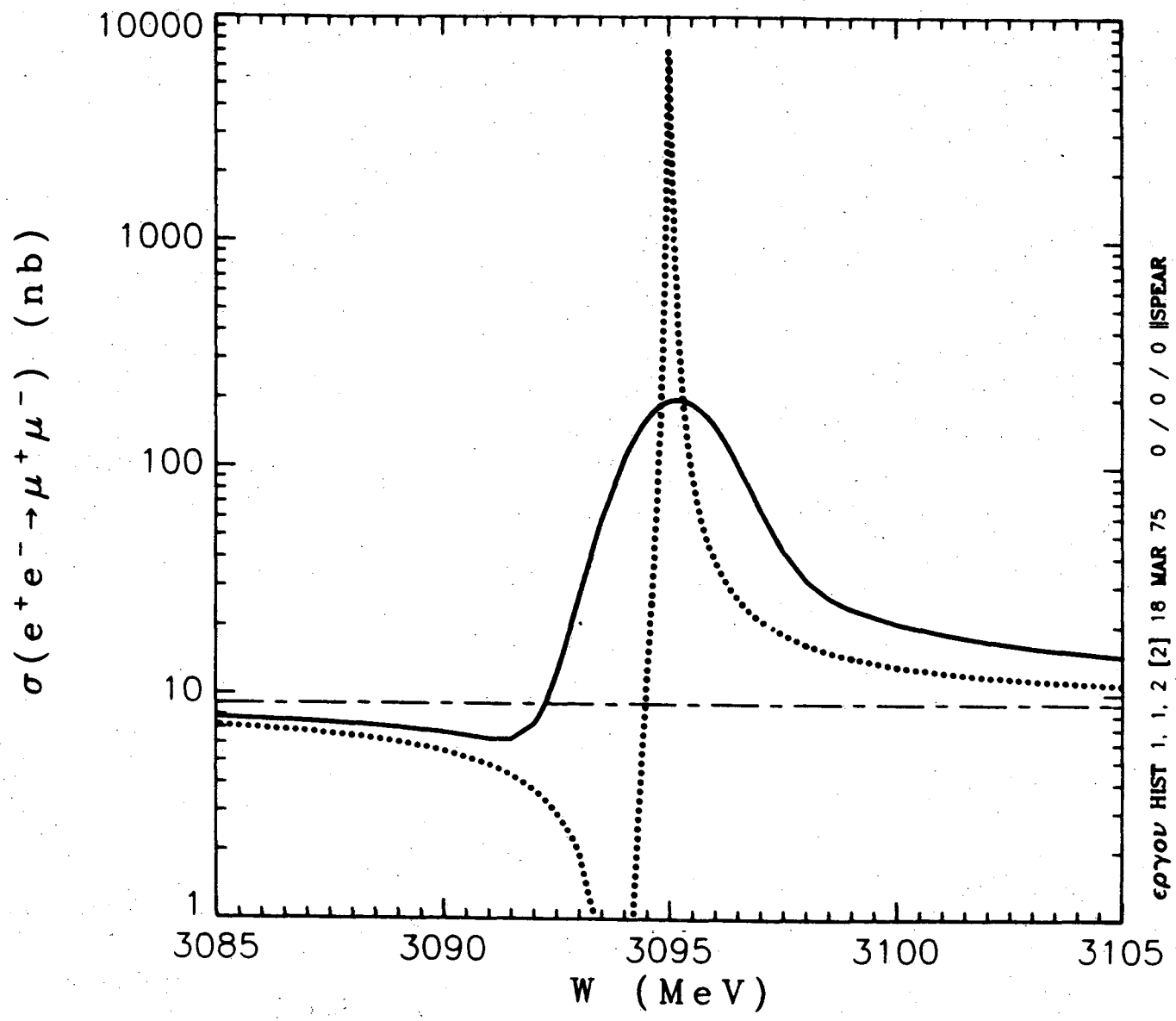

Fig. 7

0 0 0 0 4 3 0 1 9 7 8

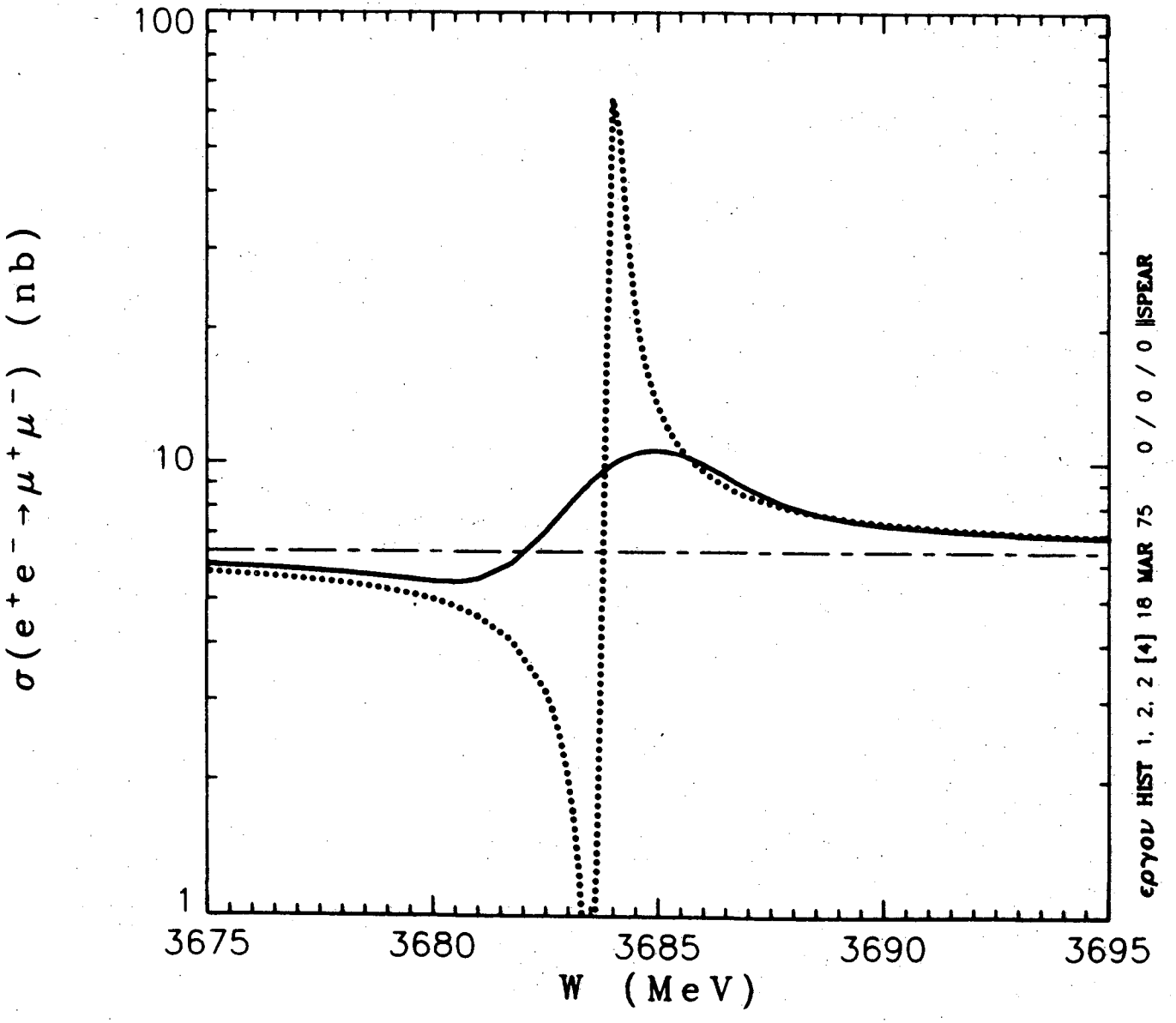
100
10
1
3675
3680
3685
3690
3695
W (MeV)
σ(e⁺e⁻→μ⁺μ⁻) (nb)
εργον HIST 1, 2, 2 [4] 18 MAR 75 0 / 0 / 0 ||SPEAR

Fig. 8

0 0 0 0 4 3 0 1 9 7 9

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