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M.K. Gaillard

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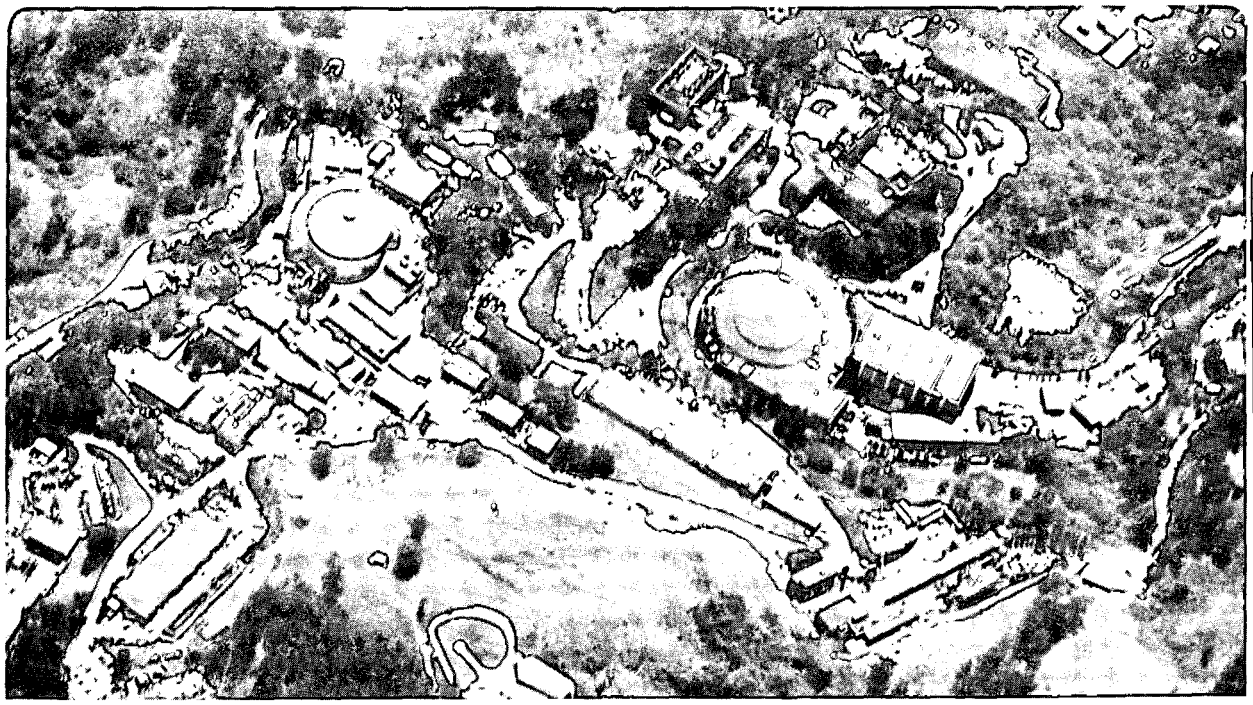

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Pauli-Villars Regularization of Supergravity and Field Theory Anomalies* †

Mary K. Gaillard

Department of Physics and Theoretical Physics Group, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

Abstract

A procedure for Pauli-Villars regularization of locally and globally supersymmetric theories is described. Implications for specific theories, especially those obtained from superstrings, are discussed with emphasis on the role of field theory anomalies.

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Participation in this conference implies a belief that supersymmetry is relevant to particle physics. Since gravity is an established force of nature, we are inevitably led to take supergravity seriously as an effective, nonrenormalizable theory valid in some energy range. As such, we should be able to treat it at the quantum level, which means that we need a regularization procedure consistent with the symmetries of the theory. In this talk I will describe a procedure [1] for regulating the one-loop quadratic divergences of a general supergravity theory [2], with only the restriction that the gauge kinetic function is diagonal in gauge indices: $f_{ab}(Z) = \delta_{ab} k_a f(Z)$. In the case of global supersymmetry, the procedure has been generalized [3] to regulate all one-loop ultraviolet divergences for a general gauged nonlinear σ-model with no dilaton. Following some preliminaries to establish notation, specify the gauge fixing, *etc.*, I will describe the Pauli-Villars field content and couplings. Finally I will mention several implications for and applications to physical issues that are especially relevant to effective supergravity theories from strings.

The tree-level supergravity lagrangian [2] I adopt, with $f(z) = x + iy$, is

$$\begin{aligned}
\frac{1}{\sqrt{g}}\mathcal{L} &= \frac{1}{2}r - \frac{x}{4}F_{\mu\nu}F^{\mu\nu} - \frac{y}{4}\tilde{F}_{\mu\nu}F^{\mu\nu} + K_{i\bar{m}}\mathcal{D}^\mu z^i \mathcal{D}_\mu \bar{z}^{\bar{m}} - V \\
&+ \frac{ix}{2}\bar{\lambda}\,\not{D}\lambda + iK^{i\bar{m}}\left(\bar{\chi}_L^{\bar{m}}\,\not{D}\chi_L^i + \bar{\chi}_R^i\,\not{D}\chi_R^{\bar{m}}\right) + e^{-K/2}\left(\frac{1}{4}f_i\bar{A}^i\bar{\lambda}_R\lambda_L - A_{ij}\bar{\chi}_R^i\chi_L^j + \text{h.c.}\right) \\
&+ \frac{1}{4}\bar{\psi}_\mu\gamma^\nu(i\,\not{D} + M)\gamma^\mu\psi_\nu - \frac{1}{4}\bar{\psi}_\mu\gamma^\mu(i\,\not{D} + M)\gamma^\nu\psi_\nu \\
&- \left[\frac{x}{8}\bar{\psi}_\mu\sigma^{\nu\rho}\gamma^\mu\lambda_a F^a_{\nu\rho} + \bar{\psi}_\mu\,\not{\mathcal{D}}\bar{z}^{\bar{m}}K_{i\bar{m}}\gamma^\mu L\chi^i \frac{1}{4}\bar{\psi}_\mu\gamma^\mu\gamma_5\lambda^a\mathcal{D}_a + i\bar{\psi}_\mu\gamma^\mu L\chi^i m_i + \text{h.c.}\right] \\
&+ \left(i\bar{\lambda}_R^a\left[2K_{i\bar{m}}(T_a\bar{z})^{\bar{m}} - \frac{1}{2x}f_i\mathcal{D}_a - \frac{1}{4}\sigma_{\mu\nu}F_a^{\mu\nu}f_i\right]\chi_L^i + \text{h.c.}\right) + 4 \text{ fermion terms}, \qquad (1)
\end{aligned}$$

where

$$\begin{aligned}
V &= \hat{V} + \mathcal{D}, \quad \hat{V} = e^{-K}(A_i\bar{A}^i - 3A\bar{A}), \quad \mathcal{D} = \frac{1}{2x}\mathcal{D}_a\mathcal{D}^a, \quad \mathcal{D}_a = K_i(T^a z^i), \\
\bar{M} &= (M)^\dagger = e^{K/2}\left(WR + \overline{W}L\right), \quad A = e^K W, \quad \bar{A} = e^K\overline{W}, \quad m_i = e^{-K/2}A_i. \qquad (2)
\end{aligned}$$

$K(z,\bar{z})$ is the Kähler potential, $W(z)$ is the superpotential, T^a is a generator of the gauge group, and

$$A_{i_1\cdots i_n} = D_{i_1}\cdots D_{i_n}A, \quad \bar{A}^{i_1\cdots i_n} = D^{i_1}\cdots D^{i_n}\bar{A}, \qquad D^i = K^{i\bar{m}}D_{\bar{m}}, \qquad (3)$$

with D_i the scalar field reparameterization covariant derivative, and $K^{i\bar{m}}$ the inverse Kähler metric. The one-loop effective action is determined from the quadratic quantum action:

$$\begin{aligned}
\mathcal{L}_{quad}(\Phi,\Theta,c) &= \frac{1}{2}\hat{\phi}^I\hat{\phi}^J\left(\partial_I\partial_J + (A_I)_J^K\partial_K\right)S + \mathcal{L}_{gf} + \mathcal{L}_{gh} = \\
&-\frac{1}{2}\Phi^T Z^\Phi\left(\hat{D}_\Phi^2 + H_\Phi\right)\Phi + \frac{1}{2}\bar{\Theta}Z^\Theta\left(i\,\not{D}_\Theta - M_\Theta\right)\Theta + \frac{1}{2}\bar{c}Z^c\left(\hat{D}_c^2 + H_c\right)c + O(\psi_{cl}), \quad (4)
\end{aligned}$$

where $\phi^I = \Phi^I,\ \Theta^I,\ \partial_I = \partial/\partial\phi^I$, and the column vectors,

$$\Phi^T = (h_{\mu\nu}, \hat{\mathcal{A}}^a, \hat{z}^i, \hat{z}^{\bar{m}}), \qquad \Theta^T = (\psi_\mu, \lambda^a, \chi_L^i, \chi_R^{\bar{m}}, \alpha), \qquad c^T = (c_\nu, c^a, c_\alpha),$$

represent the bose, fermion and ghost quantum degrees of freedom, respectively, with $\alpha = -C\bar{\alpha}^T$ an auxiliary field introduced [4] to implement gravitino gauge fixing. The connection $(A_I)_J^K$ is chosen so as to preserve all bosonic symmetries, and also to simplify matrix elements

involving the graviton. In particular the quantum variables $\hat{z}^i, \hat{z}^{\bar{m}}$ are normal coordinates in the space of scalar fields: $(A_i)^k_j = \Gamma^k_{ij}$, the affine connection associated with the Kähler metric $K_{i\bar{m}}$, giving a scalar field reparameterization invariant expansion. In (4) ψ_{cl} represents background fermion fields that I set to zero; that is, I calculate only the one-loop bosonic action.

For the bose sector, I use a smeared gauge-fixing:

$$\mathcal{L} \to \mathcal{L} + \mathcal{L}_{gf}, \quad \mathcal{L}_{gf} = -\frac{\sqrt{g}}{2} C_A Z^{AB} C_B, \quad Z = \begin{pmatrix} \delta^{ab} & 0 \\ 0 & -g^{\mu\nu} \end{pmatrix}, \quad C = \begin{pmatrix} C_a \\ C_\mu \end{pmatrix}. \tag{5}$$

The Yang-Mills gauge-fixing term:

$$C^a = \mathcal{D}''^\mu \hat{\mathcal{A}}^a_\mu + \frac{i}{\sqrt{x}} K_{i\bar{m}} \left[(T^a)^{\bar{m}} \hat{z}^i - (T^a)^i \hat{z}^{\bar{m}} \right],$$

preserves off-shell supersymmetry [5] in the limit of global supersymmetry and coincides with the string-loop result [6] for chiral multiplet wave function renormalization. The graviton gauge-fixing term:

$$\sqrt{2} C_\mu = \left(\nabla^\nu h_{\mu\nu} - \frac{1}{2} \nabla_\mu h^\nu_\nu - 2\mathcal{D}_\mu z^I Z_{IJ} \hat{z}^J + 2\mathcal{F}^a_{\mu\nu} \hat{\mathcal{A}}^\nu_a \right),$$

is the one originally introduced by 't Hooft and Veltman [7], generalized [4] to include the Yang-Mills sector. The script quantum and classical Yang-Mills fields and field strengths are canonically normalized [8]:

$$\mathcal{A}_\mu = \sqrt{x} A_\mu, \quad \hat{\mathcal{A}}_\mu = \sqrt{x} \hat{A}_\mu, \quad \mathcal{F}_{\mu\nu} = \sqrt{x} F_{\mu\nu}, \quad \sqrt{x} \mathcal{D}_\mu A_\nu = \mathcal{D}'_\mu \mathcal{A}_\mu,$$

where $\mathcal{D}_\mu$ is the gauge and general covariant derivative, and $\mathcal{D}'_\mu = \mathcal{D}_\mu - \partial_\mu x/2x$, $\mathcal{D}''_\mu = \mathcal{D}_\mu + \partial_\mu x/2x$. In the earlier literature two gravitino gauge fixing procedures have been used: a) a Landau-type gauge [9, 10] $\gamma \cdot \psi = 0$, implemented by the introduction of an auxiliary field, and b) a smeared gauge [11] $\mathcal{L} \to \mathcal{L} - \bar{F} \mathcal{M} F$, $F = \gamma \cdot \psi$, $\mathcal{M} = \frac{1}{4}(i \not{D} + 2M_\psi)$ supplemented with Nielsen-Kallosh ghosts. Here I use an unsmeared gauge $G = 0$, with the gauge-fixing function [4]

$$G = -\gamma^\nu (i \not{D} - \bar{M}) \psi_\nu - 2(\not{D} z^i K_{i\bar{m}} R \chi^{\bar{m}} + \not{D} \bar{z}^{\bar{m}} K_{i\bar{m}} L \chi^i) + \frac{x}{2} \sigma^{\nu\rho} \lambda_a F^a_{\nu\rho} + 2i m_I \chi^I - \gamma_5 \mathcal{D}_a \lambda^a, \tag{6}$$

where D_μ contains the spin and chiral Kähler connections. The quantum Lagrangian is obtained by the introduction of an auxiliary field α: $\delta(G) = \int d\alpha \exp(i\alpha G)$, and a shift in the gravitino field: $\psi' = \psi + \gamma\alpha$, $\bar{\psi}' = \bar{\psi} + \bar{\alpha}\gamma$, so as to diagonalize the gravitino kinetic energy term. The ghost and ghostino determinants are obtained in the usual way as, respectively:

$$\left(\hat{D}^2_c + H_c \right)^A_B = \frac{\partial}{\partial \epsilon_A} \delta C_B, \quad A, B = a, \mu, \quad \left(\hat{D}^2_c + H_c \right)^\alpha_\beta = \frac{\partial \delta G^\alpha}{\partial \epsilon^\beta},$$

where $\hat{D}_\mu$ is related to $\mathcal{D}_\mu$ or D_μ by additional connections. With these choices the one-loop bosonic action takes a very simple form:

$$S_1 = \frac{i}{2} \mathrm{Tr} \ln \left(\hat{D}^2_\Phi + H_\Phi \right) - \frac{i}{2} \mathrm{Tr} \ln (-i \not{D}_\Theta + M_\Theta) + \frac{i}{2} \mathrm{STr} \ln \left(\hat{D}^2_c + H_c \right), \tag{7}$$

where

$$\mathrm{STr} \ln \left(\hat{D}^2_c + H_c \right) = 2\mathrm{Tr} \ln \left(\hat{D}^2_c + H_c \right)_{c_\alpha} - 2\mathrm{Tr} \ln \left(\hat{D}^2_c + H_c \right)_{c_{a,\mu}},$$

which just reduces to determinants of the form of those for scalars and spin-$\frac{1}{2}$ fermions. Moreover the ghost and non-ghost sectors have separately supersymmetric quantum spectra, except for the Yang-Mills fields:

$$\frac{1}{2}\left(\mathrm{Tr}\ 1\right)_{\Theta} = \left(\mathrm{Tr}\ 1\right)_{\Phi} - 2N_G = 2N + 2N_G + 10. \quad \left(\mathrm{Tr}\ 1\right)_{c_\alpha} = 4, \quad \left(\mathrm{Tr}\ 1\right)_{c_{a,b}} = 4 + N_G,$$

where $N(N_G)$ is the number of chiral (gauge) supermultiplets. To evaluate (7) I separate [10, 12] the fermion determinant into helicity-even and -odd parts:

$$-\frac{i}{2}\mathrm{Tr}\ln(-i\not{D} + M_\Theta) \equiv -\frac{i}{2}\mathrm{Tr}\ln\mathcal{M}(\gamma_5) = T_- + T_+, \tag{8}$$

where here D_μ contains all fermion connections, and

$$\begin{aligned} T_- &= -\frac{i}{4}\left[\mathrm{Tr}\ln\mathcal{M}(\gamma_5) - \mathrm{Tr}\ln\mathcal{M}(-\gamma_5)\right], \qquad T_+ = -\frac{i}{4}\left[\mathrm{Tr}\ln\mathcal{M}(\gamma_5) + \mathrm{Tr}\ln\mathcal{M}(-\gamma_5)\right], \\ \mathcal{M} &= \gamma_0(-i\not{D} + M_\Theta) = \begin{pmatrix} \sigma_+^\mu D_\mu^+ & M^+ \\ M^- & \sigma_-^\mu D_\mu^- \end{pmatrix}, \qquad \sigma_\pm^\mu = (1, \pm\vec{\sigma}). \end{aligned} \tag{9}$$

Then defining

$$\hat{D}_\Theta^2 + H_\Theta \equiv (-i\not{D}_\Theta + M_\Theta)(i\not{D}_\Theta + M_\Theta),$$

The one-loop bosonic action (7) reduces to:

$$S_1 = \frac{i}{2}\mathrm{STr}\ln\left(\hat{D}^2 + H\right) + T_-.$$

The helicity-odd term T_- is at most logarithmically divergent, and is finite [4, 12] in the absence of a dilaton, that is, for $f(Z) = g^{-2} + i\theta/8\pi^2 =$ constant. It does not contribute to the effective actions considered in this talk.

First consider the case $f(Z) =$ constant. The quadratically divergent $[O(\Lambda^2)]$ contributions to the one-loop bosonic action are:

$$\begin{aligned} \mathrm{STr}H_{grav} &= -10V - 2M_\psi^2 - \frac{r}{2} - \frac{x}{2}F^2 + 4K_{i\bar{m}}\mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}, \\ \mathrm{STr}H_\chi &= 2N\left(\hat{V} + M_\psi^2 - \frac{r}{4}\right) + 2x^{-1}\mathcal{D}_a D_i (T^a z)^i - 2R_{i\bar{m}}\left(e^{-K}\bar{A}^i A^{\bar{m}} + \mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\right), \\ \mathrm{STr}H_{YM} &= 2\mathcal{D} + \frac{x}{2}F^2 + N_G\frac{r}{2}. \end{aligned} \tag{10}$$

First note the cancellation of the terms containing the squared Yang-Mills field strength F^2. Assuming that the generators of the gauge group are traceless: $\mathrm{Tr}T^a = 0$, the z-dependent terms can by regulated by the introduction Pauli-Villars chiral supermultiplets: $Z_\alpha^I = (\bar{Z}_\alpha^{\bar{I}})^\dagger$, $Z_\alpha'^{\bar{I}} = (\bar{Z}_\alpha'^I)^\dagger$, $\varphi^A = (\bar{\varphi}^A)^\dagger$, where Z^I transforms like z^i under the gauge group, and $Z'^{\bar{I}} \sim \bar{z}^{\bar{i}}$ transforms according to the conjugate representation, with Kähler potential:

$$K(Z, \bar{Z}, \varphi, \bar{\varphi}) = \sum_{\alpha, I=i, M=m} K_{i\bar{m}}(z, \bar{z})\left(Z_\alpha^I \bar{Z}_\alpha^{\bar{M}} + \bar{Z}_\alpha'^I Z_\alpha'^{\bar{M}}\right) + \sum_A e^{\alpha_A K}\varphi^A\bar{\varphi}^A,$$

superpotential:

$$W(Z, \varphi) = \sum_{A,I}\mu_I^\alpha Z_\alpha^I Z_\alpha'^{\bar{I}} + \sum_A \mu_A\left(\varphi^A\right)^2,$$

and signature $\eta^{\alpha,A} = \pm 1$, which determines the sign of the contribution to the supertrace relative to that of a physical supermultiplet (*e.g.*, ghosts have signature -1). The Pauli-Villars contribution to $\mathrm{STr}H_\chi$ is:

$$\begin{aligned}
\mathrm{STr}H_\chi^{PV} &= 2\sum_{\alpha,A}(2\eta_\alpha+\eta_A)\left(\hat{V}+M_\psi^2-\frac{r}{4}\right) \\
&\quad +4\sum_{\alpha,J}\eta_\alpha\left[x^{-1}\mathcal{D}^a D_{(\alpha J)}(T_a z)^{(\alpha J)} - R^{(\alpha J)}_{(\alpha J)i\bar{m}}\left(\bar{A}^i A^{\bar{m}} e^{-K} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\right)\right] \\
&\quad +2\sum_{A}\eta_A\left[x^{-1}\mathcal{D}^a D_A(T_a z)^A - R^A_{Ai\bar{m}}\left(\bar{A}^i A^{\bar{m}} e^{-K} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\right)\right],
\end{aligned} \tag{11}$$

with:

$$\begin{aligned}
\Gamma^A_{Bk} &= \alpha_A \delta^A_B K_k \quad R^A_{Bk\bar{m}} = \alpha_A \delta^A_B K_{k\bar{m}}, \qquad \Gamma^{(I\alpha)}_{(J\beta),k} = \delta^\alpha_\beta \Gamma^i_{jk}, \quad R^{(I\alpha)}_{(J\beta)k\bar{m}} = \delta^\alpha_\beta \delta^I_J R^i_{jk\bar{m}}, \\
D_A(T_a z)^A &= \alpha_A K_i (T_a z)^i, \quad D_I(T_a z)^I = D_i (T_a z)^i.
\end{aligned}$$

To regulate the term proportional to the space-time curvature r, I add $U(1)$ gauge supermultiplets W^a with signature η^a, chiral multiplets $e^{\theta^a} = \left(e^{\bar{\theta}^a}\right)^\dagger$ with signature η^a, $U(1)_b$ charge $q_a\delta_{ab}$, and Kähler potential: $K(\theta,\bar{\theta}) = \frac{1}{2}\sum_a \nu_a e^{\alpha_a K}(\theta_a+\bar{\theta}_a)^2$ which is invariant under $U(1)_b$: $\delta_b\theta_a = -\delta_b\bar{\theta}_a = iq_a\delta_{ab}$. The fields $(\theta^a+\bar{\theta}^a)/\sqrt{2}$ combine with the W^a to form vector supermultiplets with squared mass $\mu_a^2 = (2x)^{-1}q_a^2 e^{\alpha_a K}\nu_a$. The chiral supermultiplets contribute in the same way as φ^A with $\alpha_A \to \alpha_a$; the vector supermultiplets contribute only to the r-term, with the opposite sign. Then the overall contribution from light and heavy modes is:

$$\begin{aligned}
\mathrm{STr}H' &= 2\hat{V}\left[N\left(1+2\sum_\alpha\eta_\alpha\right)+\sum_A\eta_A(1-\alpha_A)+\sum_a\eta_a(1-\alpha_a)-5\right] \\
&\quad +2M_\psi^2\left[N\left(1+2\sum_\alpha\eta_\alpha\right)+\sum_A\eta_A(1-3\alpha_A)+\sum_a\eta_a(1-3\alpha_a)-1\right] \\
&\quad -\frac{r}{2}\left[N\left(1+2\sum_\alpha\eta_\alpha\right)+\sum_A\eta_A+1-N_G\right] \\
&\quad +2\left(1+2\sum_\alpha\eta_\alpha\right)\left[\frac{1}{x}\mathcal{D}_a(T^a z)^i - R_{i\bar{m}}\left(e^{-K}\bar{A}^i A^{\bar{m}} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\right)\right] \\
&\quad +2\left(K_{i\bar{m}}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} - 2\mathcal{D}\right)\left(2-\sum_A\eta_A\alpha_A-\sum_a\eta_a\alpha_a\right).
\end{aligned} \tag{12}$$

The finiteness condition $\mathrm{STr}H' = 0$ requires:

$$0 = 1+2\sum_\alpha\eta_\alpha = \sum_A\eta_A+\sum_a\eta_a-7 = \sum_A\eta_A+1-N_G = 2-\sum_A\eta_A\alpha_A-\sum_a\eta_a\alpha_a. \tag{13}$$

Note that there are only four independent conditions possible, but these are sufficient to cancel eight *a priori* independent quadratically divergent contributions. Next, taking, for example, $q_a = 1$, $\mu_I^\alpha = \beta_\alpha^Z\mu_I$, $\mu_A = \beta_A\mu_\varphi$, $\nu_a = x\beta_a^2|\mu_\theta|^2$, cancellation of the $O(\mu^2\ln\Lambda^2)$ terms requires the additional conditions: $\sum_\alpha\eta_\alpha^Z\left(\beta_\alpha^Z\right)^2 = \sum_A\eta_A(\beta_A)^2 = \sum_a\eta_a(\beta_a)^2 = 0$. The result for the $O(\mu^2)$ part of $S_0+S_1 = \int d^4x\,(\mathcal{L}_0+\mathcal{L}_1)$ is:

$$\begin{aligned}
\mathcal{L}_0(g^0_{\mu\nu},K)+\mathcal{L}_1 &= \mathcal{L}_0(g_{\mu\nu},K+\delta K), \qquad g_{\mu\nu} = g^0_{\mu\nu}(1+\epsilon), \\
\epsilon &= -\sum_P\frac{\lambda_P}{32\pi^2}e^{-K}A_{PQ}\bar{A}^{PQ}, \qquad \delta K = \sum_P\frac{\lambda_P}{32\pi^2}\left(e^{-K}A_{PQ}\bar{A}^{PQ}-4\mathcal{K}^P_P\right),
\end{aligned} \tag{14}$$

where P, Q = represent all heavy fields, and [13] $\lambda_P = 2\sum_p \eta_p^P \left(\beta_p^P\right)^2 \ln \beta_P^p$. This result can be expressed in terms of effective cut-offs Λ_Φ:

$$\epsilon = \sum_\Phi \frac{\lambda_\Phi \Lambda_\Phi^2}{32\pi^2}\zeta'_\Phi, \qquad \delta K = \sum_\Phi \frac{\lambda_\Phi \Lambda_\Phi^2}{32\pi^2}\zeta_\Phi, \tag{15}$$

where:

$$\begin{aligned}
\lambda_\Phi &= 2\sum_p \eta_p^\Phi \left(\beta_p^\Phi\right)^2 \ln \beta_\Phi^p, \qquad \zeta_Z = \zeta_\varphi = \zeta'_Z = \zeta'_\varphi = 1, \quad \zeta_\theta = -4\,\zeta'_\theta = 0, \\
\Lambda_{ZI}^2 &= \sum_{m=M} e^K \left(K^{i\bar{m}}\right)^2 \mu_I \bar{\mu}_{\bar{M}}, \qquad \Lambda_\varphi^2 = e^{K(1-2\alpha_\varphi)}|\mu_\varphi|^2, \qquad \Lambda_\theta^2 = |\mu_\theta|^2 e^{\alpha_\theta K}.
\end{aligned}$$

Note however that if there are three or more terms in the sum over p, the sign of λ_Φ is indeterminate [13], so caution should be used in making conclusions about the implications of these terms for the stability of the effective potential.

Before including the dilaton, I note that there is an ambiguity in the separation (9) of the fermion determinant into helicity-even and -odd contributions. For example, if $D_\mu = \partial_\mu + V_\mu + iA_\mu\gamma_5$, $M = m + m'\gamma_5$, we would identify $D_\mu^\pm = \partial_\mu + V_\mu \pm iA_\mu$, $M^\pm = m \mp m'$. However terms even and odd in γ_5 can be interchanged by the use of the identities:

$$\gamma_5 = (i/24)\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma, \qquad \sigma_{\mu\nu} = i\gamma_5\sigma_{\mu\nu}, \qquad etc.$$

The choice is generally dictated by gauge or Kähler covariance. However there is an off-diagonal gaugino-α mass term:

$$M_{\alpha\lambda^a} = -\sqrt{\frac{x}{2}}F_a^{\mu\nu}\sigma_{\mu\nu} = -\sqrt{\frac{x}{2}}\left(\alpha F_a^{\mu\nu} + i\beta\gamma_5\tilde{F}_a^{\mu\nu}\right)\sigma_{\mu\nu}, \qquad \alpha + \beta = 1. \tag{16}$$

The one-loop action S_1 is invariant under the choice of α only if the integrals are finite. For an arbitrary choice we obtain, instead of (10)

$$\mathrm{STr}H_{grav} \ni \frac{x}{2}F^2(\alpha^2 - \beta^2 - 2), \quad \mathrm{STr}H_{YM} \ni \frac{x}{2}F^2(\alpha^2 - \beta^2).$$

The choice $\alpha = 1$ is the "supersymmetric" one, in that it matches analogous matrix elements in bose and ghost sectors, resulting in the cancellation of the F^2 terms in (10). We now include a dilaton, that is we take $f_{ab} = \delta_{ab}f$, $f = x + iy \neq$ constant (which is trivially generalized to $f_{ab} = \delta_{ab}k_a f$, k_a = constant, and so includes all known string models). There is a dilatino-gaugino mass term:

$$M_{\chi^i\lambda^a} = -i\frac{f_i}{4\sqrt{x}}\left(\gamma F_a^{\mu\nu} + i\delta\gamma_5\tilde{F}_a^{\mu\nu}\right)\sigma_{\mu\nu}, \qquad \gamma + \delta = 1, \quad f_i = \partial_i f,$$

and an additional gaugino connection:

$$A^\mu_{\lambda^a\lambda^b} = -\delta_{ab}\frac{\partial^\mu y}{2x}\left(i\epsilon\gamma_5 - \zeta\frac{\epsilon^{\lambda\nu\rho\sigma}}{24}\gamma_\lambda\gamma_\nu\gamma_\rho\gamma_\sigma\right), \qquad \epsilon + \zeta = 1,$$

that give the additional contributions to the supertraces:

$$\begin{aligned}
\mathrm{STr}H_{YM} &\ni \frac{f_i\bar{f}^i}{4x}F^2(\gamma^2 - \delta^2) - N_G\left(2M_\lambda^2 + \frac{1}{2x^2}\left[\partial_\mu x\partial^\mu x + (3 - 2\zeta^2)\partial_\mu y\partial^\mu y\right]\right), \\
\mathrm{STr}H_{grav} &\ni \frac{f_i\bar{f}^i}{4x}F^2(\gamma^2 - \delta^2) - \frac{f_i\bar{f}^i}{2x^2}\mathcal{D}, \qquad \mathrm{STr}H_\chi \ni \frac{f_i\bar{f}^i}{2x^2}\mathcal{D},
\end{aligned} \tag{17}$$

where $M_\lambda^2 = (2x)^{-2}e^{-K}f_i\bar{f}^j A_j\bar{A}^i$, $\bar{f}^i = K^{i\bar{m}}\bar{f}_{\bar{m}}$. In this case, the "supersymmetric" choices that match matrix elements in the bosonic and ghost sectors are $\gamma = \delta = \frac{1}{2}$, $\epsilon = 0$, $\zeta = 1$, and the F^2 terms again cancel; the remaining terms:

$$\mathrm{STr}H \ni -N_G\left(2M_\lambda^2 + \frac{1}{2x^2}\left[\partial_\mu x\partial^\mu x + \partial_\mu y\partial^\mu y\right]\right), \tag{18}$$

can be regulated by the introduction of additional Pauli-Villars chiral multiplets: $\pi^\alpha = (\bar{\pi}^\alpha)^\dagger$ with

$$K(\pi,\bar{\pi}) = \sum_\alpha (f+\bar{f})|\pi^\alpha|^2, \quad W(\pi) = \sum_\alpha \beta_\pi^\alpha \mu_\pi(z)(\pi^\alpha)^2, \quad \eta_\alpha^\pi = \pm 1.$$

Finiteness requires: $\sum_\alpha \eta_\alpha^\pi = +N_G$, $\sum_\alpha \eta_\alpha^\pi(\beta_\pi^\alpha)^2 = 0$, and the results (12–15) are modified accordingly; the sums in (14,15) are extended to include, respectively $P = \pi_\alpha$, $\Phi = \pi$, with:

$$\Lambda_\pi^2 = \frac{e^K}{4x^2}|\beta_\alpha^\pi\mu_\pi|^2\delta_\beta^\alpha(\beta_\alpha^\pi)^2, \qquad \zeta_\pi = \zeta'_\pi = 1.$$

The result $\zeta = 1$ has implications for chiral/linear multiplet duality. In this case the y-axion contribution to the gaugino connection is $A_\mu = 2xh^{\nu\rho\sigma}\gamma_{[\mu}\gamma_\nu\gamma_\rho\gamma_{\sigma]}$, where $h^{\nu\rho\sigma} = \epsilon^{\nu\rho\sigma\mu}\partial_\mu y/4x^2$ is the three-form that is dual to y-axion in the absence of interactions; there is a similar term in the vector connection. This suggests that the linear supermultiplet formalism [14, 15] is the natural framework for describing the dilaton supermultiplet. It has been shown [16] for several models that axion/three-form duality holds at the quantum level up to finite topological anomalies. In the presence of fermion couplings to the dilaton, there are additional anomalies; interchanging terms in T_+ and T_- is analogous to shifting the integration variable. The linearly divergent triangle diagram leads to an ill-defined finite chiral anomaly that is fixed by imposing local gauge invariance; in the present case supersymmetry resolves the ambiguity. In addition, with the choice $\zeta = 1$, the dilaton supermultiplet contributes a purely "vector-like" gaugino connection, and there is no y-axion analogue of the modular anomaly (the $\mathrm{Im}tF\tilde{F}$ term induced at one loop). This result coincides with the conclusion of [17], where it was argued that a $yF\tilde{F}$ term is inconsistent with the linearity constraint of the linear multiplet formulation.

To fully regulate the theory, including all logarithmic divergences, requires additional Pauli-Villars fields and/or couplings. For example, to regulate the Yang-Mills sector we must include in the set φ^A N_G chiral multiplets $\varphi_a^A = (\bar{\varphi}_a^A)^\dagger$ that transform according to the adjoint representation of the gauge group, with $\sum_{A,a}\eta_A^a = 3N_G$. In this case the effective cut-off has been determined [17] by imposing supersymmetric matching of chiral and conformal anomalies, giving $\alpha_A^a = \frac{1}{3}$. Imposing the full finiteness conditions on $\mathrm{STr}H'^2$ may constrain the other parameters $\alpha_A, \alpha_a, \alpha_\alpha$. This program has been carried out [3] only in the case of global supersymmetry with no dilaton. In this case only Pauli-Villars chiral multiplets are needed. The full superpotential and Kähler potential are, respectively:

$$\begin{aligned}
W(z, Z_\alpha, Z'_\alpha, \varphi_\beta) &= W(z) + \sum_{\alpha,I}\mu_I^\alpha Z_\alpha^I Z'^I_\alpha + \sum_{a,\beta}\mu_\beta\varphi_a^\beta\varphi_\beta^a + \frac{1}{2}\sum_\alpha a^\alpha W(Z)_{ij}Z_\alpha^I Z_\alpha^J \\
&\quad + g^2\sum_\gamma b^\gamma\varphi_\gamma^a(Z'_\gamma T_a Z), \\
K(z^i, \bar{z}^{\bar{\imath}}, \phi_{PV}) &= K(z^i, \bar{z}^{\bar{\imath}}) + \sum_\beta \bar{\varphi}_a^\beta\varphi_\beta^a + \sum_\alpha K_{i\bar{m}} \\
&\quad (z,\bar{z})\left(Z_\alpha^I\bar{Z}_\alpha^{\bar{M}} + \bar{Z}_\alpha'^I Z_\alpha'^{\bar{M}}\right) + \sum_{\alpha;\, I,J=i,j}\beta_\alpha K_{ij}(z,\bar{z})Z_\alpha^I Z_\alpha^J + O\left(\phi_{PV}^2\right),
\end{aligned}$$

with

$$\alpha = 1,\cdots,N_I, \quad \beta = 1,\cdots,N_\varphi, \quad \gamma = 1,\cdots,N_{I\varphi}, \quad N_{I\varphi} \le \min\{N_I, N_\varphi\},$$

and additional constraints on $a^\alpha, b^\gamma, \beta_\alpha$ are imposed by the finiteness requirement.

Some of the cut-offs have a straightforward physical interpretation in the context of string theory. For chiral matter in torus compactifications or the untwisted sector of orbifold compactifications, the cut-off (in reduced Planck units and assuming a common *vev* for the moduli), $\Lambda_{Z^I}|_{\rm vac} = \Lambda_{\rm comp} = 1/R$, is just the inverse radius of compactification. The cut-off for the Yang-Mills sector, $\Lambda_{\rm gauge} = g^{-\frac{2}{3}}\Lambda_{\rm comp}$, can be interpreted in terms of the same cut-off, but incorporating the 2-loop correction to the β-function, which assures the supersymmetric relation between chiral and conformal anomalies. For twisted chiral matter, the interpretation is less transparent: $\Lambda^I_{\rm twisted}\Big|_{\rm vac} = \Lambda_{\rm comp}\left(g^{-4}\Lambda^4_{\rm comp}\right)^{3q_I-1}$. The twisted sector must be included to assure both the cancellation of the modular anomaly and consistency with string loop calculations [18, 19]. Including the Green-Schwarz counterterm [14], matching field theory anomalies–as calculated with the above prescriptions–to string loop calculations, and using the all-loop supersymmetric renormalization group invariant function of [20], it was shown [17] that the two-loop gauge unification scale (as conventionally defined by phenomenologists) is the string scale. The same conclusion has been reached in [21] where the same RGE invariant function, but a different calculational procedure, was used.

The extension of the procedure described here to a full regularization of supergravity is in progress [22]. Two cases are of special interest. For a compact σ-model coupled to gravity, the classical ratio f_π/m_{Pl} is fixed by the Bagger-Witten quantization condition [23], and one may ask whether this condition persists at the quantum level, which would require $\delta K = 0$ in (15). Since (in a class of σ-models) the scalar Ricci tensor $R_{i\bar{m}}$ is proportional to the Kähler metric $K_{i\bar{m}}$, the cut-offs in (15) can be chosen such that the quadratically divergent parts of gravity and chiral loops cancel in δK. The possibility of a full cancellation of the ultraviolet divergences as well as of the chiral anomaly is under investigation. More interesting for string theory are the noncompact σ-models that arise in torus and orbifold compactifications. These models possess classical noncompact, nonlinear symmetries that contain a discrete subgroup of modular transformations. The same considerations hold at the one-loop level as for the compact case. The possibility of a regularization procedure that respects the full continuous symmetry as well as the discrete modular symmetry has potentially important implications for phenomenology. The parameters introduced above to specify the couplings of the PV sector are in general field-dependent: $\mu = \mu(z)$, $\nu = \nu(z,\bar{z})$. For example, in superstring theory there is an invariance under a modular transformation: $K \to K + F(z) + \bar{F}(\bar{z})$, $W \to e^{-F(z)}W$, that is unbroken by perturbative quantum corrections. Thus Z^I_α has the same modular weight as z^i and φ^A has modular weight $-\alpha_A/2$; the z-dependence of $\mu(z)$, $\nu_a(z,\bar{z})$ must be chosen accordingly; this field dependence can be interpreted as threshold effects from integration over heavy modes. Typically, one expects: $\mu(z) \propto \eta(T)^p$, where T is a modulus and $\eta(T)$ is the Dedekind η-function. Such terms break the continuous classical symmetry, thereby destroying the no-scale structure of these models ($\langle V\rangle = 0$) and the protection of a mass hierarchy $\Delta m_{SUSY} \ll m_{\tilde{G}}$ that is desirable both for phenomenology and cosmology, as has been discussed in several lectures at this conference. These problems might be avoided if an anomaly free regularization is possible.

Full Pauli-Villars regularization of supergravity including the dilaton can be investigated once the ultraviolet divergences have been fully determined for this case [12].

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LAWRENCE BERKELEY NATIONAL LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL & ELECTRONIC INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720