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Effective Transmissivity of Two-Dimensional Fracture Networks

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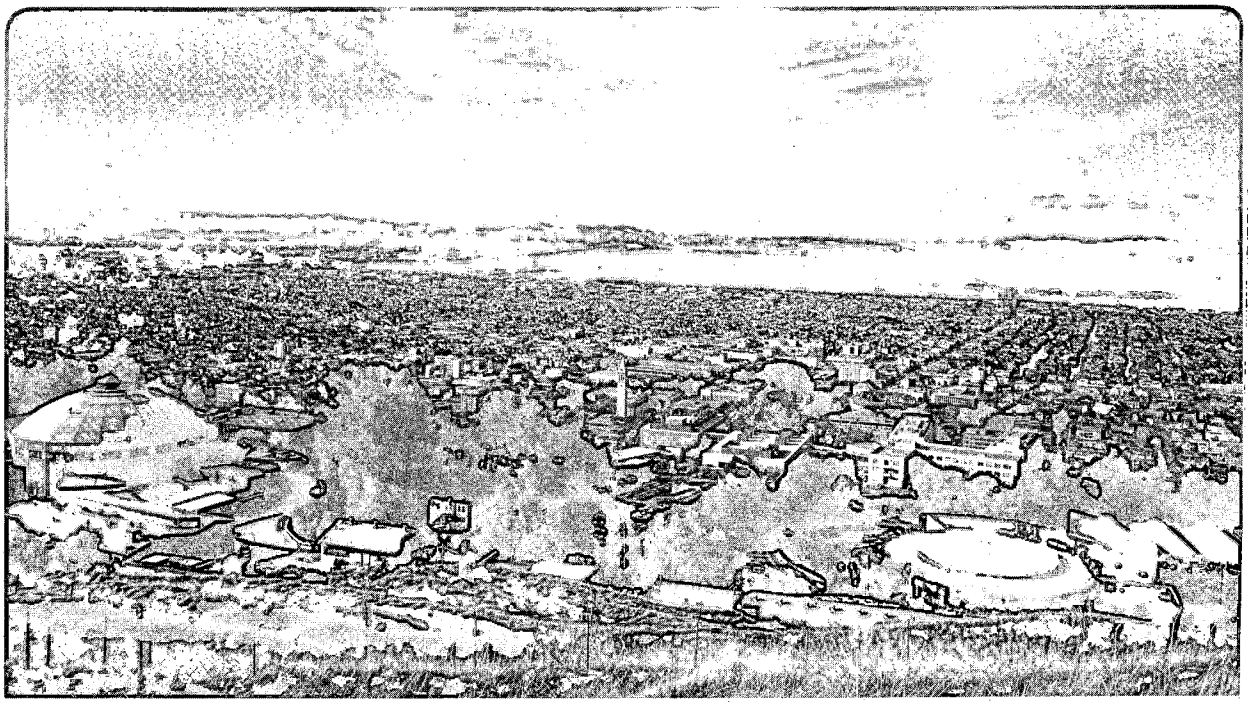
April 1995



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Effective Transmissivity of Two-Dimensional Fracture Networks

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April 1995

This work was carried out under Department of Energy Contract No. DE-AC03-76SF00098 for the Director, Office of Civilian Radioactive Waste Management, Office of Geologic Disposal, and was administered by the Nevada Operations Office, U. S. Department of Energy, in cooperation with the U. S. Geological Survey, Denver.

INTRODUCTION

Many of the sites that have been proposed as potential locations of underground radioactive waste repositories contain fractured rocks. For example, both the saturated zone and the unsaturated zone at Yucca Mountain, Nevada, contain many hydrogeologic units that are extensively fractured [1,2]. When modeling the hydrological behavior of these sites, for either the purpose of site characterization of performance assessment, computational gridblocks are often used that contain large numbers of fractures [3]. In order to treat these gridblocks as equivalent continua, it is necessary to develop a procedure for relating the hydraulic properties of the individual fractures, and the topology of the fracture network, to the overall gridblock-scale permeability [4,5]. One aspect of this problem is that of determining the *in situ* hydraulic properties of the individual fractures. Another aspect is to reconstruct the three-dimensional geometry of the fracture network based on borehole or outcrop measurements. The final stage in the problem is that of taking a network of known geometry (which may be stochastically generated), and determining its effective gridblock-scale conductivity. The purpose of this paper is to describe a simple procedure for solving this latter problem, and to demonstrate its use in cases of both saturated and unsaturated flow.

NETWORK MODEL

We will assume that a two-dimensional fracture network can be represented by a network of ''conductive elements'' that are connected to each other at nodes. The nodes are assumed to have zero volume, and to offer no additional hydraulic resistance. A length of fracture connecting two nodes will be referred to as a fracture segment. The conductance C_i of the ith individual fracture segment depends on geometrical features such as the mean aperture, the fracture wall roughness, the amount of contact area of the two rock faces, and the length of the segment [6-9]. If the rock is only partially saturated, the conductance will also depend on the degree of liquid saturation [10,11]. In the present work, we consider only the problem of estimating

the macroscopic conductivity of a network of fractures, assuming that the individual fracture segment conductances are known.

If the values of all of the conductances were known exactly, and the topology of the network were also known (*i.e.,* which conductors were connected to which nodes), evaluation of the overall macroscopic conductance of the network would require the solution of a system of algebraic equations. These equations arise by applying the equation $Q = C\Delta H$ to each conductor, where Q is the volumetric flowrate, C is the hydraulic conductance, and H is the hydraulic potential (defined here as $H = P + \rho g z$), and then invoking the fact the the sum of the fluxes into each node must be zero in order for mass to be conserved. For a network of saturated fractures, this procedure is analogous to finding the effective conductivity of a network of electrical resistors, and leads to a system of linear algebraic equations [12]. For unsaturated flow, however, the analogy between fluid flow and electrical flow through a linear resistor network breaks down, and the governing equations become nonlinear. In these cases the conductivity depends on the potential, through the hydraulic conductivity (i.e., relative permeability) function, and the conductivities of the individual fracture segments must be found as part of the solution procedure. This can be accomplished with a numerical simulator such as the TOUGH code [13], as was done by Kwicklis and Healy [14]. This procedure is tedious to implement, however, since it requires the construction of a computational grid for the entire fracture network.

In order to reduce the problem of solving the full set of network equations to a problem that is much easier to solve, several steps will be taken. There are two aspects of the network-flow problem that render it awkward to solve: the broad distribution of the values of the individual fracture segment conductances, and the irregular topology of the fracture network. The first step is therefore to replace the conductance of each fracture segment by some suitable effective conductance, $\overline{C}$. An approximate expression for $\overline{C}$, in terms of the individual conductances C_i of the fracture segments

and the mean coordination number of the network, can be found using the effective medium approximation of Kirkpatrick [15]. The next step in our procedure is to replace the actual irregular network by a square lattice of conductors (Fig. 1). The accuracy of this type of replacement procedure has been studied in detail by, among others, Hestir and Long [16]. We will merely assume that this replacement can be made, and judge its accuracy by the extent to which we can predict the actual overall conductivities. The overall conductivity of this square lattice of equal conductors, in either of the two orthogonal lattice directions, say x_1, is then trivially found to be equal to $N\bar{C}$, where N is the number of conductors oriented in the x_1 direction, per unit length in the x_2 direction. The fact that the actual coordination number may be less than four, in which case all of the conductive elements of the square lattice are not actually present, can also be accounted for using the effective medium approximation (see below).

EFFECTIVE MEDIUM APPROXIMATION

The purpose of the so-called effective medium approximation is to allow a network having a distribution of individual conductances C_i to be approximated by a geometrically identical network in which every bond has the same conductance $\bar{C}$ (Fig. 1). In general, the behavior of the original network and the effective network will differ locally, in the sense that the potential at a given node will not usually be the same in the two networks. Likewise, the flux through a given conductor will, in general, be changed if all the conductances are replaced by some value $\bar{C}$. This replacement process is intended to leave the macroscopic behavior of the original network unchanged, in some sense. One method of finding a suitable $\bar{C}$ is that proposed by Kirkpatrick [15], who used the criterion that if a single conductor C_i is replaced by a conductor with conductance $\bar{C}$, the resulting perturbations in the potentials at the nearby nodes should average out to zero. Application of various theorems of network

analysis led Kirkpatrick to the following equation that implicitly defines $\bar{C}$:

$$\sum_{i=1}^{N} \frac{\bar{C} - C_i}{(z/2-1)\bar{C} + C_i} = 0 \,, \tag{1}$$

where the coordination number z is the number of conductors that meet at each node, and the sum is taken over each individual conductor in the network. Eq. (1) was originally derived under the assumption that the coordination number of each node was the same. It was later shown [16], however, that eq. (1) can be used for topologically irregular networks if z is defined to be the average coordination number of all the nodes in the network.

In the trivial case when each conductance has an identical value $C_i = C$, eq. (1) correctly predicts that $\bar{C} = C$, regardless of the coordination number. The effective medium approximation is also exact in the two limiting cases of $z \rightarrow 2$ and $z \rightarrow \infty$, regardless of the type of distribution of the individual conductances. These two values of z correspond to series and parallel networks, respectively. For $z = 2$, eq. (1) reduces to $\bar{C}(2) = N/\sum(1/C_i)$, while for $z = \infty$, eq. (1) yields $\bar{C}(\infty) = (1/N)\sum C_i$. $\bar{C}(z)$ is generally an increasing function of z, so that it is always the case that $\bar{C}(2) < \bar{C}(z) < \bar{C}(\infty)$ for an arbitrary value $2 < z < \infty$ [16]. This fact is useful when attempting to solve eq. (1) numerically, since it allows $\bar{C}(z)$ to be bounded *a priori*, and then solved for using the bisection method.

As might be expected, the accuracy of the effective medium approximation generally decreases as the distribution of the values of the individual conductances becomes broader. Koplik [17] and David et al. [18] studied the accuracy of the effective medium approximation for two-dimensional square, triangular, and hexagonal networks. They found eq. (1) to have errors of less than 10% for narrow ''peak-like'' distributions of conductances, as well as for conductances that are uniformly distributed between some minimum and maximum values. However, eq. (1) underpredicted

$\bar{C}$ by about 50% for exponentially-decreasing distributions. Koplik [17] also found that fairly large errors would occur for log-uniform distributions of the form $p(C) = constant/C$ for $C_o < C < 1/C_o$, if C_o is close to zero. The common factor in all cases where eq. (1) breaks down is not broadness of the conductance distribution *per se*, but the existence of a relatively large number of conductors with conductances that are very close to zero (in the sense of being much smaller than the mean). The conductances of rock fractures are often found to follow lognormal distributions [19], which go to zero rapidly as the conductance goes to zero. Hence, it seems that actual conductivity distributions will likely be of the form for which eq. (1) is fairly accurate.

CONDUCTANCE OF A SQUARE LATTICE

Once the effective medium approximation has been used, we then assume that the conductors are arranged on a square lattice (see Fig. 1). A detailed examination of various methods that have been proposed to replace an irregular network with a square network has been made by Hestir and Long [16], under the assumption that all the fractures have the same conductivity per unit length. We expect, however, that the uncertainty in the gridblock-scale conductivity due to the variation in conductivities among the individual fractures will be much larger than that which arises due to the difference between a square and irregular network. In this regard we note that David et al. [18] presented exact calculations of the effective conductances of square, hexagonal, and other regular lattices, and found that topology was of secondary importance, as compared to the distribution of individual conductivities. Now imagine a rectangular gridblock, such as would be used in a numerical finite-difference simulation, that contains this square lattice of equal-strength conductors. If a potential difference ΔH were imposed from top to bottom across this block, it follows from symmetry considerations that each of the N_x vertical connections would carry the same amount of flow, and that there would be no lateral flow through any of the horizontal conductors.

If the conductance of each element is $\bar{C}$, then the total flow through the block would be $Q = N_x \bar{C} \Delta H$. According to Darcy's law, the flux would be $Q = K L_x L_y \Delta H / L_z$, where L_x is width of the block normal to the flow direction, L_y is the thickness in the third direction ("into the page"), and L_z is the length of the block in the flow direction. Hence, the continuum conductivity is given by $K = \bar{C} N_x L_z / L_x L_y$.

Finally, however, we must account for the fact that the actual mean coordination number z of the network will usually be less than four, in which case the "equivalent" square lattice should have only $z/4$ of its bonds intact. This can be done by again invoking the effective medium approximation, eq. (1). We first take the lattice shown in Fig. 1c, and "fill in" the missing bonds with bonds that have zero conductance; this procedure clearly will not effect the overall conductance. The new lattice, shown in Fig. 1d, now has a coordination number of 4. A fraction $f = 1 - (z/4)$ of its conductors have conductance $C = 0$, whereas a fraction $1 - f = z/4$ of its conductors have $C = \bar{C}$. Application of eq. (1) to this lattice yields, after some algebraic manipulation, the result $\bar{\bar{C}} = (z/2 - 1)\bar{C} = (1 - 2f)\bar{C}$. According to this result, if 10% of the bonds are "missing", the effective conductance will be decreased by 20%; when 50% of the bonds are removed, the effective conductance drops to zero. This critical fraction $f^* = 1/2$ (i.e., $z^* = 2$) can be explained heuristically by noting that if fluid enters a node from one bond, there must be at least one other bond connected to that node in order for the fluid to continue through the network. Hence, the critical coordination number needed for the network to be fully conducting should be $z^* = 2$; this is in fact rigorously true for a square lattice [20]. However, more precise analysis using percolation theory shows that the relationship between $\bar{\bar{C}}$ and $\bar{C}$ is not a linear function of z near the critical value $z = 2$, but varies as $\bar{\bar{C}}/\bar{C} = const \times (z - 2)^t$, where the percolation exponent t is equal to about 1.3 [21,22]. However, we will use the simpler expression $\bar{\bar{C}} = \bar{C}(z/2 - 1)$, which should be sufficiently accurate as long as the fracture network is not near the percolation limit $z = 2$.

APPLICATION TO A SATURATED FRACTURE NETWORK

We have used the procedure described above to predict the overall gridblock conductivity of the fracture network that was analyzed by Priest [12] under conditions of saturated flow (Fig. 2). A square region of size $10\,\mathrm{m} \times 10\,\mathrm{m}$ was covered with twelve "randomly" oriented and located fractures of varying lengths and apertures. Five of these fractures, along with parts of the seven others (shown as dotted lines in Fig. 2) can be removed because they are clearly not part of the network that will actually conduct fluid. As there are four nodes at which four segments meet (nodes 4,6,10,13), and six nodes at which three segments meet (nodes 5,7,9,12,14), the mean coordination number of the conducting network is $z = (4 \cdot 4 + 6 \cdot 3)/(4+6) = 3.40$.

For saturated flow, the conductance of a fracture of length L in the direction of flow, aperture b, and length w in the transverse direction, is given by the so-called "cubic law" [12]:

$$C = \frac{\rho g b^3 w}{12 \mu L}, \tag{2}$$

where μ is the viscosity of the fluid and ρ is the density. The fractures shown in Fig. 2 are assumed to have unit depth in the third direction, *i.e.*, $w = 1\,\mathrm{m}$. The apertures and lengths of the fifteen fracture segments contained in the conducting network are shown in Table 1, along with the computed conductances, which varied by a factor of about 290:1 from largest to smallest. Note that as the only parameter that enters into the flow calculations for a given fracture is the conductance, C, the present example does not actually depend in any fundamental way on our use of the cubic law to relate conductance to aperture. The conductances in Table 1 are then used in eq. (1) to find $\bar{C} = 0.3316 \times 10^{-6}\,\mathrm{m^2/s}$. This value must then be corrected by the multiplicative factor $(z/2) - 1 = 0.70$, to account for the "missing" bonds in the square lattice, yielding $\bar{\bar{C}} = 0.2321 \times 10^{-6}\,\mathrm{m^2/s}$. Next we must decide upon the number of rows and columns

of fractures that should be used in the equivalent square lattice. Again, a detailed discussion of various approaches to this question is given by Hestir and Long [16]; here we take the following simple approach. As there are eight intersections of the fracture network with the four sides of the square outer boundary of the gridblock, there are an average of two intersections per side, which corresponds to a square lattice that contains two horizontal and two vertical fractures. The overall conductivity in the x direction is then found to be

$$K_x = \frac{\bar{\bar{C}} N_x L_z}{L_x L_y} = \frac{(0.2321 \times 10^{-6}\,\mathrm{m^2/s})(2)(10\,\mathrm{m})}{(10\,\mathrm{m})(1\,\mathrm{m})} = 0.4642 \times 10^{-6}\,\mathrm{m/s}. \qquad (3)$$

The volumetric flowrate through the block is found by substituting this conductivity, along with the head drop of 2 m, into Darcy's law:

$$Q = \frac{K L_x L_y \Delta H}{L_z} = \frac{(0.4642 \times 10^{-6}\,\mathrm{m/s})(10\,\mathrm{m})(1\,\mathrm{m})(2\,\mathrm{m})}{(10\,\mathrm{m})} = 0.9284 \times 10^{-6}\,\mathrm{m^3/s} \qquad (4)$$

per unit thickness in the third direction. This value is within 2% of the exact value $0.947 \times 10^{-6}\,\mathrm{m^3/s}$ that was found by Priest [12] by solving the full set of twenty simultaneous algebraic equations.

APPLICATION TO AN UNSATURATED FRACTURE NETWORK

We have also applied our procedure to unsaturated flow in the 2-D fracture network (Fig. 3) that was studied by Kwicklis and Healy [14]. They constructed a fracture network consisting of 9 fractures transecting a 5 m×5 m square region, which resulted in 51 fracture segments. The fractures consisted of two sets, a subvertical set with a mean dip angle of 74 degrees, and a subhorizontal set with a mean dip angle of 23 degrees; these values were chosen to be consistent with the fracture data from

Yucca Mountain, Nevada, described by Wang and Narasimhan [1]. In the simulations that we will consider, the subvertical fractures had a mean aperture of 125 μm, and the subhorizontal fractures had a mean aperture of 25 μm. This would seem to imply a certain degree of anisotropy; however, the network was constructed such that no constant-aperture continuous path exists between the inflow (top) and outflow (bottom) boundaries. We interpret this fact, coupled with the fact that the fracture orientations of the two sets exhibited large scatter about the mean dip angle, as providing some justification our use of an essentially isotropic model to analyze flow in the network.

After dead-end segments were removed, this network had a mean coordination number of 3.43. For the ''mixed aperture'' simulation, 56% of the fracture segments had a mean aperture of 125 μm, and 44% of the fracture segments had a mean aperture of 25 μm. The hydraulic conductivity functions (Fig. 4) of the two different fracture types were computed using a variable-aperture flow model [10], in conjunction with some assumptions concerning the distribution of apertures within each fracture plane. For cases such as this in which the conductivities follow a pure bi-modal distribution, eq. (1) reduces to a quadratic equation, which is easily solved. As the fractures intersect the four gridblock faces in ten locations (see Fig. 3), we took our equivalent square lattice as having $10/4 = 2.5$ vertical and horizontal fractures per gridblock. The gridblock-scale conductivities were computed using the procedure described above, over a range of hydraulic heads from -0.01 to -0.3 m. The results, shown in Fig. 5, agree fairly well with the conductivities that were computed by Kwicklis and Healy [14]. They used the numerical simulator TOUGH [13], and associated each computed permeability with the mean value of the two potentials at the top and bottom edges of the block. Our method has the advantage, however, of requiring the solution of only a single equation for each value of the pressure head, as opposed to the 51 simultaneous equations that were solved [14] when computing the actual flow field (the nine fracture segments were each discretized into more than one computational elements; see Fig.

3). Moreover, our proposed upscaling method does not require the construction of a numerical grid.

CONCLUSIONS

A major issue in the modeling of fluid flow in fractured rock masses is the determination of appropriate gridblock-scale conductivities. We have used Kirkpatrick's effective medium approximation [15] to develop a method for predicting the gridblock-scale hydraulic conductivity of a two-dimensional fracture network. The method requires knowledge of the mean coordination number of the fracture network, the mean fracture spacing, and the hydraulic conductivity functions of the individual fractures. However, in contrast to methods that solve for the detailed flow field through the entire network, the proposed upscaling procedure requires solution of only a single nonlinear equation. The method has been tested against numerical simulations conducted by Priest [12] on saturated flow through a fracture network, and also against the unsaturated flow simulations of Kwicklis and Healy [14]. In both cases the proposed upscaling method gave accurate predictions of the macroscopic hydraulic conductivity. Furthermore, the method is expected to be more accurate for gridblocks that contain larger numbers of conductive segments [16,17], such as would typically be encountered in modeling the performance of an underground radioactive waste repository located in fractured rock.

ACKNOWLEDGEMENTS

This work was carried out under Department of Energy Contract No. DE-AC03-76SF00098 for the Director, Office of Civilian Radioactive Waste Management, Office of Geologic Disposal, administered by the Nevada Operations Office, U. S. Department of Energy, in cooperation with the U. S. Geological Survey, Denver. The authors thank Christine Doughty and Kenzi Karasaki of LBL for reviewing this paper.

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Table 1. Hydraulic properties of the fracture segments from the network shown in Fig. 2 (after Priest [12]).

Start (Node #)	End (Node #)	Aperture (μm)	Length (m)	Conductance (10^{-6} m^2/s)
1	4	70	2.85	0.098
2	4	160	2.30	1.455
3	6	240	5.30	2.132
4	5	70	0.85	0.330
4	12	160	3.95	0.847
5	6	70	3.25	0.086
5	9	180	2.50	1.906
6	7	70	1.95	0.144
6	10	240	0.95	11.892
7	8	70	1.80	0.156
9	10	90	2.85	0.209
9	13	180	1.45	3.287
10	7	90	1.60	0.372
10	15	240	2.55	4.430
11	12	60	4.35	0.041
12	13	60	1.25	0.141
13	14	180	3.65	1.306
14	15	60	2.00	0.088
14	18	130	2.70	0.665
15	16	60	2.20	0.080

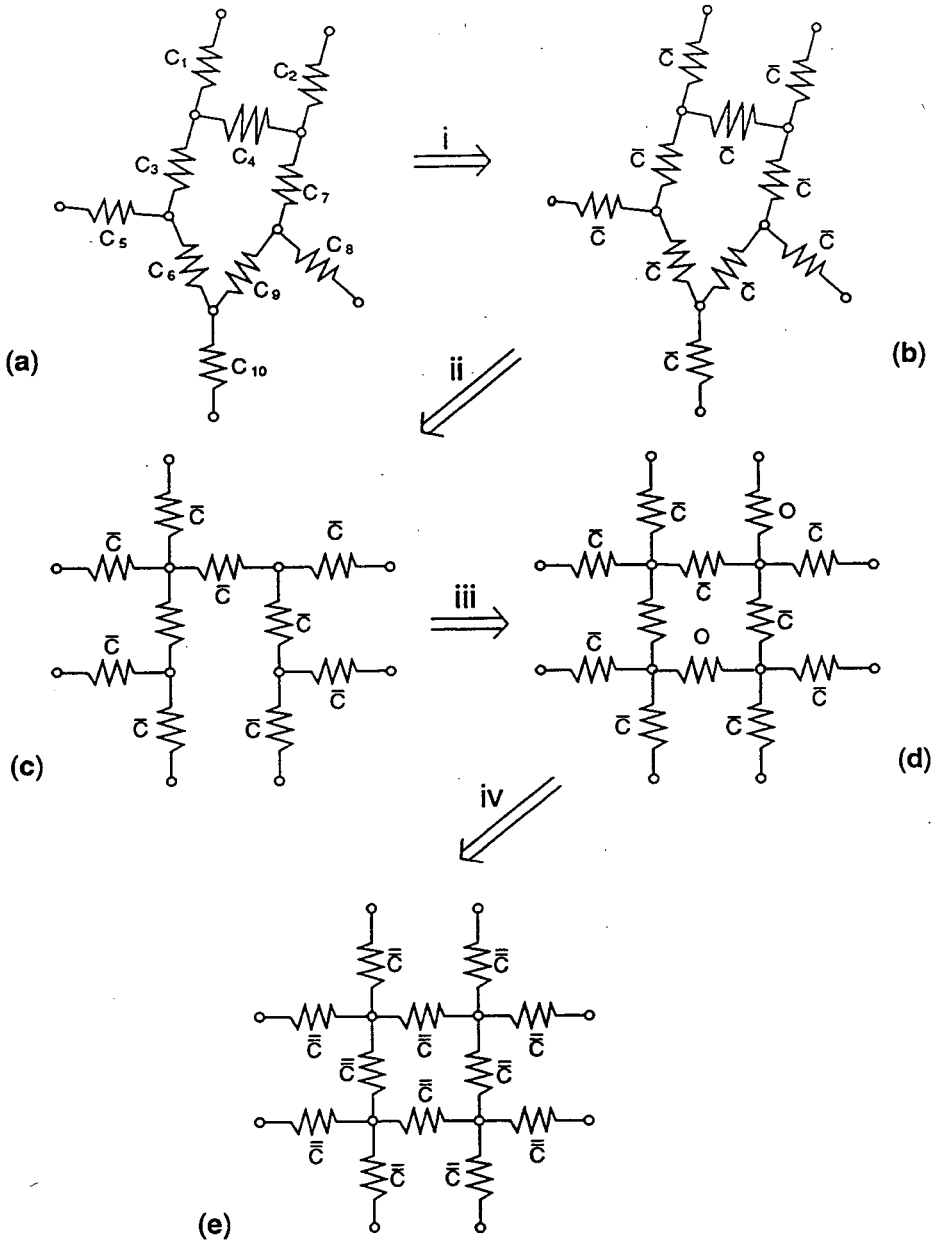

Fig. 1. Schematic diagram illustrating the homogenization process. First (i) we use eq. (1) to replace each conductance C_i with $\bar{C}$. We then (ii) replace the irregular lattice with a quasi-square lattice having the same mean coordination number and mean fracture spacing. Next (iii) we fill in the "missing" bonds with bonds that have zero conductance. Finally (iv) we use eq. (1) again to replace each of the bonds of this heterogeneous square lattice with a bond having conductivity $\bar{\bar{C}}$. The overall conductance of this final network is then trivial to compute.

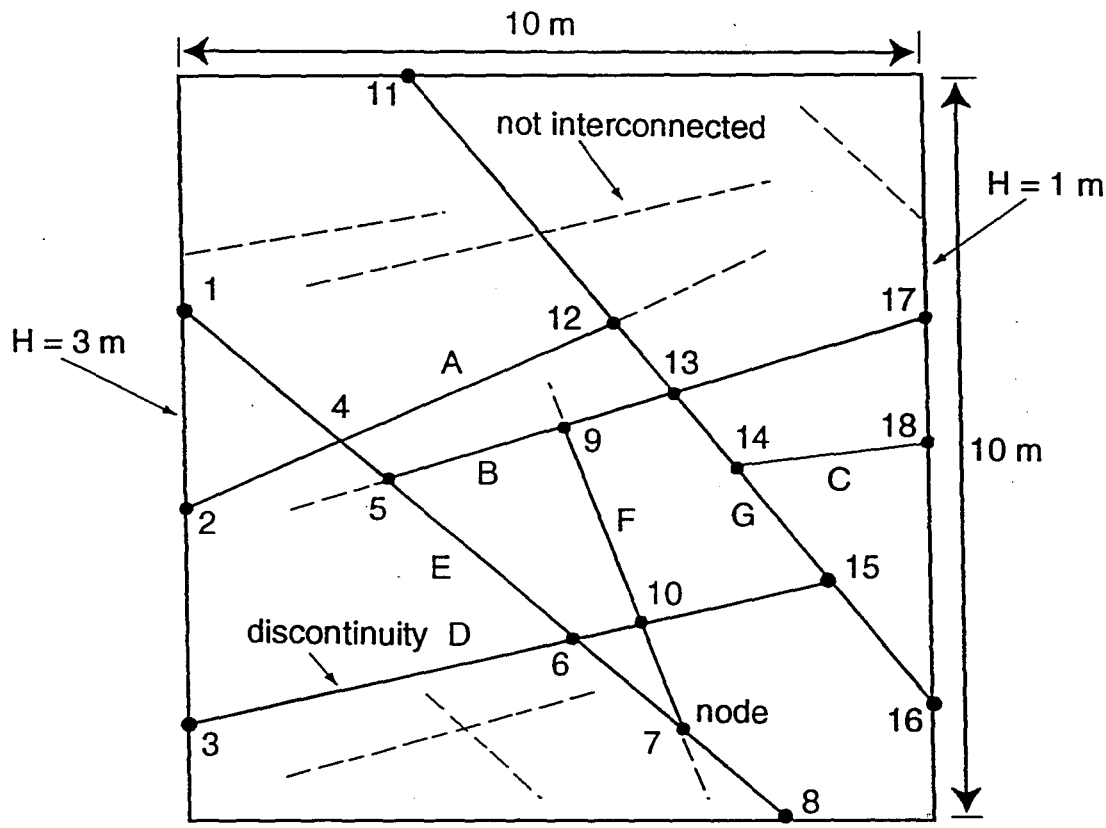

Fig. 2. Fracture network used in the saturated flow simulations conducted by Priest (1993). The left and right faces are held at uniform (but different) potentials, and the top to bottom faces have a linear potential gradient imposed along their lengths.

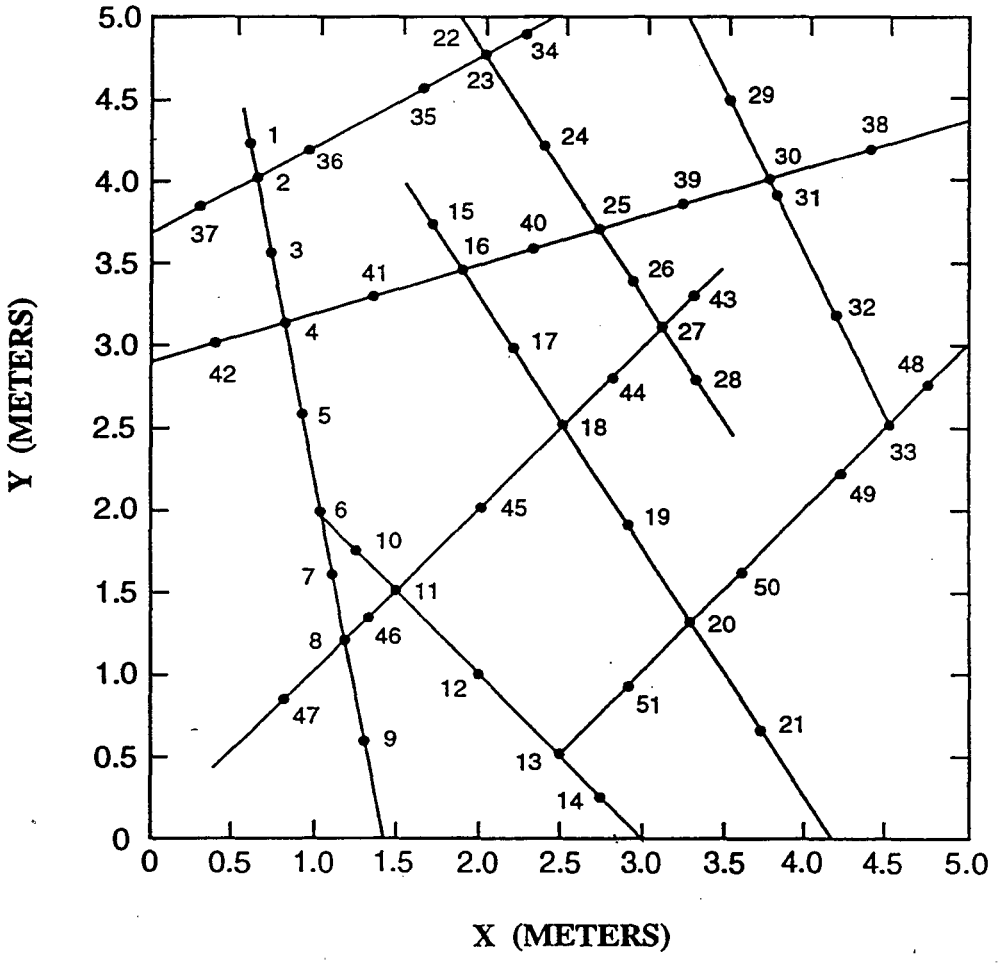

Fig. 3. Fracture network used in unsaturated flow simulations conducted by Kwicklis and Healy (1993). The top and bottom faces are held at the same pressure, so that the fluid flows from top to bottom due to gravity; the lateral sides are assumed to be impermeable.

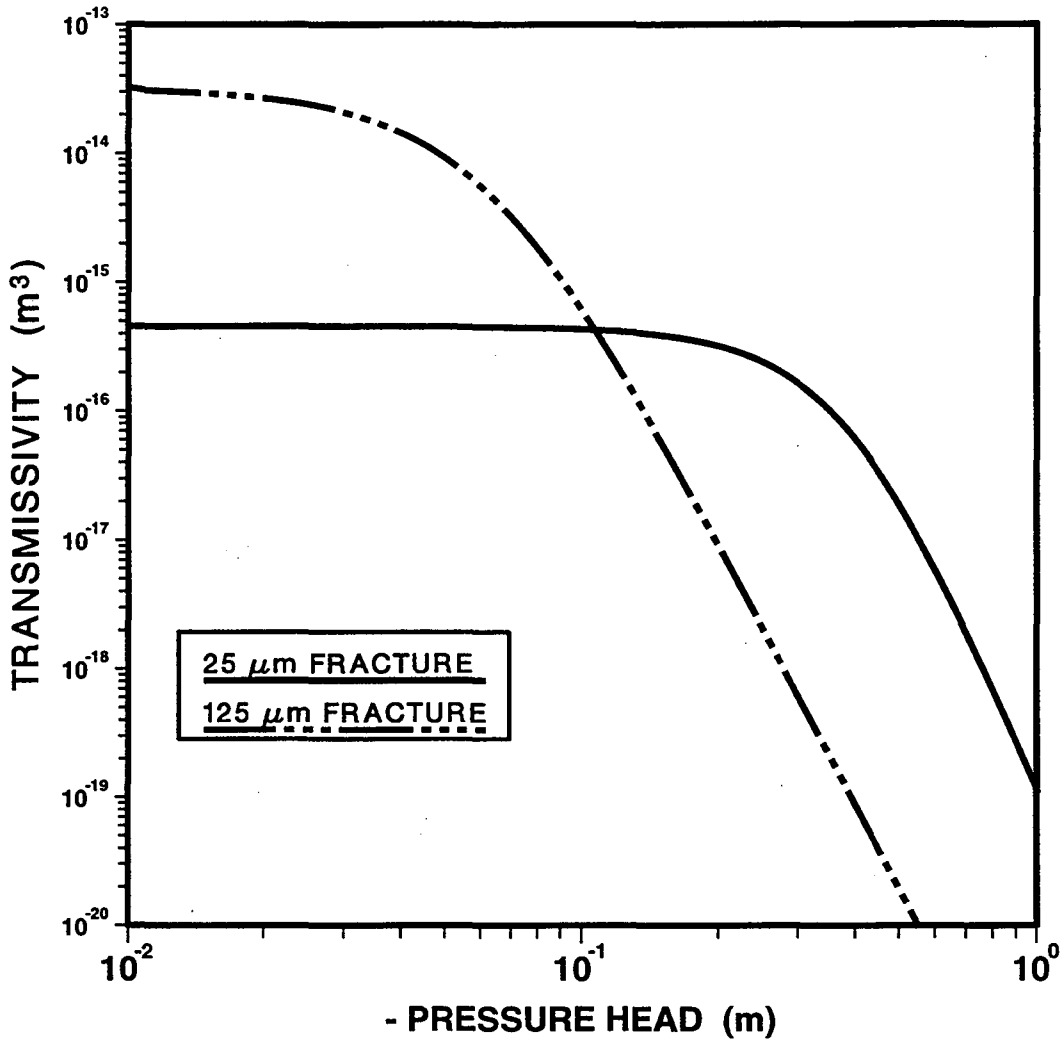

Fig. 4. Hydraulic conductivity functions for the two types of fractures used in the network shown in Fig. 2. The subvertical set of fractures have a mean aperture of 125 μm, and the subhorizontal set have a mean aperture of 25 μm.

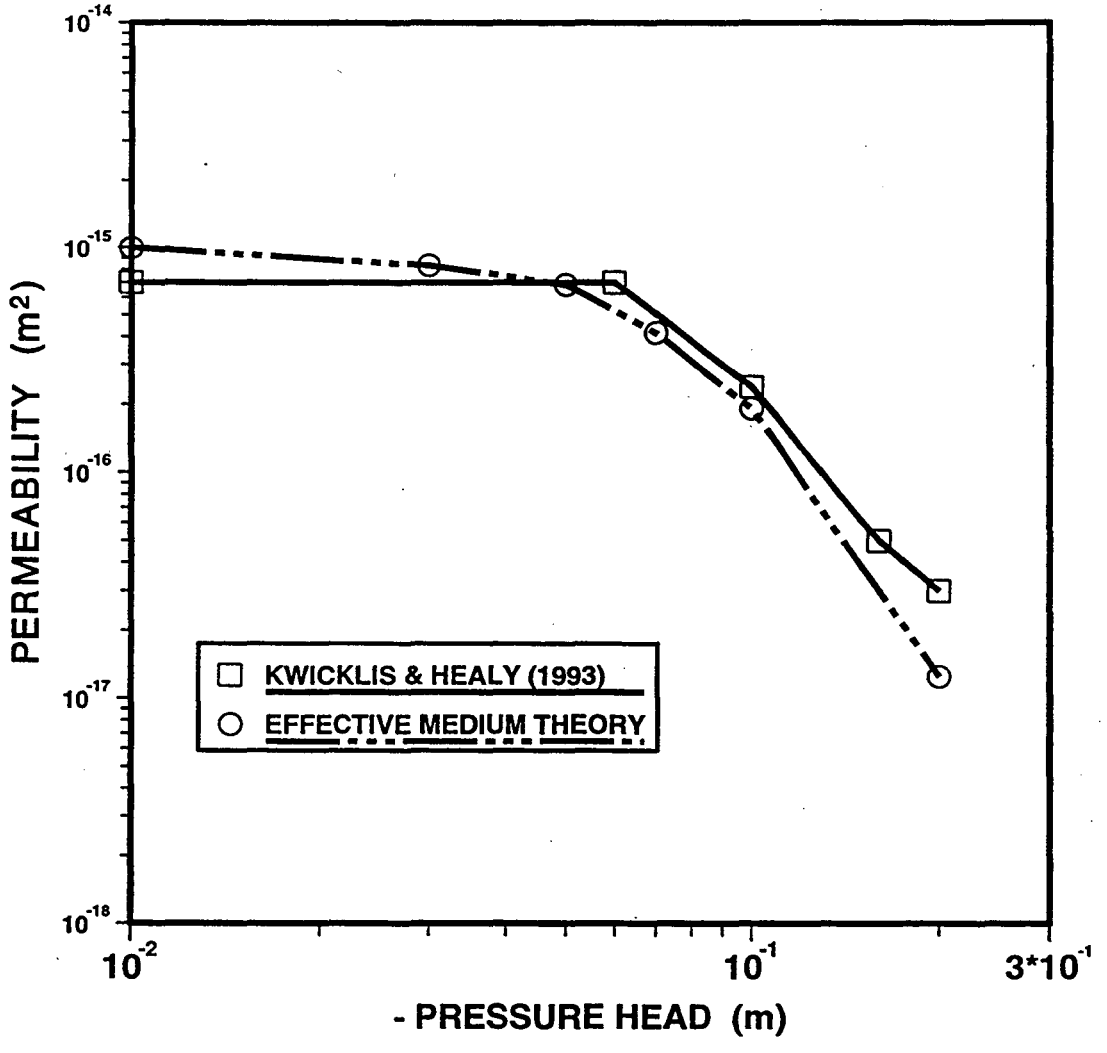

Fig. 5. Gridblock-scale hydraulic conductivity of the fracture network shown in Fig. 2, as a function of hydraulic potential, as calculated by numerical solution of the flow equations, and by using the proposed upscaling procedure.

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