LBL-37343
UC-414

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Submitted to Physical Review D

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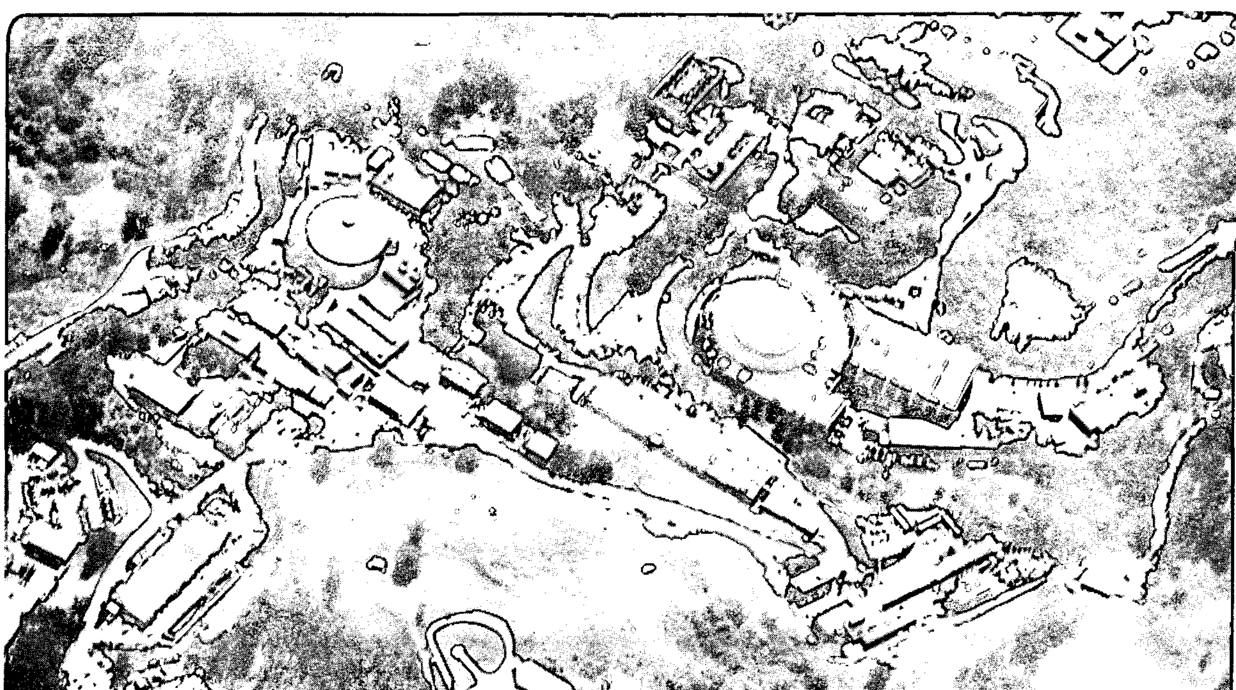
August 1995



Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

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August 11, 1995

LBL-37343
UCB-PTH-95/17

Flavor Mixing Signals For Realistic Supersymmetric Unification. *

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Abstract

The gauge interactions of any supersymmetric extension of the standard model involve new flavor mixing matrices. The assumptions involved in the construction of minimal supersymmetric models, both $SU(3) \times SU(2) \times U(1)$ and grand unified theories force a large degree of triviality on these matrices. However, the requirement of realistic quark and lepton masses in supersymmetric grand unified theories, forces these matrices to be non-trivial. This leads to important new dominant contributions to the neutron electric dipole moment and to the decay mode $p \to K^o \mu^+$, and suggests that there may be important weak scale radiative corrections to the Yukawa coupling matrix of the up quarks. The lepton flavor violating signal $\mu \to e\gamma$ is studied in these theories when $\tan\beta$ is sufficiently large that radiative effects of couplings other than λ_t must be

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-90-21139.

included. The naive expectation that large $\tan\beta$ will force sleptons to unacceptably large masses is not borne out: radiative suppressions to the leptonic flavor mixing angles allow regions where the sleptons are as light as 300 GeV, provided the top Yukawa coupling in the unified theory is near the minimal value consistent with m_t.

I. Introduction

It has recently been demonstrated that flavor and CP violation provide an important new probe of supersymmetric grand unified theories [1-4]. These new signals, such as $\mu \rightarrow e\gamma$ and the electron electric dipole moment d_e, are complementary to the classic tests of proton decay, neutrino masses and quark and charged lepton mass relations. The classic tests are very dependent on the flavor interactions and symmetry breaking sector of the unified model: it is only too easy to construct models in which these signals are absent or unobservable. However, they are insensitive to the hardness scale, Λ_H, of supersymmetry breaking.[1] On the other hand, the new flavor and CP violating signals are relatively insensitive to the form of the flavor interactions and unified gauge symmetry breaking, but are absent if the hardness scale, Λ_H, falls beneath the unified scale, M_G. The signals are generated by the unified flavor interactions leaving an imprint on the form of the soft supersymmetry breaking operators [5], which is only possible if supersymmetry breaking is present in the unified theory at scales above M_G.

The flavor and CP violating signals have been computed in the minimal $SU(5)$ and $SO(10)$ models for leptonic [1-3] and hadronic processes [4], for moderate values of $\tan\beta$, the ratio of the two Higgs vacuum expectation values. While rare muon decays provide an important probe of $SU(5)$, it is the $SO(10)$ theory which is most powerfully tested. If the hardness scale for supersymmetry breaking is large enough, as in the popular supergravity models, it may be possible for the minimal $SO(10)$ theory to be probed throughout the interesting range of superpartner masses by searches for $\mu \rightarrow e\gamma$ and d_e.

The flavor changing and CP violating probes of $SO(10)$ are sufficiently powerful to warrant an exploration of consequences for non-minimal models, which is the subject of this paper. In particular, we study $SO(10)$ theories in which

(I) *The Yukawa interactions are non-minimal.*

In the minimal model the quarks and leptons lie in three 16's and the two Higgs doublets H_U and H_D lie in two 10 dimensional representations 10_U and 10_D. The quark and charged lepton masses are assumed to arise from the

[1]This is the highest scale at which supersymmetry breaking squark and gluino masses appear in the theory as local interactions.

interactions $16\boldsymbol{\lambda}_U 16\ 10_U + 16\boldsymbol{\lambda}_D 16\ 10_D$. This model is a useful fiction: it is very simple to work with, but leads to the mass relation $m_e/m_\mu = m_d/m_s$, which is in error by an order of magnitude. It is clearly necessary to introduce a mechanism to insert $SO(10)$ breaking into the Yukawa interactions. The simplest way to achieve this is to assume that at the unification scale, M_G, some of the Yukawa interactions arise from higher dimensional operators involving fields A which break the $SO(10)$ symmetry group. This implies that $\boldsymbol{\lambda}_{U,D} \to \boldsymbol{\lambda}_{U,D}(A)$. Every realistic model of $SO(10)$ which has been constructed has this form; hence one should view this generalization of the minimal model as a necessity.

(II) *The ratio of electroweak VEV's,* $\tan\beta = v_U/v_D$, *is allowed to be large,* $\approx m_t/m_b$.

This is certainly not a necessity; to the contrary, a simple extrapolation of the results of [2] to such large values of $\tan\beta$ suggests that it is already excluded by the present limit on $\mu \to e\gamma$. The case of large $\tan\beta$ in $SO(10)$ has received much attention [6, 7, 8, 9] partly because it has important ramifications for the origin of $m_t/m_b = (\lambda_t/\lambda_b)\tan\beta$. To what extent is this puzzling large ratio to be understood as a large hierarchy of Yukawa couplings, and to what extent in terms of a large value for $\tan\beta$? If the third generation masses arise from a single interaction of the form $16_3 16_3 10$ it is possible to predict m_t using m_b and m_τ as input [6], providing the theory is perturbative up to M_G. The prediction is 175 ± 10 GeV [7], and requires $\tan\beta \approx m_t/m_b$. In this paper we investigate whether this intriguing possibility is excluded by the $\mu \to e\gamma$ signal; or, more correctly, we determine whether it requires a soft origin for supersymmmetry breaking, making it incompatible with the standard supergravity scenario [10].

In the next section we show that $SO(10)$ models with $\boldsymbol{\lambda} \to \boldsymbol{\lambda}(A)$ possess new gaugino mixing matrices in the up-quark sector, which did not arise in the minimal models. In section III we set our notation for the supersymmetric standard model with arbitrary gaugino mixing matrices, and we show which mixing matrices are expected from unified models according to the gauge group and the value of $\tan\beta$. In section IV we describe the new phenomenological signatures which are generated by the gaugino mixing matrices in the up sector; these signatures are generic to all models with Yukawa interactions generated from higher dimensional operators. The consequences of large $\tan\beta$ for the flavor and CP violating signatures are analyzed analytically in section V and numerically in section VI. The analysis of the first five sections applies to a wide class of models. In section VII we illustrate the results in the particular models introduced by Anderson et al [9]. As well as providing illustrations, these models have features unique to themselves. Conclusions are drawn in section VIII.

II. New Flavor Mixing in the Up Sector

In [1-4] flavor and CP violating signals are studied in minimal $SU(5)$ and $SO(10)$ models with moderate $\tan\beta$. In these models the radiative corrections to the scalar mass matrices are dominated by the top quark Yukawa coupling λ_t of the unified theory, so the scalar mass matrices tend to align with the up-type Yukawa coupling matrix and all non-trivial flavor mixing matrices are simply related to the KM matrix. However, as mentioned above, the minimal models do not give realistic fermion masses. One has to insert $SO(10)$ breaking into the Yukawa interactions. The simplest way to achieve this is to assume that the light fermion masses come from the non-renormalizable operators

$$\lambda'_{ij} 16_i \frac{A_1}{M_1}\frac{A_2}{M_2}\cdots\frac{A_\ell}{M_\ell}10\frac{A_{\ell+1}}{M_{\ell+1}}\cdots\frac{A_n}{M_n}16_j, \tag{2.1}$$

where the 16_i's contain the three low energy families, 10 contains the Higgs doublets, and A's are adjoint fields with vacuum expectation values (VEV's) which break the $SO(10)$ gauge group. After substituting in the VEV's of the adjoints, they become the usual Yukawa interactions with different Clebsch factors associated with Yukawa couplings of fields with different quantum numbers. For example in the models introduced by Anderson et al. [9], (hereafter referred to as ADHRS models)

$$\boldsymbol{\lambda}_U = \begin{pmatrix} 0 & z_u C & 0 \\ z'_u C & y_u E & x_u B \\ 0 & x'_u B & A \end{pmatrix}, \boldsymbol{\lambda}_D = \begin{pmatrix} 0 & z_d C & C \\ z'_d & y_d E & x_d B \\ 0 & x'_d B & A \end{pmatrix}, \boldsymbol{\lambda}_E = \begin{pmatrix} 0 & z_e C & 0 \\ z'_e C & y_e E & x_e B \\ 0 & x'_e B & A \end{pmatrix}, \tag{2.2}$$

where the x, y, z's are Clebsch factors arising from the VEV's of the adjoint fields. Thus realistic fermion masses and mixings can be obtained.

The radiative corrections to the soft SUSY-breaking operators above M_G are now more complicated. From the interactions (2.1) the following soft supersymmetry breaking operators are generated:

$$\lambda^\dagger_{ik}(A) m^2_{k\ell}(A) \lambda_{\ell j}(A)\; \phi^\dagger_i \phi_j, \tag{2.3}$$

where ϕ_i, ϕ_j are scalar components of the superfields, and $\lambda_{ij}(A)$ are adjoint dependent couplings, $\lambda(A) = \lambda' \frac{A_1}{M_1}\cdots\frac{A_n}{M_n}$. After the adjoints take their VEV's, the $m^2_{k\ell}(A)$ become the usual soft scalar masses. If we ignore the wavefunction renormalization of the adjoint fields (which is valid in the one-loop approximation), this is the same as if we had replaced the adjoints by their VEV's all the way up to the ultraheavy scale where the ultraheavy fields are integrated out, and treated these nonrenormalizable operators as the usual Yukawa interactions and scalar mass operators. This is a convenient way of thinking and we will use it in the rest of the paper.

Above the GUT scale, in addition to the Yukawa interactions which give the fermion masses

$$Q\lambda_U U^c H_U,\ Q\lambda_D D^c H_D,\ E^c\lambda_E L H_D, \tag{2.4}$$

the operators (2.1) also lead to

$$\begin{aligned} &Q\lambda_{qq} Q H_{U_3},\ E^c\lambda_{eu} U^c H_{U_3},\ N\lambda_{nd} D^c H_{U_3}, \\ &Q\lambda_{q\ell} L H_{D_3},\ U^c\lambda_{ud} D^c H_{D_3}, N\lambda_{n\ell} L H_U, \end{aligned} \tag{2.5}$$

where H_{U_3}, H_{D_3} are the triplet partners of the two Higgs doublets H_U and H_D. Each Yukawa matrix has different Clebsch factors associated with its elements, so they can not be diagonalized in the same basis. The scalar mass matrices receive radiative corrections from Yukawa interactions of both (2.4) and (2.5), which, in the one-loop approximation, take the form

$$\begin{aligned} \Delta \mathbf{m}_Q^2 &\propto \lambda_U\lambda_U^\dagger + \lambda_D\lambda_D^\dagger + 2\lambda_{qq}\lambda_{qq}^\dagger + \lambda_{q\ell}\lambda_{q\ell}^\dagger, \\ \Delta \mathbf{m}_U^2 &\propto 2\lambda_U^\dagger\lambda_U + \lambda_{eu}^\dagger\lambda_{eu} + 2\lambda_{ud}\lambda_{ud}^\dagger, \\ \Delta \mathbf{m}_D^2 &\propto 2\lambda_D^\dagger\lambda_D + \lambda_{nd}^\dagger\lambda_{nd} + 2\lambda_{ud}^\dagger\lambda_{ud}, \\ \Delta \mathbf{m}_L^2 &\propto \lambda_E^\dagger\lambda_E + 3\lambda_{q\ell}^\dagger\lambda_{q\ell} + \lambda_{n\ell}^\dagger\lambda_{n\ell}, \\ \Delta \mathbf{m}_E^2 &\propto 2\lambda_E\lambda_E^\dagger + 3\lambda_{eu}\lambda_{eu}^\dagger. \end{aligned} \tag{2.6}$$

In the minimal $SO(10)$ model, scalar mass renormalizations above M_G arise from a single matrix λ_U. It is therefore possible to choose a "U-basis" in which the scalings are purely diagonal. This is clearly not possible in the general models. All scalar mass matrices and Yukawa matrices are in general diagonalized in different bases. Therefore, flavor mixing matrices should appear in all gaugino vertices, including in the up-quark sector (where they are trivial in the minimal models studied in [1-4]). The up-type quark-squark-gaugino flavor mixing is a novel feature of the general models. Its consequences will be discussed in Sec. IV. Also, the flavor mixing matrices are no longer simply the KM matrix. They are model dependent and are different for different types of quarks and charged leptons, and are fully described in the next section.

III. Flavor Mixing Matrices in General Superymmetric Standard Models.

In this section we set our notation for the gaugino flavor mixing matrices in the supersymmetric theory below M_G, taken to have minimal field content. We also give general expectations for these matrices in a wide variety of unified theories.

The most general scalar masses are 6×6 matrices for squarks and charged sleptons and 3×3 matrix for sneutrinos,

$$\begin{aligned} \mathbf{m}_U^2 &= \begin{pmatrix} \mathbf{m}_{U_L}^2 & (\zeta_U + \lambda_U \mu \cot\beta) v_U \\ (\zeta_U^\dagger + \lambda_U^\dagger \mu \cot\beta) v_U & \mathbf{m}_{U_R}^2 \end{pmatrix}, \\ \mathbf{m}_D^2 &= \begin{pmatrix} \mathbf{m}_{D_L}^2 & (\zeta_D + \lambda_D \mu \tan\beta) v_D \\ (\zeta_D^\dagger + \lambda_D^\dagger \mu \tan\beta) v_D & \mathbf{m}_{D_R}^2 \end{pmatrix}, \\ \mathbf{m}_E^2 &= \begin{pmatrix} \mathbf{m}_{E_L}^2 & (\zeta_E + \lambda_E \mu \tan\beta) v_D \\ (\zeta_E^\dagger + \lambda_E^\dagger \mu \tan\beta) v_D & \mathbf{m}_{E_R}^2 \end{pmatrix}, \\ \mathbf{m}_\nu^2 &= \left(\mathbf{m}_{\nu_{ij}}^2\right), \end{aligned} \tag{3.1}$$

where $\mathbf{m}_{U_L}^2, \mathbf{m}_{D_L}^2, \mathbf{m}_{U_R}^2, \mathbf{m}_{D_R}^2, \mathbf{m}_{E_L}^2, \mathbf{m}_{E_R}^2$ are 3×3 soft SUSY-breaking mass matrices for the left-handed and right-handed squarks and sleptons, and $\zeta_U, \zeta_D, \zeta_E$ are the trilinear soft SUSY-breaking terms. To calculate flavor violating processes, such as $\mu \to e\gamma$, one can diagonalize the mass matrix $\mathbf{m}_E^2$ by the 6×6 unitary rotation matrix V_E and $\mathbf{m}_\nu^2$ by the 3×3 unitary rotation V_ν,

$$\mathbf{m}_E^2 = V_E \overline{\mathbf{m}}_E^2 V_E^\dagger, \qquad \mathbf{m}_\nu^2 = V_\nu \overline{\mathbf{m}}_\nu^2 V_\nu^\dagger, \tag{3.2}$$

where $\overline{\mathbf{m}}_E^2, \overline{\mathbf{m}}_\nu^2$ are diagonal. The amplitude for $\mu \to e\gamma$ is given by the diagrams in Fig. 1, summing up all the internal scalar mass eigenstates.

If the entries in the scalar mass matrices are arbitrary, they generally give unacceptably large rates for flavor violating processes. From the experimental limits one expects that the first two generation scalar masses should be approximately degenerate and the chirality-changing mass matrices ζ_A should be approximately proportional to the corresponding Yukawa coupling matrices λ_A. In this paper we treat the chirality-conserving mass matrices and chirality-changing mass matrices separately, i.e., the mass eigenstates are assumed to be purely left-handed or right-handed, and the chirality-changing mass terms are treated as a perturbation. This may not be a good approximation for the third generation where the Yukawa couplings are large, the correct treatment will be used in the numerical studies of Sec. VI. The superpotential contains

$$W \supset Q^T \lambda_U U^c H_U + Q^T \lambda_D D^c H_D + E^{cT} \lambda_E L H_D, \tag{3.3}$$

where $\lambda_U, \lambda_D, \lambda_E$ are the Yukawa coupling matrices which are diagonalized by the left and right rotations,

$$\begin{aligned} \lambda_U &= V^*_{U_L} \overline{\lambda}_U V^\dagger_{U_R}, \\ \lambda_D &= V^*_{D_L} \overline{\lambda}_D V^\dagger_{D_R}, \\ \lambda_E &= V^*_{E_R} \overline{\lambda}_E V^\dagger_{E_L}. \end{aligned} \tag{3.4}$$

The soft SUSY-breaking interactions contain

$$\begin{aligned} \widetilde{Q}^\dagger \mathbf{m}^{2*}_Q \widetilde{Q} &+ \widetilde{U}^{c\dagger} \mathbf{m}^2_U \widetilde{U}^c + \widetilde{D}^{c\dagger} \mathbf{m}^2_D \widetilde{D}^c + \widetilde{L}^\dagger \mathbf{m}^2_L \widetilde{L} + \widetilde{E}^{c\dagger} \mathbf{m}^{2*}_E \widetilde{E}^c \\ &+ \widetilde{Q}^T \zeta_U \widetilde{U}^c H_U + \widetilde{Q}^T \zeta_D \widetilde{D}^c H_D + \widetilde{E}^{cT} \zeta_E \widetilde{L} H_D. \end{aligned} \tag{3.5}$$

Because the trilinear terms should be approximately proportional to the Yukawa couplings, we write

$$\zeta = \zeta_0 + \Delta\zeta = A\lambda + \Delta\zeta. \tag{3.6}$$

The soft-breaking mass matrices are diagonalized by:

$$\begin{aligned} \mathbf{m}^{2*}_Q &= U_Q \overline{\mathbf{m}}^{2*}_Q U^\dagger_Q, \;\; \mathbf{m}^2_U = U_U \overline{\mathbf{m}}^2_U U^\dagger_U, \;\; \mathbf{m}^2_D = U_D \overline{\mathbf{m}}^2_D U^\dagger_D, \\ \mathbf{m}^2_L &= U_L \overline{\mathbf{m}}^2_L U^\dagger_L, \;\; \mathbf{m}^{2*}_E = U_E \overline{\mathbf{m}}^{2*}_E U^\dagger_E, \end{aligned} \tag{3.7}$$

$$\Delta\zeta_U = V'^*_{U_L} \Delta\overline{\zeta}_U V'^\dagger_{U_R}, \;\; \Delta\zeta_D = V'^*_{D_L} \Delta\overline{\zeta}_D V'^\dagger_{D_R}, \;\; \Delta\zeta_E = V'^*_{E_R} \Delta\overline{\zeta}_E V'^\dagger_{E_L}. \tag{3.8}$$

In the mass eigenstate basis the rotation matrices V, U appear in the gaugino couplings,

$$
\begin{aligned}
\mathcal{L}_g &= \sqrt{2}g' \sum_{\pi=1}^{4} \Big[-\frac{1}{2}\bar{e}_L W^\dagger_{E_L} \tilde{e}_L N_n (H_{n\widetilde{B}} + \cot\theta_W H_{n\widetilde{w}_3}) + \bar{e}^c_L W^\dagger_{E_R} \tilde{e}_R N_n H_{n\widetilde{B}} \\
&+ \frac{1}{2}\cot\theta_W \bar{\nu}_L \tilde{\nu}_L N_n H_{n\widetilde{w}_3} \\
&+ \bar{u}_L W^\dagger_{U_L} \tilde{u}_L N_n (\frac{1}{6} H_{n\widetilde{B}} + \frac{1}{2}\cot\theta_W H_{n\widetilde{w}_3}) + \bar{d}_L W^\dagger_{D_L} \tilde{d}_L N_n (\frac{1}{6} H_{n\widetilde{B}} - \frac{1}{2}\cot\theta_W H_{n\widetilde{w}_3}) \\
&- \frac{2}{3}\bar{u}^c_L W^\dagger_{U_R} \tilde{u}_R N_n H_{n\widetilde{B}} + \frac{1}{3}\bar{d}^c_L W^\dagger_{D_R} \tilde{d}_R N_n H_{n\widetilde{B}} + h.c.\Big] \\
&+ g\sum_{c=1}^{2} [\bar{e}_L W^\dagger_{E_L} \tilde{\nu}_L (\chi_c K_{c\widetilde{w}}) + \bar{\nu}_L \tilde{e}_L (\chi^\dagger_c K^*_{c\widetilde{w}}) \\
&+ \bar{d}_L W^\dagger_{D_L} \tilde{u}_L (\chi_c K_{c\widetilde{w}}) + \bar{u}_L W^\dagger_{U_L} \tilde{d}_L (\chi^\dagger_c K^*_{c\widetilde{w}}) + h.c.] \\
&+ \sqrt{2}g_3 [\bar{u}_L W^\dagger_{U_L} \tilde{u}_L \tilde{g} + \bar{d}_L W^\dagger_{D_L} \tilde{d}_L \tilde{g} + \bar{u}^c_L W^\dagger_{U_R} \tilde{u}_R \tilde{g} + \bar{d}^c_L W^\dagger_{D_R} \tilde{d}_R \tilde{g} + h.c.], \qquad (3.9)
\end{aligned}
$$

where[2] the neutralino and chargino mass eigenstates are related to the gauge eigenstates by e.g. $\widetilde{B} = \sum_{n=1}^4 H_{n\widetilde{B}} N_n$, $\widetilde{w}_3 = \sum_{n=1}^4 H_{n\widetilde{w}_3} N_n$, $\widetilde{w}^+ = \sum_{c=1}^2 K_{c\widetilde{w}} \chi_c$, and

$$
\begin{aligned}
W_{E_L} &= U^\dagger_L V_{E_L},\ W_{E_R} = U^\dagger_E V_{E_R},\ W_{U_L} = U^\dagger_Q V_{U_L},\ W_{D_L} = U^\dagger_Q V_{D_L}, \\
W_{U_R} &= U^\dagger_U V_{U_R},\ W_{D_R} = U^\dagger_D V_{D_R}.
\end{aligned}
$$

There are also non-diagonal chirality-changing mass terms

$$
\begin{aligned}
-\mathcal{L}^{n.d}_m &= \tilde{e}^T_R W^*_{E_R} (A_E + \mu\tan\beta) \lambda_E W^\dagger_{E_L} \tilde{e}_L v_D + \tilde{e}^T_R U^T_E \Delta\zeta_E U_L \tilde{e}_L v_D \\
&+ \tilde{d}^T_L W^*_{D_L} (A_D + \mu\tan\beta) \lambda_D W^\dagger_{D_R} \tilde{d}_R v_D + \tilde{d}^T_L U^T_Q \Delta\zeta_D U_D \tilde{d}_R v_D \\
&+ \tilde{u}^T_L W^*_{U_L} (A_U + \mu\cot\beta) \lambda_U W^\dagger_{U_R} \tilde{u}_R v_U + \tilde{u}^T_L U^T_Q \Delta\zeta_U U_U \tilde{u}_R v_U \\
&+ h.c. \qquad (3.10)
\end{aligned}
$$

The lepton flavor violating couplings are summarized in Fig. 2.

In the rest of this section we discuss the flavor mixing matrices in the minimal supersymmetric standard model, minimal and general $SU(5)$ and $SO(10)$ models, with moderate or large $\tan\beta$. The results are summarized in Table 1.

[2] Neutrino masses are not discussed here and we choose the neutrino to be in the sneutrino mass eigenstate basis.

For the minimal supersymmetric standard model, the radiative corrections to the soft masses only come from the Yukawa interactions of the MSSM:

$$\begin{aligned}
\Delta\mathbf{m}_Q^2 &\propto \boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + \kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger, \\
\Delta\mathbf{m}_U^2 &\propto 2\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U, \\
\Delta\mathbf{m}_D^2 &\propto 2\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D, \\
\Delta\mathbf{m}_L^2 &\propto \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E, \\
\Delta\mathbf{m}_E^2 &\propto 2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger.
\end{aligned} \tag{3.11}$$

We have assumed a boundary condition on the scalar mass matrices $\mathbf{m}_A^2 \propto I$ at M_{PL}, and $\kappa \neq 1$ represents the possibility that the proportionality constants are not universal. For moderate $\tan\beta, \lambda_t \gg \lambda_b$ so that the radiative corrections are dominated by λ_t. Thus one can neglect the $\boldsymbol{\lambda}_D$ contribution and the only nontrivial mixing is W_{D_L}. For large $\tan\beta, \lambda_t$ and λ_b are comparable, so $\mathbf{m}_Q^2$ will lie between $\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger$ and $\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger$. Therefore both W_{U_L} and W_{D_L} are non-trivial. with KM matrix and one can ignore them.

For the minimal $SU(5)$ model, there are only two Yukawa matrices, $\boldsymbol{\lambda}_U = \boldsymbol{\lambda}_{10}, \boldsymbol{\lambda}_D = \boldsymbol{\lambda}_E = \boldsymbol{\lambda}_5$, and

$$\begin{aligned}
\Delta\mathbf{m}_Q^2 &\propto 3\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + 2\kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger, \\
\Delta\mathbf{m}_U^2 &\propto 3\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U + 2\kappa\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D, \\
\Delta\mathbf{m}_D^2 &\propto 4\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D, \\
\Delta\mathbf{m}_L^2 &\propto 4\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D, \\
\Delta\mathbf{m}_E^2 &\propto 3\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + 2\kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger.
\end{aligned} \tag{3.12}$$

For moderate $\tan\beta, \lambda_t \gg \lambda_b$, we have non-trivial mixings for W_{D_L} and W_{E_R}, as found in [1, 2]. For large $\tan\beta$, $\boldsymbol{\lambda}_D$ can not be ignored, giving non-trivial mixings for W_{U_L} and W_{U_R}.

For the minimal $SO(10)$ model considered in [2, 3],

$$\begin{aligned}
\Delta\mathbf{m}_Q^2 &\propto 5\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + 5\kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger, \\
\Delta\mathbf{m}_U^2 &\propto 5\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U + 5\kappa\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D,
\end{aligned}$$

$$\Delta \mathbf{m}_D^2 \propto 5\boldsymbol{\lambda}_U^\dagger \boldsymbol{\lambda}_U + 5\kappa \boldsymbol{\lambda}_D^\dagger \boldsymbol{\lambda}_D,$$
$$\Delta \mathbf{m}_L^2 \propto 5\boldsymbol{\lambda}_U^\dagger \boldsymbol{\lambda}_U + 5\kappa \boldsymbol{\lambda}_D^\dagger \boldsymbol{\lambda}_D,$$
$$\Delta \mathbf{m}_E^2 \propto 5\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger + 5\kappa \boldsymbol{\lambda}_D \boldsymbol{\lambda}_D^\dagger. \tag{3.13}$$

We have non-trivial mixings $W_{D_L}, W_{D_R}, W_{E_L}$, and W_{E_R} for moderate $\tan\beta$ and non-trivial mixings for all W's for large $\tan\beta$.

For the general $SU(5)$ or $SO(10)$ models, defined in the last section, we get non-trivial mixings for all mixing matrices in general. However, in $SU(5)$ models with moderate $\tan\beta$, the splittings among $\mathbf{m}_D^2$ and $\mathbf{m}_L^2$ are too small (because they are generated by the small $\boldsymbol{\lambda}_5(A)$) to give significant flavor changing effects.

One might expect that the mixing in the W_U's are smaller than those in the W_D's because of the larger hierarchy in $\boldsymbol{\lambda}_U$ compared with $\boldsymbol{\lambda}_D$. However, a given W is the product of a $U^\dagger$ (which diagonalizes the scalar mass matrix) and a V (which diagonalizes the Yukawa matrix). Even if the mixings in V_U's are smaller than those in V_D's because of the larger hierarchies in $\boldsymbol{\lambda}_U$, we do not have a general argument for the size of mixings in U matrices. This is because U diagonalizes (appropriate combinations of) known Yukawa matrices and unknown Yukawa matrices appearing above the GUT scale, (2.5). The mixings in $U^\dagger$ and V can add up or cancel each other. Our only general expectation is that these new Yukawa matrices have similar hierarchical patterns as $\boldsymbol{\lambda}_U$ or $\boldsymbol{\lambda}_D$. Without a specific model, one can at most say that all non-trivial W's are expected to be comparable to V_{KM}; the argument that the mixings in W_U's should be smaller than is W_D's is not valid.

In the minimal models at moderate $\tan\beta$, the leading contributions to flavor changing processes, such as $\mu \to e\gamma$, involve diagrams with a virtual scalar of the third generation. Although such contributions are highly suppressed by mixing angles, they dominate because they have large violations of super-GIM[11]: the top Yukawa coupling makes $m_{\tilde{\tau}}$ very different from $m_{\tilde{e}}, m_{\tilde{\mu}}$. At large $\tan\beta$, the strange/muon Yukawa couplings get enhanced, so the splitting between $m_{\tilde{e}}$ and $m_{\tilde{\mu}}$ increases, leading to potentially competitive contributions to flavor changing processes which do not involve the third generation. The importance of these new diagrams can be estimated by comparing the contributions to Δm_{21}^2 (in a basis where gaugino vertices are diagonal) when the super-GIM cancellation is between scalars of the first two generations (2-1) and third generations (3-1):

$$\frac{\Delta m^2_{21}(2\text{-}1)}{\Delta m^2_{21}(3\text{-}1)} \simeq \frac{V_{cd}\lambda_2^2}{V_{td}V_{ts}\lambda_t^2} \simeq \begin{cases} 10^{-2}, & \text{for } \lambda_2 = \lambda_c, \\ \left(\frac{\tan\beta}{60}\right)^2, & \text{for } \lambda_2 = \lambda_s. \end{cases} \tag{3.14}$$

We can see that for large $\tan\beta$ (or any $\tan\beta$ with small λ_s coming from the mixing of Higgs at M_G i.e., $\lambda_s(M_G) = \frac{\tan\beta}{60}\lambda_2(M_G)$), this could be comparable to the flavor violating effects from the large splitting of the third generation scalar masses. However, for the $\mu \to e\gamma$ in $SO(10)$ models, it does not contribute to diagrams which are proportional to m_τ, (because it does not involve the third generation scalars), the dominant contributions are still those diagrams considered in [2]. For flavor changing processes which do not need chirality flipping, such as $K - \overline{K}$ mixing, and all flavor changing processes in $SU(5)$ models, this non-degeneracy between the first two generations is important. The above discussion is summarized in Table 1.

Table 1

		$SU(5)$		$SO(10)$	
	MSSM	Minimal	general	minimal	general
δm_3^2	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
δm_2^2	$\bullet$	$\bullet$	$\circ$	$\bullet$	$\circ$
W_{U_L}	$\bullet$	$\bullet$	$\surd$	$\bullet$	$\surd$
W_{D_L}	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
W_{U_R}	—	$\bullet$	$\surd$	$\bullet$	$\surd$
W_{D_R}	—	—	$\surd^*$	$\surd$	$\surd$
W_{E_L}	—	—	$\surd^*$	$\surd$	$\surd$
W_{E_R}	—	$\surd$	$\surd$	$\surd$	$\surd$

Table 1: Summary table for the flavor mixing matrices:

δm_3^2 : important effects due to some third generation scalars not degenerate with those of first two generations.

δm_2^2 : non-negligible effects due to non-degeneracy of the scalars; of the first two generations.

W_i : fermion i and scalar $\tilde{i}$ are rotated differently to get to mass basis.

$\surd$: present for any value of $\tan\beta$.

$\bullet$: present only for large $\tan\beta$.

$\circ$: present for large $\tan\beta$, but model dependent for moderate $\tan\beta$.

–: not present.

$*$: although present, its effect for moderate $\tan\beta$ on flavor violation is small due to the small non-degeneracy among different generation scalars.

IV. Phenomenology from up-type mixing

As discussed in the previous section, unlike the minimal models with moderate $\tan\beta$ studied in [1, 2, 3, 4] in generic GUT's (for any $\tan\beta$) and even for minimal GUT's (at large $\tan\beta$), we expect mixing matrices in the up sector. Having motivated an origin for non-trivial up mixing matrices $W_{U_{L(R)}} \neq 1$, we consider some effects they produce. In the following we simply assume some $W_{u_{L(R)}}$ at the weak scale and consider their phenomenological consequences. (See however Section V and the appendix for a discussion of the scaling of mixing matrices from GUT to weak scales.) In particular we discuss $D-\bar{D}$ mixing, corrections to up-type quark masses, contributions to the neutron electric dipole moment (e.d.m.) and the possibility of different dominant proton decay modes than those expected from minimal models.

IVa. $D-\bar{D}$ mixing:

To get an idea for the contribution of up-type mixing matrices to $D-\bar{D}$ mixing, we follow [12, 13] and employ the mass insertion approximation. The bounds obtained from $D-\bar{D}$ mixing on the 6×6 up-squark mass matrix $m_U^2 = \begin{pmatrix} m^2_{U_{LL}} & m^2_{U_{LR}} \\ m^2_{U_{RL}} & m^2_{U_{RR}} \end{pmatrix}$ (in the basis where gluino and Yukawa couplings are diagonal) are summarized in [13]. For average up-squark mass of $\widetilde{m} = 1$ TeV, they are

$$\sqrt{\frac{m^2_{U_{LL12}}}{\widetilde{m}^2}\frac{m^2_{U_{RR12}}}{\widetilde{m}^2}} \leq .04, \tag{4.1}$$

$$\frac{m^2_{U_{LR12}}}{\widetilde{m}^2} \leq .06. \tag{4.2}$$

Consider first (4.1). In the last section we estimated that the contribution to m^2_{12} from the slight non-degeneracy between the first two generation scalars is generically at most comparabale to that from the non-degeneracy between the first two and third generation scalars. Thus, for our calculation, we only consider the contribution from the splitting between first two and third generation scalars. Then, for $A = L, R$

$$\left|\frac{m^2_{AA12}}{\widetilde{m}^2}\right| = \left|W_{U_{A13}} W^\dagger_{U_{A32}}\right| \left|\frac{m^2_{\tilde{t}_A} - m^2_{\tilde{u}_A}}{\widetilde{m}^2}\right| \leq \left|W_{U_{A13}} W^\dagger_{U_{A32}}\right|. \tag{4.3}$$

We see that for W's of the same size as the corresponding KM matrix elements, the LHS of (4.1) is of order 4×10^{-4}, and the bound is easily satisfied.

Turning to (4.2), note that if $\zeta_U = A\lambda_U$, $m^2_{U_{LR12}} = 0$. However, we expect $\zeta_U = A\lambda_U + \Delta\zeta_U$, with $\Delta\zeta_U$ induced in running from M_{PL} to M_G having primarily a third generation component in the gauge eigenstate basis. If all relevant mixing matrix elements are of order the KM matrix elements, we expect $\left|\frac{m^2_{U_{LR12}}}{m^2}\right| = O\left(\left|\frac{Am_t}{m^2} V_{td} V_{ts}\right|\right)$. Again, we see that the bound (4.2) is generically easily satisfied, and thus we do not in general expect significant contributions to $D - \bar{D}$ mixing.

IVb. Weak-scale corrections to up-type quark masses:

It is well known that there are important weak-scale radiative corrections to the down quark mass matrix proportional to $\tan\beta$ [7, 8, 14, 15, 16]. In general unified models, with non-zero W_U, there are also importrant weak scale corrections to the up quark mass matrix.

From the diagram in Fig. 3, we have a contribution to up-type masses proportional to m_t. We find, again assuming degeneracy between the scalars of the first two generations,

$$\begin{aligned}\Delta m_u^{ij} &= \frac{8}{3}\left(\frac{\alpha_s}{4\pi}\right) m_t \left(\frac{A + \mu\cot\beta}{M_{\tilde{g}}}\right) W_{U_{L3i}} W_{U_{R3j}} \left[h(x_{t_L}, x_{t_R}) - h(x_{t_L}, x_{u_R})\right. \\ &\quad \left. - h(x_{u_L}, x_{t_R}) + h(x_{u_L}, x_{u_R})\right],\end{aligned} \tag{4.4}$$

where

$$x_i \equiv \frac{\widetilde{m}_i^2}{M_{\tilde{g}}^2}, \quad h(x,y) = \frac{1}{x-y}\left[\frac{x\log x}{1-x} - \frac{y\log y}{1-y}\right]. \tag{4.5}$$

The largest fractional change in the mass occurs for the up quark. If $W_{U_{L(R)31}}$ is comparable to the corresponding KM matrix element, the contribution to $\frac{\Delta m_u}{m_u}$ is not significant. However, if each of the $W_{U_{L(R)31}}$ are a factor 3 larger than the corresponding KM elements we can get sizable contributions. In Fig. 4, we plot $\frac{\Delta m_u}{m_u}$ in $\frac{m_{\tilde{u}}}{M_{\tilde{g}}} - \frac{m_{\tilde{t}}}{m_{\tilde{u}}}$ space, where we have assumed $m_{\tilde{u}_L} = m_{\tilde{u}_R} \equiv m_{\tilde{u}}$; $m_{\tilde{t}_L} = m_{\tilde{t}_R} \equiv m_{\tilde{t}}$, and we have put $|W_{U_{L31}}| = |W_{U_{R31}}| = 1/30$, $(A + \mu\cot\beta)/m_{\tilde{t}} = 3$. Any deviations from these values can simply be multiplied in $\Delta m_u/m_u$. In some regions of the parameter space it is possible to get the entire up quark mass as a radiative effect.

IVc. Neutron e.d.m.:

If we attach a photon in all possible ways to the diagram giving the contribution to u-quark mass, we get a contribution to the u-quark e.d.m., which is proportional to m_t for any value of $\tan\beta$. Evaluating the diagram, we find

$$d^u = e|F|\sin\phi_u \tag{4.6}$$

where

$$F = \frac{8}{3}\left(\frac{\alpha_s}{4\pi}\right) m_t \frac{A+\mu\cot\beta}{M_{\tilde{g}}^3} W_{U_{L31}} W^*_{U_{L33}} W_{U_{R31}} W^*_{U_{R33}}$$
$$\times\left[\tilde{G}_2(x_{t_L}, x_{t_R}) - \tilde{G}_2(x_{t_L}, x_{u_R}) - \tilde{G}_2(x_{t_R}, x_{u_L}) + \tilde{G}(x_{u_L}, x_{u_R})\right], \tag{4.7}$$

where

$$\tilde{G}_2(x,y) = \frac{g(x)-g(y)}{x-y}, \quad g(x) = \frac{1}{2(x-1)^3}[x^2-1-2x\log x] \tag{4.8}$$

and

$$Im\left[m_t W_{U_{L31}} W^*_{U_{L33}} W_{U_{R31}} W^*_{U_{R33}}\right] \equiv |m_t W_{U_{L31}} W^*_{U_{R33}} W_{U_{R31}} W^*_{U_{R33}}|\sin\phi_u. \tag{4.9}$$

In general we expect a large non-zero $\sin\phi_u$. If the combination of W's appearing in the above is comparable to the combination giving a down quark e.d.m., the u-quark contribution will dominate over the d-quark contribution to the neutron e.d.m. considered in [3] by a factor $\frac{m_t}{4m_b\tan\beta}$, (the factor 4 comes from the quark model result $d_n = 4/3d_d - 1/3d_u$). Hence, the neutron e.d.m. may be competitive with $\mu\to e\gamma$ and d_e as the most promising flavor changing signal for supersymmetric unification.

IVd. Proton decay:

Finally we turn briefly to the relevance of up-type mixing matrices for proton decay; in particular to the important question of the charge of the lepton in the final state. We know that upon integrating out the superheavy Higgs triplets we can generate the baryon number violating operators $\frac{1}{2M_H}(QQ)(QL)$ and $\frac{1}{M_H}(EU)(DU)$ in the superpotential. These operators must subsequently be dressed at the weak scale in order to obtain four-fermion operators leading to proton decay. The dressing may be done with neutralinos, charginos or gluinos where possible. Since the dressed operator grows with gauge couplings and vanishes for vanishing neutralino/chargino/gluino mass, one might naively expect gluino dressing to be most important. However, if the up-type mixing matrices are trivial, gluino dressed operators can only lead to proton decay with a neutrino in the final state. To see this, we examine each operator separately: $(\dot{e}u_a)(d_b u_c)\epsilon^{abc}$ (where a, b, c are color indices) must involve u's from two different generations because of the ϵ^{abc}. One of them has to be a u, so the other is a c or a t. If there is no up mixing, the up flavor does not change in the dressing process, so the final state would have to contain a c or a t. Since $m_t, m_c > m_p$, this can not happen. Next, consider $(QQ)(QL) = u_L^a d_L^b (u_L^c e_L - d_L^c v_L)\epsilon_{abc}$. By exactly the same argument as the above, the $u_L^a d_L^b u_L^c e_L \epsilon_{abc}$ operator can not contribute to proton decay. Thus, we see that in the absence of mixing in the up sector, gluino dressing can only give neutrinos in the final state. However, the above arguments break down if up-mixing matrices are non-trivial, since gluino dressed diagrams give a significant contribution to the branching ratio for charged lepton modes in proton decay. A detailed study of flavor mixing in the up sector [17] concludes that, whether the wino or gluino dressings are dominant, the muon final state in proton decay is of greatly enhanced importance. Without the mixings, one expects $\frac{\Gamma(p \to K^o \mu^+)}{\Gamma(p \to K^+ \bar{\nu})} \approx 10^{-3}$. The up mixing in general models increases this by $O(100)$ making the mode $p \to K^o \mu^+$ a favorable one for discovery of proton decay.

Section V. Large $\tan\beta$: Analytic Treatment

The large $\tan\beta$ scenario is interesting for a number of reasons. For moderate $\tan\beta$, the only way to understand $m_t \gg m_b, m_\tau$ is to have $\lambda_t \gg \lambda_b, \lambda_\tau$ at the weak scale. This gives us little hope of attributing a common origin for third generation Yukawa couplings at a higher scale. However, for large $\tan\beta \sim O\left(\frac{m_t}{m_b}\right)$, the weak scale $\lambda_t, \lambda_b, \lambda_\tau$ are comparable and the above hope is restored. (In fact it is realized in $SO(10)$ models like the ADHRS example outlined in section VII). For us, this is sufficient motivation to study the large $\tan\beta$ case in more detail. Also, this case was not studied in [2]. We shall see that unexpected new features arise in the large $\tan\beta$ limit.

The largest contribution to the $\mu \to e\gamma$ amplitude comes from the diagram with $L-R$ scalar mass insertion (Fig. 5). In the $L-R$ insertion approximation, the amplitude for $\mu_{L(R)}$ decay is

$$\begin{aligned} F_{L(R)} &= \frac{\alpha}{4\pi\cos^2\theta_W} m_\tau W_{E_{L(R)32}} W_{E_{R(L)31}} W^*_{E_{L(R)33}} W^*_{E_{R(L)33}} (A_E + \mu\tan\beta) \\ &\times \left[G_2(m^2_{\tilde{\tau}_L}, m^2_{\tilde{\tau}_R}) - G_2(m^2_{\tilde{e}_L}, m^2_{\tilde{\tau}_R}) - G_2(m^2_{\tilde{\tau}_L}, m^2_{\tilde{e}_R}) + G_2(m^2_{\tilde{e}_L}, m^2_{\tilde{e}_R}) \right], \end{aligned}$$

where

$$\begin{aligned} G_2(m_1^2, m_2^2) &= \frac{G_2(m_1^2) - G_2(m_2^2)}{m_1^2 - m_2^2}, \\ G_2(m^2) &= \sum_{n=1}^{4} (H_{n\tilde{B}} + \cot\theta_W H_{n\tilde{w}_3}) g_2\left(\frac{m^2}{M_n^2}\right). \end{aligned} \tag{5.1}$$

Note, however, that for large $\tan\beta$ the $L-R$ insertion approximation may be a bad one, since the chirality changing mass for the third generation becomes comparable to the chirality conserving masses. A correct treatment will be used for the numerical analysis in the next section. We still expect, however, that the amplitude to be proportional to $W_{E_{32}} W_{E_{31}}$ because of the unitarity of the mixing matrices: the sum of contributions from the first two generations is proportional to $W_{1i}W^*_{1j} + W_{2i}W^*_{2j} = -W_{3i}W^*_{3j}$ for $i \neq j$, and the contribution from the third generation is itself proportional to $W_{3i}W^*_{3j}$.

Two simplifications in the dependence of the $\mu \to e\gamma$ rate on parameter space occur for large $\tan\beta$. First, since the dominant diagram involves the $L-R$ insertion $(A + \mu\tan\beta)m_t$, and since $\tan\beta$ is large, the amplitude does not depend on the weak scale parameter A. Second, in the large tanβ limit, the chargino mass matrix is

$$M_{\chi} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & -\mu \end{pmatrix} \longrightarrow \begin{pmatrix} M_2 & \sqrt{2}M_W \\ 0 & -\mu \end{pmatrix}, \qquad (5.2)$$

and the parameters M_2, μ have a direct interpretation as the chargino masses. (Note that this assures us that $\mu \tan\beta$ will likely always be much bigger than A; for a $\tan\beta$ of 50, the LEP lower bound on chargino mass of 45 GeV tells us that $\mu \tan\beta > 2$ TeV, so for A to be comparable to $\mu \tan\beta$ we must have $A > 2$ TeV.)

In considering $\mu \to e\gamma$ for large $\tan\beta$, two factors come immediately to mind which tend to (perhaps dangerously) enhance the rate over the case with moderate $\tan\beta$.

(i) As we have already mentioned, the dominant contribution to $\mu \to e\gamma$ grows with $\tan\beta$; the diagram in Fig. 5 is proportional to $\tan\beta$, a factor of 900 in the rate for $\tan\beta = 60$ compared to $\tan\beta = 2$.

(ii) For large $\tan\beta$, λ_τ can be $O(1)$ and we can not neglect its contribution to the running of the slepton mass matrix from M_G to M_S (soft SUSY breaking scale). This scaling generally splits the third generation slepton mass even further from the first two generations, meaning a less effective super-GIM mechanism and a larger amplitude for $\mu \to e\gamma$.

While both of the above effects certainly exist, there are also two sources of *suppression* of the amplitude for large $\tan\beta$, which can together largely compensate for the above factors:

(i)$'$ Large $\tan\beta$ allows λ_t to be smaller than for moderate tanβ. There are two reasons for this. First, large $\tan\beta$ allows v_U to be larger and so λ_t can be smaller to reproduce the top mass. Secondly, $b - \tau$ unification [18] is achieved with a smaller λ_τ in the large $\tan\beta$ regime [7, 8]. Since λ_t is smaller, a smaller non-degeneracy between the third and first two generations is induced in running from M_{PL} to M_G, suppressing the amplitude compared to the moderate $\tan\beta$ case.

(ii)$'$ In comparing large and moderate tanβ, we must know how the mixing matrices $W_{L,R_{3i}}$ (appearing at the vertices of the diagrams responsible for $\mu \to e\gamma$) compare in these two cases. In the moderate tanβ minimal models discussed in [2], $W_{L,R_{3i}}$ were equal to the corresponding KM matrix elements V_{KM3i} at M_G, and this equality was approximately maintained in running form M_G to M_S. As discussed in the previous sections, for more general models one expects that the $W_{L(R)_{3i}}$ at M_G are equal to V_{KM3i} at M_G up to some combination of Clebsches. One might then expect (as in the minimal models) that this relationship continues to approximately hold at lower scales. In fact for large $\tan\beta$ this expectation is false. We find that often, the $W_{L(R)3i}$ *decrease* from M_G to M_S, overcompensating for the increased non-degeneracy between the third and first two generation slepton masses induced by large λ_τ (point (ii) above).

In the following, we examine the scaling of these mixing matrices in detail. Consider first the lepton sector. The renormalization group equation (RGE) for $\boldsymbol{\lambda}_E$ (in the following $t = \frac{log\mu}{16\pi^2}$) is

$$-\frac{d\boldsymbol{\lambda}_E}{dt} = \boldsymbol{\lambda}_E[3\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E + Tr(3\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D + \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E) - 3g_2^2 - \frac{9}{5}\, g_1^2] \tag{5.3}$$

giving

$$-\frac{d}{dt}\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E = 6\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E + 2\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E Tr(3\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D + \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E) - (6g_2^2 + \frac{18}{5}\, g_1^2)\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E \tag{5.4}$$

$$-\frac{d}{dt}\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger = 6\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger + 2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger Tr(3\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D + \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E) - (6g_2^2 + \frac{18}{5}\, g_1^2)\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger. \tag{5.5}$$

These in turn imply that the basis in which $\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E$ is diagonal, and the (in general different) basis where $\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger$ is diagonal, do not change with scale. Consider now the evolution of the left handed slepton mass matrix $\mathbf{m}_L^2$. The RGE for $\mathbf{m}_L^2$ is

$$-\frac{d}{dt}\mathbf{m}_L^2 = (\mathbf{m}_L^2 + 2m_{H_d}^2)\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E + 2\boldsymbol{\lambda}_E^\dagger\mathbf{m}_E^2\boldsymbol{\lambda}_E + \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E\mathbf{m}_L^2 + 2\zeta_E^\dagger\zeta_E + \text{ Gaugino terms.} \tag{5.6}$$

In the basis where $\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E$ is diagonal, keeping only the λ_τ contribution, the $3i$ entry ($i \neq 3$) becomes:

$$-\frac{d}{dt}m_{L3i}^2 = \lambda_\tau^2 m_{L3i}^2 + 2(\zeta_E^\dagger\zeta_E)_{3i}. \tag{5.7}$$

In this basis, we have $\mathbf{m}_L^2 = W_L^\dagger\overline{\mathbf{m}}_L^2 W_L$. (Here and in the remainder of this section, we abbreviate $W_{E_{L(R)}} \to W_{L(R)}$). Assuming degeneracy between scalars of the first two generations, $m_{L_{3i}}^2 = W_{L3i}W_{L33}^\dagger(m_{\tau_L}^2 - m_{e_L}^2) \equiv W_{L3i}W_{L33}^\dagger\Delta m_L^2$. Then (5.7) becomes

$$-\frac{d}{dt}(W_{L3i}W_{L33}^\dagger\Delta m_L^2) = \lambda_\tau^2(W_{L3i}W_{L33}^\dagger\Delta m_L^2) + 2(\zeta_E^\dagger\zeta_E)_{3i}. \tag{5.8}$$

For now, we ignore the $(\zeta_E^\dagger \zeta_E)_{3i}$ term in (5.8), yielding the solution:

$$(W_{L3i} W_{L33}^\dagger \Delta m_L^2)(M_S) = e^{-I_\tau}(W_{L3i} W_{L33}^\dagger \Delta m^2)(M_G), \tag{5.9}$$

where

$$I_i \equiv \int_0^{\log \frac{M_G}{M_S}} \frac{dt}{16\pi^2} \lambda_i^2(t). \tag{5.10}$$

Thus,

$$W_{L3i} W_{L33}^\dagger(M_S) = e^{-I_\tau} \frac{\Delta m_R^2(M_G)}{\Delta m_R^2(M_S)} W_{L3i}^\dagger W_{L_{33}}(M_G). \tag{5.11}$$

Similarly, we find

$$W_{R3i} W_{R33}^\dagger(M_S) = e^{-2I_\tau} \frac{\Delta m_R^2(M_G)}{\Delta m_R^2(M_S)} W_{R3i} W_{R_{33}}^\dagger(M_G). \tag{5.12}$$

Note that, generically the quantities $\frac{\Delta m_{L(R)}^2(M_G)}{\Delta m_{L(R)}^2(M_S)}$ are smaller than one, since the third generation mass gets split even further from the first two generations in running from M_G to M_S. Thus, we find that the $W_{L(R)_{3i}}$ get smaller in magnitude as we scale from M_G to M_S, in contrast with the KM matrix elements V_{KM3i}, which scale as

$$V_{KM3i}(M_S) = e^{(I_t+I_b)} V_{KM3i}(M_G). \tag{5.13}$$

Suppose that at M_G the $W_{L(R)}$ are related to V_{KM} though some combination of Clebsches determined by the physics above the GUT scale.

$$W_{L(R)33}^\dagger W_{L(R)3i}(M_G) = z_{iL(R)} V_{KM3i}(M_G). \tag{5.14}$$

This relationship is not maintained at lower scales; instead we have:

$$W_{L33}^\dagger W_{L3i}(M_S) = \frac{\Delta m_L^2(M_G)}{\Delta m_L^2(M_S)} e^{-(I_\tau+I_t+I_b)} z_{i_L} V_{KM3i}(M_S), \tag{5.15}$$

$$W_{R33}^\dagger W_{R3i}(M_S) = \frac{\Delta m_R^2(M_G)}{\Delta m_R^2(M_S)} e^{-(2I_\tau+I_t+I_b)} z_{i_R} V_{KM3i}(M_S). \tag{5.16}$$

The dominant contribution to the $\mu \to e\gamma$ rate is proportional to $|W_{L33}^\dagger W_{L32} W_{R33}^\dagger W_{R31}(M_S)|^2 + |W_{L33}^\dagger W_{L31} W_{R33}^\dagger W_{R32}(M_S)|^2$, giving

$$\begin{aligned} Br(\mu \to e\gamma) \;&=\; \left[\frac{\Delta m_L^2(M_G)}{\Delta m_L^2(M_S)} \frac{\Delta m_R^2(M_G)}{\Delta m_R^2(M_S)}\right]^2 e^{-(6I_\tau+4I_t+4I_b)} \times (|z_{2_L} z_{1_R}|^2 + |z_{1_L} z_{2_R}|^2) \\ &\times Br(\mu \to e\gamma, W_{L(R)3i} W_{L(R)33}^\dagger(M_S) \to V_{KM3i}(M_S)) \\ &\equiv \epsilon(|z_{2_L} z_{1_R}|^2 + |z_{1_L} z_{2_R}|^2) \times Br(\mu \to e\gamma, W_{L(R)33}^\dagger W_{L(R)33}(M_S) \to V_{KM3i}(M_S)). \end{aligned} \tag{5.17}$$

This ϵ represents a possibly significant suppression of the rate for large $\tan\beta$.

At this point, the reader may object: it is true that the $W_{L(R)3i}$ decrease from M_G to M_S, but as already mentioned, the non-degeneracy between the third and first two generating is increasing. Which effect wins? We argue that in general there is a net suppression. This is easiest to see if in computing the $\mu \to e\gamma$ amplitude, we use the mass insertion approximation rather than mixing matrices at the the vertices (Fig. 6). Although this may be a poor approximation, it serves to illustrate our point. (Of course no such approximation is made in our numerical work.) From the diagram it is clear that the amplitude is proportional to $m^2_{L32}m^2_{R31}(M_S)$. From (5.7), we see that the rate scales as

$$\left(m^2_{L32}m^2_{R31}\right)^2(M_S) = e^{-(6I_\tau+4I_t+4I_b)}\left(m^2_{L_{32}}m^2_{R_{31}}\right)(M_G), \qquad (5.18)$$

a net suppression. In the mass insertion approximation, then, the terms $\frac{\Delta m^2(M_G)}{\Delta m^2(M_S)}$ in (5.17) serve to exactly compensate for the increased non-degeneracy between $m^2_{\tilde{e}_L}$ and $m^2_{\tilde{\tau}_L}$; what remains is still a suppression. This, together with (i)′ invalidates the naive expectation that the theory is ruled out in most regions of parameter space due to the enhancing factors (i) and (ii), (although there are still stringent constraints on the parameter space).

The above analysis suggests that individual lepton number conservation is an infrared fixed point of the MSSM (whereas individual quark number conservation is an ultraviolet fixed point). A more complete analysis of scaling for the lepton sector and a discussion of scaling in the quark sector is presented in the appendix.

Section VI. Large $\tan\beta$: Numerical Results

The amplitude for $\mu \to e\gamma$ depends on the 6×6 slepton mass matrix $\mathbf{M}^2$. In the basis where $\mathbf{m}_L^2, \mathbf{m}_E^2$ are diagonal, we have

$$\mathbf{M}^2 = \begin{pmatrix} \overline{\mathbf{m}}_{E_L}^2 + \mathbf{D}_L & \mathbf{k} \\ \mathbf{k}^\dagger & \overline{\mathbf{m}}_{E_R}^2 + \mathbf{D}_R \end{pmatrix} \tag{6.1}$$

where in the large $\tan\beta$ limit, $D_i = -(T_{3i} - Q_i \sin^2\theta_W)M_Z^2$ is the D-term contribution, and $k_{ij} = \mu m_\tau \tan\beta W_{L3i} W_{R3j}$. The amplitude from Fig. 1 for μ_L decay is

$$F_L = \frac{\alpha}{4\pi\cos^2\theta_W} W_{Li2}^\dagger G_2(\mathbf{M}^2)_{LRij} W_{Rj1}^\dagger$$

where

$$G_2(\mathbf{M}^2) \equiv \begin{pmatrix} G_2(M^2)_{LL} & G_2(M^2)_{LR} \\ G_2(M^2)_{RL} & G_2(M^2)_{RR} \end{pmatrix}, \tag{6.3}$$

In [2], $\mathbf{M}^2$ was approximately diagonalized by the $\mu m_\tau \tan\beta$ insertion approximation, and $G_2(\mathbf{M}^2)$ was calculated using this approximate diagonalization. Since here $\tan\beta$ is large, we wish to avoid making such an approximation, and numerically diagonalize the full 6×6 $\mathbf{M}^2$.

Faced with a rather large parameter space, we must decide which parameters to use in our numerical work. We have firstly decided to do our analysis only for large $\tan\beta$, since the moderate $\tan\beta$ scenario has been covered in [2]. Secondly, we choose to present our results in a different way than in [2], where the rates for $\mu \to e\gamma$ were plotted against a combination of Planck scale and weak scale parameters. In our work, we compute $\mu \to e\gamma$ entirely in terms of weak scale parameters. In particular, we assume that the necessary condition for a significant $\mu \to e\gamma$ rate exist at the weak scale, namely non-trivial mixing matrix $W_{L,R_{3i}}$ and non-degeneracy between third and first two generation slepton masses. In the previous sections, we have shown a possible way in which these ingredients may be produced. Our plots for $\mu \to e\gamma$ rates are made against low energy parameters, and we separately plot the regions in low energy parameter space predicted by our particular scenario for generating $\mu \to e\gamma$. This way, our plots are in terms of experimentally accessible quantities and can be thought of as constraining the parameter space of the effective 3-2-1 softly broken supersymmetric theory resulting from the spontaneous breakdown of a GUT. (We use the GUT to relate weak scale gaugino masses.) Our low energy plots have no dependence on the physics above the GUT scale, all the model dependence comes into the predictions for low energy parameters the GUT makes. If the predicted region of low energy parameters corresponds to a $\mu \to e\gamma$ rate exceeding experimental bounds, the theory is ruled out.

There is a more practical reason for working directly with low-energy parameters specific to large $\tan\beta$: the well known difficulty in achieving electroweak symmetry breaking in this regime. Working with high energy parameters, and imposing universal scalar masses necessitates a fine-tune to achieve $SU(2)\times U(1)$ breaking. However, we have nowhere in our analysis made the assumption of universal scalar masses, hence the Higgs masses and squark/slepton masses are independent in our analysis, and therefore the μ parameter is not tightly constrained by squark/slepton masses. Working with weak scale parameters allows us to assume that the desired breaking has occurred without having to know the details of the breaking.

With the aforementioned assumption about the existence of a GUT, and assuming degeneracy between the first two generations, the rate for $\mu \to e\gamma$ depends on the weak scale parameters $\mu, \tan\beta, M_2, m^2_{\tilde{e}_L}, m^2_{\tilde{\tau}_L}, m^2_{\tilde{e}_R}, m^2_{\tilde{\tau}_R}, W_{L3i}, W_{R3i}$. We know that the amplitude depends on $W_{L(R)3i}$ simply through the product $W_{L3i}W_{R3j}$, so for normalization in our plots we put $W_{L(R)3i} = V_{KM3i}$. Any deviation from this can be simply multiplied into the rate. We also fix $\tan\beta = 60$, and put $m_{\tilde{\tau}_{L(R)}} = m_{\tilde{e}_{L(R)}} - \Delta_{L(R)}$. Next, we use some high energy bias to relate $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$: we assume that their difference is proportional to M_2 (as would be the case if they started out degenerate and were split only through different gauge interactions), so we put $m_{\tilde{e}_L} = m_{\tilde{e}_R} - rM_2$. In all specific models we have looked at, r is small (less than about .2). We find that, as long as r is small, the rate has little dependence on its exact value, so we put $r = 0, m_{\tilde{e}_L} = m_{\tilde{e}_R} \equiv \overline{m}_{\tilde{e}}$. We also found that as long as $\frac{\Delta_L}{\Delta_R}$ is close to 1, there is little dependence on its actual value either, so we put $\Delta_L = \Delta_R \equiv \Delta$.

Now, the $\mu \to e\gamma$ rate depends only on $\mu, M_2, \overline{m}_{\tilde{e}}$ and Δ, and we have the large $\tan\beta$ interpretation of μ and M_2 as chargino masses. Fixing $\overline{m}_{\tilde{e}} = 300$ GeV, we make contour plots of $Br(\mu \to e\gamma)$. The rate scales roughly as $\overline{m}_{\tilde{e}}^{-4}$ and μ^2 for scalar masses heavy compared with gaugino masses. In Fig. 7, we fix μ and plot in $M_2 - \Delta$ space. In Fig. 8, we fix Δ and plot in $\mu - M_2$ space. In Fig. 9, we plot the values of Δ predicted by the GUT against M_2, for various values of $\lambda_t(M_G)$ and $A_e(M_S)$ and for two values of b_5, the gauge beta function coefficient above the GUT scale. In Fig. 10, we plot the suppression factor ϵ for the same parameter set as in Fig. 9. We see that, over a significant region in parameter space, ϵ is small, between 0.2 and 0.01.

It is clear from Fig. 7 that, with no suppression, a typical value for Δ of 0.3 ($\times$300GeV) would give rise to rates above the current bound of $Br(\mu \to e\gamma) < 4.9 \times 10^{-11}$[19]. However, from Fig. 10, the suppression from ϵ is seen to be typically 20, allowing Δ's of up to 0.45 ($\times$300GeV). We see that ϵ is crucial in giving the GUT more breathing room, as Δ's of less than 0.45 are more common. From Fig. 8 it is also clear that regions of small μ and M_2 (that is, light chargino masses) are preferred. Smaller μ is preferred because it decreases the L-R mass $\mu m_\tau \tan\beta$, small M_2 is preferred because in the limit that the neutralino mass tends to zero, the diagrams Fig. 5 vanish. We also note that smaller μ, M_2 are preferred for electroweak symmetry breaking[7, 8].

If μ and M_2 are both small, the lightest supersymmetric particle (LSP) can be quite light, (but where it has significant higgsino component, it must be heavier than 45 GeV in order to be consistent with the precise measurement of the Z width), and it annihilates (primarily through its higgsino components) through a Z into fermion antifermion pairs much like a heavy neutrino. The contribution of the LSP to energy density of the universe Ωh^2 then just depends on its mass, and the size of its higgsino components, both of which only depend on μ and M_2 in the large $\tan\beta$ limit. In Fig. 11, we make a plot of Ωh^2 in $\mu - M_2$ space. We see that it is possible to get $\Omega \sim O(1)$ in some region of the parameter space.

VII. The Example of ADHRS Models

In this section, we study the ADHRS models [9] which are known to give realistic fermion masses and mixing patterns. These models are specific enough for us to do calculations and make some real predictions. Although not necessarily correct, they are good representatives of general GUT models. We believe that by studying them, one can see in detail the general features of generic realistic GUT models and the differences between them and the minimal $SU(5)$ or $SO(10)$ models.

As mentioned in Sec. II, in ADHRS models, the three families of quarks and leptons lie in three 16 dimensional representations of $SO(10)$, and the two low energy Higgs doublets lie in a single 10 dimensional representation. Only the third generation Yukawa couplings come from a renormalizable interaction

$$\lambda_{33} 16_3 16_3 10. \tag{7.1}$$

All other small Yukawa couplings come from nonrenormalizable interactions after integrating out the heavy fields. These interactions can be written in general as

$$16_i \lambda_{ij}(A_a) 16_j 10. \tag{7.2}$$

The A_a's are fields in the adjoint representation of $SO(10)$ and their vevs break $SO(10)$ down to the Standard Model gauge group. Therefore, these Yukawa couplings can take different values for fermions of the same generation with different quantum numbers under $SU(3)\times SU(2)\times U(1)$ and a realistic fermion mass pattern and nontrivial KM matrix can be generated. In ADHRS models, the minimal number (four) of operators is assumed to generate the up, down-type quark and charged lepton Yukawa coupling matrices $\boldsymbol{\lambda}_U, \boldsymbol{\lambda}_D$ and $\boldsymbol{\lambda}_E$, and they take the form at M_G

$$\boldsymbol{\lambda}_U = \begin{pmatrix} 0 & z_u C & 0 \\ z'_u C & y_u E & x_u B \\ 0 & x'_u B & A \end{pmatrix}, \boldsymbol{\lambda}_D = \begin{pmatrix} 0 & z_d C & 0 \\ z'_d C & y_d E & x_d B \\ 0 & x'_d B & A \end{pmatrix}, \boldsymbol{\lambda}_E = \begin{pmatrix} 0 & z_e C & 0 \\ z'_e C & y_e E & x_e B \\ 0 & x'_e B & A \end{pmatrix}, \tag{7.3}$$

where the x, y, z's are Clebsch factors arising from the VEV's of the adjoint Higgs fields A_a. This form is known to give the successful relations $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$ and $V_{td}/V_{ts} = \sqrt{m_d/m_s}$ [20] so it is well motivated. Strictly speaking, the interaction (7.2) become the usual Yukawa form only after the adjoints A_a take their VEV's at the GUT scale. However, as we explained in Sec. II, they can be treated as the usual Yukawa interactions up to the ultraheavy scale (which we will assume to be M_{PL}) where the ultraheavy fields are integrated out if the wavefunction renormalizations of A_a's are ignored. In the one-loop approximation which we use later in calculating radiative corrections from M_{PL} to M_G, they give the same results, because the wavefunction renormalizations of the adjoints A_a only contribute at the two-loop order. This makes our analysis much easier. Above the GUT scale, in addition to the Yukawa interactions (2.4) which give the fermion masses, we have the interactions (2.5) as well. Each Yukawa matrix has different Clebsch factors x, y, z associated with its elements. All the Yukawa matrices have the ADHRS form

$$\lambda_I = \begin{pmatrix} 0 & z_I C & 0 \\ z_I' C & y_I E & x_I B \\ 0 & x_I' B & A \end{pmatrix}, \; I = qq, eu, ud, q\ell, nd, n\ell. \tag{7.4}$$

If each entry of the Yukawa matrices is generated dominantly by a single operator, like in the ADHRS models, then the phases of the same entries of all Yukawa matrices are identical. One can remove all but the λ_{22} phases by rephasing the operators. After phase redefinition only E is complex and is responsible for CP violation. In order to generate the realistic fermion mass and mixing pattern, one expects the following hierarchies,

$$\begin{aligned} \frac{B}{A} &\sim V_{cb} &&\sim \epsilon^2, \\ \frac{E}{A} &\sim \frac{m_s}{m_b} &&\sim \epsilon^2, \\ \frac{C}{E} &\sim \sin\theta_c \sim \epsilon, && \text{where } \epsilon \sim 0.2. \end{aligned} \tag{7.5}$$

The hierarchical Yukawa matrices can be diagonalized approximately [20], the unitary rotation matrices which diagonalize them at the GUT scale can be approximately written as

$$\lambda = \begin{pmatrix} 0 & zC & 0 \\ z'C & y\,|E|\,e^{\tilde{\phi}} & xB \\ 0 & x'B & A \end{pmatrix} = V_F^* \bar{\lambda} V_B^\dagger, \tag{7.6}$$

$$V_F \simeq \begin{pmatrix} e^{i\phi} & S_{F_1}e^{i\phi} & 0 \\ -S_{F_1} & 1 & S_{F_2} \\ S_{F_1}S_{F_2} & -S_{F_2} & 1 \end{pmatrix}, \tag{7.7}$$

$$V_B \simeq \begin{pmatrix} 1 & S_{B_1} & 0 \\ -S_{B_1}e^{-i\phi} & e^{-i\phi} & S_{B_2} \\ S_{B_1}S_{B_2}e^{-i\phi} & -S_{B_2}e^{-i\phi} & 1 \end{pmatrix}, \tag{7.8}$$

where

$$\begin{aligned} S_{F_2} &= \frac{xB}{A},\ S_{B_2} = \frac{x'B}{A}, \\ S_{F_1} &= \frac{zC}{E'},\ S_{B_1} = \frac{z'C}{E'}, \\ E' &= |yE - S_{F2}S_{B2}A| = \left|y|E|e^{i\tilde{\phi}} - \frac{x'xB^2}{A}\right|, \\ \tilde{\phi} &= arg(E),\ \phi = arg(yE - \frac{x'xB^2}{A}). \end{aligned}$$

The soft SUSY-breaking scalar masses for the three low energy generations and trilinear A terms are assumed to be universal[3] at Planck scale M_{PL} as in [2]. Beneath M_{PL}, the radiative corrections from the Yukawa couplings destroy the universalities and render the mixing matrices non-trivial. In the one-loop approximation, the radiative corrections to the soft SUSY-breaking parameters at M_G are simply related to the Yukawa coupling matrices and therefore the relations between general mixing matrix elements and KM matrix elements are also simple. This allows us to see the similar hierarchies in the general mixing matrices and the KM matrix very clearly. Although the one-loop approximation may not be a good approximation for quantities involving third generation Yukawa couplings, we will be satisfied with it since it simplifies things a lot and the uncertainties in other quantities such as Clebsch factors are probably much

[3] If the nonrenormalizable operators already appear in the superpotential of the underlying supergravity theory, the A terms will be different for different dimensional operators, and will induce unacceptably large $\mu \to e\gamma$ rate because the triscalar interactions and the Yukawa interactions can not be diagonalized in the same basis for the first two generations. In theories where the nonremormalizable operators come from integrating out heavy fields at M_{PL} and all the relevant interactions have the same A term, the resulting nonrenormalizable operators will also have the same A term.

bigger than the errors made in the one-loop approximation. The RG equations, for $\mathbf{m}_E^2$ as an example, from M_{PL} to M_G are

$$
\begin{aligned}
\frac{d}{dt}\mathbf{m}_E^2 &= \frac{1}{16\pi^2}\Big[2(2\boldsymbol{\lambda}_E\mathbf{m}_L^2\boldsymbol{\lambda}_E^\dagger + 2\boldsymbol{\lambda}_E m_{H_D}^2\boldsymbol{\lambda}_E^\dagger + \mathbf{m}_E^2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger + \boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger\mathbf{m}_E^2 + 2\zeta_E\zeta_E^\dagger) \\
&\quad + 3(2\boldsymbol{\lambda}_{eu}\mathbf{m}_U^2\boldsymbol{\lambda}_{eu}^\dagger + 2\boldsymbol{\lambda}_{eu} m_{H_{U_3}}^2\boldsymbol{\lambda}_{eu}^\dagger + \mathbf{m}_E^2\boldsymbol{\lambda}_{eu}\boldsymbol{\lambda}_{eu}^\dagger + \boldsymbol{\lambda}_{eu}\boldsymbol{\lambda}_{eu}^\dagger\mathbf{m}_E^2 + 2\zeta_{eu}\zeta_{eu}^\dagger) \\
&\quad - \text{gaugino mass contribution}\Big]. \qquad (7.9)
\end{aligned}
$$

In the one-loop approximation, the gaugino mass contributions are diagonal and the same for all three generations, so they can be absorbed into the common scalar masses and do not affect the diagonalization. The corrections to scalar masses at M_G have the following leading flavor dependence

$$
\begin{aligned}
\Delta\mathbf{m}_E^2 &\propto 2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger + 3\boldsymbol{\lambda}_{eu}\boldsymbol{\lambda}_{eu}^\dagger \\
&= 5\begin{pmatrix} \overline{z_e^2}C^2 & \overline{z_e y_e}CE^* & \overline{z_e x_e}CB \\ \overline{z_e y_e}CE & \overline{z_e'^2}C^2 + \overline{y_e^2}\,|E|^2 + \overline{x_e^2}B^2 & \overline{y_e x_e'}EB + \bar{x}_e BA \\ \overline{z_e x_e'}CB & \overline{y_e x_e'}E^*B + \bar{x}_e BA & \overline{x_e'^2}B^2 + A^2 \end{pmatrix}, \qquad (7.10)
\end{aligned}
$$

where the overline represents the weighted average of the Clebsch factors, $\overline{z_e^2} = \frac{1}{5}(2z_e^2 + 3z_{eu}^2)$ and so on. Because $\Delta\mathbf{m}_E^2$ is hierarchical, assuming no big x, y Clebsches (ADHRS models have some big z Clebsches), the rotation matrix which diagonalizes it can be given approximately as

$$
U_E(M_G) \simeq \begin{pmatrix} 1 & \bar{S}_{E_1} & \bar{S}_{E3} \\ -\bar{S}_{E_1}e^{-i\widetilde{\phi}} & e^{-i\widetilde{\phi}} & \bar{S}_{E3} \\ \bar{S}_{E_1}\bar{S}_{E_2}e^{-i\widetilde{\phi}} - \bar{S}_{E3} & -\bar{S}_{E2}e^{-i\widetilde{\phi}} & 1 \end{pmatrix}, \qquad (7.11)
$$

where

$$
\begin{aligned}
\bar{S}_{E_2} &= \frac{\bar{x}_e B}{A}, \quad \bar{S}_{E_3} = \frac{\overline{z_e x_e'}CB}{A^2} \\
\bar{S}_{E_1} &= \frac{\overline{z_e y_e}C\,|E|}{\overline{z_e'^2}C^2 + \overline{y_e^2}\,|E|^2 + (\overline{x_e^2} - \bar{x}_e^2)B^2}, \quad e^{-i\widetilde{\phi}} = \frac{E}{|E|}.
\end{aligned}
$$

Similarly, for other scalar masses the leading flavor dependent corrections at M_G are

$$
\Delta\mathbf{m}_L^2 \propto \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E + 3\boldsymbol{\lambda}_{q\ell}^\dagger\boldsymbol{\lambda}_{q\ell} + \boldsymbol{\lambda}_{n\ell}^\dagger\boldsymbol{\lambda}_{n\ell},
$$

$$
\begin{aligned}
\Delta\mathbf{m}_Q^2 &\propto \boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + \boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger + 2\boldsymbol{\lambda}_{qq}\boldsymbol{\lambda}_{qq}^\dagger + \boldsymbol{\lambda}_{q\ell}\boldsymbol{\lambda}_{q\ell}^\dagger,\\
\Delta\mathbf{m}_U^2 &\propto 2\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U + \boldsymbol{\lambda}_{eu}^\dagger\boldsymbol{\lambda}_{eu} + 2\boldsymbol{\lambda}_{ud}\boldsymbol{\lambda}_{ud}^\dagger,\\
\Delta\mathbf{m}_D^2 &\propto 2\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D + \boldsymbol{\lambda}_{nd}^\dagger\boldsymbol{\lambda}_{nd} + 2\boldsymbol{\lambda}_{ud}^\dagger\boldsymbol{\lambda}_{ud},
\end{aligned}
\tag{7.12}
$$

and the rotation matrices which diagonalize them are given by expressions similar to (7.11) with Clebsches replaced by the appropriate ones. Then, the mixing matrices appearing at the lepton-slepton-gaugino vertices are given by

$$
\begin{aligned}
W_{E_L}(M_G) &= U_L^\dagger V_{E_L}\\
&\simeq \begin{pmatrix} 1 & S_{E_{L1}} - S_{L1}e^{i(\widetilde{\phi}-\phi_e)} & \bar{S}_{L_1}(\bar{S}_{L_2} - S_{E_{L_2}})e^{i\widetilde{\phi}} - \bar{S}_{L_3} \\ (\bar{S}_{L_1} - \bar{S}_{E_{L_1}})e^{-i(\widetilde{\phi}-\phi_e)} & e^{i(\widetilde{\phi}-\phi_e)} & -(\bar{S}_{L_2} - S_{E_{L_2}})e^{i\widetilde{\phi}} \\ -S_{E_{L_1}}(\bar{S}_{L_2} - S_{E_{L_2}})e^{-i\phi_e} + \bar{S}_{L_3} & (\bar{S}_{L_2} - S_{E_{L_2}})e^{-i\phi_e} & 1 \end{pmatrix},
\end{aligned}
\tag{7.13}
$$

$$
\begin{aligned}
W_{E_L}(M_G) &= U_E^\dagger V_{E_R}\\
&\simeq \begin{pmatrix} e^{i\phi_e} & S_{E_{R1}}e^{i\phi}e - \bar{S}_{E_1}e^{i\widetilde{\phi}} & \bar{S}_{E_1}(\bar{S}_{E_2} - S_{E_{R2}})e^{i\widetilde{\phi}} - \bar{S}_{E_3} \\ \bar{S}_{E_1}e^{i\phi_e} - S_{E_{R1}}e^{i\widetilde{\phi}} & e^{i\widetilde{\phi}} & -(\bar{S}_{E_2} - S_{E_{R_2}})e^{-i\phi} \\ -S_{E_{R1}}(\bar{S}_{E_2} - S_{E_{R2}}) + \bar{S}_{E_3}e^{i\phi_e} & \bar{S}_{E_2} - S_{E_{R2}} & 1 \end{pmatrix},
\end{aligned}
\tag{7.14}
$$

where

$$
\begin{aligned}
S_{E_{L_1}} &= \frac{z'_eC}{E'_e},\ S_{E_{R_1}} = \frac{z_eC}{E'_e},\ E'_e = |y_eE - \frac{x'_ex_eB^2}{A}|,\\
S_{E_{L_2}} &= \frac{x'_eB}{A},\ S_{E_{R_2}} = \frac{x_eB}{A},\ \phi_e = arg\left(y_e\,|E|\,e^{i\widetilde{\phi}} - \frac{x'_ex_eB^2}{A}\right)\\
&\qquad\qquad\simeq \widetilde{\phi}\ \ (\text{if } y_e \sim x_e, x'_e),\\
\bar{S}_{E_1} &= \frac{\overline{z_ey_e}C|E|}{\overline{z_e'^2}C^2 + \overline{y_e^2}\,|E|^2 + (\overline{x_e^2} - \bar{x}_e^2)B^2},\ \bar{S}_{E_2} = \frac{\bar{x}_eB}{A},\ \bar{S}_{E_3} = \frac{\overline{z_ex'_e}CB}{A^2},\\
\bar{S}_{L_1} &= \frac{\widehat{z'_ey_e}C|E|}{\widehat{z_e^2}C^2 + \widehat{y_e^2}|E|^2 + (\widehat{x_e^2} - \hat{x}_e^2)B^2},\ \bar{S}_{L_2} = \frac{\widehat{x'_e}B}{A},\ \bar{S}_{L_3} = \frac{\widehat{z'_ex_e}CB}{A^2},
\end{aligned}
$$

and

$$
\bar{x}_e = \frac{1}{5}(2x_e + 3x_{eu}),\ \ \widehat{x'_e} = \frac{1}{5}(x'_e + 3x'_{g\ell} + x'_{n\ell})\ \text{ etc.}
$$

Note that

$$\bar{S}_{L_3} \sim \text{ Clebsch } \times \frac{CB}{A^2},\ S_{E_{L_1}}(\bar{S}_{L_2} - \bar{S}_{E_{L_2}}) = \text{Clebsch} \times \frac{CB}{EA}. \tag{7.15}$$

If there is no very big or small Clebsch involved and no accidental cancellation, $\bar{S}_{L_3}, \bar{S}_{E_3}$ can be neglected in W's.

Compared with V_{KM},

$$V_{KM}(M_G) = V_{U_L}^{\dagger} V_{D_L}$$

$$\simeq \begin{pmatrix} 1 & S_{D_{L_1}} - S_{U_{L_1}} e^{-i(\phi_d - \phi_u)} & -S_{U_{L_1}}(S_{D_{L_2}} - S_{U_{L_2}})e^{i\phi_u} \\ S_{U_{L_1}} - S_{D_{L_1}} e^{-i(\phi_d - \phi_u)} & e^{-i(\phi_d - \phi_u)} & (S_{D_{L_2}} - S_{U_{L_2}})e^{i\phi_u} \\ S_{D_{L_1}}(S_{D_{L_1}} - S_{U_{L_2}})e^{-i\phi_d} & -(S_{D_{L_2}} - S_{U_{L_2}})e^{-i\phi_d} & 1 \end{pmatrix}, \tag{7.16}$$

where

$$\begin{aligned} S_{U_{L1}} &= \frac{z_u C}{E'_u},\ E'_u = \left| y_u E - \frac{x_u x'_u B^2}{A} \right|, \phi_u = arg\left(y_u E - \frac{x_u x'_u B^2}{A} \right), \\ S_{D_{L1}} &= \frac{z_d C}{E'_d},\ E'_d = \left| y_d E - \frac{x_d x'_d B^2}{A} \right|,\ \phi_d = arg\left(y_d E - \frac{x_d x'_d B^2}{A} \right), \\ S_{U_{L2}} &= \frac{x_u B}{A}, \\ S_{D_{L2}} &= \frac{x_d B}{A}, \end{aligned}$$

we can see that the W's and V_{KM} do have similar hierarchical patterns, but have different Clebsch factors associated with their entries.

When a specific model is given, one can calculate all the Clebsch factors and make some definite predictions for that particular model. For example, the ADHRS Model 6, which gives results in good agreement with the experimental data, has the following four effective fermion mass operators

$$\begin{aligned} O_{33} &= 16_3\ 10\ 16_3, \\ O_{23} &= 16_2\ \frac{A_Y}{A_X}\ 10\ \frac{A_Y}{A_X} 16_3, \\ O_{23} &= 16_2 \frac{A_X}{M}\ 10\ \frac{A_{B-L}}{A_X}\ 16_2 \quad \text{or other 5 choices}, \\ O_{12} &= 16_1 \left(\frac{A_X}{M}\right)^3\ 10\ \left(\frac{A_X}{M}\right)^3\ 16_2, \end{aligned} \tag{7.17}$$

where A_X, A_Y, A_{B-L} are adjoint's of $SO(10)$ with VEV's in the $SU(5)$ singlet, hypercharge, and $B-L$ directions. There are six choices of O_{22} operators which give the same predictions for the fermion masses and mixings, but different Clebsches for other operators appearing above M_G. Fortunately, they do not enter the leading terms of the most important mixing matrix elements $W_{E_{L32}}, W_{E_{L31}}, W_{E_{R32}}, W_{E_{R31}}$, which appear in the leading contributions to the amplitudes of LFV processes and the electric dipole moment.

The magnitude of the mixing matrix elements $V_{KM32}, V_{KM31}, W_{E_{L32}}, W_{E_{L31}}, W_{E_{R32}}, W_{E_{R31}}$, and the relevant Clebsch factors are listed in Tables 2 and 3.

Table 2

	u	d	e	eu	$q\ell$	$n\ell$
x	-1	$-\frac{1}{6}$	$\frac{3}{2}$	-6	$\frac{1}{4}$	0
x'	-1	$-\frac{1}{6}$	$\frac{3}{2}$	-6	$\frac{1}{4}$	0
y	0	1	3	-	-	-
z	$-\frac{1}{27}$	1	1	$-\frac{1}{27}$	1	125
z'	$-\frac{1}{27}$	1	1	$-\frac{1}{27}$	1	125
$\widehat{x'_e} = \frac{9}{20}$, $\overline{x_e} = -3$						

Table 2: Clebsch factors for Yukawa coupling matrices in ADHRS model 6.

Table 3

	ADHRS models	Model 6	Relevant process
$\left\lvert W_{E_{L32}}/V_{ts}\right\rvert$	$\left\lvert\frac{\widehat{x'_e}-x'_e}{x_d-x_u}\right\rvert$	1.26	
$\lvert W_{E_{R32}}/V_{ts}\rvert$	$\left\lvert\frac{\bar{x}_e-x_e}{x_d-x_u}\right\rvert$	5.4	
$\lvert W_{E_{L31}}/V_{td}\rvert$	$\left\lvert\frac{z'_e y_d(\widehat{x'_e}-x'_e)}{z_d y_e(x_d-x_u)}\right\rvert$	0.42	
$\lvert W_{E_{R31}}/V_{td}\rvert$	$\left\lvert\frac{z_e y_d(\bar{x}_e-x_e)}{z_d y_e(x_d-x_u)}\right\rvert$	1.8	
$\left\lvert\frac{W_{E_{L32}} W_{E_{R31}}}{V_{ts}V_{td}}\right\rvert$	$\left\lvert\frac{z_e y_d(\bar{x}_e-x_e)(\widehat{x'_e}-x'_e)}{z_d y_e(x_d-x_u)^2}\right\rvert$	2.268	$\mu \to e\gamma$ amplitude
$\left\lvert\frac{W_{E_{R32}} W_{E_{L31}}}{V_{ts}V_{td}}\right\rvert$	$\left\lvert\frac{z'_e y_d(\bar{x}_e-x_e)(\widehat{x'_e}-x'_e)}{z_d y_e(x_d-x_u)^2}\right\rvert$	2.268	$\mu \to e\gamma$ amplitude
$\left\lVert\left(\frac{\sqrt{2}W_{E_{L31}} W_{E_{R32}}}{\sqrt{\lvert W_{E_{L32}} W_{E_{R31}}\rvert^2+\lvert W_{E_{R32}} W_{E_{L31}}\rvert^2}}\right) / \left(\frac{V_{td}}{V_{ts}}\right)\right\rVert$	$\left\lvert\frac{\sqrt{2}z'_e z_e y_d}{\sqrt{(z_e^2+z_e'^2)z_d y_e}}\right\rvert$	$\frac{1}{3}$	d_e

Table 3: The relevant Clebsch factors for $\mu \to e\gamma$ and d_e in ADHRS model 6.

In ADHRS models $\tan\beta$ is large. The $\mu \to e\gamma$ rate for large $\tan\beta$ has been calculated in Sec. V and VI for $W_{E_{L32}} = W_{E_{R32}} = V_{ts}$ and $W_{E_{L31}} = W_{E_{R31}} = V_{td}$. To obtain the predictions of ADHRS models we only have to multiply the results by the suitable Clebsch factors. The relevant Clebsch factors for Model 6 are listed in Table 3. For a generic realistic GUT model with small $\tan\beta$, for example the modified ADHRS models in which the down type Higgs lies predominantly in some fields which do not interact with the three low energy generations and contain only a small fraction of the doublets in the 10 which interact with the low energy generations [21], most of analysis should still hold. In this case the leading contributions to $\mu \to e\gamma$ are the same ones as in the minimal $SO(10)$ model of Ref. [2] (Fig. 10 $b_{L,R}, c_{L,R}, c'_{L,R}$ of [2]). The diagrams c_{LR}, c'_{LR} involve the corrections to the trilinear scalar couplings.

In the one-loop approximation the leading corrections to ζ_E at M_G contain pieces proportional to $\boldsymbol{\lambda}_E, \boldsymbol{\lambda}_E(\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E + 3\boldsymbol{\lambda}_{q\ell}^\dagger\boldsymbol{\lambda}_{q\ell} + \boldsymbol{\lambda}_{n\ell}^\dagger\boldsymbol{\lambda}_{n\ell}), (2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger + 3\boldsymbol{\lambda}_{eu}\boldsymbol{\lambda}_{eu}^\dagger)\boldsymbol{\lambda}_E$ respectively. The piece proportional to $\boldsymbol{\lambda}_E$ can be absorbed into ζ_{E_0} by a redefinition of A_E, the other two pieces are proportional to the product of $\boldsymbol{\lambda}_E$ and the corrections to the scalar masses,

$$\begin{aligned}
\Delta\zeta_E &= \Delta\zeta_{E_R} + \Delta\zeta_{E_L} \\
\Delta\zeta_{E_R} &= \frac{1}{\mu_{E_R}}\Delta\mathbf{m}_E^2\,\boldsymbol{\lambda}_E \\
\Delta\zeta_{E_L} &= \frac{1}{\mu_{E_L}}\boldsymbol{\lambda}_E\,\Delta\mathbf{m}_L^2
\end{aligned} \tag{7.18}$$

where μ_{E_L}, μ_{E_R} are proportional constants ($\mu_{E_R} = \mu_{E_L} = \frac{6m_0^2+A_0^2}{3A_0}$ in one-loop approximation). The LFV couplings in Fig. 2e, $\tilde{e}_R^T U_E^T \Delta\zeta_E U_L \tilde{e}_L v_D$, now can be written as

$$\begin{aligned}
&\frac{1}{\mu_{E_R}}\tilde{e}_R^T U_E^T \Delta\mathbf{m}_E^2 \boldsymbol{\lambda}_E U_L \tilde{e}_L v_D + \frac{1}{\mu_{E_L}}\tilde{e}_R^T U_E^T \boldsymbol{\lambda}_E \Delta\mathbf{m}_L^2 U_L \tilde{e}_L v_D \\
&= \frac{1}{\mu_{E_R}}\tilde{e}_R^T \Delta\overline{\mathbf{m}}_E^2 W_{E_R}^* \bar{\boldsymbol{\lambda}}_E W_{E_L}^\dagger \tilde{e}_L v_D + \frac{1}{\mu_{E_L}}\tilde{e}_R^T W_{E_R}^* \bar{\boldsymbol{\lambda}}_E W_{E_L}^\dagger \Delta\overline{\mathbf{m}}_L^2 \tilde{e}_L v_D,
\end{aligned} \tag{7.19}$$

where the overline means that the matrix is diagonal. Again, the amplitudes are given by the same formulas as in [2] (eqn. 29, 30), except that $V_{32}^e V_{31}^e (V_{33}^{e*})^2$ has to be replaced by $W_{E_{L32}} W_{E_{R31}} W_{E_{L33}}^* W_{E_{R33}}^*$, and $W_{E_{R32}} W_{E_{L31}} W_{E_{R33}}^* W_{E_{L33}}^*$, and $\frac{5}{7} I_G'$ by $\frac{1}{\mu_{E_R}}\Delta\overline{m}_{E33}^2$ and $\frac{1}{\mu_{E_L}}\Delta\overline{m}_{L33}^2$. The results in [2] are only modified by some multiplicative factors and therefore represent the central values for the LFV processes.

It was pointed out in [2, 3] that the electric dipole moment of the electron (d_e) constitutes an independent and equally important signature for the $SO(10)$ unified theory as $\mu \to e\gamma$ does. The diagrams which contribute to the electric dipole moment of the electron are the same as the ones which contribute to $\mu \to e\gamma$, with $\mu_L(\mu_L^c)$ replaced by $e_L(e_L^c)$. Thus a simple relation between d_e and the $\mu \to e\gamma$ rate was obtained in the minimal $SO(10)$ model [2],

$$\Gamma(\mu \to e\gamma) = \frac{\alpha}{2} m_\mu^3 |F_2|^2, \tag{7.20}$$

$$|d_e| = e|F_2| \left|\frac{V_{td}}{V_{ts}}\right| \sin\phi = e\sqrt{\frac{2\Gamma(\mu \to e\gamma)}{\alpha m_\mu^3}} \left|\frac{V_{td}}{V_{ts}}\right| \sin\phi, \tag{7.21}$$

where ϕ is an unknown new CP violating phase defined by

$$Im[m_\tau (V^e_{31})^2 (V^{e*}_{33})^2] = |m_\tau (V^e_{31})^2 (V^{e*}_{33})^2| \sin\phi.$$

In a more generic $SO(10)$ model, such as the ADHRS model, we still have this simple relation but the mixing matrix elements have to be replaced by the W's:

$$|d_e| = e\sqrt{\frac{2\Gamma(\mu \to e\gamma)}{\alpha m_\mu^3}} \frac{\sqrt{2}|W_{E_{L31}} W_{E_{R31}}|}{\sqrt{|W_{E_{L32}} W_{E_{R31}}|^2 + |W_{E_{R31}} W_{E_{L31}}|^2}} \sin\phi', \qquad (7.22)$$

where ϕ' is defined by

$$Im[m_\tau W_{E_{L31}} W_{E_{R31}} W^*_{E_{R33}} W^*_{E_{L33}}] = |m_\tau W_{E_{L31}} W_{E_{R31}} W^*_{E_{L33}} W^*_{E_{R33}}| \sin\phi'.$$

In particular, in ADHRS models there is only one CP violating phase, so the phase ϕ' can be related to the phase appeared in the KM matrix of the Standard Model. From eqn. (7.13) (7.14) (7.16) we can see that $\phi' \approx \phi_e, \phi_e \approx \phi_d \approx \tilde{\phi}, \phi_u = 0$ (because $y_u = 0$). The rephrase invariant quantity J of the KM matrix is given by

$$\begin{aligned} J &= Im V_{ud} V_{tb} V^*_{td} V^*_{ub} \\ &\simeq -S_{U_{L1}} S_{D_{L1}} (S_{D_{L2}} - S_{U_{L2}})^2 \sin\phi_d. \end{aligned} \qquad (7.23)$$

Therefore the CP violating phase appeared in d_e related to the CP violation in the Standard Model by

$$\sin\phi' \simeq \frac{J}{|V_{td}||V_{ub}|}. \qquad (7.24)$$

Finally, as mentioned in the Sec. III, we consider the possibility that the slight non-degeneracy between the first two generation scalar masses could give a significant contribution to the flavor changing processes because of the larger mixing matrix elements. We still use ADHRS models as an example to estimate this contribution to the LFV process $\mu \to e\gamma$. For an order of magnitude estimate, the mass insertion approximation in the super-KM basis employed in [1] will serve as a convenient method. After rotating the $\Delta\mathbf{m}^2_E$ in eqn. (7.10) to the charged lepton mass eigenstate basis, the contribution from the first two generations to $\Delta m^2_{E_{21}}$ is

$$\Delta m^2_{E_{21}}(2\text{-}1) \simeq V_{E_{R22}} V^*_{E_{R21}} \Delta m^2_{E22} + V_{E_{R12}} V^*_{E_{R11}} \Delta m^2_{E_{11}} + V_{E_{R22}} V^*_{E_{R11}} \Delta m^2_{E_{21}}$$

$$\simeq [-\frac{z_e C}{E'_e}\frac{\overline{z_e'^2}C^2+\overline{y_e^2}|E|^2+\overline{x_e^2}B^2}{A^2}+\frac{z_e C}{E'_e}\frac{\overline{z_e^2}C^2}{A^2}+e^{-i\phi_e}\overline{z_e y_e}\frac{CE}{A^2}]\Delta m^2_{E_{33}}$$

$$\simeq -\frac{z_e}{y_e}\left[\frac{\overline{y_e^2}C|E|+\overline{x_e^2}\frac{CB^2}{|E|}+\overline{z_e y_e}C|E|}{A^2}\right]\Delta m^2_{E_{33}}$$

$$\text{(assume } z_e = z'_e \text{ as in ADHRS model).} \quad (7.25)$$

Compared with the result found in [1] for minimal $SU(5)$:

$$\begin{aligned}\Delta m^2_{E_{21}}(BH) &= V^*_{ts}V_{td}\Delta m^2_{E_{33}}\\ &\simeq -\frac{z_d C}{E'_d}\frac{(x_d-x_u)^2B^2}{A^2}\Delta m^2_{E_{33}}\\ &\simeq -\frac{z_d}{y_d}\frac{(x_d-x_u)^2\frac{CB^2}{|E|}}{A^2}\Delta m^2_{E_{33}},\end{aligned} \quad (7.26)$$

we can see that if the Clebsch factors are $O(1)$, this contribution is comparable to that of the minimal $SU(5)$ model. In order for this contribution to be competitive with the dominant diagrams (Fig. 10 $b_{L,R}, c_{L,R}, c'_{L,R}$ of [2]) which are enhanced by $\frac{m_\tau}{m_\mu}$, large Clebsch factors are required. While it is possible to have large Clebsch factors, we consider them as model dependent, not generic to all realistic unified theories.

VIII. Conclusions

In supersymmetric theories, the Yukawa interactions which violate flavor symmetries not only generate the quark and lepton mass matrices, but necessarily also lead to radiative breaking of flavor symmetries in the squark and slepton mass matrices, leading to a variety of flavor signals. While such effects have been well studied in the MSSM and, more recently, in minimal unified models, the purpose of this paper has been to explore these phenomena in a wide class of grand unified models which have realistic fermion masses.

We have argued that, if the hardness scale Λ_H is above M_G, the expectation for all realistic grand unified supersymmetric models is that non-trivial flavor mixing matrices should occur at *all* neutral gaugino vertices. These additional, weak scale, flavor violations are expected to have a form similar to the Kobayashi-Maskawa matrix. However, the precise values of the matrix elements are model dependent and have renormalization group scalings which differ from those of the Kobayashi-Maskawa matrix elements.

It is the non-triviality of the flavor mixing matrices of neutral gaugino couplings in the up quark sector which strongly distinguishes between the general and minimal unified models, as shown in Table 1. Although the minimal unified models provide a simple approximation to flavor physics, they are not realistic, so we stress the important new result that flavor mixing in the up sector couplings of neutral gauginos is a necessity in unified models. this leads to four important phenomenological consequences. While the $D^0 - \bar{D}^0$ mixing induced by this new flavor mixing is generally not close to the present experimental limit, it could be much larger than that predicted in the standard model.

The new mixing in the up-quark sector implies that there may be significant radiative contributions to the up quark mass matrix which arise when the superpartners are integrated out of the theory. This is illustrated in Figure 4, where the new mixing matrix elements have been taken to be a factor of three larger than the corresponding Kobayashi-Maskawa matrix elements. In this case the entire up quark mass could be generated by such a radiative mechanism: above the weak scale the violation of up quark flavor symmetries lies in the squark mass matrix.

The electric dipole moment of the neutron, d_n, is a powerful probe of the neutral gaugino flavor mixing induced by unified theories. In the minimal $SO(10)$ theory, d_n arises from the flavor mixing in the down sector, which leads to a down quark dipole moment, d_d. However, in realistic models the flavor mixing in the up quark sector leads to a d_u which typically provides the dominant contribution to d_n. Thus the neutron electric dipole moment is a more powerful probe of unified supersymmetric theories than previously realized.

The presence of flavor mixing in the up sector plays a very important role in determining the branching ratio for a proton to decay to $K^0\mu^+$. In the minimal models, without such mixings, this branching ratio is expected to be about 10^{-3}: the charged lepton mode will not be seen and experimental efforts must concentrate on the mode containing a neutrino, $K^+\nu$. However, including these mixings the charged lepton branching ratio is greatly increased to about 0.1. While this number is very model dependent, we nevertheless think that this effect greatly changes the importance of searching for the charged lepton mode.

These four phenomenologocal consequences are sufficiently interesting that we stress once more that they appear as a necessity in a wide class of unified theories. The absence of mixing in the up sector is a special feature of the minimal models. Since the flavor sectors of the minimal models must be augmented to obtain realistic fermion masses, any conclusions based on the absence of flavor mixings in the up sector are specious.

A second topic addressed in this paper is the effect of large $\tan\beta$ on the lepton process, $\mu \to e\gamma$ which is expected in unified supersymmetric $SO(10)$ models. The amplitude for this process has a contribution proportional to $\tan\beta$. In this paper, we have found that the naive expectation that large $\tan\beta$ in supersymmetric $SO(10)$ is excluded by $\mu \to e\gamma$ is incorrect, at least for all values of the superpartner masses of interest. Contour plots for the $\mu \to e\gamma$ branching ratio are shown in Figures 7 and 8. It depends sensitively on the parameter Δ, which is the mass splitting between the scalar electron and scalar tau, and is plotted in Figure 9. Lower values of the top quark Yukawa coupling, which for large $\tan\beta$ still give allowed predictions for the b/τ mass ratio, give a much reduced value for Δ, thereby reducing the $\mu \to e\gamma$ rate and partially compensating the $\tan^2\beta$ enhancement. A further significant suppression of an order of magnitude is induced by the renormalization group scaling of the leptonic flavor mixing angles, and is shown in Figure 10. The net effect is that while the case of $\tan\beta \approx m_t/m_b$ is not excluded in SO(10), the $\mu \to e\gamma$ rate is still typically larger than for moderate $\tan\beta$, so that this process provides a more powerful probe of the theory as $\tan\beta$ increases.

For large $\tan\beta$, μ and M_2 become the physical masses of the two charginos. The $\mu \to e\gamma$ contours of Figure 8 show that μ and M_2 should not be too large, providing an important limit to the chargino masses in the large $\tan\beta$ limit. Furthermore, this constrains the LSP mass to be quite small. We find that in this region it is still possible for the LSP to account for the observed dark matter, and even to critically close the universe, as can be seen from Figure 11. However, the requirement that the LSP mass be larger than 45 GeV suggests that the two light charginos will not be light enough to be discovered at LEP II.

As an example of theories with both a realistic flavor sector and large $\tan\beta$ we studied the models introduced by Anderson et al. The flavor sectors of these theories are economical: the free parameters can all be fixed from the known quark and lepton masses and mixings. Hence the flavor mixing matrices at all neutral gaugino vertices can be calculated. These are shown for the lepton sector of model 6 in Table 3. The Clebsch factors enhance the $\mu \to e\gamma$ amplitude by a factor of 2.3, and suppress d_e by a factor of 3. Even taking the top quark Yukawa coupling to have its lowest value the rate for $\mu \to e\gamma$ in this theory is very large. Another interesting feature of these theories is that the flavor sectors contain just a single CP violating phase. This means that the phase which appears in the result for d_n and d_e can be computed: since it is closely related to the phase of the Kobayashi-Maskawa matrix it is not very small. That which appears in d_e is given in eqn. (7.24) and is numerically about 0.2. We have computed the radiative corrections to m_u in the ADHRS models and have found that the new mixing matrices in the up sector are not large enough to yield sizable contributions: thus the ADHRS analysis of the quark mass matrices is not modified. Furthermore, due to a cancellation special to these theories, there is no contribution to d_n from the up quark at one loop.

Acknowledgements

The authors would like to thank Hitoshi Murayama for many useful discussions. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under contract No. DE-AC03-76SF00098 and in part by the National Science Foundation under Grant No. PHY-90-21139. The work of N.A-H was supported by an NSERC '67 fellowship.

Note Added:

While finalizing this work, we received a preprint by Ciafaloni, Romanio and Strumia[22], where the large $\tan\beta$ scenario is also considered. However, unlike this work, they assume strict universality in soft scalar masses, such that imposing electroweak symmetry breaking leads them into a region of parameter space with a high mass (1 TeV) for the sleptons. In their discussion of general models, they do not include flavor violating RG scaling of scalar masses above M_G.

Appendix

In this appendix, we first give a more complete treatment of mixing matrix scaling in the lepton sector, and then give a treatment for the quark sector.

Let us return to (5.7) and consider the effect of including the $(\zeta_E^\dagger \zeta_E)_{3i}$ term. In general the scaling from M_{PL} to M_G will generate a $\zeta_E^\dagger \zeta_E$ not diagonal in the same basis as $\lambda_E^\dagger \lambda_E$, so we expect some non-zero $(\zeta_E^\dagger \zeta_E)_{3i}$. From the RGE for ζ_E, neglecting gauge couplings,

$$-\frac{d}{dt}\zeta_E = \zeta_E[5\lambda_E^\dagger \lambda_E + Tr(3\lambda_D^\dagger \lambda_D + \lambda_E^\dagger \lambda_E)] + \lambda_E[4\lambda_E^\dagger \zeta_E + Tr(6\zeta_D \lambda_D^\dagger + 2\zeta_E \lambda_E^\dagger)]. \qquad (A.1)$$

We have

$$\begin{aligned} -\frac{d}{dt}(\zeta_E^\dagger \zeta_E) &= 5[\zeta_E^\dagger \zeta_E \lambda_E^\dagger \lambda_E + \lambda_E^\dagger \lambda_E \zeta_E^\dagger \zeta_E] + 2Tr(3\lambda_D^\dagger \lambda_D + \lambda_E^\dagger \lambda_E)\zeta_E^\dagger \zeta_E \\ &+ 8\zeta_E^\dagger \lambda_E \lambda_E^\dagger \zeta_E + (\zeta_E^\dagger \lambda_E + \lambda_E^\dagger \zeta_E) Tr(6\zeta_D \lambda_D^\dagger + 2\zeta_E \lambda_E^\dagger). \qquad (A.2) \end{aligned}$$

Then, to first order in the off diagonal parts of $\zeta_E^\dagger \zeta_E$ and $\zeta_E \zeta_E^\dagger$, and keeping only third generation Yukawa couplings we have

$$-\frac{d}{dt}(\zeta_E^\dagger \zeta_E)_{3i} = (\zeta_E^\dagger \zeta_E)_{3i}[17\lambda_\tau^2 + 6\lambda_b^2 + 6\eta \lambda_b \lambda_\tau], \qquad (A.3)$$

where $\eta \equiv \frac{\zeta_{D_{33}}}{\zeta_{E_{33}}}$. Because of the large numerical coefficient in front of $\lambda_\tau^2, \lambda_b^2$ in the above equation, $(\zeta_E^\dagger \zeta_E)_{3i}$ is driven to zero more rapidly than W_{L3i}, after which it ceases to have any effect on the running of W_{L3i}. More explicitly, from (5.7) we have that

$$\frac{d}{dt}(m_{L3i}^2(t) e^{\int_0^t dt' \lambda_\tau^2(t')}) = -2(\zeta_E^\dagger \zeta_E)_{3i}(t) e^{\int_0^t dt' \lambda_\tau^2(t')}. \qquad (A.4)$$

Solving (A.3) for $(\zeta_E^\dagger \zeta_E)_{3i}(t)$ and inserting into (A.4) we get

$$-\frac{d}{dt}\left(m_{L3i}^2(t) e^{\int_0^t dt' \lambda_\tau^2(t')}\right) = 2(\zeta_E^\dagger \zeta_E)_{3i}(M_G) e^{-\int_0^t dt'[16\lambda_\tau^2 + 6\lambda_b^2 + 6\eta\lambda_b\lambda_t](t')}. \qquad (A.5)$$

Integrating (A.5), we find

$$\begin{aligned} m_{L3i}^2(M_S) e^{I_\tau} - m_{L3i}^2(M_G) &= -2\int_0^{\frac{1}{16\pi^2}\log\frac{M_G}{M_S}} dt e^{-\int_0^t dt'[16\lambda_\tau^2 + 6\lambda_b^2 + 6\eta\lambda_b\lambda_\tau](t')} \\ &\times (\zeta_E^\dagger \zeta_E)_{3i}(M_G) \\ &\equiv \delta(\zeta_E^\dagger \zeta_E)_{3i}(M_G). \qquad (A.6) \end{aligned}$$

So, we have

$$m^2_{L3i}(M_S) = e^{-I_\tau}[m^2_{L3i}(M_G) + \delta(\zeta^\dagger_E\zeta_E)(M_G)]. \tag{A.7}$$

We expect m^2_{L3i} and $(\zeta^\dagger_E\zeta_E)_{3i}$ to be related by some combination of Clebsches x at M_G as follows:

$$(\zeta^\dagger_E\zeta_E)_{3i} = \frac{A_0^2}{m_0^2} x m^2_{L3i} \tag{A.8}$$

Where A_0, m_0^2 are the universal A parameter and scalar mass at M_{PL}, respectively. Then, we have from (A.7)

$$W^\dagger_{L33}W_{L3i}(M_S) = e^{-I_\tau}\frac{\Delta m^2(M_G)}{\Delta m^2(M_S)}[1 + \delta\frac{A_0^2}{m_0^2}x]W^\dagger_{L33}W_{L3i}(M_G). \tag{A.9}$$

Clearly if $\delta\frac{A_0^2}{m_0^2}x \ll 1$, inclusion of the $(\zeta^\dagger_E\zeta_E)_{3i}$ term in (5.7) do not change any of our results. If $\delta\frac{A_0^2}{m_0^2}x \sim 1$ or $\gg 1$, we can still of course use (A.9), but the suppression effect may disappear. A simple estimate shows, however, that δ itself is already small $\sim \frac{1}{10}$, and so we are only in trouble if $\frac{A_0^2}{m_0^2}x$ is big. To see this, replace λ_τ, λ_b and η by some average values $\bar\lambda_\tau, \bar\lambda_b$ and $\bar\eta$ in the expression (A.6) for δ. Then,

$$\begin{aligned}\delta &= -2\int_0^{\frac{1}{16\pi^2}\log\frac{M_G}{M_S}} e^{-t(16\bar\lambda_\tau^2+6\bar\lambda_b^2+6\bar\lambda_b\bar\lambda_t\bar\eta)} \\ &= -\frac{1}{8\bar\lambda_t^2 + 3(\bar\lambda_b^2 + \bar\eta\bar\lambda_b\bar\lambda_\tau)}\left[e^{-\frac{1}{16\pi^2}\log\frac{M_G}{M_S}(16\bar\lambda_\tau^2+6\bar\lambda_b^2+6\bar\eta\bar\lambda_b\bar\lambda_\tau)}\right].\end{aligned} \tag{A.10}$$

So,

$$|\delta| < \frac{1}{8\bar\lambda_\tau^2 + 3(\bar\lambda_b^2 + \bar\eta\bar\lambda_b\bar\lambda_\tau)}. \tag{A.11}$$

For the $\bar\lambda$'s between 0.5 and 1, and $\bar\eta \sim 1, |\delta|$ ranges from $\frac{1}{3}$ to $\frac{1}{15}$.

How can we qualitatively understand the above results for the scaling of mixing matrices? The renormalization group equations try to align the soft supersymmetry breaking flavor matrices with whatever combination of flavor matrices responsible for their renormalization. However, because a given coupling can only be renormalized by harder couplings, there is a hierarchy in which flavor matrices affect the running of others. The Yukawa matrices, being dimensionless, can only be affected by other Yukawa matrices. In the lepton sector, this is the reason that the basis in which e.g. $\boldsymbol{\lambda}^\dagger_E\boldsymbol{\lambda}_E$ is diagonal does not change. Next, the soft trilinear terms, having mass dimension one, can only be affected by other trilinear terms and Yukawa couplings. Again in the lepton sector this means that e.g. $\zeta_E^\dagger \zeta_E$ tries to align itself with $\lambda_E^\dagger \lambda_E$. Finally, the scalar mass, having dimension two, are affected by everything: $\mathbf{m}_L^2$ tries to align with $\lambda_E^\dagger \lambda_E$, but suffers interference from $\zeta_E^\dagger \zeta_E$, unless $\zeta_E^\dagger \zeta_E$ is diagonal in the same basis as $\lambda_E^\dagger \lambda_E$. Even if $\zeta_E^\dagger \zeta_E$ is not diagonal in the same basis as $\lambda_E^\dagger \lambda_E$, it is trying to align itself with $\lambda_E^\dagger \lambda_E$, so $\mathbf{m}_L^2$ will still tend to align with $\lambda_E^\dagger \lambda_E$.

From the above discussion, it is clear that the situation is slightly complicated in the quark sector. In the lepton sector, there was a fixed direction in flavor space given by λ_E, with which the soft matrices aligned. In the quark sector, we have both λ_U and λ_D, and $\lambda_U \lambda_U^\dagger$, $\lambda_D \lambda_D^\dagger$ are misaligned ($V_{KM} \neq 1$). This complicates the analysis for W_{U_L}, W_{D_L} so we discuss them last. Let us now examine the scaling of W_{U_R}, W_{D_R}. (Throughout the following, we assume degeneracy between first two generation scalar masses, we neglect all Yukawa coupling matrix eigenvalues except those of the third generation, and we do not include the effect of trilinear soft terms in the scaling. The last assumption is made for simplicity; we can make similar arguments about the importance of these neglected trilinear terms as we did above in the lepton sector.)

First, we show that the basis in which $\lambda_U^\dagger \lambda_U$ is diagonal remains fixed. The RGE for $\lambda_U^\dagger \lambda_U$ is

$$-\frac{d}{dt}\lambda_U^\dagger \lambda_U = 6(\lambda_U^\dagger \lambda_U)^2 + 2\lambda_U^\dagger \lambda_D \lambda_D^\dagger \lambda_U + 2(3Tr\lambda_U\lambda_U^\dagger - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2)\lambda_U^\dagger \lambda_U. \tag{A.12}$$

Working in a basis where $\lambda_U^\dagger \lambda_U$ is diagonal, let us see if $\frac{d}{dt}\lambda_U^\dagger \lambda_U$ has off-diagonal components. We have, (recalling that in this basis $\lambda_D^\dagger \lambda_D = V_{KM}\bar{\lambda}_D^2 V_{KM}^\dagger$),

$$\begin{aligned} -\frac{d}{dt}(\lambda_U^\dagger \lambda_U)_{\substack{ij \\ i\neq j}} &= 2(\bar{\lambda}_U V_{KM}\bar{\lambda}_D^2 V_{KM}^\dagger \bar{\lambda}_U)_{ij} \\ &= 2\bar{\lambda}_{U_i} V_{KM_{i\ell}} \bar{\lambda}_{D_\ell}^2 V_{KM_{\ell j}}^\dagger \bar{\lambda}_{U_j} \\ &= 0 \text{ for } i,j \neq 3 \end{aligned} \tag{A.13}$$

since we neglect all Yukawa's except the third generation. Similarly, the basis in which $\lambda_D^\dagger \lambda_D$ is diagonal does not change. Thus, the discussion for the scaling of W_{U_R}, W_{D_R} is completely analogous to that in the lepton sector, and we find

$$W_{U_{R3i}} W_{U_{R33}}^\dagger (M_S) = e^{-2I_t}\frac{\Delta m_U^2(M_G)}{\Delta m_U^2(M_S)} W_{U_{R3i}} W_{U_{R33}}^\dagger (M_G), \tag{A.14}$$

$$W_{D_{R3i}} W^\dagger_{D_{R33}}(M_S) = e^{-2I_b} \frac{\Delta m^2_D(M_G)}{\Delta m^2_D(M_S)} W_{D_{R3i}} W^\dagger_{D_{R3}}(M_G). \qquad (A.15)$$

We now turn to W_{U_L}, W_{E_L}. Let $V^*_{U_L}(t)$ be the matrix diagonalizing $\lambda_U \lambda^\dagger_U(t)$:

$$\lambda_U \lambda^\dagger_U(t) = V^*_{U_L}(t) \bar{\lambda}^2_U(t) V^{*\dagger}_{U_L}(t). \qquad (A.16)$$

In the superfield basis in which $\lambda_U \lambda^\dagger_U$ is diagonal, the squark mass matrix is $\widetilde{\mathbf{m}}^{2*}_{Q3i} = V^\dagger_{U_L} \mathbf{m}^{2*}_Q V_{U_L}$. Note as before that $\widetilde{\mathbf{m}}^{2*}_{Q3i} = (W^\dagger_{U_L} \overline{\mathbf{m}}^{2*}_Q W_{U_L})_{3i} = W_{U_{L3i}} W^\dagger_{U_{L33}} \Delta m^2_Q$, so we are interested in $\frac{d}{dt} \widetilde{\mathbf{m}}^{2*}_{Q3i}$. Now,

$$\begin{aligned} \frac{d}{dt} \widetilde{\mathbf{m}}^{2*}_Q = \frac{d}{dt}(V^\dagger_{U_L} \mathbf{m}^{2*}_Q V_{U_L}) &= \left(\frac{d}{dt} V^\dagger_{U_L}\right) \mathbf{m}^{2*}_Q V_{U_L} + V^\dagger_{U_L} \frac{d}{dt} \mathbf{m}^{2*}_Q V_{U_L} + V^\dagger_{U_L} \mathbf{m}^{2*}_Q \frac{d}{dt} V_{U_L} \\ &= \left[\widetilde{\mathbf{m}}^{2*}_Q, V^\dagger_{U_L} \frac{d}{dt} V_{U_L}\right] + V^\dagger_{U_L} \frac{d}{dt} \mathbf{m}^{2*}_Q V_{U_L}. \qquad (A.17) \end{aligned}$$

The second term is the analogue of what we have already seen in the lepton and right-handed quark sector; using the RGE for $\mathbf{m}^{2*}_Q$ we find to leading order

$$\left(V^\dagger_{U_L} \frac{d}{dt} \mathbf{m}^{2*}_Q V_{U_L}\right)_{3i} = -(\lambda^2_t + \lambda^2_b) \widetilde{\mathbf{m}}^2_{Q3i}. \qquad (A.18)$$

Now, $V^\dagger_{U_L} \frac{d}{dt} V_{U_L}$ is obtained from the RGE for $\lambda_U \lambda^\dagger_U$. Actually, note that

$$V^\dagger_{U_L} \left(\frac{d}{dt} \lambda_U \lambda^\dagger_U\right) V_{U_L} = \left[V^\dagger_{U_L} \frac{d}{dt} V_{U_L}, \bar{\lambda}^2_U\right] + \frac{d}{dt} \bar{\lambda}^2_U, \qquad (A.19)$$

so that only $[V^\dagger_{U_L} \frac{d}{dt} V_{U_L}, \bar{\lambda}^2_U]$ is determined. (This is a reflection of the fact that V_{U_L} is not unique: let $X(t)$ be any unitary transformation leaving $\overline{\mathbf{m}}^2_Q(t)$ invariant: $\overline{\mathbf{m}}^2_Q(t) = X^\dagger(t) \overline{\mathbf{m}}^2_Q(t) X(t)$. In our case, $X(t)$ is most generally a $U(2)$ matrix in the first two generation subspace. Then, if V_{U_L} diagonalizes $\mathbf{m}^{2*}_Q$, so does $V_{U_L} X$. Under this change, $V^\dagger_{U_L} \frac{d}{dt} V_{U_L}$ is not invariant, but $[V^\dagger_{U_L} \frac{d}{dt} V_{U_L}, \bar{\lambda}^2_U]$ *is* invariant). Further, since we neglect first two generations Yukawa eigenvalues, $[V^\dagger_{U_L} \frac{d}{dt} V_{U_L}, \bar{\lambda}^2_U]_{ij} = 0$ for $i,j = 1,2$, and only $[V^\dagger_{U_L} \frac{d}{dt} V_{U_L}, \bar{\lambda}^2_U]_{3i(i3)} = (\mp) \lambda^2_t V^\dagger_{U_L} \frac{d}{dt} V^\dagger_{U_{Li3(3i)}}$ is determined, and we can choose all other components of $V^\dagger_{U_L} \frac{d}{dt} V_{U_L}$ to vanish. From the RGE for $\lambda_U \lambda^\dagger_U$,

$$\begin{aligned} -\frac{d}{dt}(\lambda_U \lambda^\dagger_U) &= 6(\lambda_U \lambda^\dagger_U)^2 + 2(3Tr \lambda_U \lambda^\dagger_U - \frac{16}{3} g^2_3 - 3g^2_2 - \frac{13}{15} g^2_1) \lambda_U \lambda^\dagger_U \\ &+ \{\lambda_U \lambda^\dagger_U, \lambda_D \lambda^\dagger_D\}, \qquad (A.20) \end{aligned}$$

we find

$$-\left(V^{\dagger}_{U_L}(\frac{d}{dt}\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^{\dagger})V_{U_L}\right)_{3i} = \{\bar{\boldsymbol{\lambda}}_U^2, V_{KM}\bar{\boldsymbol{\lambda}}_D^2 V_{KM}^{\dagger}\}_{3i}$$
$$= \lambda_t^2\lambda_b^2 V_{KM33}V^{\dagger}_{KM3i}, \qquad (A.21)$$

and thus

$$\left(V^{\dagger}_{U_L}\frac{d}{dt}V_{U_{L3i}}\right) = -\lambda_b^2 V^{\dagger}_{KM3i}V_{KM33}. \qquad (A.22)$$

Thus to leading order

$$[V^{\dagger}_{U_L}\mathbf{m}_Q^2 V_{U_L}, V^{\dagger}_{U_L}\frac{d}{dt}V_{U_L}]_{3i} = -\Delta m_Q^2\,\lambda_b^2\,V^{\dagger}_{KM3i}V_{KM33}, \qquad (A.23)$$

and finally we have

$$-\frac{d}{dt}(W_{U_{L3i}}W^{\dagger}_{U_{L33}}\Delta m_Q^2) = (\lambda_t^2+\lambda_b^2)W_{U_{L3i}}W^{\dagger}_{U_{L33}}\Delta m_Q^2 + \lambda_b^2 V^{\dagger}_{KM3i}V_{KM33}\Delta m_Q^2. \qquad (A.24)$$

Similarly we find

$$-\frac{d}{dt}(W_{D_{L3i}}W^{\dagger}_{D_{L33}}\Delta m_Q^2) = (\lambda_t^2+\lambda_b^2)W_{D_{L3i}}W^{\dagger}_{D_{L33}}\Delta m_Q^2 + \lambda_t^2 V^{\dagger}_{KM3i}V_{KM33}\Delta m_Q^2. \qquad (A.25)$$

We can formally solve the above equations, e.g.

$$W_{U_{L3i}}W^{\dagger}_{U_{L33}}(M_S) = e^{-\left(I_t+I_b+\int_{M_S}^{M_G}dt'\lambda_b^2\frac{V^{\dagger}_{KM3i}V_{KM33}}{W_{U_{L3i}}W^{\dagger}_{U_{L33}}}\right)}\frac{\Delta m_Q^2(M_G)}{\Delta m_Q^2(M_S)}W_{U_{L3i}}W^{\dagger}_{U_{L33}}(M_G), \qquad (A.26)$$

and, to a good approximation, given that $W_{U_{L3i}}$ does not scale very significantly, we can replace

$$\int_{M_S}^{M_G}dt'\lambda_b^2\frac{V^{\dagger}_{KM3i}V_{KM33}}{W_{U_{L3i}}W^{\dagger}_{U_{L33}}} \approx I_b\frac{V^{\dagger}_{KM3i}V_{KM33}}{W_{U_{L3i}}W^{\dagger}_{U_{L33}}}(M_G). \qquad (A.27)$$

So, an approximate solution of the RGE for W_{U_L}, W_{D_L} is

$$W_{U_{L3i}}W^{\dagger}_{U_{L33}}(M_S) \approx e^{-\left(I_t+I_b\left(1+\frac{V^{\dagger}_{KM3i}V_{KM33}}{W_{U_{L3i}}W^{\dagger}_{U_{L33}}}(M_G)\right)\right)}\frac{\Delta m_Q^2(M_G)}{\Delta m_Q^2(M_G)}W_{D_{L3i}}W^{\dagger}_{U_{L33}}(M_G), \qquad (A.28)$$

and similarly

$$W_{D_{L3i}} W^{\dagger}_{D_{L33}}(M_S) \approx e^{-\left(I_b + I_t\left(1+\frac{V^{\dagger}_{KM3i} V_{KM33}}{w_{U_{L3i}} w^{\dagger}_{U_{L33}}}(M_G)\right)\right)} \frac{\Delta m^2_Q(M_G)}{\Delta m^2_Q(M_S)} W_{D_{L3i}} W^{\dagger}_{D_{L33}}(M_G). \tag{A.29}$$

The above results are in agreement with qualitative expectations; the extra terms in the exponential of (A.28) and (A.29) are a reflection of the fact that the bases in which $\lambda_U \lambda_U^{\dagger}$ and $\lambda_D \lambda_D^{\dagger}$ are diagonal change with scale. For moderate $\tan\beta$, however, we expect that the basis in which $\lambda_U \lambda_U^{\dagger}$ is diagonal should not change with scale, and in this limit the extra term drops out of (A.28).

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Figure Captions

Fig. 1. Feynman diagrams contributing to $\mu \to e\gamma$.

Fig. 2 Lepton flavor violating couplings in general supersymmetric Standard Models.

Fig. 3 Corrections to the up-type quark mass matrix, proportional to m_t.

Fig. 4 Contours for $\frac{\Delta m_u}{m_u}$ in $\frac{m_{\tilde{u}}}{M_{\tilde{g}}} - \frac{m_{\tilde{t}}}{m_{\tilde{u}}}$ plane, assuming $m_{\tilde{u}_L} = m_{\tilde{u}_R} \equiv m_{\tilde{u}}$, $m_{\tilde{t}_L} = m_{\tilde{t}_R} \equiv m_{\tilde{t}}$, $W_{U_{L31}} = W_{U_{R31}} = \frac{1}{30}$, $\frac{A+\mu\cot\beta}{m_{\tilde{t}}} = 3$.

Fig. 5 The diagram which gives the dominant contribution to $\mu \to e\gamma$ in the large $\tan\beta$ limit. A photon is understood to be attached to the diagram in all possible ways.

Fig. 6 The dominant diagram (for $\mu \to e\gamma$) in the mass insertion approximation.

Fig. 7 Contours for $Br(\mu \to e\gamma)$ in $M_2 - \Delta$ plane with $m_{\tilde{e}_{L(R)}} = 300GeV$, $\Delta \equiv m_{\tilde{e}_{L(R)}} - m_{\tilde{t}_{L(R)}}$, $W_{E_{L(R)32}} = 0.04$, $W_{E_{L(R)31}} = 0.01$, for (a) $\mu = 100$ GeV, (b) $\mu = 300$ GeV. Contours for negative μ are virtually identical. To get $Br(\mu \to e\gamma)$ prediction from a GUT, multiply by appropriate Clebsch, and ϵ factor (Fig. 10).

Fig. 8 Contours for $Br(\mu \to e\gamma)$ in $\mu - M_2$ plane for (a) $\Delta = 0.25$, (b) $\Delta = 0.5$, with other parameters same as in Fig. 7. The blacked out regions are ruled out by the LEP bound of 45 GeV on chargino masses. The thick dashed lines are contours for a 45 GeV LSP mass.

Fig. 9 Plots of the averaged difference between the third and the first two generations charged slepton masses $\Delta \equiv \frac{\Delta_L+\Delta_R}{2}$, $\Delta_{L(R)} \equiv m_{\tilde{e}_{L(R)}}$ (at M_S), against M_2, for $\frac{1}{2}(m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R}) = (300\text{ GeV})^2$, $\lambda_t = \lambda_b = \lambda_\tau$ (at M_G) $=$ 0.5, 0.8, 1.1, $A_e(M_S) = 1, 0, -1$, two values of the gauge beta function coefficient b_5 between M_G and M_{PL}, (a) $b_5 = 3$ (asymptotically free), (b) $b_5 = -20$. Scalar masses are assumed degenerate at $M_{PL} = 2.4 \times 10^{18}$ GeV. M_G is taken to be 2.7×10^{16} GeV.

Fig 10 Plots of the suppression factor ϵ against M_2, with the same parameters as in Fig. 9.

Fig. 11 Contours for Ωh^2 in $\mu - M_2$ plane in the large $\tan\beta$ limit. Dashed lines are LSP mass contours of 30, 45, and 60 GeV. For all regions of $m_{LSP} < 45$ GeV in this plot, the Higgsino components of LSP are too big and therefore they are ruled out by the Z width.

Fig. 1

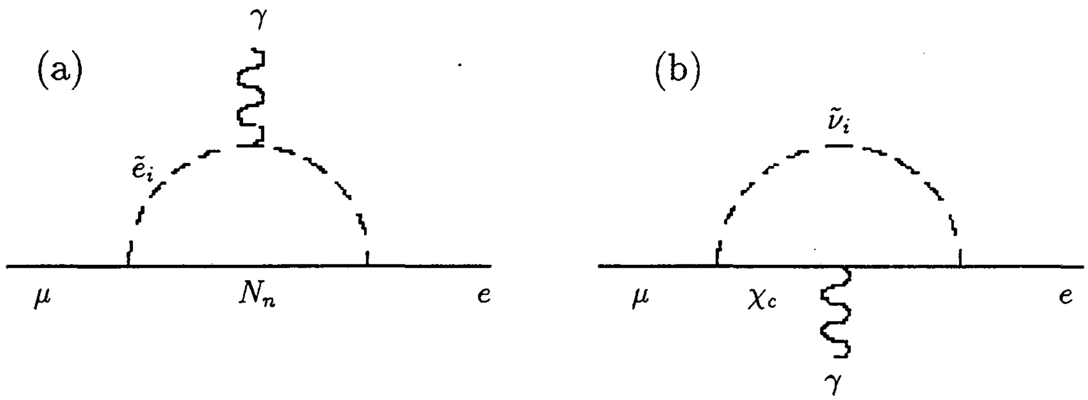


Fig. 2

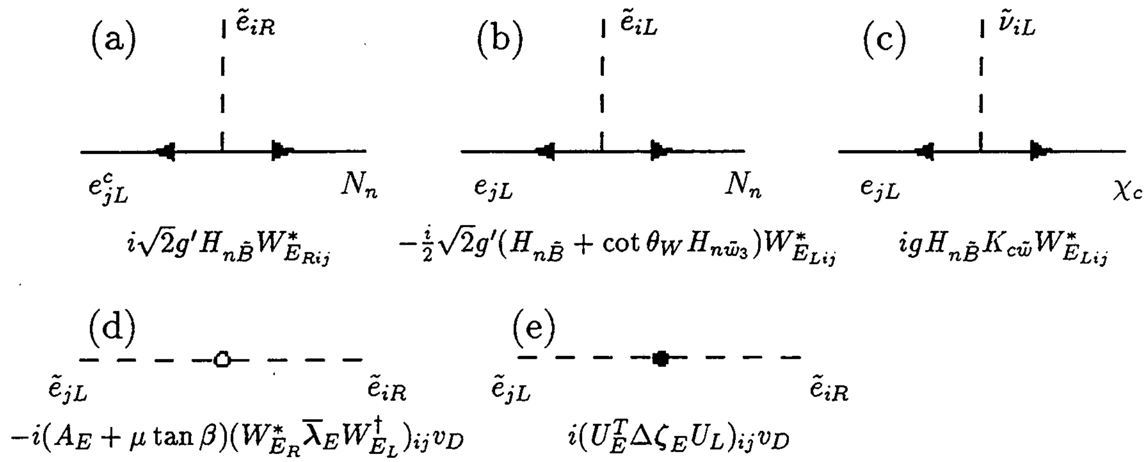


Fig. 3

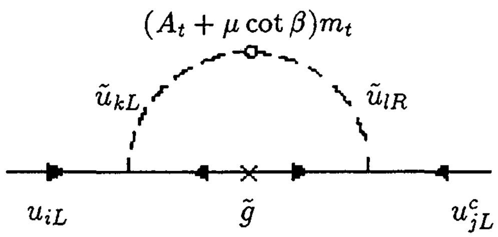


Fig. 5

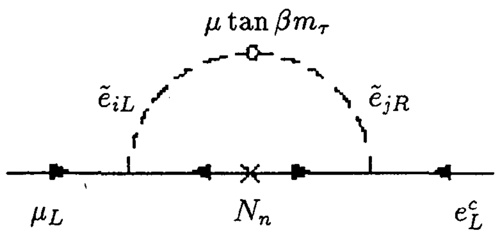


Fig. 6

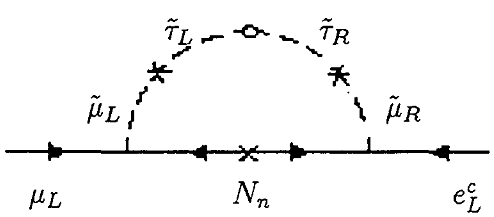
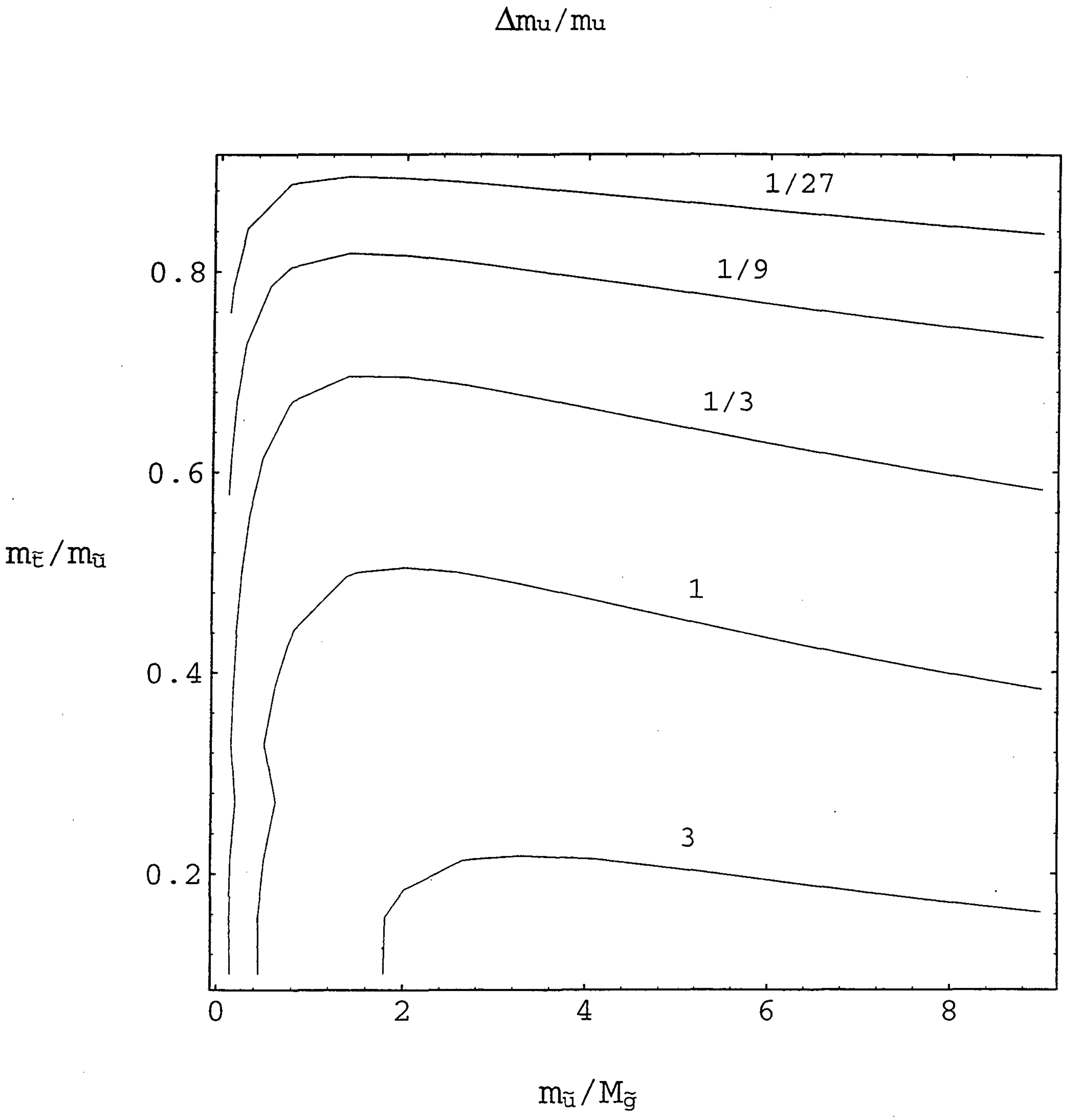


Fig. 4


$\Delta m_u / m_u$
1/27
1/9
1/3
1
3
0.8
0.6
0.4
0.2
0
2
4
6
8
$m_{\tilde{t}} / m_{\tilde{u}}$
$m_{\tilde{u}} / M_{\tilde{g}}$

Br(μ->eγ)

Fig. 7(a)

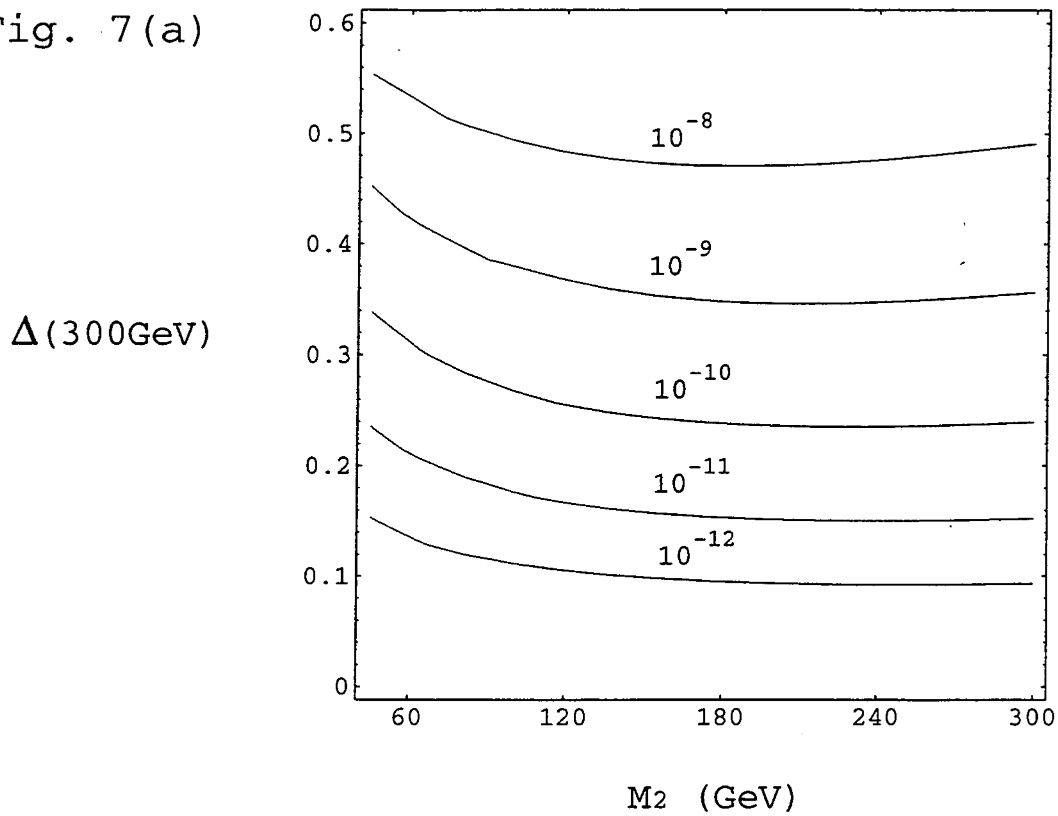
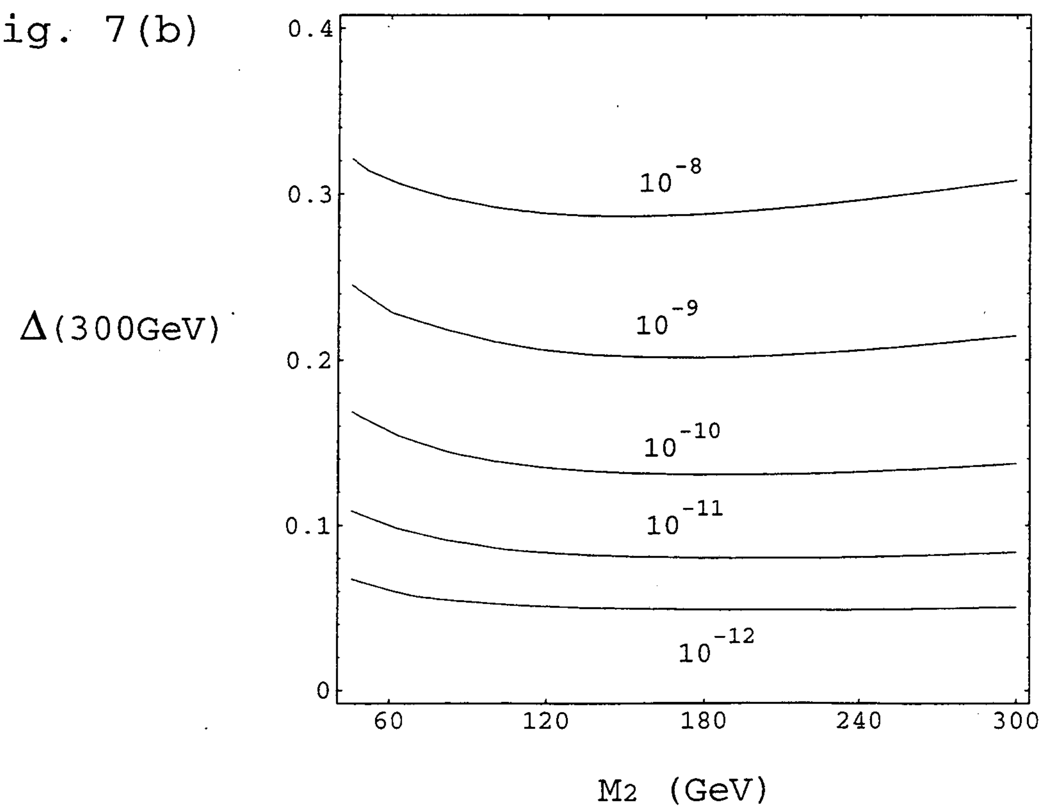


Fig. 7(b)



Br(μ->eγ)

Fig. 8(a)

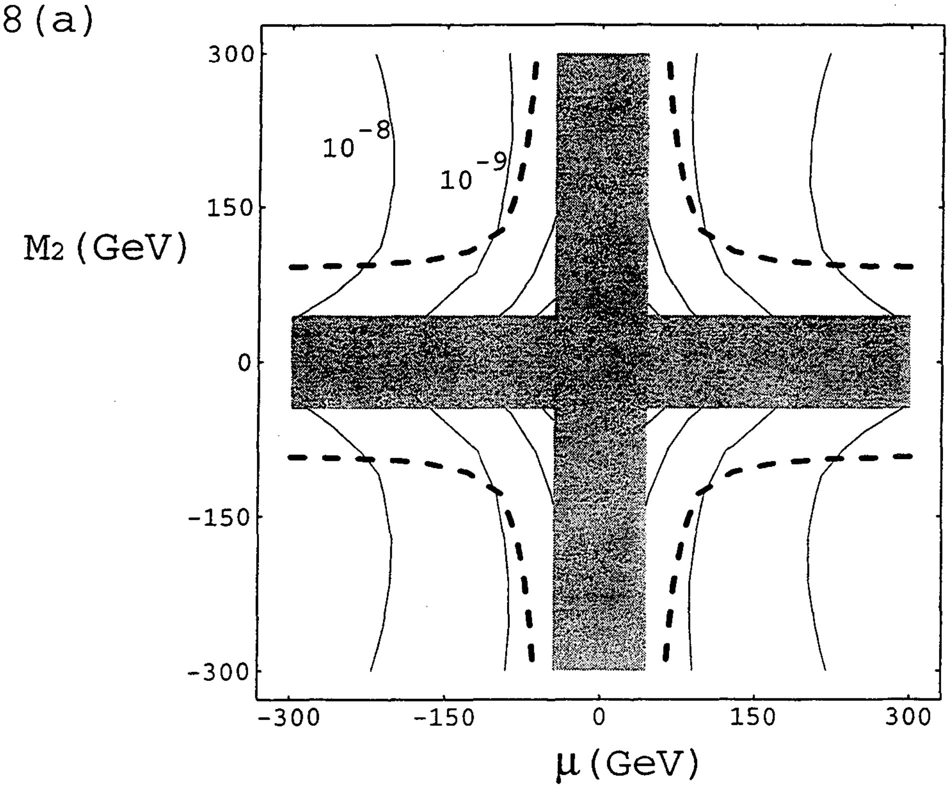


Fig. 8(b)

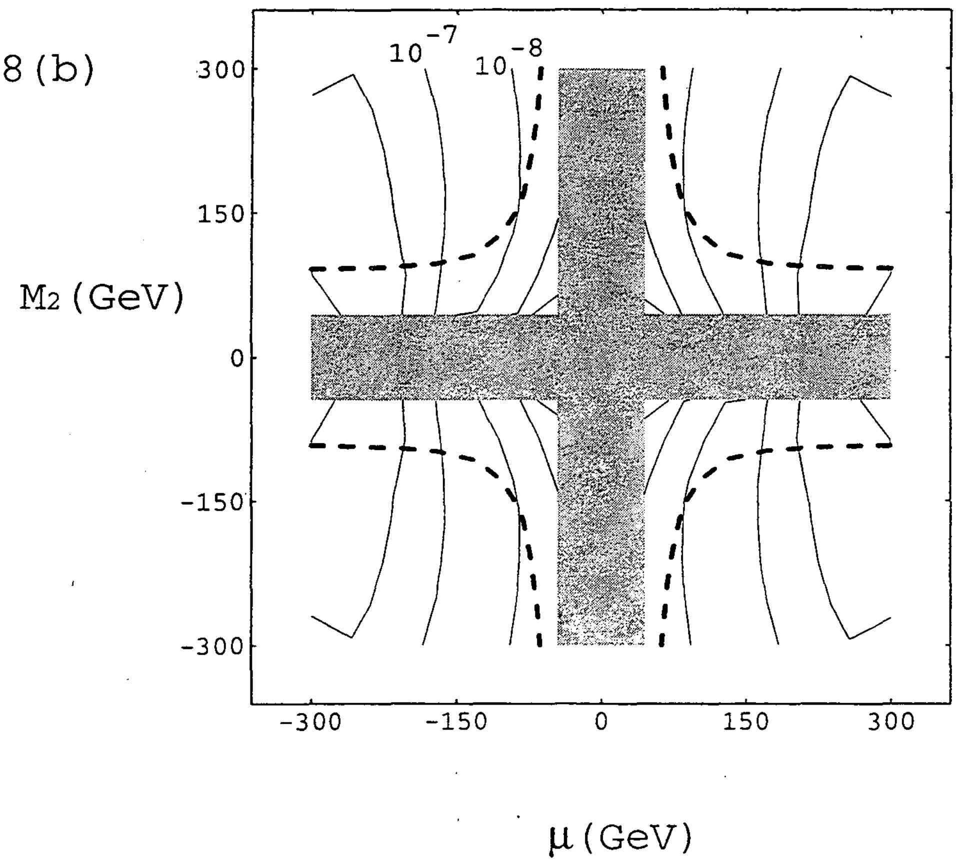
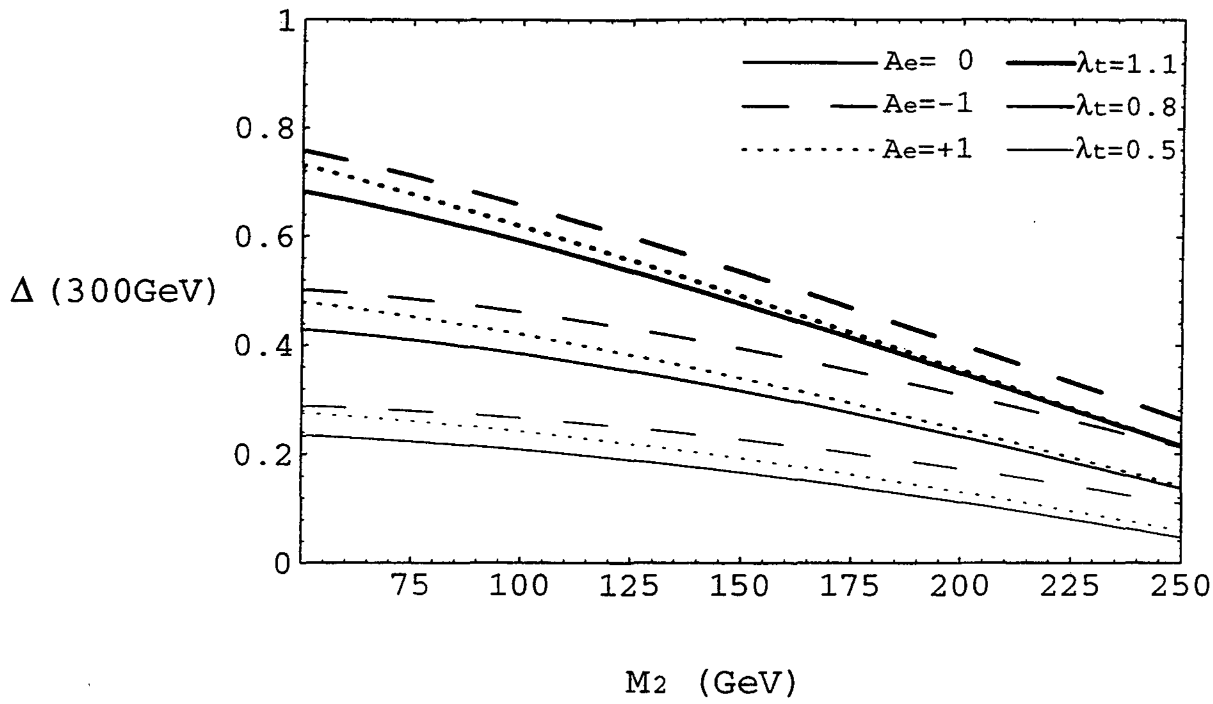
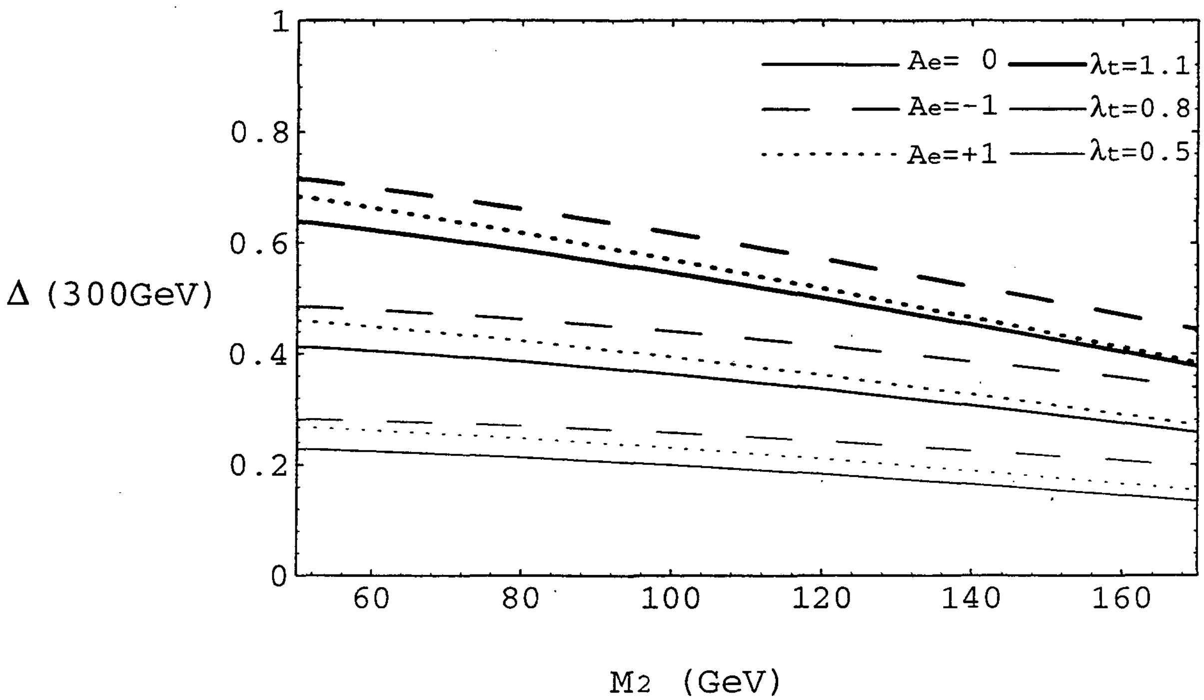


Fig. 9(a)



1
0.8
0.6
0.4
0.2
0
Δ (300GeV)
Ae= 0
Ae=-1
Ae=+1
λt=1.1
λt=0.8
λt=0.5
75
100
125
150
175
200
225
250
M2 (GeV)

Fig. 9(b)



1
0.8
0.6
0.4
0.2
0
Δ (300GeV)
Ae= 0
Ae=-1
Ae=+1
λt=1.1
λt=0.8
λt=0.5
60
80
100
120
140
160
M2 (GeV)

Fig. 10(a)

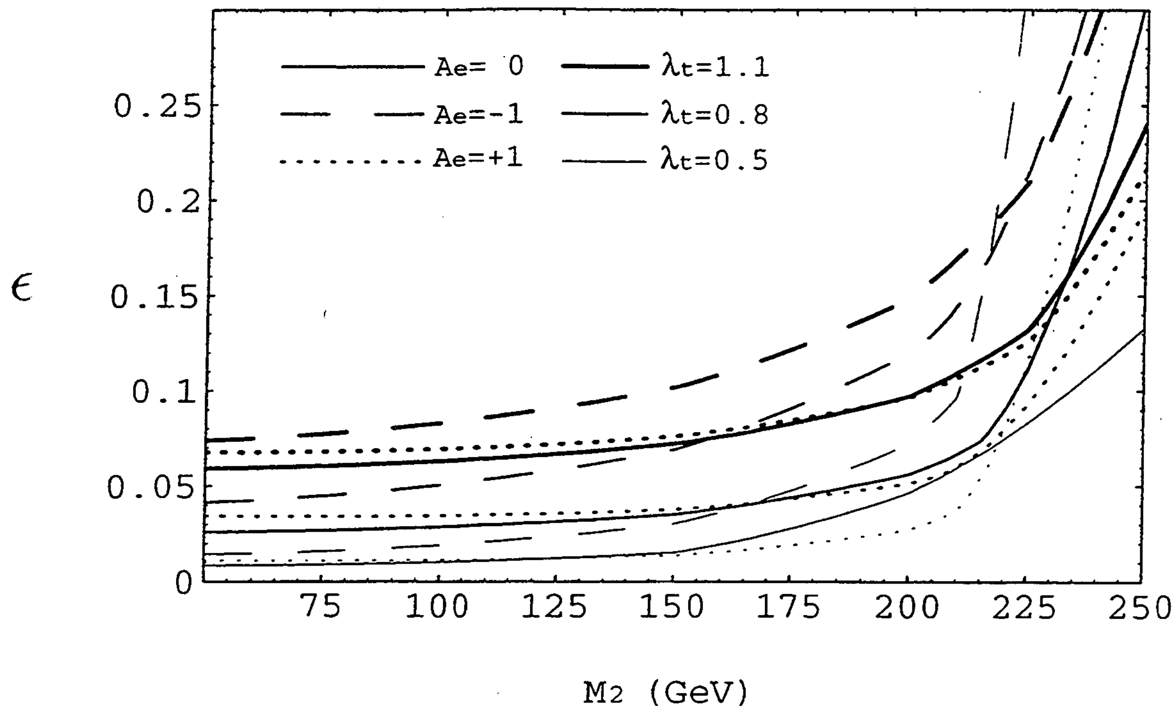


Fig. 10(b)

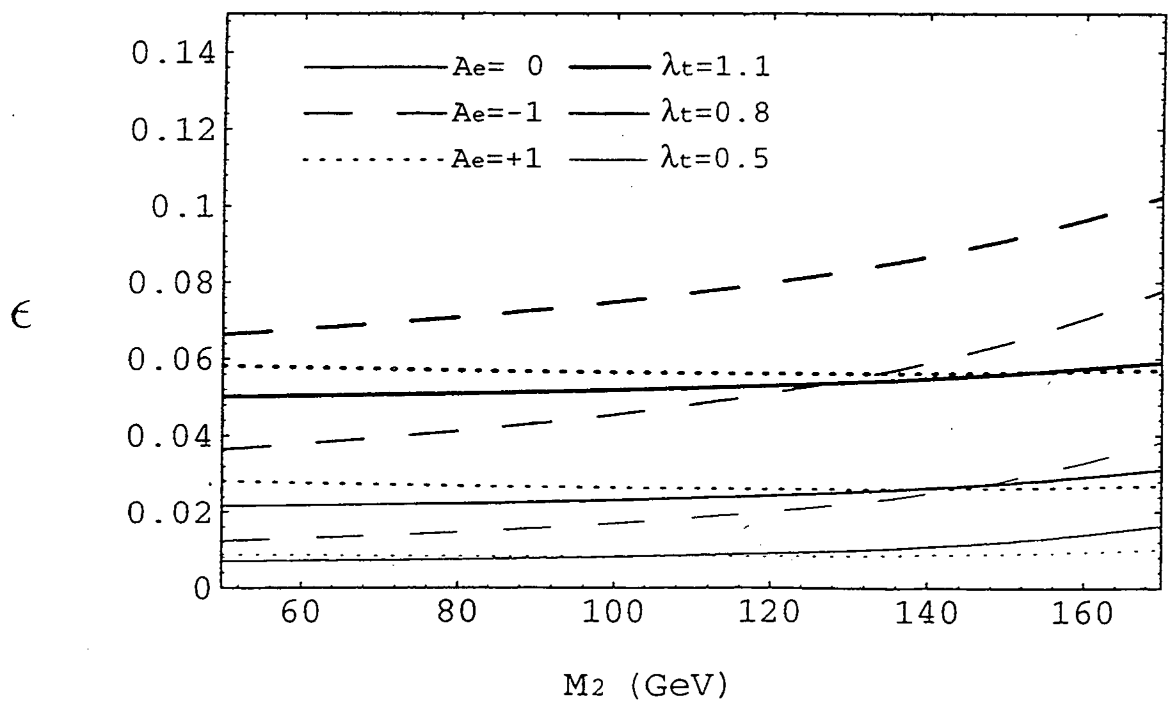
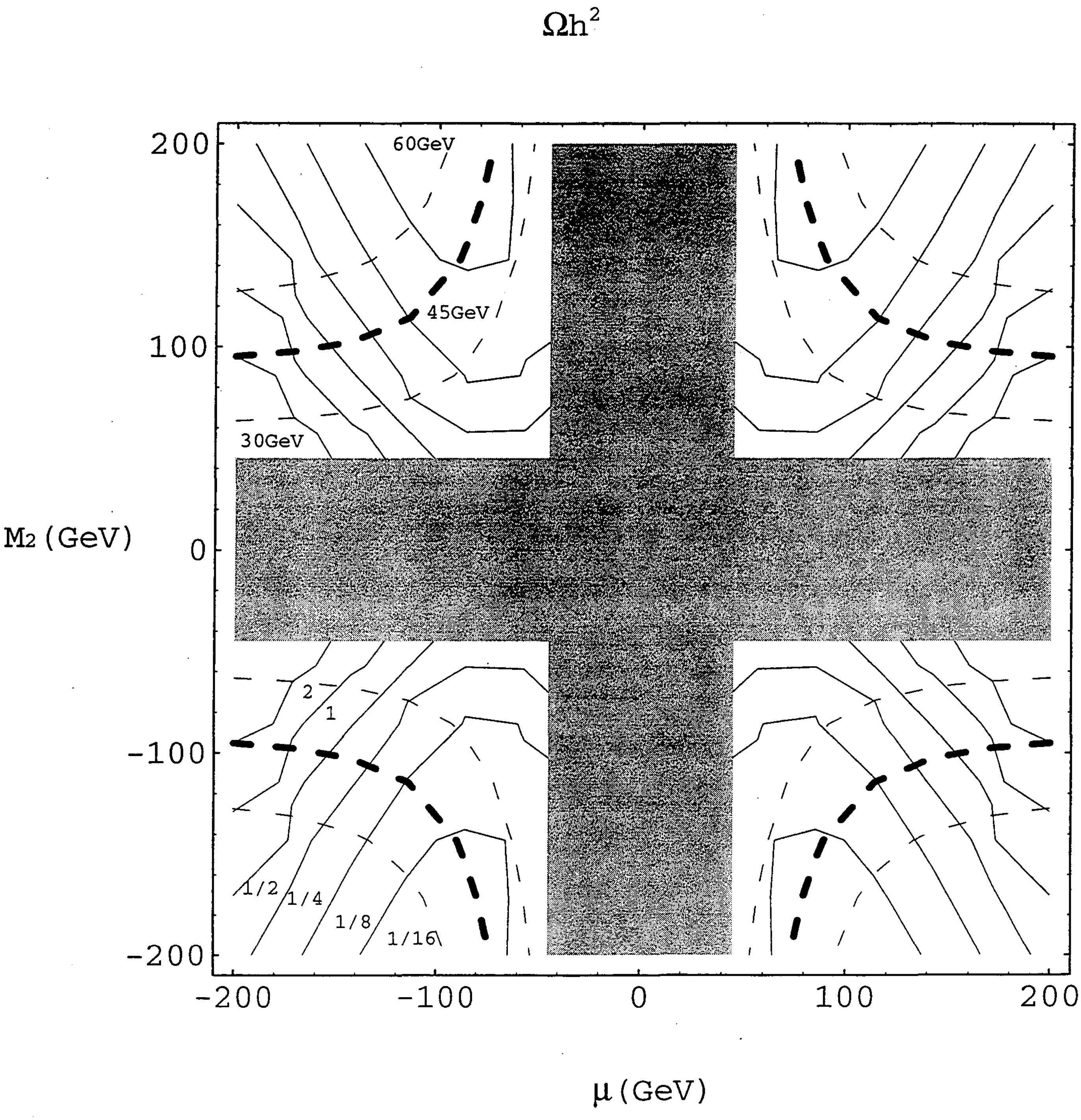


Fig. 11



Ωh^2
60GeV
45GeV
30GeV
2
1
1/2
1/4
1/8
1/16
200
100
0
-100
-200
M2 (GeV)
μ (GeV)