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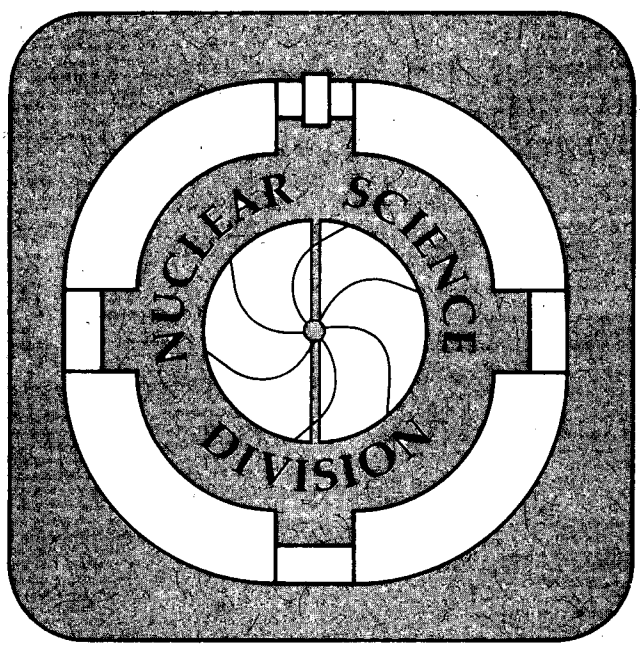
The Congruence Energy: A Contribution to Nuclear Masses and Deformation Energies

W.D. Myers and W.J. Swiatecki

June 1995



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The Congruence Energy: A Contribution to Nuclear Masses and Deformation Energies*

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Abstract

The difference between measured binding energies and those calculated using a shell- and pairing-corrected Thomas-Fermi model can be described approximately by $C(I) = -10\exp(-4.2|I|)$ MeV. Our interpretation of this extra binding is in terms of the granularity of quantal nucleonic density distributions, which leads to a stronger interaction for a neutron and proton with congruent nodal structures of their wave functions. The predicted doubling of this congruence energy in fission is supported by an analysis of measured fission barriers and by a study of wave functions in a dividing Hill-Wheeler box potential. A semi-empirical formula for the shape-dependent congruence energy is described.

• Evidence for a Kink in Nuclear Bindings

Figure 1 shows the difference between measured binding energies and those calculated using a shell- and pairing-corrected Thomas-Fermi model.[1)] The plot refers to the 28 even isobars A = 6, 8...60 that straddle the locus N = Z in the chart of nuclei. An approximate representation of this kinked anomaly, the 'Congruence Energy,' is shown by the curve, given by

$$C(I) = -10\exp(-4.2|I|)\ \text{MeV} \quad , \tag{1}$$

where $I = (N - Z)/A$.

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* This research was presented at a poster session at the ENAM 95 International Conference on Exotic Nuclei and Atomic Masses, Arles, France, June 19–23, 1995.

- **Physics of the Congruence Energy**

In our interpretation, the kink at $N = Z$ is related to the quantal granularity of nucleonic densities. The density distribution of a quantized particle in a potential well consists of cushion-like bumps between a latticework of the wave function's nodal surfaces. A pair of nucleons with congruent nodal structures, say a neutron and a proton, will interact more strongly (in the case of short-range forces) than a pair with uncorrelated density modulations. Since the number of neutron-proton pairs is the lesser of N and Z, i.e., $\frac{1}{2}(N+Z)-\frac{1}{2}|N-Z|$, and since each pair interacts with a strength inversely proportional to the nuclear volume, i.e., to A^{-1}, the congruence energy—the extra binding not present in a statistical Thomas-Fermi model—would be expected to contain a negative term independent of A, modified by a positive term proportional to $|N-Z|/A$. (A contribution to nuclear masses proportional to $|I|$ is often referred to as a "Wigner term.")

- **A Curious Doubling During Fission**

A remarkable feature of this congruence energy, in addition to its telltale dependence on $|I|$, is that, being independent of A, it has the same value for a fissioning nucleus as for each of the resulting fission fragments. Hence the total congruence energy must somehow double as the fissioning nucleus deforms into a necked-in scission shape. Is there empirical evidence for such doubling?

- **Evidence from Fission Barriers**

Figure 2 shows the fission barriers of 36 nuclei as a function of a fissility parameter $Z^2/A(1-2{\cdot}2I^2)$. The curves connect points calculated using the abovementioned Thomas-Fermi model. The upper one assumed that the congruence energy is the same at the saddle point as in the ground state, the lower that it has doubled. There is a fascinating hint in the fact that the almost perfect agreement with the upper curve for Radium and heavier elements, gives place to an approach to the lower curve for lighter nuclei. The Radium region is precisely where saddle-point shapes develop (rather suddenly) a pronounced neck!

Recently, four additional symmetric fission barriers became available for ^{75}Br and $^{90,94,98}Mo$.[2] Figure 3 shows what appears to be a dramatic confirmation of the doubling of the congruence energy for the very necked-in saddle-point shapes in question.

- **Shape Dependence of the Congruence Energy**

To gain insight into this question, consider the schematic 'Hill-Wheeler' box potential filled with particles whose wave functions are products of three sines. Let the particles interact by a δ-function potential. Verify that the ratio of the interaction between fully congruent and fully non-congruent particles is given by

$$\left[\int_0^\pi \sin^4 nx\, dx \Big/ \int_0^\pi \sin^2 nx \sin^2 mx\, dx\right]^3 = 27/8 \ , \tag{2}$$

independently of the quantum numbers n, m specifying the relevant wave function components, and independently of the shape of the box. So no shape dependence? Wait...

- **Fission of the Hill-Wheeler Box**

In order to simulate fission, introduce a semi-transparent δ-function partition parallel to one of the faces and dividing the box into two regions with relative volumes in the ratio ζ:(1–ζ). Denote the transparency of the partition by T (T = 1 means the uncut box, T = 0 means two non-communicating pieces). Verify that the relative congruence energy $\omega(T) \equiv C(T)/C(1)$ for a pair of particles is given by the parametric relation

$$\omega(n) = n\pi \frac{\theta - \frac{2}{3}\sin 2\theta + \frac{1}{12}\sin 4\theta + \beta^4\left(\phi - \frac{2}{3}\sin 2\phi + \frac{1}{12}\sin 4\phi\right)}{\left[\theta - \frac{1}{2}\sin 2\theta + \beta^2\left(\phi - \frac{1}{2}\sin 2\phi\right)\right]^2} \ , \tag{3}$$

$$T(n) = \left[1 + \tfrac{1}{4}(\cot\theta + \cot\phi)^2\right]^{-1} \ , \tag{4}$$

where $\theta = n\pi\zeta$, $\phi = n\pi - \theta$, $\beta = \sin\theta/\sin\phi$, and $\cot\theta + \cot\phi$ is required to be ≤ 0. Here n is now a parameter, no longer an integer, except when T = 1, in which case it becomes the quantum number n = 1, 2, 3, ... specifying the number of antinodes in the original sinusoidal wave function component that is being cut by the partition.

Figure 4 shows the resulting $\omega(T)$ for pairs of particles in 12 consecutive wave functions, in the case when $\zeta = 0.37$. As T decreases from 1 to 0, four of the wave functions end up in the smaller fragment. Its relative volume is 0.37, and so the intercept of $\omega(T)$ at T = 0 is $(0.37)^{-1} = 2.703$. Eight wave functions end up in the larger volume, with intercept $(0.63)^{-1} = 1.587$. Between T = 0 and T = 1 some complicated things are going on, but look at the dependence on T of the *average* of the 12 functions

$\omega(T)$. The points in Fig. 5 show the result. They are closely represented by the simple function $2-\sqrt{T}$! Thus the anticipated doubling of the congruence energy for $T = 0$ is confirmed. (In Fig. 5 the twelve states in question were those with n in the range $n \approx 1001$–1012, but the result of taking $n \approx 1$–12, or $n \approx 1$–100, was similar.) The conjecture, not yet proven algebraically, presents itself that, on the average, the doubling of the relative congruence energy follows a law involving the square root of the degree of communication between the fragments of the fissioning box. In the case of a necked-in fissioning *nucleus*, one might tentatively identify the degree of communication T with the relative degree of neck opening, i.e. the relative neck area. This leads to an expression of the type

$$\begin{aligned}\omega &= 2-\sqrt{(\text{Neck area})/(\text{Mean fragment cross-section})} \\ &= 2-R_n/\overline{R}_f, \quad \text{for necked-in shapes,}\end{aligned} \tag{5}$$

$$\omega = 1, \quad \text{for convex shapes,} \tag{6}$$

where R_n is an effective neck radius and $\overline{R}_f$ is a mean of the effective transverse radii of the two nascent fragments.

- **Suggested Formula for the Congruence Energy**

Thus we arrive at a tentative semi-empirical expression for the congruence energy of necked-in shapes:

$$C = \left[-C_o \exp(-c|I|/C_o)\right]\left(2-R_n/\overline{R}_f\right) , \tag{7}$$

where C_o and c are adjustable parameters with the approximate values $C_o \approx 10$ MeV, $c \approx 42$ MeV.

Acknowledgments

We would like to thank L.G. Moretto, K.X. Jing and G.J. Wozniak for providing us with the fission barriers for Br and Mo. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, and by the Office of Basic Energy Sciences, Division of Nuclear Sciences, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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2. L.G. Moretto, K.X. Jing and G.J. Wozniak, *Phys. Rev. Let.* **74** (1995) 3557, and private communication 1995.

Figure Captions

Fig. 1. The difference between measured binding energies and those calculated using the shell- and pairing-corrected Thomas-Fermi model of Ref. 1. The points refer to the 28 isobars with A = 6, 8...60 that straddle the locus N = Z in the chart of nuclei. The curve is the semi-empirical fit $C(I) = -10\exp(-4.2|I|)$ MeV.

Fig. 2. Calculated fission barriers (open symbols) and measurements corrected for ground-state shell effects. The open diamonds assume that the congruence energy at the saddle point is the same as in the ground state, the open squares assume that it has doubled. "Fissility" is defined as $Z^2/A(1 - 2{\cdot}2\, I^2)$ (Ref. 1).

Fig. 3. Same as Fig. 2, but extended down to ^{8}Be (Ref. 1). The four points around fissility 16–20 refer to ^{75}Br and ^{90,94,98}Mo (Ref. 2).

Fig. 4. The relative congruence energies for pairs of particles described by 12 consecutive wave functions in a Hill-Wheeler box potential that is being cut by a partition of transparency T, at a location that divides the box into pieces with relative volumes 0.37:0.63.

Fig. 5. The average of the 12 curves in Fig. 4 (circles) and the conjectured formula $2 - \sqrt{T}$ describing the doubling of the average congruence energy with loss of communication between the two nascent fragments.

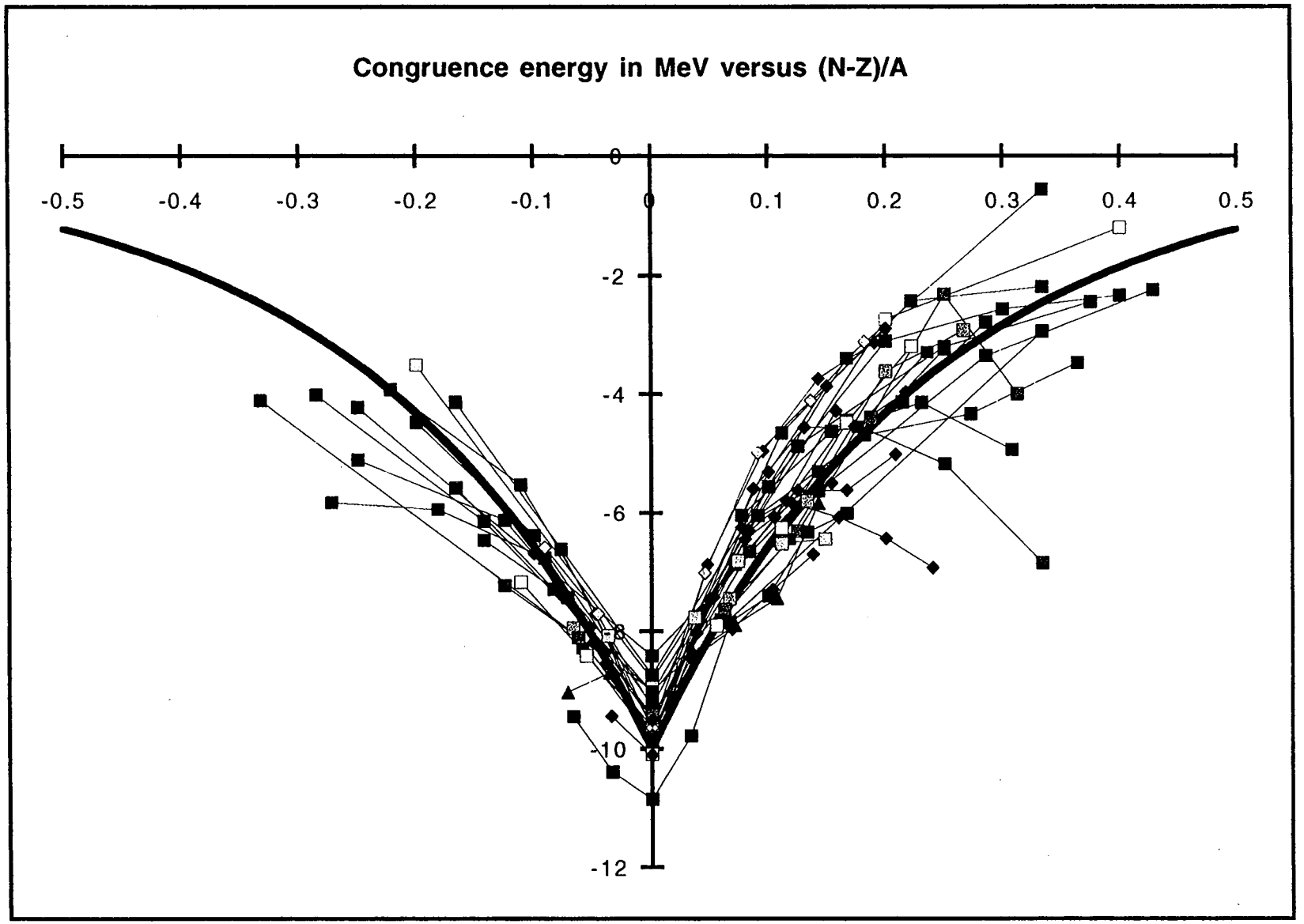
Congruence energy in MeV versus (N-Z)/A
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0
-2
-4
-6
-8
-10
-12

Figure 1

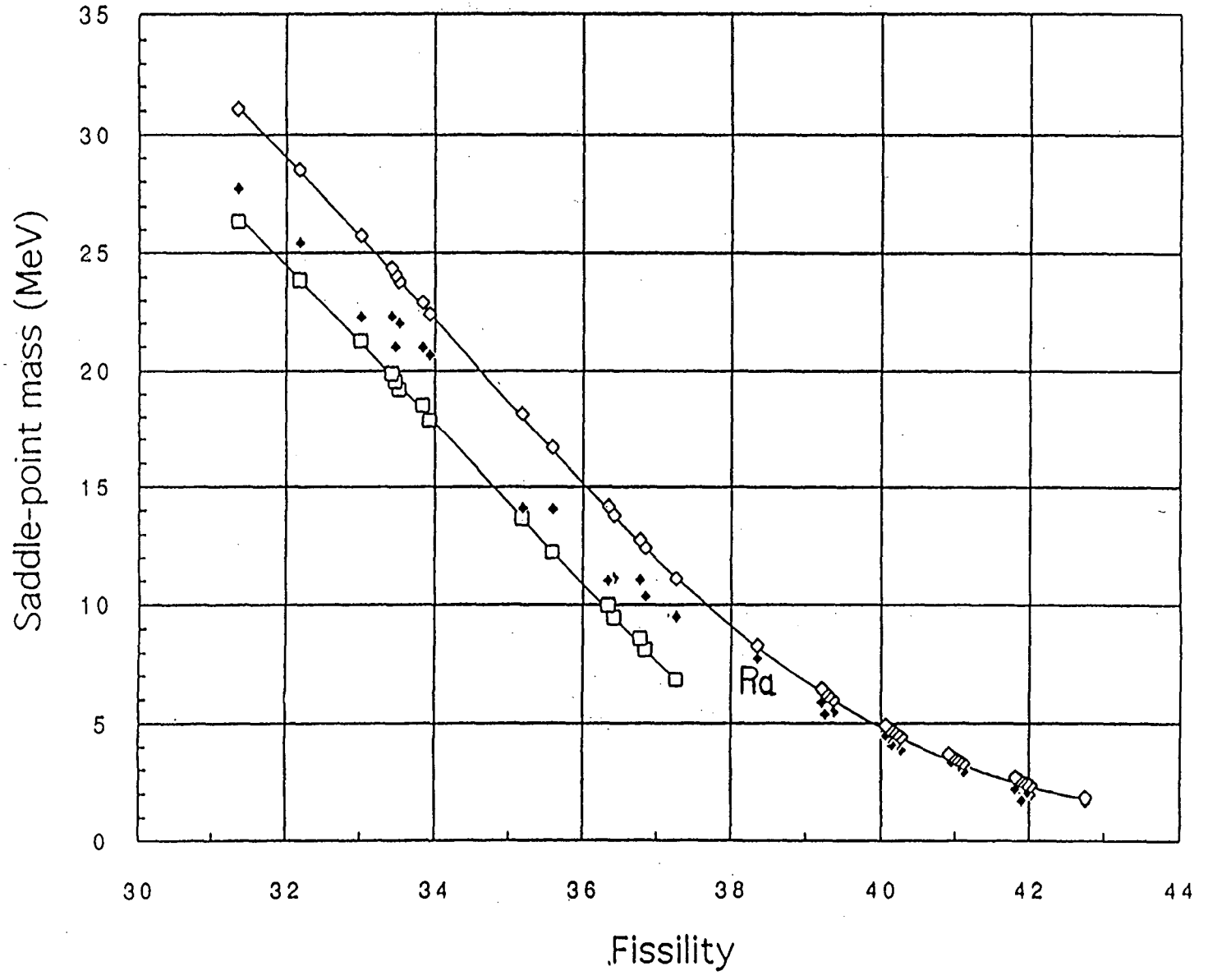
Saddle-point mass (MeV)
35
30
25
20
15
10
5
0
30
32
34
36
38
40
42
44
Fissility
Ra

Figure 2

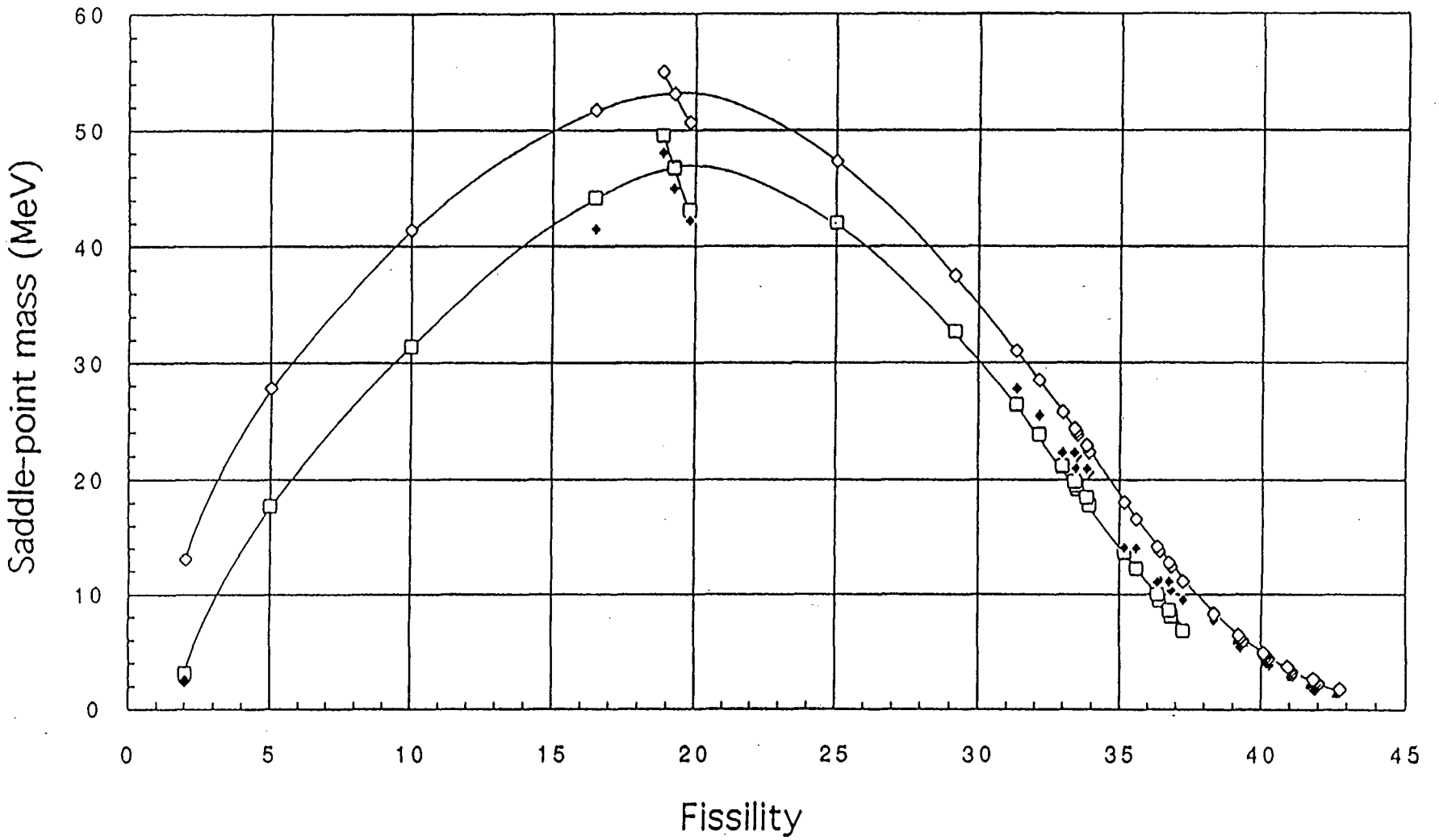
Saddle-point mass (MeV)
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
45
Fissility

Figure 3

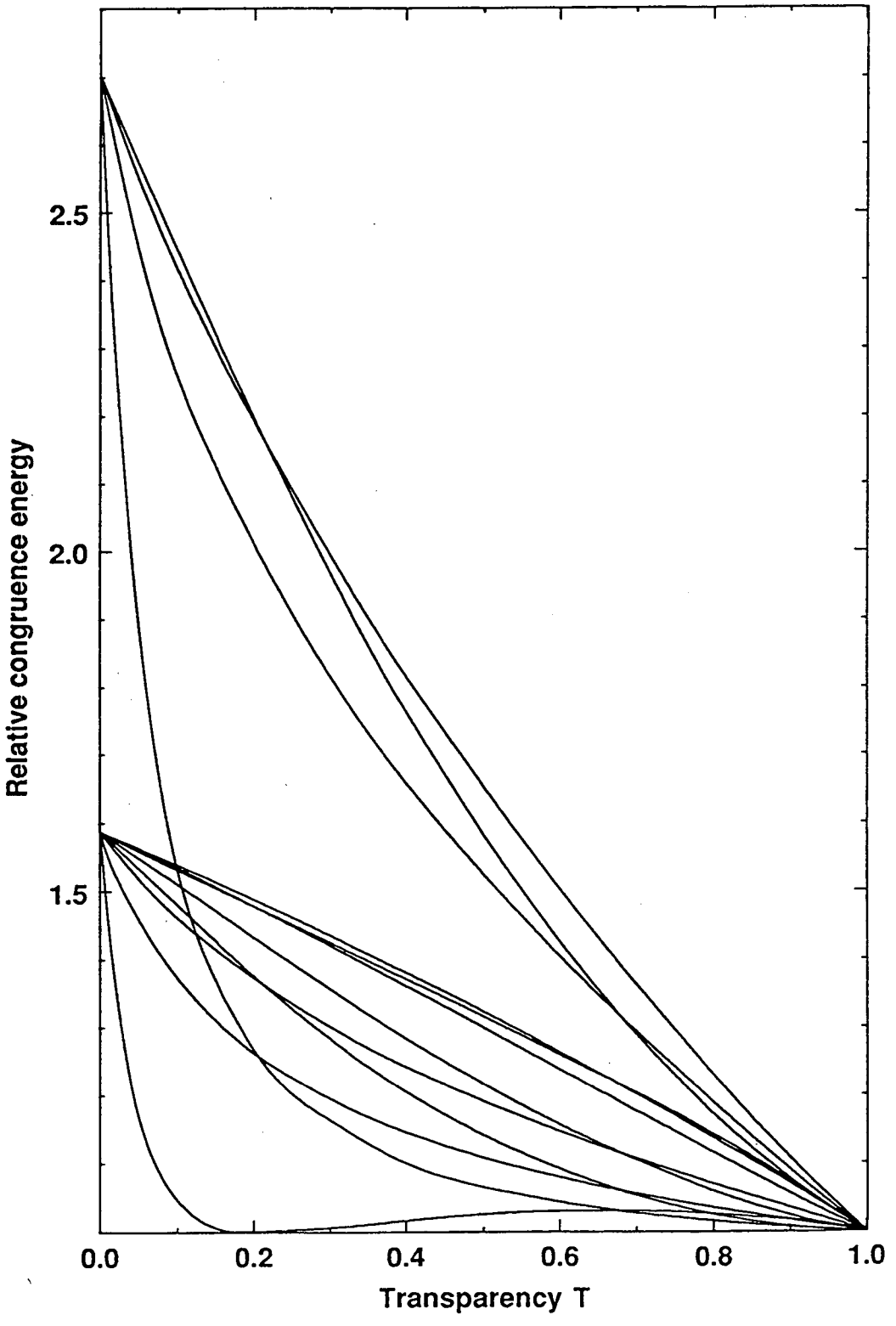
2.5
2.0
1.5
Relative congruence energy
0.0
0.2
0.4
0.6
0.8
1.0
Transparency T

Figure 4

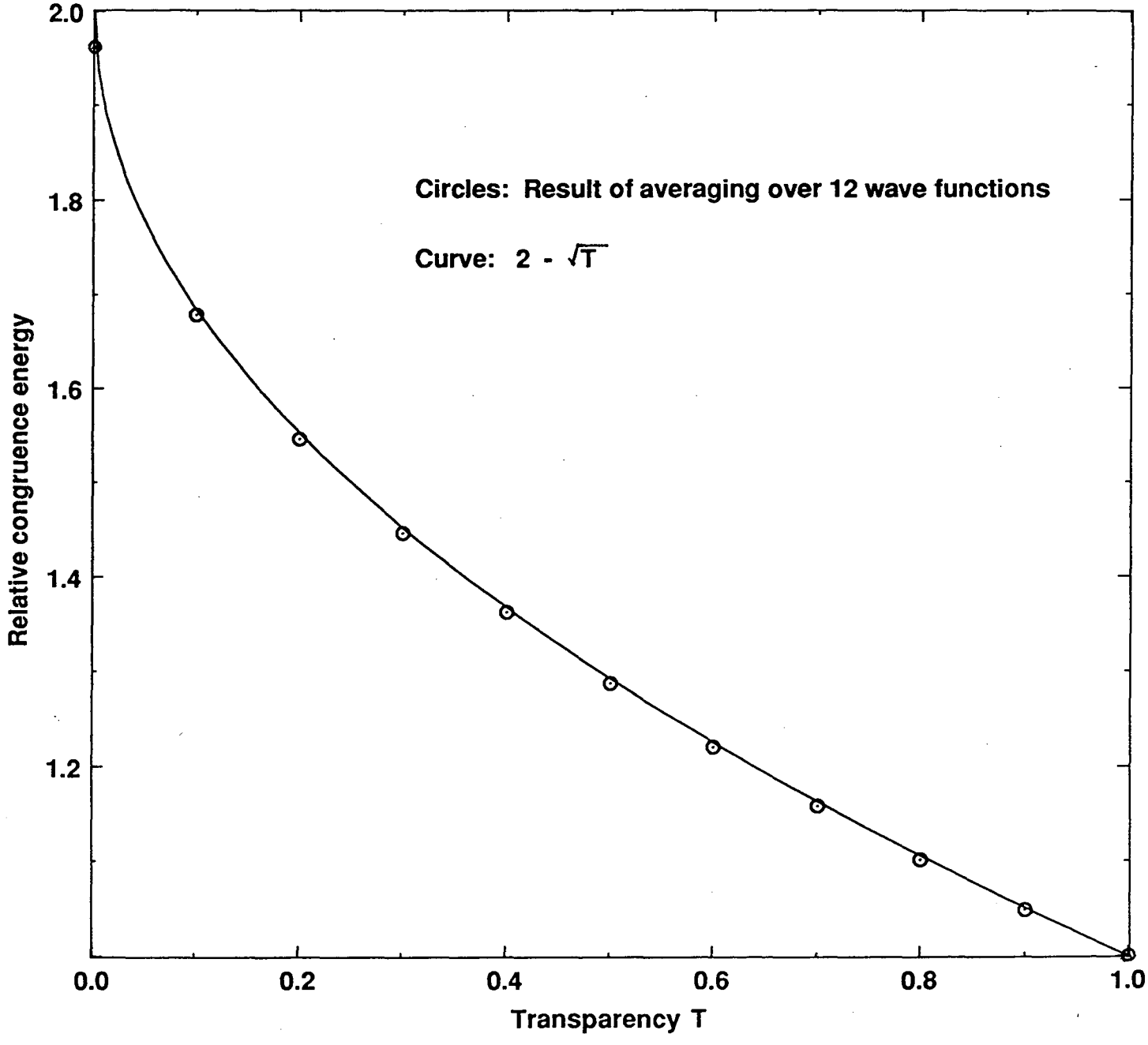
Circles: Result of averaging over 12 wave functions
Curve: 2 - √T
Relative congruence energy
Transparency T
2.0
1.8
1.6
1.4
1.2
0.0
0.2
0.4
0.6
0.8
1.0

Figure 5

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