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Effective Field Theories of Baryons and Mesons, or, What Do Quarks Do?

G.L. Keaton
(Ph.D. Thesis)

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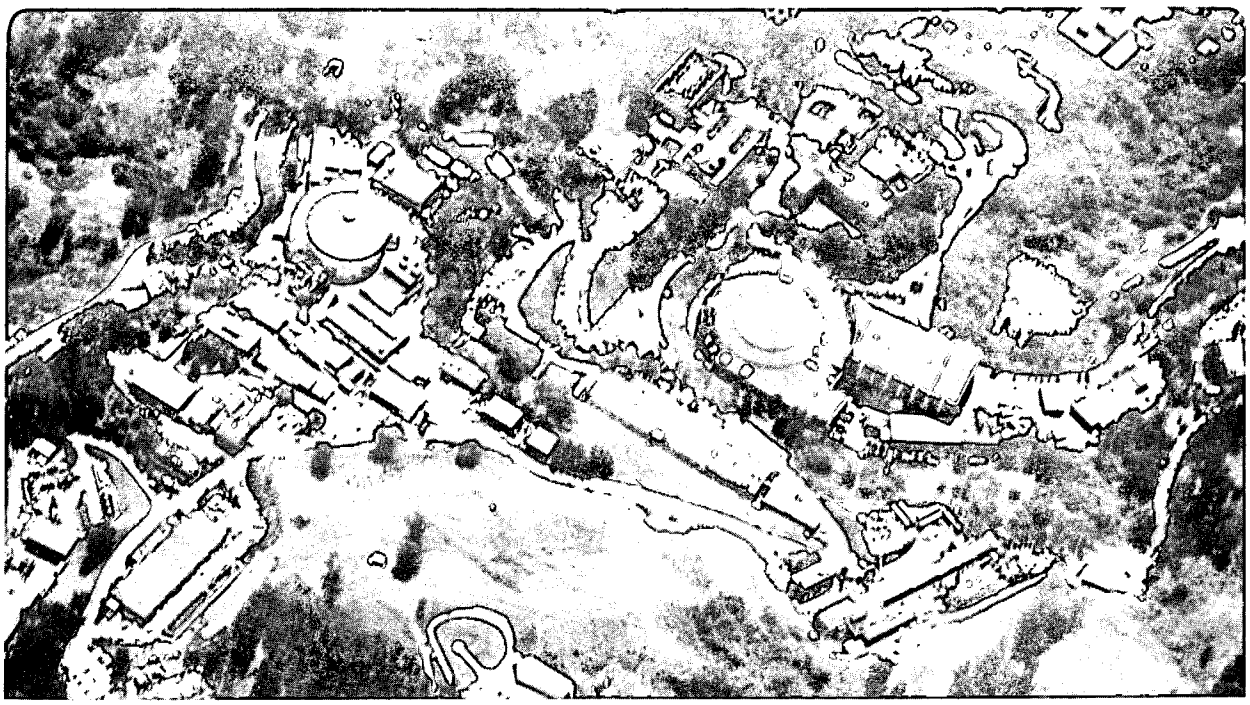

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Effective Field Theories of Baryons and Mesons, Or, What Do Quarks Do? *

(Ph.D. Thesis)

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Effective Field Theories of Baryons and Mesons
Or,
What Do Quarks Do?

by

Gregory Lee Keaton

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Abstract

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Doctor of Philosophy in Physics
University of California at Berkeley

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This thesis is an attempt to understand the properties of the protons, pions and other hadrons in terms of their fundamental building blocks. In the first chapter, I review several of the approaches that have already been developed. The Nambu–Jona-Lasinio model offers the classic example of a derivation of meson properties from a quark Lagrangian. The chiral quark model encodes much of the intuition acquired in recent decades. I also discuss the non-linear sigma model, the Skyrme model, and the constituent quark model, which is one of the oldest and most successful models.

In the constituent quark model, the constituent quark appears to be different from the "current" quark that appears in the fundamental QCD Lagrangian. Recently it was proposed that the constituent quark is a topological soliton. In chapter 2 I investigate this soliton, calculating its mass, radius, magnetic moment, color magnetic moment, and spin structure function. Within the approximations used, the magnetic moments and spin structure function cannot simultaneously be made to agree with the constituent quark model. Some discussion of what to expect from better approximations is included.

In Chapter 3 I use a different plan of attack. Rather than trying to model the constituents of the baryon, I begin with an effective field theory of baryons and mesons, with couplings and masses that are simply determined phenomenologically. Meson loop corrections to baryon axial currents are then computed

in the $1/N$ expansion. It is already known that the one-loop corrections are suppressed by a factor $1/N$; here it is shown that the two-loop corrections are suppressed by $1/N^2$. To leading order, these corrections are exactly the same as would be calculated in the constituent quark model. This method therefore offers a different approach to the constituent quark.

The appendices give some calculational details omitted in the text, including the strange fractal-like behaviour encountered while integrating some of the differential equations used in Chapter 2. The epilogue is an endeavor to synthesize many of the ideas presented here.

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Chapter 1

Tools of the Trade

One of the main questions facing particle phenomenology today is, how can the properties of the baryons and mesons be derived from QCD? The QCD Lagrangian is written in terms of the quark and gluon fields:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\psi - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + (\text{ghosts}) + (\text{gauge fixing})$$

Since these quarks and gluons compose hadrons, it seems reasonable that the QCD Lagrangian could give rise to an "effective" Lagrangian whose dynamical degrees of freedom are not quarks and gluons, but baryons and mesons. The coupling constants and masses in this proposed effective theory are in principle calculable from the underlying QCD Lagrangian.

Atomic physics provides the ideal example of such a project, where a knowledge of the underlying force (the Coulomb force) allows us to predict the all the properties of the states of the hydrogen atom. One could also imagine making an effective Lagrangian for hydrogen atoms, treating them as a single particles experiencing the Van der Waals force of attraction. The strength of this force can be calculated from the fundamental theory. Such an approach works when the atoms are separated by distances which are large compared to the atomic radius. At shorter distances, however, or at energies greater than 13.6 eV, the detailed structure of the atoms comes into play, and we can no longer overlook their composite structure.

If such an effective theory is possible in atomic physics, albeit within the

above limitations, can we construct a parallel effective theory of hadrons, operable within analogous perimeters? We immediately encounter a problem: the strong force is much stronger than the Coulomb force. This means that inside the proton, which ostensibly consists of three quarks, the forces are so great that the energy contained in the gluon field strength is enough to create additional quark–anti-quark pairs and gluons. As a result it is not even known how many quarks, anti-quarks, and gluons to include in the Fock space of the proton, let alone how to calculate the dynamics of these constituents.

It has therefore proven impossible—so far—to derive an effective theory of mesons and baryons from QCD. However, I think it is a good idea to have in mind an example of what such a derivation would look like, if it *were* possible. The clearest example comes from the Nambu–Jona-Lasinio model. In this model the QCD interactions have been simplified (or approximated) to the point where an effective theory of pions can actually be derived. The calculations are relatively easy—almost pretty—and it gives us some idea of what to expect from effective theories in general. The model is not perfect, but once it is clear what goes wrong, it can be generalized in certain ways to better accommodate the data (at the price of being less predictive).

This chapter will therefore begin with a summary of the Nambu–Jona-Lasinio model, and then go on to review other related approaches to baryon-meson effective field theories.

1.1 The Nambu–Jona-Lasinio Model

The Nambu–Jona-Lasinio model [1, 2] begins with the following Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/)\psi + \frac{g^2}{\Lambda^2}\left\{(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau^a\psi)^2\right\} \tag{1.1}$$

This is essentially a relativistic generalization of the BCS Lagrangian used in the theory of superconductivity [3]. The hope is that somehow the quarks' interactions with each other through gluons can be roughly approximated by the above four-quark interactions. This is already, then, a kind of effective theory. QCD should in principle give us the coupling g as well as the scale

Λ above which the Lagrangian is not expected to be a good approximation. I discuss how these might be obtained from QCD later; for now I just take g and Λ to be unknown constants to be fitted phenomenologically to the data at the end of the calculation.

The Lagrangian (1.1) is invariant under the following "chiral transformation":

$$\begin{aligned} \delta\psi &= i\epsilon_b\tau^b\gamma_5\psi \\ \delta\bar{\psi} &= \bar{\psi}i\epsilon_b\tau^b\gamma_5 \end{aligned} \tag{1.2}$$

where the τ^b are the Pauli matrices for the $SU(2)$ flavor group. This invariance is important because the QCD Lagrangian is also invariant under this symmetry (except for the quark mass terms, which are small). However, in Nature the chiral symmetry seems to be broken spontaneously by the vacuum state, and the pions are the "almost"-Goldstone bosons of the broken symmetry ("almost" because the pions are not quite massless; they acquire a small mass due to the explicit symmetry breaking of the quark masses).

In this treatment, I take the mass of the quarks to be approximately zero. If the Nambu–Jona-Lasinio model is realistic, we should expect to see that chiral symmetry is somehow spontaneously broken, and massless pions result.

The path integral greatly simplifies the analysis of this model. We begin with the generating functional Z with external scources j and j_5^a:

$$\begin{aligned} Z[j,j_5^a] &= \int D\psi D\bar{\psi} \exp i \int d^4x \left\{ \bar{\psi}i\partial\!\!\!/\psi + \frac{g^2}{\Lambda^2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau^a\psi)^2] \right. \\ &\qquad \left. + j\bar{\psi}\psi + j_5^a\bar{\psi}\gamma_5\tau^a\psi \right\} \end{aligned} \tag{1.3}$$

Taking derivatives of Z with respect to the currents will give Green functions:

$$-\frac{\delta^2 Z}{\delta j_5^a(x)\delta j_5^b(y)} = < T\left(\bar{\psi}(x)\gamma_5\tau^a\psi(x)\right)\left(\bar{\psi}(y)\gamma_5\tau^b\psi(y)\right) > \tag{1.4}$$

When x is well separated from y we expect that the disturbance created by $\bar{\psi}\gamma_5\tau^a\psi$ will travel mainly in the form of a pion, so the Green function should be roughly proportional to a pion propagator. Similarly,

$$-\frac{\delta^2 Z}{\delta j(x)\delta j(y)} = < T\left(\bar{\psi}(x)\psi(x)\right)\left(\bar{\psi}(y)\psi(y)\right) >$$

is roughly proportional to the propagator of a scalar particle, if such a particle exists.

The main trick involved in the Nambu–Jona-Lasinio model is the following: the path integral of Eq. (1.3) can be rewritten using auxiliary fields σ and π^a as

$$\begin{aligned} Z &= \int D\psi D\bar{\psi} D\sigma D\pi^a \exp i \int d^4x \left\{ \bar{\psi} i\partial\!\!\!/ \psi + \frac{g^2}{\Lambda^2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau^a\psi)^2] \right. \\ &\quad + j\bar{\psi}\psi + j_5^a\bar{\psi}\gamma_5\tau^a\psi - \left(\frac{\Lambda}{2g}\sigma + \frac{g}{\Lambda}\bar{\psi}\psi + \frac{\Lambda}{2g}j\right)^2 \\ &\quad \left. - \left(\frac{\Lambda}{2g}\pi^a + \frac{ig}{\Lambda}\bar{\psi}\gamma_5\tau^a\psi + \frac{\Lambda}{2ig}j_5^a\right)^2 \right\} \end{aligned}$$

This equation can be verified by noting that the $D\sigma$ and $D\pi^a$ integrals can be performed—they are simply Gaussians—and just contribute to the normalization constant in front of the integral (which we are ignoring). However, rather than doing these integrals, it is more advantageous to multiply out the terms, which have been arranged so that all the four-quark terms cancel:

$$\begin{aligned} Z &= \int D\psi D\bar{\psi} D\sigma D\pi^a \exp i \int d^4x \left\{ \bar{\psi}(i\partial\!\!\!/ - \sigma - i\gamma_5\tau^a\pi^a)\psi - \frac{\Lambda^2}{4g^2}[\sigma^2 + \pi^2] \right. \\ &\quad \left. - \frac{\Lambda^2}{4g^2}[2j\sigma - 2ij_5^a\pi^a + j^2 - j_5^2] \right\} \end{aligned} \tag{1.5}$$

Now the action is only quadratic in the quark fields, so the fermionic integral can be evaluated (using $\pi^a\tau^a \equiv \boldsymbol{\pi}$):

$$\begin{aligned} Z &= \int D\sigma D\pi^a \det(i\partial\!\!\!/ - \sigma - i\gamma_5\boldsymbol{\pi}) \\ &\qquad \cdot \exp i \int d^4x \frac{-\Lambda^2}{4g^2}\{\sigma^2 + \pi^2 + 2j\sigma - 2ij_5^a\pi^a + j^2 - j_5^2\} \\ &= \int D\sigma D\pi^a \exp\left\{ \operatorname{tr}\log(p\!\!\!/ - \sigma - i\gamma_5\boldsymbol{\pi}) \right. \\ &\qquad \left. + i\int d^4x \frac{-\Lambda^2}{4g^2}[\sigma^2 + \pi^2 + 2j\sigma - 2ij_5^a\pi^a + j^2 - j_5^2] \right\} \end{aligned} \tag{1.6}$$

Here the term $\not{p}-\sigma-i\gamma_5\boldsymbol{\pi}$ is treated as a matrix with Dirac, flavor, and spacetime indices, and I have nonchalantly taken the logarithm and trace of it. For more details on this peculiar kind of matrix, see Appendix A.

We now have an action written entirely in terms of the σ and π^a fields. The interpretation of these fields comes from the Green functions:

$$\frac{\delta^2 Z}{\delta j_5^a(x)\delta j_5^b(y)} = \left(\frac{\Lambda^2}{2g^2}\right)^2 < T\pi^a(x)\pi^b(y) > +i\frac{\Lambda^2}{2g^2}\delta(x-y)$$

As argued after Eq. (1.4), this Green function should be roughly proportional to the pion propagator if x is well separated from y. Here we see that it is proportional to the propagator of the π^a field. We therefore make the identification that π^a is proportional to the physical pion field. Similarly, σ is proportional to a physical scalar particle (the experimental status of which is dubious—see later).

This identification of the physical fields was made possible by introducing the currents into the Lagrangian. We will not need the currents any more, however, so I drop them.

Up to this point, there have been only formal—and exact—manipulations of the original Nambu–Jona-Lasinio path integral. Now it is time to make a huge approximation, which is that the action appearing in Eq. (1.6) can be used at tree level. That is, we hope that the σ and π^a loops are not important. Such a drastic measure is somewhat justified in the large-N expansion, where it is assumed that the original Lagrangian (1.1) contains N identical copies of up- and down-quarks. In this case the coupling is scaled, $g \to g/\sqrt{N}$, in order to keep the theory finite as $N \to \infty$. Then the effective action of Eq. (1.6) becomes

$$\Gamma = -iN\,\mathrm{tr}\log(\not{p}-\sigma-i\gamma_5\boldsymbol{\pi}) - \int d^4x \frac{N\Lambda^2}{4g^2}(\sigma^2+\pi^2) \tag{1.7}$$

If N is large, the action is large, and so the classical approximation is justified. In QCD, N=3, so we make the classical approximation without much justification.

The fields σ and π assume whatever values minimize the energy. Choose

$\pi^a = 0$ and $\sigma =< \sigma >=$ const.[1] Then

$$0 = \frac{\partial \Gamma}{\partial \sigma} = \mathrm{tr}\,\frac{i}{\not{p} - \sigma} - \frac{\Lambda^2}{2g^2}\sigma$$

Explicitly we have (including a factor 2 for the flavor trace and 4 for the Dirac trace; see also Appendix A),

$$2 \cdot 4i \int \frac{d^4p}{(2\pi)^4} \frac{\sigma}{p^2 - \sigma^2} = \frac{\Lambda^2}{2g^2}\sigma \tag{1.8}$$

Eq. (1.8) is a condition on $< \sigma >$ and is the analog of the gap equation in superconductivity. One solution is $\sigma = 0$; the other is given by

$$\frac{1}{g^2} = \frac{1}{\pi^2}\left(1 - \frac{\sigma^2}{\Lambda^2}\log\frac{\Lambda^2}{\sigma^2}\right)$$

If a solution to this transcendental equation exists, g must be greater than π. Chiral symmetry is broken only if the coupling is strong enough; then σ can take on a vacuum expectation value v. If $g < \pi$, however, the only solution is $< \sigma >= 0$.

The rest of the analysis of the Nambu–Jona-Lasinio model proceeds as follows. Assume that the σ-field takes on a vacuum expectation value v. Then expand around this value: $\sigma = v + s$. The logarithm appearing in the effective action (1.7) can be written,

$$\begin{aligned} &\mathrm{tr}\log\left(\not{p} - v - (s + i\gamma_5\pi)\right) \\ &\quad = \mathrm{tr}\log\left(1 - \frac{1}{\not{p} - v}(s + i\gamma_5\pi)\right) + \mathrm{tr}\log(\not{p} - v) \end{aligned}$$

The last term just contributes an overall constant and can therefore be dropped. The remaining logarithm can then be expanded,

[1] Since π^a represents a pseudo-scalar field and σ represents a scalar field, and since the vacuum should be invariant under the parity transformation, this is the logical choice. If, however, we chose (say) $< \pi^1 >=< \sigma >=$ const, we could simply rotate our coordinate frame (and with it our definition of the parity operation) so that in the new system, $< \pi^a >= 0$ and $< \sigma >=$ const.

$$\begin{aligned}
&\mathrm{tr}\log\left(1-\frac{1}{\not p-v}(s+i\gamma_5\boldsymbol{\pi})\right)\\
&=\mathrm{tr}\left\{\frac{-1}{\not p-v}(s+i\gamma_5\boldsymbol{\pi})-\frac{1}{2}\frac{1}{\not p-v}(s+i\gamma_5\boldsymbol{\pi})\frac{1}{\not p-v}(s+i\gamma_5\boldsymbol{\pi})\right.\\
&\left.-\frac{1}{3}\frac{1}{\not p-v}(s+i\gamma_5\boldsymbol{\pi})\frac{1}{\not p-v}(s+i\gamma_5\boldsymbol{\pi})\frac{1}{\not p-v}(s+i\gamma_5\boldsymbol{\pi})+\ldots\right\} \qquad (1.9)
\end{aligned}$$

This expansion gives rise to an infinite number of meson vertices, each one of which is calculable (see Appendix A). The pion is massless, as expected, because the two terms that might give the pion mass cancel exactly, thanks to the gap equation, (1.8).

To leading order in p^2/Λ^2 and v^2/Λ^2, the effective Lagrangian turns out to be

$$\begin{aligned}
\mathcal{L} \;=\; & \frac{N}{2}\frac{\log(\Lambda^2/v^2)}{4\pi^2}[(\partial_\mu s)^2+(\partial_\mu\pi^a)^2]-2Nv^2\frac{\log(\Lambda^2/v^2)}{4\pi^2}s^2\\
& -vN\frac{\log(\Lambda^2/v^2)}{2\pi^2}s(s^2+(\pi^a)^2)-\frac{N\log(\Lambda^2/v^2)}{8\pi^2}(s^2+(\pi^a)^2)^2\\
& +\ldots \qquad (1.10)
\end{aligned}$$

This Lagrangian must be renormalized, in order that the kinetic energy terms have the canonical normalization. Hence

$$s=Rs_{phys} \quad ; \quad \pi^a=R\pi^a_{phys}$$

where

$$R=\frac{2\pi}{\sqrt{N\log(\Lambda^2/v^2)}}$$

One encouraging result is that, because the fields are renormalized by a factor $\propto 1/\sqrt{N}$, the three-meson vertices are $\sim 1/\sqrt{N}$, the four-meson vertices are $\sim 1/N$; and each additional meson costs a factor of $1/\sqrt{N}$ (cf. Eq.(1.10)). This is exactly the result obtained by 't Hooft [4] for the coupling of mesons in the limit of a large number of colors for QCD.

Evidently the Nambu–Jona-Lasinio model has the right qualitative behaviour. Quantitatively, the model has had some success as well. It can be generalized to include three flavors, and to include vector and axial vector mesons.

(This generalization is often called the "extended Nambu–Jona-Lasinio model.") In such a model, decay rates and scattering lengths can be calculated. These calculations match the experimental numbers with an accuracy of 10% to 20% or better. This success must be examined critically, however. The masslessness of the pions is guaranteed by the Goldstone theorem, and the $\pi - \pi$ scattering amplitudes are fixed by current algebra to order p^2. The Nambu–Jona-Lasinio model correctly predicts these quantities, but since they are model independent results, they cannot be taken as successes of the Nambu–Jona-Lasinio model itself.

In order to genuinely test the Nambu–Jona-Lasinio model, other parameters must be calculated, such as the order p^4 contributions to scattering amplitudes, and the masses of the mesons in the extended model. Many of the p^4 amplitudes fit the data within the experimental uncertainties, but the masses of the particles are not so well predicted [1, 2]. In fact, there is no hard evidence that the scalar particle even exists.

We therefore need to find some way to generalize and improve the model. The following sections illustrate some of the approaches that are popular.

1.2 The Non-Linear Sigma Field

One of the major embarrassments of the above model is that there is no convincing candidate for the σ particle. So we should either assume it is very heavy and integrate it out of the Lagrangian, or else not include it in the first place. This section examines the latter option.

We begin with some notation. Let Σ be defined by

$$\Sigma = \frac{1}{v}(\sigma + i\pi^a\tau^a)$$

How does Σ transform under the chiral transformation (1.2)? Taking our cue from the previous section we take $\sigma \sim -\bar{\psi}\psi$ and $\pi^a \sim -i\bar{\psi}\gamma_5\tau^a\psi$. Then under the chiral transformation,

$$\begin{aligned} \delta\sigma &= 2\epsilon_b\pi^b \\ \delta\pi^a &= -2\epsilon_a\sigma \end{aligned}$$

Therefore Σ transforms as

$$\Sigma \rightarrow e^{-i\epsilon_b\tau_b}\Sigma e^{-i\epsilon_b\tau_b} \tag{1.11}$$

Now we are ready to eliminate the σ field. We assume, as the evidence suggests, that there are only three meson degrees of freedom, not four (for two flavors at low energies). One way to enforce this is to require

$$\sigma^2 + \pi^2 = v^2$$

This requirement can be rewritten in terms of Σ,

$$\det \Sigma = 1,$$

which means that Σ, previously a $U(2)$ matrix by its definition, must be an $SU(2)$ matrix. Accordingly, Σ can be written

$$\Sigma = e^{i\theta^a\tau^a/v}$$

where now the θ^a parameterize the physical degrees of freedom. We might as well identify these with the pions; some general theorems guarantee that this is legitimate [5]. We therefore have the following recipe for eliminating the σ field:

$$\sigma + i\boldsymbol{\pi} \rightarrow ve^{i\boldsymbol{\pi}/v} \quad \text{and} \quad \sigma + i\gamma_5\boldsymbol{\pi} \rightarrow ve^{i\gamma_5\boldsymbol{\pi}/v}$$

1.3 The Chiral Quark Model

Armed now with the non-linear sigma field, we return to Eq. (1.5), which occurred half-way through the derivation of the meson Lagrangian in the Nambu–Jona-Lasinio model. Assuming that we could somehow reach this point from QCD, and assuming that the pion field we would end up with would be non-linear, the quark part of the Lagrangian would be

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - ve^{i\pi^a\tau^a\gamma_5/v})\psi$$

Actually it is too presumptuous to assume that we would know the coupling of the pions to the quarks, so we should add an unknown coupling constant g_1:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - g_1 ve^{i\pi^a\tau^a\gamma_5/v})\psi$$

This Lagrangian can also be written using the left- and right-handed components of the quark fields, and $\Sigma = \exp(i\tau^a\pi^a/v)$,

$$\mathcal{L} = \bar{\psi}i\partial\!\!\!/\psi - g_1 v\bar{\psi}_L\Sigma\psi_R - g_1 v\bar{\psi}_R\Sigma^\dagger\psi_L \tag{1.12}$$

Here $g_1 v$ is equal to the mass of the quark. We started with massless quarks, but now because chiral symmetry is broken ($< \Sigma >= 1$ spontaneously breaks the symmetry), the quarks acquire a mass $m = g_1 v$. This mass is to be identified with the *constituent* mass of the quarks, about 330 MeV according to constituent quark models of the nucleon.

Georgi and Manohar [19] envision the following scheme for getting to Eq. (1.12) from QCD. Imagine picking some scale Λ, and integrating out of the path integral all of the modes that have an energy higher than Λ, leaving behind an effective Lagrangian for the modes of energy less than Λ. If Λ is less than or equal to some threshold value Λ_χ, the quarks' interactions with each other through the gluons somehow spontaneously break chiral symmetry, resulting in the creation of Goldstone bosons (the pions). If we set $\Lambda = \Lambda_\chi$, therefore, we expect that the resulting effective Lagrangian will contain quarks, pions, and gluons. The Lagrangian is:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\partial\!\!\!/ - gA\!\!\!/)\psi - m\bar{\psi}_L\Sigma\psi_R - m\bar{\psi}_R\Sigma^\dagger\psi_L \\ &\quad -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \frac{v^2}{4}\mathrm{tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma)\end{aligned} \tag{1.13}$$

The domain of validity for this Lagrangian is expected to be between the chiral symmetry breaking scale Λ and the confinement scale Λ_{QCD} (below which the quarks cannot be considered as dynamical degrees of freedom because they are too tightly bound). Georgi and Manohar estimate the Λ to be about 1 Gev; Λ_{QCD} is somewhere in the neighborhood of 100–300 MeV.

There is some discussion as to whether the kinetic energy for Σ, the last term of Eq.(1.13), should be included. Georgi and Manohar seem to think it should. Ian Aitchison (private communication) maintains that it should not be included, since the pions will acquire kinetic energy anyway when the quarks are integrated out (just as in the Nambu–Jona-Lasinio model). An alternative description is that the virtual quark loops give the pions kinetic energy, so it

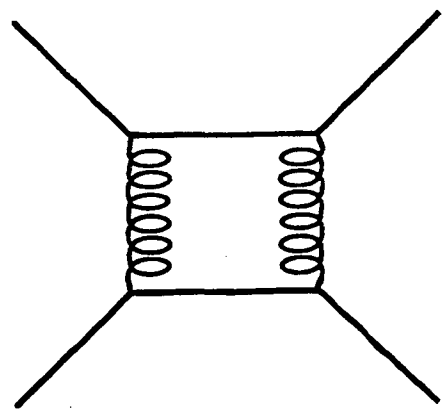

Figure 1.1: Diagram that generates a Nambu–Jona-Lasinio type interaction.

need not be included at tree level. I think it is not incorrect to include the kinetic energy in Eq. (1.13), as long as we remember that it is renormalized by the quark loops.

How close are we to actually deriving Eq.(1.13) from QCD? I looked at the box diagram of Fig. (1.1), which gives rise to four-quark interactions of the type that appear in the Nambu–Jona-Lasinio model. Integrating from Λ to ∞, and assuming that the QCD coupling constant α is to be evaluated at the scale Λ, I find that this diagram contributes a term

$$\frac{\alpha^2}{\Lambda^2}\left(\frac{N^2-2}{N}\right)[(\bar{\psi}\psi)^2-(\bar{\psi}\gamma_5\tau^a\psi)^2]+\text{other terms}$$

where, as usual, N is the number of colors. If we compare this result to the N-color Nambu–Jona-Lasinio model,

$$\mathcal{L}_{int}=\frac{g^2}{N\Lambda^2}[(\bar{\psi}\psi)^2-(\bar{\psi}\gamma_5\tau^a\psi)^2]$$

and impose the condition $g>\pi$ in order for chiral symmetry breaking to occur, we find the condition

$$\alpha>\frac{\pi}{\sqrt{N^2-2}}\approx 1.2$$

To obtain such a value of α, Λ must be roughly twice Λ_{QCD}. If $\Lambda_{QCD}\approx 200$ MeV, then $\Lambda\approx 400$ MeV, and the range of validity of the chiral quark model

(Eq. (1.13)) is too small to be of much use. This analysis seems to imply that chiral symmetry breaking and confinement cannot really be treated separately.

However, there are other points of view. Diakonov and Petrov [6] attempted to integrate out the gluons of QCD by integrating over the classical saddle points known as instantons. They end up with the chiral quark model, Eq. (1.13), except that the gluon terms are absent. They successfully predict many low energy parameters, such as the pion decay constant f. And yet, a Lagrangian like Eq. (1.13) without the gluons which cause confinement would be totally incapable of describing what we see. This deficiency makes the entire derivation suspect.

Therefore Eq.(1.13), the chiral quark model, must simply be taken as a model. A satisfactory derivation does not exist. I will return to this question in the Epilogue.

The chiral quark model is often written in a more convenient form. Eq. (1.13) is inconvenient because the mass terms mix the right- and left-handed states. It is more advantageous to write the Lagrangian in terms of the mass eigenstates. Define

$$\xi = e^{i\pi^a\tau^a/2v}$$

so that

$$\xi\xi = \Sigma$$

Now change variables,

$$\begin{aligned} \psi'_R &= \xi\psi_R \\ \psi'_L &= \xi^\dagger\psi_L \end{aligned} \tag{1.14}$$

Then the quark part of Eq. (1.13) becomes

$$\mathcal{L} = \bar{\psi}'\{i\not{D} + \not{V} + \not{A}\gamma_5\}\psi' - m\bar{\psi}'\psi'$$

where

$$\begin{aligned} V_\mu &= \frac{i}{2}(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi) \\ A_\mu &= \frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi) \end{aligned}$$

and D_μ is the covariant derivative with respect to the gluon field. The coupling of the quarks to the vector field V_μ remains unchanged under renormalization, due to the conservation of vector current. However, the coupling to the axial field A_μ may be renormalized in a way that we will not presume to know how to calculate; we therefore include an extra constant g_a in the Lagrangian,

$$\mathcal{L} = \bar{\psi}'\{i\not{D} + \not{V} + g_a\not{A}\gamma_5\}\psi' - m\bar{\psi}'\psi'$$

There is one very peculiar thing that happens when the quarks undergo the transformation of Eq. (1.14). The Jacobian of the transformation is not unity, and the terms $D\psi$ and $D\bar{\psi}$ in the path integral pick up a phase under the transformation. This phase contributes an extra term to the Lagrangian, known as the Wess-Zumino-Witten [7] anomaly term.

The anomaly term can be obtained in the following round-about way. The transformation (1.14) can be realized by performing a series of infinitesimal transformations. Defining

$$\xi_\lambda = e^{i\lambda\pi/2v}$$

the infinitesimal transformation is

$$T(\lambda) = \frac{d\xi}{d\lambda}$$

The total transformation of ψ_R is therefore

$$\psi_R' = \psi_R + \int_0^1 d\lambda\, T(\lambda)\, \psi_R$$

So far, of course, nothing has been said except that the integral of a derivative is the thing itself. But now rename λ to be a time-like coordinate x_5; define $\tilde{\Sigma} = \exp(ix_5\pi/v)$, and $\mathrm{L}_i = \tilde{\Sigma}^\dagger(\partial/\partial x^i)\tilde{\Sigma}$ where $i = 0, 1, 2, 3$ or 5. Then the contribution to the action due to the Jacobian of the change of variables is [8]

$$\Gamma_{WZW} = \frac{iN}{240\pi^2}\int d^5x\,\epsilon_{ijklm}\,\mathrm{tr}(L^iL^jL^kL^lL^m) \tag{1.15}$$

This term is zero when there are only two flavors, because of the properties of the Pauli matrices. However, if Σ is generalized to include three flavors, the term is non-vanishing. Curiously, the anomaly term cannot be written as a local term to be integrated over the four-dimensional space-time in which we live—it gives a non-local contribution to the action.

1.4 The Non-Linear Sigma Model

To obtain a Lagrangian for mesons, we could start with the chiral quark model and integrate out the quarks and gluons. Unfortunately we have no idea how to carry out such a project, but we can get some idea of what we *would* get, if we could do the integration. There would be a term giving kinetic energy to the pions, and other terms giving interactions; in fact any term that has the correct Lorentz and chiral symmetry should appear, with a coefficient which is determined in principle by the underlying theory, but which in practice we cannot calculate. So we end up with the Lagrangian [8]

$$\begin{aligned}\mathcal{L} &= \frac{v^2}{4}\,\mathrm{tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma) + c_1\,\mathrm{tr}(\partial_\mu\Sigma\partial_\nu\Sigma^\dagger)\cdot\mathrm{tr}(\partial^\mu\Sigma\partial^\nu\Sigma^\dagger)\\ &\quad + c_2\left[\mathrm{tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma)\right]^2 + c_3\,\mathrm{tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\partial_\nu\Sigma\partial^\nu\Sigma^\dagger) + \ldots \qquad (1.16)\end{aligned}$$

The first coefficient must be $v^2/4$ to obtain the canonical normalization for the pion kinetic energy. The other coefficients c_1, c_2, and c_3 must be fitted to experiment.

The quantity v can be obtained from experiment in the following way. We can compute the Noether current associated with the chiral transformation of Eq. (1.11) and compare it to the PCAC relation

$$j^a_{5\mu} = -f\partial_\mu\pi^a$$

where f is the pion decay constant, 92 MeV. This procedure yields

$$v = f$$

The Wess-Zumino-Witten anomaly term should also be included in the above effective Lagrangian, Eq. (1.16). In the last section we saw how this term could come about from transforming the quark fields. Actually it can be shown from very general arguments [7]that this term should exist, independent of the intermediate steps taken on the road to the effective Lagrangian. Evidently, then, this term would arise in the `tr log` term when the quarks are integrated out, even if they do not first undergo the transformation (1.14). The coefficient

of the anomaly term is not arbitrary, but is fixed at the value given in Eq.(1.15) [7, 8]. This is the only term for which a low-energy coupling is successfully predicted from the fundamental theory.

1.5 The $1/N$ Expansion

The $1/N$ expansion [4, 50] is a tool that has become increasingly applied to the study of mesons and baryons. It is based on the following idea, already alluded to: the color $SU(3)$ group is generalized to $SU(N)$. Then the baryon contains N quarks instead of 3. The gluon self-energy is $\propto g^2N$. In order to keep this quantity finite for arbitrarily large N, the coupling constant g must be rescaled: $g \to g/\sqrt{N}$. This simple rescaling has a number of consequences; among them, the pion decay constant f becomes proportional to $\sqrt{N}$.

This program leads to several simplifications if N is large. For example, because of the numerous quarks existing in the large-N proton, a Hartree-Fock approach is justified, since the field that a particular quark experiences is the average field produced by $N-1$ other quarks. If one of the other quarks fluctuates away from the average, its contribution is only a fraction $1/N$ of the total, so this fluctuation, to leading order in an expansion of the Hartree field in powers of $1/N$, can be ignored.

This example illustrates both the strength and the weakness of the $1/N$ expansion. The strength is that many hadronic properties become simpler if only the first one or two terms of the $1/N$ expansion need to be kept. The weakness is that at the end of the calculation, we need to set $N = 3$, so it is not clear how useful the expansion is in actual practice. We would have to suffer from unbelievably bad luck if the expansion were not useful for *any* of the hadronic properties; it would seem overly optimistic to assume that the expansion would work for *all* properties.

1.6 The Skyrme Model

It is perhaps most natural to think of the non-linear sigma model, Eq.(1.16), as a tool for perturbatively calculating pion interactions. In the perturbative regime, the magnitude of the pion field is small. However, large pion fields might occur, and we might try to see if the Lagrangian (1.16) holds in this case also. One such non-perturbative field configuration is the Skyrme configuration, which turns out to be a model of the nucleon.

In the Skyrme model [9, 10], a special choice of c_1, c_2, and c_3 are taken so that the effective action is:

$$\mathcal{A} = \int d^4x \left\{ \frac{f^2}{4} \mathrm{tr}(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) + \frac{1}{32e^2} \mathrm{tr}([L_\mu, L_\nu]^2) \right\} + \Gamma_{WZW}$$

where $L_\mu \equiv \Sigma^\dagger \partial_\mu \Sigma$, f is the pion decay constant, and e is some unknown constant. In practice both f and e will be allowed to vary to fit the data.

Suppose the Σ field is in the following configuration:

$$\Sigma_0 = e^{iF(r)\hat{r}^i \tau^i} \tag{1.17}$$

where $F(0) = \pi$ and $F(\infty) = 0$. This is a peculiar situation because the space and isospace coordinates find themselves married together in the same dot product. Therefore the configuration is not rotationally invariant, but under a *combined* space and isospace rotation, Σ_0 is invariant.

There is another property of this so-called "hedgehog" configuration. In order for the energy of any Σ to be finite, we must require that $\Sigma(\mathbf{r}) \to 0$ as $\mathbf{r} \to \infty$ in any direction. This effectively reduces our three dimensional space R^3 to be topologically a 3-sphere, S^3. The flavor space $SU(2)$ is also topologically S^3, so Σ maps $S^3 \to S^3$. The particular map Σ_0 gives a one-to-one correspondence between the two spheres, and so has winding number one. The configuration is stable because it cannot "unwind." It is therefore called a topological soliton.

So far we have said nothing about the function $F(r)$. F should be whatever function minimizes the total energy. This leads to a differential equation for F which can be solved numerically. The function begins at π when $r = 0$; it

initially descends linearly, then curves to form a long tail that approaches zero as $1/r^2$.

The configuration (1.17) in general does not remain static, but can rotate. We can account for this by taking

$$\Sigma = A(t)\Sigma_0(r)A^\dagger(t)$$

where $A^\dagger \dot{A} = i\omega_a \tau_a$ gives the frequency and direction of rotation. The ω's will then be quantized, resulting in a wavefunction $\Psi(A)$. This quantization procedure is equivalent to the quantization of a top.

This configuration inherits two rather curious properties from the Wess-Zumino-Witten term. Actually, as mentioned before, the anomaly term vanishes if there are only two flavors, so the field Σ is typically embedded in a 3×3 matrix before the anomaly term is evaluated.

The first property is the quark number. Using the same transformation whose Noether current is $\bar{\psi}\gamma_\mu\psi$ in the chiral quark model, we can compute the quark number current of the hedgehog configuration Σ_0. Only the anomaly term contributes, and it turns out that [10]

$$\int d^3x j_0(x) = N$$

That is, there are N quarks in this configuration. This is strange, because we started with pions, which are quark–anti-quark pairs and which therefore have quark number equal to zero. Now, however, we count N quarks. Perhaps, then, the nucleon is described by this kind of field configuration.

The second anomalous property is even stranger than the first. The Wess-Zumino-Witten term, once evaluated, gives a contribution to the action

$$-i\int dt \frac{N}{2}\,\mathrm{tr}(YA^\dagger\dot{A}) \tag{1.18}$$

where Y is the hypercharge matrix,

$$Y = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Since Eq. (1.18) is linear in the time derivative rather than quadratic, it offers an equation of constraint rather than an equation of motion. The momentum conjugate to the hypercharge coordinate is constrained to be $N/3$, so that a rotation of 3π in the hypercharge coordinate gives Ψ a phase of $\exp(i\pi N)$. However, because of the close connection between isospace and space coordinates in Σ_0, a rotation of 3π in the hypercharge coordinate is equivalent to a rotation of 2π in ordinary space. Therefore, the wavefunction picks up a phase

$$(-1)^N \quad \text{under a rotation of } 2\pi$$

So, for example, if N=3, the configuration must have half-integral spin. This makes it a serious candidate for the nucleon. The discovery of this relation by Witten [11] in 1983 generated a flurry of activity. (For an excellent review see [10].)

How does this model compare with experiment? As with the Nambu–Jona-Lasinio model, it is qualitatively correct but quantitatively not very accurate. Nucleon properties such as the magnetic moment and charge radius are typically predicted to within only about 50% [21]. In order to fit the observed mass difference between the nucleon and the delta, the pion decay constant must be set to 64.5 MeV, as opposed to the experimental value, $f = 92$ MeV.

The model can be extended by including the strange baryons, and by adding vector and axial vector mesons to the low-energy Lagrangian; however, these modifications do not noticeably improve the accuracy [12]. It is thought that perhaps the Skyrme model becomes exact in the large number of colors limit. If so, then the real world (as we would expect) deviates somewhat from this limit.

1.7 The Constituent Quark Model

Another model of the baryon is the constituent quark model. This model is the "Bohr atom" of particle physics—a model based on intuition and simple physical arguments, but which lacks rigorous justification.

In this model [13, 14, 15], the up and down quarks have effective masses of roughly 350 MeV, and the strange quark has a mass of about 500 MeV. The

baryon wavefunctions can be written in terms of quark wavefunctions, completely antisymmetrized in color (and therefore symmetric in spin-flavor indices). Then the mass of the baryon can be computed by adding the mass of its constituents, together with a color-magnetic dipole-dipole interaction:

$$M_n = \sum_i m_i + A \sum_{i<j} < n|\boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\sigma}^{(j)}|n > \tag{1.19}$$

The constant A is a free parameter; once it is fitted to the data, the spectrum of Eq. (1.19) fits the observed spectrum to within a few percent.

The magnetic moments of the baryons can also be predicted in this model,

$$\boldsymbol{\mu}_n =< n| \sum_i \frac{q_i \mathbf{S}_i}{m_i} |n >$$

This prediction fits the baryon octet data with a typical accuracy of 10%.

The constituent quark model works surprisingly well, especially in light of its simplicity. One difficulty, however, is that there is no explanation for why the up and down quarks should have a mass of 350 MeV. Since the pions are nearly massless, and their mass (squared) is proportional to the quark masses, typical estimates place the up quark mass between 2 and 8 MeV, and the down mass between 5 and 15 MeV [16]. Similarly, the strange quark is estimated to have a mass between 100 and 300 MeV, as opposed to the constituent quark value of $\approx$ 500 MeV.

One explanation for the source of this mass comes from the chiral quark model, or its predecessor, the Nambu–Jona-Lasinio model. In those models, the quarks acquire a mass through their coupling to the vacuum expectation value of $< \bar{\psi}\psi >$, which breaks chiral symmetry.

Since the chiral quark model is a constituent quark model that includes couplings to pions, we can push this model further by using it to calculate pion-loop corrections to baryon properties. The diagrams of Fig. 1.2 give some of the diagrams that will contribute to the renormalization of the baryon axial current (marked by an "x"). Figs. 1.2(a) and (b) are divergent, because of the forms of the propagators involved. Fig. 1.2(c) and (d), however, are less divergent, because gluons must transfer momentum from one quark to another, and the

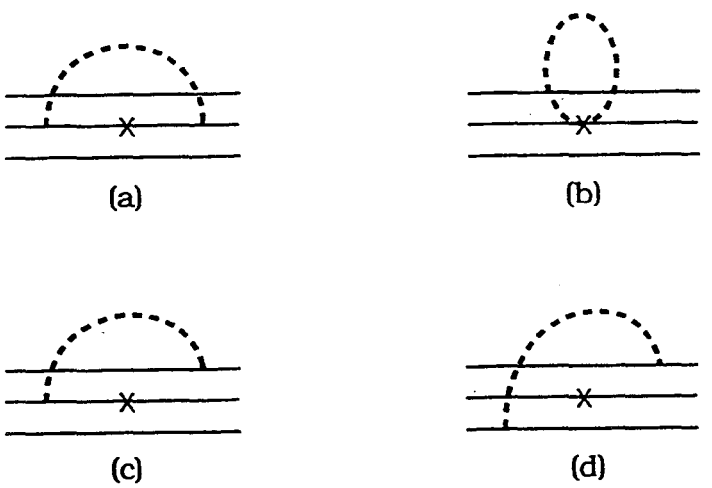

Figure 1.2: Diagrams which contribute to the renormalization of the baryon axial current.

additional gluon propagators soften the divergences[2]. Therefore Figs.1.2(a) and (b) give the largest contributions. When only these diagrams are taken into account, the model agrees well with the data on the weak semi-leptonic decays of baryons [47].

1.8 What Next?

The successes of the constituent quark model merit further exploration. The chiral quark model presents one scenario for how a quark of ≈ 4 MeV can acquire a mass of ≈ 350 MeV. However, in light of the problematic foundations of the chiral quark model, one might look for other possible explanations. One scheme is based on the following idea: we know that the vacuum somehow spontaneously breaks chiral symmetry, resulting in a quark condensate $< \bar{\psi}\psi >$. The pions

[2]However, if the gluons act on a time scale that is very short compared to the time it takes the pion to complete its loop, the quarks will reassemble themselves into a baryon while the pion is away. Then baryon propagators must be used everywhere, and Figs. 1.2 (c) and (d) are just as divergent as Fig.1.2(a). In the chiral quark model, we are assuming that the gluons act slowly, so Figs. 1.2(c) and (d) are suppressed.

π^a are regarded as small disturbances in this condensate, undulations caused by local rotations of the condensate in different flavor directions. The question then arises, could such rotations be possible in *color* space, as well? If so, these rotations could be described by a new degree of freedom, Π^a, where a is a color index instead of a flavor index. Presumably this new degree of freedom would obey an effective Lagrangian similar to the non-linear sigma model, and would then sustain a topological soliton configuration, the analog of the Skyrmion. Could this soliton, then, be the heavy constituent quark?

This question is addressed in the next chapter. It turns out that not only is the mass large, it is *too* large. Also, most of the angular momentum of this soliton is orbital angular momentum, but we know from experiments that most of the constituent quark's angular momentum comes from its spin.

So we are left with the question, why does the constituent quark model work? Perhaps the model is not literally true, but its predictions are accurate due to some underlying symmetry at work. In fact, the large number of colors ("large-N") expansion partially explains the success of the constituent quark model. This model begins with an effective theory of baryons and mesons, with *none* of the couplings or masses derived from QCD; they are all fitted phenomenologically. Then it is found than many of the baryon properties follow the constituent quark pattern, with corrections often of order $1/N^2$. (see [56] and other references given in Chapter 3). In Chapter 3 I look at the renormalization of baryon properties from pion loops, using the large-N expansion. I find that the constituent quark pattern indeed continues to hold, even during the renormalization procedure.

After these two rather involved chapters, the Epilogue closes with some speculation as to what can be learned from all of this work.

Chapter 2

The Constituent Quark as a Topological Soliton

2.1 Introduction

Why does the constituent quark model work? This chapter[1] examines one possible answer to that question.

The constituent quark model is puzzling because all the clues from QCD and current algebra [17] indicate that the nucleon is made of a swarm of gluons and nearly massless quarks and anti-quarks, all interacting strongly. According to the constituent quark model, however, the nucleon is composed of nothing but three massive quarks, interacting weakly. The model works well: baryon [14, 15] and heavy meson [18] masses can be fitted to within a few percent, and most baryon magnetic moments [17, 15] to within 30%.

One explanation for the success of this model is that the nucleon really does contain three weakly interacting components. In this picture, the nearly massless "current quarks" are fundamental particles, and through their strong interactions each is able to draw around itself a cloak of virtual gluons and quark–anti-quark pairs, resulting in the collective excitation called a "constituent quark". The constituent quark has the same spin and flavor as the original current quark,

[1] Most of this chapter has previously appeared in G.L. Keaton, Nucl. Phys. B 425 (1994) 595

but is heavier and less strongly interacting.

To explain how the constituent quark might arise as a collective excitation, two models have been proposed. The first is Manohar and Georgi's chiral quark model [19] (which is closely related to the Nambu–Jona-Lasinio model [1]). In this model the current quark increases its mass by coupling to the quark condensate that forms when chiral symmetry is broken. The resulting constituent quark then automatically has the same spin and flavor as the original current quark.

The second model is the quark soliton model proposed by Kaplan [20]. It is based on a very simple and appealing idea: the quark condensate may undergo rotations in color space as well as flavor space. The flavor rotations give rise to the usual non-linear sigma model Lagrangian used in pion physics. One form of this Lagrangian, the Skyrme Lagrangian [9, 21, 10], will support a topological soliton that is a model of the nucleon. Similarly, the color rotations of the condensate can be described by a Lagrangian that also supports a soliton. This soliton is a candidate for the constituent quark. The topological properties of such a soliton with winding number 1 ensure that it has spin 1/2 and baryon number 1/3, just as the original current quark.

This soliton has been analyzed in two dimensions [22]. In four dimensions, its mass and radius have been computed [23], and it was found that either the soliton's mass or its radius (or both) must be larger than expected for the constituent quark. However, Ref. [23] argues that this is not a fatal flaw in the model.

I have used a different technique to study the mass and radius, and have reached similar conclusions. I have gone on to evaluate the soliton's magnetic moment, color magnetic moment, and spin structure function. Within the approximations used, the spin structure function and the magnetic moments cannot both be fitted simultaneously in this model. Some speculation is offered on what to expect from better approximations.

The main purpose of this chapter is to show how the static properties of the soliton may be evaluated, and to test whether the soliton's properties are compatible with the constituent quark's. Section 2.2 introduces the Lagrangian

and the soliton. Section 2.3 shows how I extract the static properties of the soliton. These properties are then computed and the results given in Section 2.4. The chapter closes with a short discussion in Section 2.5.

2.2 The Soliton

In the quark soliton model, the quark condensate can undergo rotations in color space as well as in flavor space. The color degrees of freedom are parameterized by $U = e^{2i\Pi^a T^a/f}$. The capital Π^a is used to distinguish this field from the ordinary pion field π^a; the T^a are the generators of color SU(3); the constant f is analogous to the pion decay constant f_π. With $\tilde{R}_\mu \equiv U^\dagger D_\mu U$, Kaplan [20] proposed the following one-flavor Lagrangian (extensions to more flavors are discussed below) :

$$\begin{aligned}\mathcal{L}[U, A_\mu] &= -\frac{1}{4g^2}G^{\mu\nu}_a G^a_{\mu\nu} - \frac{f^2}{4}tr(\tilde{R}_\mu\tilde{R}^\mu) + \frac{1}{32e^2}tr([\tilde{R}_\mu, \tilde{R}_\nu]^2) \\ &\quad + \frac{2}{3}f^4\nu^2 tr(T_a U T_a U^\dagger) + n\mathcal{L}_{WZW} \qquad (2.1)\end{aligned}$$

This Lagrangian is patterned after the Skyrme Lagrangian [9, 21, 10]. The first term gives kinetic energy to the gluons; the second gives kinetic energy to the chiral field U. The third term, called the "Skyrme term", is introduced to stabilize the soliton solution. If this term is absent from the ordinary (ungauged) Skyrme Lagrangian, then the soliton shrinks to zero size. The fourth term breaks color $SU(3)_L \times SU(3)_R$ symmetry, and consequently gives mass to the (as yet undiscovered) Π particles. This term is necessary because QCD interactions, whose low energy behaviour this Lagrangian is intended to model, explicitly break *color* (not flavor!) $SU(3)_L \times SU(3)_R$ symmetry. The last term of Eq.(2.1) is the Wess–Zumino–Witten term [7]. It differs from the Lagrangian which would arise from Γ_{WZW} of Eq.(1.15) in two ways: first, the chiral fields are gauged (see Ref. [24]), and second, the integer coefficient in front is $n = 1$ instead of $n = N_c = 3$.

Each of the first four terms is multiplied by an arbitrary constant, to be fitted phenomenologically. g, e, and ν are dimensionless; f has the dimensions of mass. The mass of the Π particle is equal to $2\nu f$. The soliton solutions of this

Lagrangian have baryon number n/N_c [20], and upon rotation by 2π, the soliton will acquire a phase $(-1)^n$ [11]. Therefore the soliton is a fermion with baryon number 1/3. This soliton, which Kaplan calls a "qualiton", is a candidate for the constituent quark.

One may wonder whether the Lagrangian (2.1) actually follows from QCD. Two groups have made some progress toward deriving a Lagrangian similar to (2.1) from QCD [25, 26]. Their Lagrangians differ from Eq. (2.1) in two ways. First, more flavors are included, so that U is no longer an $SU(N_c)$ matrix but an $SU(n_f N_c)$ matrix. Second, their Lagrangians contain additional terms:

$$\begin{aligned}\Delta\mathcal{L} &= a\, tr[(\tilde{R}_\mu \tilde{R}^\mu)^2] + b\, tr[(D_\mu \tilde{R}^\mu)^2] + c\, tr[G_{\mu\nu} U G^{\mu\nu} U^\dagger] \\ &\quad + d\, tr[G_{\mu\nu} D^\mu U^\dagger D^\nu U + G_{\mu\nu} D^\mu U D^\nu U^\dagger] \end{aligned} \tag{2.2}$$

The first two terms tend to destabilize the soliton [27]. They also include four powers of time derivatives, which considerably complicates the quantization of the soliton. Only the minimal qualiton model of Eq. (2.1) will be considered here. I discuss in Section 2.5 the possible effects of the above changes to the Lagrangian.

The construction of the soliton from the Lagrangian (2.1) proceeds in the three steps sketched below (see [20] for more details). Step one: construct the classical solution. The field U takes on the "hedgehog" ansatz form:

$$U_{cl} = e^{iF(r)\hat{\mathbf{r}}\cdot\boldsymbol{\tau}} \tag{2.3}$$

where the τ^i are the Pauli matrices embedded in color SU(3). In this ansatz $F(0) = \pi$ and $F(\infty) = 0$. The gauge field is given by

$$A_{i\,cl} = -A^i_{cl} = i\frac{\gamma(r)}{2r}\epsilon_{ijk}\hat{r}^j\tau^k \quad , \quad i = 1,2,3 \tag{2.4}$$

Throughout this chapter an anti-hermitian gauge field is used: $D_\mu = \partial_\mu + A_\mu$. The profile functions F and γ can be determined by minimizing the soliton's classical mass, $m_{cl} = -\int d^3x\, \mathcal{L}[U_{cl}, A_{i\,cl}]$. The resulting Euler–Lagrange equations can be solved numerically [28]. (For more information on the numerical work, see Appendix B.) It is convenient to use the dimensionless variable $\tilde{r} = 2fr$, since the Euler-Lagrange equations that determine $F(\tilde{r})$ and $\gamma(\tilde{r})$ do

not depend on the parameter f. Fig. 2.1 shows the profiles $F(\tilde{r})$ and $\gamma(\tilde{r})$ for $1/e = 0$, $g = 12.4$, and $\nu = 237$. These values were chosen for two reasons: first, the resulting soliton has the same spin structure function as expected for the constituent quark (see below). Second, this demonstrates that the soliton can be stable even when the Skyrme term is absent ($1/e = 0$). Evidently, the gauge field is sufficient to stabilize the soliton. (A similar feature has been seen in the Skyrme Lagrangian, where the Skyrmion itself is stable in the absence of the Skyrme term as long as the ρ-meson gauge field is present[29].)

Step two of constructing the soliton: make it rotate. This is done by conjugating the field U_{cl} by a matrix Ω:

$$U = \Omega U_{cl} \Omega^\dagger \tag{2.5}$$

If the gauge field were absent, Ω would be a function of time but not of space. Since the gauge field is present, however, Ω must depend on r as well as t. This can be understood as follows: the gauge field also rotates:

$$\mathbf{A} = \Omega \mathbf{A}_{cl} \Omega^\dagger - \Omega \nabla \Omega^\dagger \tag{2.6}$$

The rotation of U and $\mathbf{A}$ generates a charge density

$$j_0^a = i\frac{f^2}{2} tr[(U^\dagger T^a U - T^a)\tilde{R}_0] - \frac{i}{8e^2} tr\{[(U^\dagger T^a U - T^a), \tilde{R}^\nu][\tilde{R}_0, \tilde{R}_\nu]\} \tag{2.7}$$

The color electric fields must be given by the rotating gauge fields of Eq. (2.6), and must also satisfy Gauss's Law with the charge of Eq. (2.7). This constrains Ω to satisfy certain differential equations given in Ref. [20] (see also Appendix B). For now it is enough to know that Ω can be parameterized by three functions $\omega_1(r)$, $\omega_2(r)$, and $\omega_3(r)$. These functions are shown in Fig. 2.2 for the same set of input parameters as in Fig. 2.1. ω_1 and ω_3 are closely analogous to ordinary angular velocity, and they are smaller for smaller r. That is, the soliton must rotate more slowly in the middle than on the outside because otherwise it generates too much charge to be consistent with Gauss's Law.

As $r \to \infty$, the matrix $\Omega(r,t)$ becomes equal to a matrix $W(t)$. The Lagrangian (2.1) can be rewritten in terms of this matrix as [20]:

$$L = -m_{cl} + \frac{I_1}{2} \sum_{m=1,2,3} (iW^\dagger \dot{W})_m^2 + \frac{I_2}{2} \sum_{\alpha=4\ldots7} (iW^\dagger \dot{W})_\alpha^2 + \frac{1}{\sqrt{12}} (iW^\dagger \dot{W})_8 \tag{2.8}$$

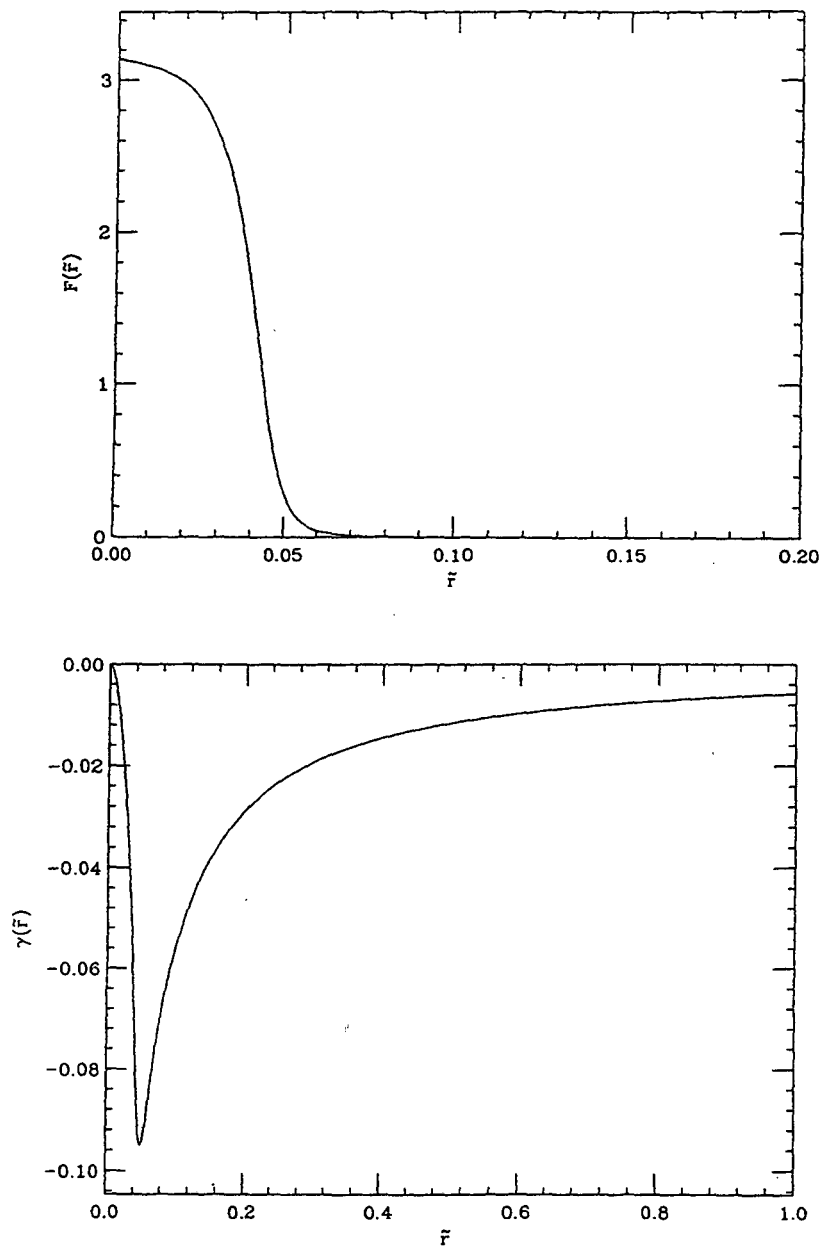

Figure 2.1: The functions $F(\tilde{r})$ and $\gamma(\tilde{r})$ for $1/e = 0$, $\alpha_s = 12.25$, and $\nu = 237$. The soliton has $\mu_c/\beta = 2.4 \times 10^{-4}$ and $s_q = 0.75$.

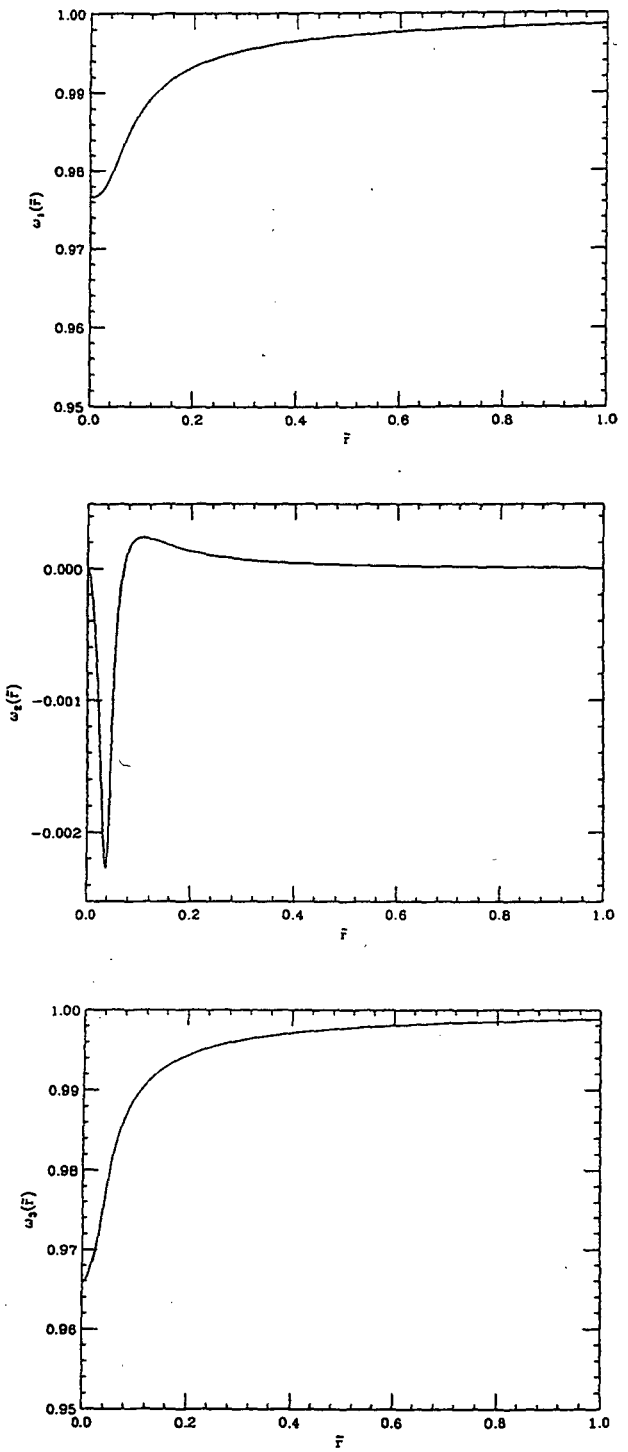

Figure 2.2: The functions ω_1, ω_2, and ω_3 for the same input parameters as in Fig. 2.1.

Here $(W^\dagger \dot{W})_a \equiv 2\, tr(T_a W^\dagger \dot{W})$. The last term in this Lagrangian comes from the Wess–Zumino term. The moments of inertia I_1 and I_2 are coefficients that can be computed once F, γ, ω_1, ω_2, and ω_3 are known.

The Lagrangian (2.8) describes a top spinning in SU(3) space. When the Hamiltonian is constructed, the canonical momenta P_a are used. Explicitly [10], $P_a = I_1(iW^\dagger \dot{W})_a$ for $a = 1, 2, 3$ and $P_a = I_2(iW^\dagger \dot{W})_a$ for $a = 4, \ldots, 7$. P_a is equal to the ordinary angular momentum Λ_a for $a = 1, 2, 3$. Since the Wess–Zumino term only contributes one power of $(W^\dagger \dot{W})_8$ to the Lagrangian (2.8), P_8 will not have an equation of motion but rather an equation of constraint: $P_8 = 1/\sqrt{12}$.

Step three in building the soliton: quantize it. Here W and P_a are no longer treated as ordinary matrices, but as operators on a Hilbert space. The total mass of the soliton is given by the resulting energy eigenvalues,

$$E = M_{TOT} = m_{cl} + j(j+1)(\frac{1}{2I_1} - \frac{1}{2I_2}) + \frac{1}{2I_2}(C_2 - \frac{1}{12}) \tag{2.9}$$

where j is the spin quantum number and C_2 is the color SU(3) Casimir operator. We are interested in the lowest energy state, which is a spin–1/2, color triplet state ($j = 1/2$ and $C_2 = 4/3$). Then

$$M_{TOT} = m_{cl} + \frac{3}{8I_1} + \frac{1}{4I_2} \tag{2.10}$$

The soliton is described by a state $|q, \sigma>$ whose wave functions are given by the SU(3) Wigner D-functions in the triplet representation [10, 30, 31]:

$$\psi_{q,\sigma}(W) =< W|q, \sigma >= \sqrt{3} D^{(3)}_{q,\sigma}(W) \tag{2.11}$$

Here q and σ are SU(3) indices: $q = (I, I_3, Y)$ gives the color isospin and hypercharge of the particle, and $\sigma = (s, -m_s, 1/3)$ gives the spin. The last entry is constrained to be 1/3 by the Wess–Zumino term.

There is one point worth mentioning now. In the above procedure, the functions F and γ are determined by minimizing the classical mass; they are then used to calculate the moments of inertia I_1 and I_2. This is called the "semi-classical" approach. A more exact procedure [32] is to view the total mass as a

functional of F and γ,

$$M_{TOT}[F,\gamma] = m_{cl}[F,\gamma] + \frac{3}{8I_1[F,\gamma]} + \frac{1}{4I_2[F,\gamma]} \tag{2.12}$$

and then find those functions F and γ which minimize the total mass, not just the classical mass. The resulting integro-differential equations have never been worked out.

In the Skyrme model the semi-classical approach is sufficient because m_{cl} is of order N_c and the moments of inertia are also of order N_c. Therefore the rotational energy does not contribute much to the total energy, and the error made in the semi-classical approximation is small. However, no such N_c–counting argument exists for the qualiton, and there is no guarantee that the semi-classical approach is enough. Still, it is worth trying.

Having constructed the soliton, the question is whether the four parameters f, e, g, and ν can be adjusted to give realistic values for the static properties of the constituent up and down quarks.

2.3 Properties of the Soliton

The static properties of the soliton are discussed in this section. For each observable, I first state what is expected from the static quark model, and then describe how to extract this quantity from the soliton model. The results are then used in Section 2.4 for numerical computations.

2.3.1 Mass and Radius

The constituent quark mass is typically taken to be ≈ 350 MeV. For definiteness, I will take Quigg's value, $m = 362$ MeV [15]. The radius of the constituent quark should be less than the radius of the nucleon. The isoscalar rms radius of the nucleon is $r_{rms} = 0.72$ fm. Therefore, in units where $\hbar = c = 1$, there is an upper bound on the dimensionless quantity $m \cdot r_{rms}$ for the constituent quark:

$$m r_{rms} \leq 1.3 \tag{2.13}$$

In the soliton model, r_{rms} can be computed using the singlet part of the anomalous Wess–Zumino vector current:

$$r_{rms}^2 = \int d^3r\, r^2\, J_{WZ}^0$$

where [24]

$$J_{WZ}^{\mu} = \frac{1}{48\pi^2}\epsilon^{\mu\alpha\beta\gamma} tr[2\tilde{R}_\alpha \tilde{R}_\beta \tilde{R}_\gamma - 3F_{\alpha\beta}(\tilde{R}_\gamma + \tilde{L}_\gamma)]$$

Here $\tilde{R}_\mu = U^\dagger D_\mu U$ and $\tilde{L}_\mu = (D_\mu U)U^\dagger$. The convention $\epsilon_{0123} = -\epsilon^{0123} = 1$ is used. Using Eqns. (2.3) - (2.6) (cf. [20]),

$$r_{rms}^2 = \frac{1}{\pi}\frac{1}{(2ef)^2}\int \tilde{r}d\tilde{r}[2F - (1+\gamma)^2 \sin 2F] \tag{2.14}$$

Given the radius from Eq.(2.14) and the mass as calculated from Eq. (2.10), can the qualiton satisfy the inequality (2.13)?

To answer this question, it is easiest to look first in the limit where $g \to 0$ and $\nu \to 0$, because the result is identical to the ordinary SU(3) Skyrmion with $N_c = 1$. In this model the total mass is

$$\begin{aligned} M &= m_{cl} + \frac{3}{8I_1} + \frac{1}{4I_2} \\ &= \hat{m}\frac{2f}{e} + \frac{3}{8}\left(\frac{2fe^3}{\hat{I}_1}\right) + \frac{1}{4}\left(\frac{2fe^3}{\hat{I}_2}\right) \end{aligned}$$

where $\hat{m}$, $\hat{I}_1$, and $\hat{I}_2$ can be calculated numerically: $\hat{m} = 36.5$, $\hat{I}_1 = 106.6$, $\hat{I}_2 = 40.6$. (Ref. [10] gets similar values.) The isoscalar rms radius is

$$r_{rms} = \frac{\hat{r}_{rms}}{2fe}$$

with $\hat{r}_{rms} = 2.12$ (cf. [21]). Therefore,

$$Mr_{rms} = \hat{r}_{rms}[\frac{\hat{m}}{e^2} + (\frac{3}{8\hat{I}_1} + \frac{1}{4\hat{I}_2})e^2]$$

Differentiating the above equation with respect to e and setting the derivative equal to zero reveals that, when $g = \nu = 0$,

$$Mr_{rms} \geq 2.52 \tag{2.15}$$

with the minimum occurring at e=7.84. Numerically, it is found that the full soliton also has a minimum $(Mr_{rms}) = 2.52$ at this "Skyrme" configuration ($e = 7.84$, $\nu = 0$, and $g = 0$); (Mr_{rms}) always increases when g and ν move away from 0.

Therefore it is not possible for the soliton to satisfy the inequality (2.13). Ref. [23] uses a different technique but comes to the same conclusion: the radius or the mass of the soliton is larger than expected for a constituent quark. Ref. [23] explores whether a large radius is a serious flaw. The problem with this approach is that it requires the confining force to be so strong that it contracts the quark to roughly half its original size. This goes against the spirit of the constituent quark model, in which the quarks are weakly interacting inside the hadron.

The alternate possibility is that the mass is large. At first it might seem that this, too, would violate the principles of the constituent quark model, since the binding energy per quark would be at least half the constituent mass. This would require the quarks to be very strongly interacting as well. However, such an argument must be treated with care in a confining theory. It is conceivable (though admittedly it seems unlikely) that a light hadron could be composed of heavy constituents, with the constituents nevertheless weakly interacting as long as they do not stray too far from the hadron center of mass. In any event, until details of the inter–quark forces are included, the relationship between the mass of the constituents and the mass of the nucleon cannot be determined. Since these details have not yet been worked out, the question of the excessive mass cannot be further addressed in this work.

2.3.2 Magnetic Moment

The magnetic moment of a particle with charge q and spin $\mathbf{S}$ can be written

$$\boldsymbol{\mu} = q\beta\mathbf{S} \tag{2.16}$$

where β is a parameter with the dimensions of length. In the constituent quark model (where the quarks have a Dirac g–factor of two), $\beta = 1/m$. Using $m = 362$ MeV, the value of β is 0.544 fm.

In the soliton model, the magnetic moment is given by

$$\boldsymbol{\mu} = q\frac{1}{2}\int d^3x\, \mathbf{r} \times \mathbf{J} \tag{2.17}$$

where $\mathbf{J}$ is the electromagnetic current per unit charge. $\mathbf{J}$ is the Noether current associated with the same $U(1)$ transformation that gives rise to the singlet Wess–Zumino current. Therefore, $\mathbf{J} = \mathbf{J}_{WZ}$. It is easiest to work in the gauge where $\mathbf{A}_i = \mathbf{A}_{i\,cl}$, $A_0 = \Omega^\dagger\dot{\Omega}$, and $U = U_{cl}$. Then,

$$J^i_{WZ} = -\frac{1}{8\pi^2}\epsilon^{ijk}\partial_k tr[A_0U^\dagger\partial_jU + A_0\partial_jUU^\dagger + A_0U^\dagger A_jU - A_0UA_jU^\dagger] \tag{2.18}$$

A_0 can be written in terms of the angular momentum $\mathbf{\Lambda}$:

$$\begin{aligned} A_0 = \Omega^\dagger\dot{\Omega} \;&=\; \frac{-i}{2I_1}[(\omega_1+\omega_2)\mathbf{\Lambda}\cdot\boldsymbol{\tau} - \omega_2(\hat{\mathbf{r}}\cdot\mathbf{\Lambda})\hat{\mathbf{r}}\cdot\boldsymbol{\tau}] \\ &\quad + \text{terms} \propto \lambda_4 \ldots \lambda_8 \end{aligned}$$

The extra terms proportional to $\lambda_4 \ldots \lambda_8$ do not contribute to the trace in Eq. (2.18). The resulting magnetic moment is

$$\begin{aligned} \boldsymbol{\mu} \;&=\; \lim_{R\to\infty} -\frac{q}{3\pi}\frac{\mathbf{\Lambda}}{I_1}\left\{\int_0^R r^2dr[\omega_1F' + \frac{1}{r}(\omega_1+\omega_2)(1+\gamma)\sin 2F]\right. \\ &\quad \left. +[r^3\omega_1F' + \frac{1}{2}r^2(\omega_1+\omega_2)(1+\gamma)\sin 2F]_{r=R}\right\} \end{aligned} \tag{2.19}$$

In Section 2.4 this formula is used for the numerical computation of β for the soliton.

2.3.3 Color Magnetic Moment

In the constituent quark model, the hyperfine mass splitting of the hadrons is given by the interaction of the color magnetic moments of the constituents:

$$\Delta E_{hfs} = -\frac{2}{3}|\psi(0)|^2\sum_{i<j} < n|\boldsymbol{\mu}^a_{(i)}\cdot\boldsymbol{\mu}^a_{(j)}|n >$$

where the sum is over the quarks i and j in the nucleon $|n>$. The color magnetic moment can be defined by

$$\boldsymbol{\mu}^a = \mu_c\mathbf{S}\lambda^a \tag{2.20}$$

where the λ^a are the Gell-Mann matrices and μ_c is a parameter with the dimensions of length. In the constituent quark model, $\mu_c = g/2m$. Using $\alpha_s = g^2/4\pi = 0.4$ and $m = 362$ MeV [15], the value of μ_c is 0.610 fm. The ratio of the constituent quark's color magnetic moment to its magnetic moment is proportional to $\mu_c/\beta = 1.12$.

The qualiton also has a color magnetic moment, which can be extracted from the asymptotic behaviour of its B-field. The standard dipole form for $\mathbf{B}$, using the normalization of Eq. (2.1), is

$$B_a^i = \frac{g\mu_a^j}{4\pi}(3\hat{r}_i\hat{r}_j - \delta_{ij})\frac{1}{r^3} \tag{2.21}$$

The B-field of the qualiton turns out to have a similar form at large r, so the coefficient μ_a^j can easily be determined.

At large radius, the B-field of the classical soliton is

$$B^i_{a\,cl} = -r_B(3\hat{r}_i\hat{r}_a - \delta_{ia})\frac{1}{r^3} \tag{2.22}$$

The constant r_B is determined by the numerical solution for γ : $\lim_{r\to\infty}\gamma(r) = -r_B/r$.

The asymptotic B-field of the quantized soliton can be calculated as follows. Under the quantization procedure, $\mathbf{B}$ becomes an operator $\hat{\mathbf{B}}$ rather than just a matrix. At large radius, $\hat{\mathbf{B}} = W\mathbf{B}_{cl}W^\dagger$. (The matrix W is defined just before Eq.(2.8).) The expectation value of $\hat{\mathbf{B}}$ with respect to the quark soliton state $|q,\sigma>$ is

$$\mathbf{B} =< q,\sigma|\hat{\mathbf{B}}|q,\sigma >=< q,\sigma|W\mathbf{B}_{cl}W^\dagger|q,\sigma >$$

Using $B^i = B^i_m T^m$,

$$B^i_n = 2 < q,\sigma|tr[T^n W T^m W^\dagger]|q,\sigma > B^i_{m\,cl} \tag{2.23}$$

In order to compute the above matrix element, we can use the wavefunctions given in Section 2.2, Clebsch–Gordan coefficients [33], and the identity [31]

$$< W|tr[T_n W T_m W^\dagger]|W >= \frac{1}{2}D^{(8)}_{nm}(W)$$

The result is

$$< q,\sigma|Tr[T^n W T^m W^\dagger]|q,\sigma >= -\frac{3}{32} < q,\sigma|\sigma^m\lambda_n|q,\sigma > \tag{2.24}$$

Combining this equation with (2.23) and (2.22) gives

$$B_n^i = \frac{3r_B}{16} < q, \sigma|\sigma^m \lambda_n|q, \sigma > (3\hat{r}_i\hat{r}_m - \delta_{im})\frac{1}{r^3} \tag{2.25}$$

Therefore, using Eq. (2.21),

$$\boldsymbol{\mu}_a = \frac{4\pi}{g}\frac{3r_B}{16}\boldsymbol{\sigma}\lambda_a \tag{2.26}$$

Using $\mathbf{S} = \boldsymbol{\sigma}/2$ and Eq. (2.20), we find that $\mu_c = (4\pi/g)(3r_B/8)$. This information is used in Section 2.4.

The above procedure is sufficient to determine the color magnetic moment, but there is another way to evaluate it which parallels the calculation of the ordinary magnetic moment. Testing whether these two methods agree serves as a useful check on the numerical computations.

The color magnetic moment should be given by

$$\boldsymbol{\mu}_a = \frac{g}{2}\int d^3r\, \mathbf{r} \times \mathbf{J}_a$$

where J_a^μ is the current which couples to the gauge field A_a^μ. This integral has already been worked out in the SU(2) case [21] for ν, $\alpha_s \to 0$. It is unchanged for SU(3), and the result is

$$\mu_a^i = -gI_1 tr[T^a W T^i W^\dagger] \tag{2.27}$$

where the trace is to be taken as a matrix element between quark states, as above. Combining this with Eq. (2.26) gives

$$r_B = \frac{\alpha_s I_1}{2} \quad (\alpha_s, \nu \ll 1)$$

This expression is indeed satisfied by the numerical computations of r_B and I_1 when α_s and ν are small.

2.3.4 Spin Structure

The spin structure function of the constituent quark is not as well established as the previous properties. In fact, it is not obvious from the recent spin

structure experiments whether the data are even consistent with the constituent quark model. They turn out to be consistent, but some explanation is required.

The Fourier transform $g_q(\mathbf{r})$ of the constituent quark spin structure function is defined as follows: for a single spin-up quark $|q\uparrow>$ and the field ψ which annihilates it,

$$g_q(\mathbf{r}) =< q\uparrow|\bar{\psi}(\mathbf{r})\gamma_z\gamma_5\psi(\mathbf{r})|q\uparrow>$$

I call the integral over all space of g_q the "spin content" s_q:

$$s_q = \int d^3r\, g_q(\mathbf{r})$$

In the non-relativistic limit one would expect $s_q = \sigma_z = 1$. However, the recently measured spin structure functions of the neutron and proton force us to change these expectations.

To begin, look at the nucleon. The contribution of the up quark spin to a spin–up proton is:

$$\Delta u = \int d^3r\ <p\uparrow|\bar{u}\gamma_z\gamma_5 u|p\uparrow> \tag{2.28}$$

If the up quark is non–relativistic, Δu is just equal to the number of up quarks with spin parallel to the proton's spin, minus the number of up quarks with spin anti-parallel. Δd and Δs are similarly defined. Several relations exist between these quantities:

$$\Delta u - \Delta d = g_A = 1.2573 \pm .0028 \tag{2.29}$$

is used in the Bjorken sum rule [34], and

$$\Delta s - \Delta d = D - F = .328 \pm .019 \tag{2.30}$$

results from an analysis of semileptonic hyperon decay [35]. Combining both of the above two equations with a third equation would give three equations and three unknowns, and so Δu, Δd, and Δs could all be determined. Actually, any one of several recent experiments could be chosen as the third input for this procedure. The EMC experiment [36] gives

$$\frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right) = 0.126 \pm 0.02 \tag{2.31}$$

	Δu	Δd	Δs
EMC	$.74 \pm .06$	$-.52 \pm .06$	$-.19 \pm .06$
E142	$.93 \pm .03$	$-.33 \pm .03$	$0.00 \pm .04$
SMC	$.75 \pm .08$	$-.51 \pm .08$	$-.18 \pm .08$
Ellis & Karliner	$.80 \pm .04$	$-.46 \pm .04$	$-.13 \pm .04$
CQM	1.33	$-.33$	0
Modified CQM	.80	$-.45$	$-.20$

Table 2.1: Quark contributions to the spin of the proton, using Eqns. (2.29) - (2.30) and EMC [36] (line 1), E142 [37] (line 2), or SMC [38] data (line 3). Line 4 gives Ellis and Karliner's analysis [39]. Line 5 gives the constituent quark model (CQM) prediction, which uses $s_q = 1$ and $\Delta g = 0$. Line 6 gives the results of the CQM with the modification that $s_q = 3/4$ and $(\alpha_s/2\pi)\Delta g = 0.2$ (see text).

The result of the E142 experiment [37] is

$$\frac{1}{2}\left(\frac{1}{9}\Delta u + \frac{4}{9}\Delta d + \frac{1}{9}\Delta s\right) = -0.022 \pm 0.011 \tag{2.32}$$

and the SMC experiment [38] gives

$$\frac{1}{4}\left(\frac{5}{9}\Delta u + \frac{5}{9}\Delta d + \frac{2}{9}\Delta s\right) = 0.023 \pm 0.025 \tag{2.33}$$

Equations (2.29) and (2.30) can be combined with either Eq. (2.31), (2.32), or (2.33). The results of all three possibilities are given in the first three lines of Table 2.1.

The various results do not agree. This discrepancy has inspired some discussion [39, 40]. Ellis and Karliner [39] show, for example, that the three experiments do agree as long as perturbative QCD corrections are taken into account. The QCD corrections will not exactly apply to the soliton model, but the results of Ellis and Karliner's analysis, including these corrections, are listed in the fourth line of Table 2.1 to give an idea of the range of values currently under discussion.

How does all of this relate to the constituent quark? In the constituent quark model, the matrix element in Eq.(2.28) can be related to the helicity of

the individual quarks and to the polarization of the gluons [41] present in the proton:

$$\Delta u = \frac{4}{3}\int d^3r \ < q\uparrow|\bar{\psi}\gamma_z\gamma_5\psi|q\uparrow> -\frac{\alpha_s}{2\pi}\Delta g$$

$$\Delta d = -\frac{1}{3}\int d^3r \ < q\uparrow|\bar{\psi}\gamma_z\gamma_5\psi|q\uparrow> -\frac{\alpha_s}{2\pi}\Delta g$$

$$\Delta s = 0 - \frac{\alpha_s}{2\pi}\Delta g$$

As before, $|q\uparrow>$ is a single quark state (of either up or down flavor), and ψ annihilates that quark. The gluon contribution Δg must be included, because the current appearing in Eq. (2.28) can interact via a quark loop with the gluon sea, which may be polarized. The prefactors 4/3 and $-1/3$ come from the constituent quark model wavefunctions [15].

In the naive constituent quark model $\int d^3r \ < q\uparrow|\bar{\psi}\gamma_z\gamma_5\psi|q\uparrow> \equiv s_q = 1$ and $\Delta g = 0$. This results in line 5 of Table 2.1, which does not agree well with experiment. However, if for some reason s_q turns out to equal 3/4, then $g_A = \Delta u - \Delta d = 5/4$, which is very close to the experimental value. If in addition $(\alpha_s/2\pi)\Delta g = 0.2$, then the values of Δu, Δd, and Δs more or less agree with experiment. These values [2] are shown in the last line of Table 2.1. Using this empirical argument we take the spin content of the soliton to be

$$s_q \equiv \int d^3r \ < q\uparrow|j^3_{(5)}(r)|q\uparrow> = \frac{3}{4} \tag{2.34}$$

In any case the spin content of the constituent quark should be of $\mathcal{O}(1)$, although its exact value is somewhat model dependent.

In the soliton model, the color singlet axial-vector current $j^\mu_{(5)}$ arises only from the Wess–Zumino anomaly term. In order to compute this current, it is necessary to start from the general Wess–Zumino Lagrangian which includes both left– and right–handed fields [24]. Then

$$j^\mu_{(5)} \ = \ \frac{1}{i}\left(\frac{\partial\mathcal{L}_{WZ}}{\partial A^R_\mu} - \frac{\partial\mathcal{L}_{WZ}}{\partial A^L_\mu}\right)_{A^L=A^R=A}$$

[2]These same values have been used in a relativistic quark model by Brodsky and Schlumpf [42]. However, in their model $|q\uparrow>$ is a pointlike quark in a relativistic bound state, and $\int d^3r \ < q\uparrow|\bar{\psi}\gamma_z\gamma_5\psi|q\uparrow>$ is reduced from its naive value of 1 by the relativistic nature of the quark wavefunction rather than by the internal structure of the quark itself.

$$= -\frac{1}{48\pi^2}\epsilon^{\mu\alpha\beta\gamma} tr[F_{\alpha\beta}(\tilde{R}_\gamma - \tilde{L}_\gamma)] \tag{2.35}$$

and

$$\int d^3r\, \mathbf{j}_{(5)} = \frac{\mathbf{\Lambda}}{18\pi^2 I_1}\int d^3r\, \frac{\sin^2 F}{r}\Big[(1+\gamma)(\omega_1' + \omega_2' + \frac{\omega_2}{r}) - \omega_1\gamma(1+\gamma)\frac{1}{r} - \gamma'(\omega_1+\omega_2)\Big] \tag{2.36}$$

where again $\mathbf{\Lambda}$ is the angular momentum of the soliton. From here, the matrix element required in Eq. (2.34) can be obtained easily.

2.4 Numerical Results

The static properties of the quark soliton can now be computed. It is easiest to look at dimensionless quantities, since these quantities are determined only by the three dimensionless parameters e, g, and ν. The fourth parameter f determines the overall scale, and can be fixed later.

The constituent quark can be described by the following two dimensionless quantities: the ratio of its color magnetic moment to its magnetic moment, $\mu_c/\beta = 1.12$, and its spin content, $s_q = 0.75$. Requiring that the soliton's ratio μ_c/β take on the physical value of 1.12 will constrain the permissible values of e, g, and ν to lie on a two–dimensional surface within the three–dimensional parameter space. Alternatively, requiring that the spin content s_q achieve its physical value of 0.75 will define a different surface within the parameter space. In general, these two surfaces will intersect to form a (one–dimensional) curve. This curve is the family of points where the soliton is a good model of the constituent quark. Unfortunately, I find that these two surfaces do not intersect.

To begin searching the parameter space for points appropriate to a constituent quark, one might first try the point suggested by Ref. [23] ($e = 5.7$, $\alpha_s = 0.28$, $\nu = 0.36$). However, this gives $\mu_c/\beta = 1.94$ and $s_q = 0.0041$ (compared to the experimental values of 1.12 and 0.75).

I searched the parameter space using a variety of techniques [43, 44]. First, I started at some point in the parameter space and minimized the function

$$\mathcal{F} = \frac{|\mu_c/\beta - 1.12|}{1.12} + \frac{|s_q - 0.75|}{0.75}$$

where $\mathcal{F}$ is a function of the input variables e,g, and ν. No matter what the starting point was, it was only possible to make either the first or the second term of $\mathcal{F}$ close to zero, but not both simultaneously.

Second, I found some points where μ_c/β acheived its physical value of 1.12. Since these points typically have a very small spin content ($s_q \lesssim 0.01$), I used a numerical routine to move in the direction of increasing s_q, keeping μ_c/β fixed. Regardless of the location of the starting point on the ($\mu_c/\beta = 1.12$) surface, I found no spin content greater than 0.06 (whereas 0.75 is required). Similarly, I could fix s_q and search for a point where μ_c/β acheived its physical value, but again I did not find an acceptable solution.

Third, I evaluated the qualiton properties for points spaced evenly in a three-dimensional lattice in parameter space. This approach confirmed the negative conclusions of the other two.

The results of the numerical work are summarized in Fig. 2.3. The shaded portion is the region allowed in the soliton model; the cross-hairs indicate the constituent quark model values. Table 2.2 gives more information on some representative points from Fig. 2.3.

Fig. 2.3 and Table 2.2 demonstrate that the spin content and the magnetic moments cannot both be simultaneously fitted in this model. Any kind of best fit would probably involve making both s_q and μ_c/β some fraction (1/4 or less) of their experimental values. This is the main result of the chapter.

2.5 Discussion

The above analysis shows that μ_c/β and s_q cannot both be of $\mathcal{O}(1)$ simultaneously. As discussed in Section 2.2, however, all of this analysis used the semi-classical approximation. This approximation is valid only if the rotational energy does not contribute much to the total mass; i.e., if $m_{cl}/M_{TOT} \approx 1$. However, for all the points listed in Table 2.2, m_{cl}/M_{TOT} is between 0.2 and 0.003. Therefore it is necessary to go beyond the semi-classical approximation.

In other words, the qualiton model does not appear to be a viable model of the constituent quark if the semi-classical approach is used. The approach

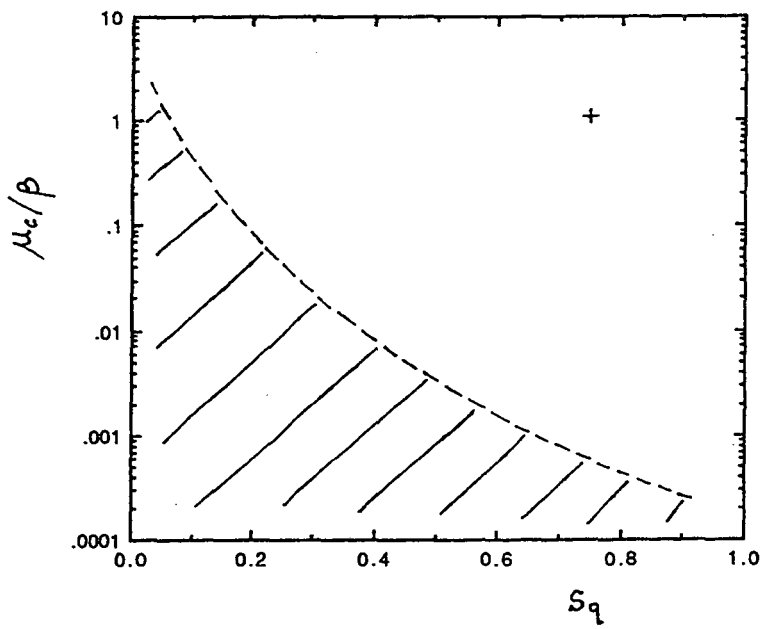

Figure 2.3: Comparison of the soliton model with experiment. The shaded region is allowed by the soliton model; the cross-hairs indicate the values required by the constituent quark model.

s_q	μ_c/β	Point where upper limit is reached
$\leq .06$	1.12 (fixed)	$e = 975,\ \alpha_s = 56,\ \nu = 84$
.2 (fixed)	$\leq .08$	$e = 970,\ \alpha_s = 27,\ \nu = 88$
.4 (fixed)	$\leq .008$	$e = 834,\ \alpha_s = 20,\ \nu = 160$
.75 (fixed)	$\leq .0006$	$e = 808,\ \alpha_s = 13,\ \nu = 232$
.75	1.12	"Experiment"

Table 2.2: The spin and magnetic properties of the soliton. The first line shows that when the input parameters are varied keeping μ_c/β fixed, s_q is always less than the given bound. The rest of the table shows that for s_q fixed, μ_c/β is bounded. These bounds are compared with the experimental values (baryonic properties interpreted through the constituent quark model).

itself is not valid in the region of parameter space where the model starts to become interesting. If a better approximation can be used, what is the hope for the future of the qualiton model?

The qualiton still faces two obstacles: its excessive mass, and its strong interactions. First, the mass: within the semi-classical approximation the product of the mass and the rms radius exceeds a plausible value, even at its minimum. While moving in parameter space away from this minimum in a direction that favors realistic magnetic moments or spin content, the moments of inertia become so small that the semi-classical approximation is suspect. Improving the approximation in the manner suggested after Eq. (2.12) may increase the moments of inertia and so lower the mass, but this improvement seems unlikely to lower the mass enough to make Mr_{rms} realistically small. Another kind of improvement on the semi-classical approximation, the inclusion of additional degrees of freedom, will only increase the mass. So it is likely that no matter what approximation is used, the mass will be larger than expected for a constituent quark. If the constituent quark is a soliton, the question is no longer what makes the constituent quark so heavy, but rather, what makes it so light?

Second, the strong coupling: in order to make s_q large enough, α_s must be large. When α_s is small ($\alpha_s \lesssim 1$), $s_q \propto \alpha_s$. In the semi-classical approach, the proportionality constant is roughly 1.7×10^{-2}, almost independent of e and ν. Unless this constant changes by more than two orders of magnitude when the semi-classical approach is discarded, α_s will have to be at least of $\mathcal{O}(1)$ if s_q is to be of $\mathcal{O}(1)$. However, any $\alpha_s \gtrsim 1$ will sabotage the constituent model because the model requires its constituents to interact perturbatively.

There is one potential solution to this problem: the gauge field surrounding the soliton may screen the particle's charge, so that even when α_s is large, the interactions between qualitons can be treated perturbatively. Preliminary calculations indicate that some screening does occur. However it remains to be seen whether, once the qualiton is fully quantized (beyond the semi-classical approximation), this screening is enough to make the qualiton model realistic.

In short, the constituent quark cannot be described by the qualiton in the semi-classical approximation. If a better approximation gives the correct spin

and magnetic properties, the qualiton's large mass and strong interactions will have to be explained.

What would be the result of starting with a more complicated Lagrangian, as suggested by Refs. [25, 26] and described above Eq. (2.2)? Including more flavors will only modify the properties of the soliton by group theoretic factors of order one, so that the results of this chapter will be qualitatively unchanged. On the other hand, adding more terms to the Lagrangian may have a significant effect. As stated above, one of the main problems of the qualiton is that the spin content is very small when α_s is small. The spin content is the expectation value of the axial current j_5^μ, and the only term in the Lagrangian that contributes to this current is the Wess–Zumino term, whose contribution is small when α_s is small. However, if the Lagrangian could be augmented by another term which contributes significantly to j_5^μ, then the qualiton might acquire a viable spin content even at low α_s.

Indeed, the first term of Eq. (2.2) (and only that term) contributes to j_5^μ, possibly resolving the spin content problem. However, as mentioned in Section 2.2, this term tends to destabilize the soliton. Ideally, the term must be large enough to provide the appropriate spin content, yet small enough that the qualiton remains stable. Unfortunately, if the term is large, its quartic powers of time derivatives make the Hamiltonian nearly intractable.

Faced with these difficulties, it is tempting to return to the chiral quark model mentioned in the introduction. One may even wonder whether constituent quarks exist at all. Perhaps the constituent quark model operators and wave functions simply have the right symmetry properties, and corrections to their matrix elements are suppressed for some reason (for example, by powers of $1/N_c$). In any case the success of the constituent quark model is not yet understood.

Chapter 3

Large-N Baryons, Chiral Loops, and the Emergence of the Constituent Quark

Having tried, and failed, at a "bottom up" approach to explain the success of the constituent quark model from more fundamental principles, I now turn to a "top down" approach. I begin with an effective theory of baryons and mesons, not claiming to have derived any of the parameters from QCD. Using this theory in the large-N and heavy-baryon limits, however, it can be shown that some of the constituent quark model results can be regained. This chapter examines how this comes about in the context of the pion loop renormalization of the baryon axial current.

3.1 Introduction

How are the baryons' properties renormalized by pion loops? This classic question gains renewed interest with the advent of each new calculational technique.

Pion loop corrections to baryon properties have been studied using the nonlinear sigma model with derivative couplings [45]. Later, Jenkins and Manohar [46, 47, 48] simplified the problem by invoking the heavy baryon approximation

Figure 3.1: The one loop vertex renormalization (a) and wavefunction renormalization (b). The "x" represents the axial current operator.

[49]. Using a Lagrangian that included the baryon octet and Goldstone boson octet, they found that the one-loop correction to the baryon axial current was large—as much as 100% of the tree-level value. However, if the baryon decuplet is also included, the total one-loop corrections are smaller, on the order of 30% of the tree-level value. That is, the loops involving decuplet states tend to cancel the loops involving only octet states. This was good news for perturbation theory, but it left unanswered the question, "What is the loop expansion parameter?" A seemingly coincidental cancellation of large corrections did not leave behind any obvious parameter that could justify, for example, the belief that the two-loop corrections should be any smaller than the one-loop corrections.

This question can be addressed within the framework of large-N techniques for baryons [50, 51], which have recently been rediscovered and greatly expanded [52, 53, 54, 55, 56, 57, 58, 59]. One of the results is that the baryon coupling to the axial current is on the order of the the number of colors,

$$g_A \sim N \tag{3.1}$$

This also means that the baryon-pion coupling is $\sim g_A k_\mu / f \sim \sqrt{N}$. However, the renormalization graph of Fig. 3.1 (a) gives a contribution of order N^2 to g_A, which if taken alone would violate Eq. (3.1) and doom perturbation theory (since the one-loop contribution would be much larger than the tree-level value). But there is another graph that must not be forgotten, the wavefunction renormalization of Fig. 3.1 (b). When both of these diagrams are included,

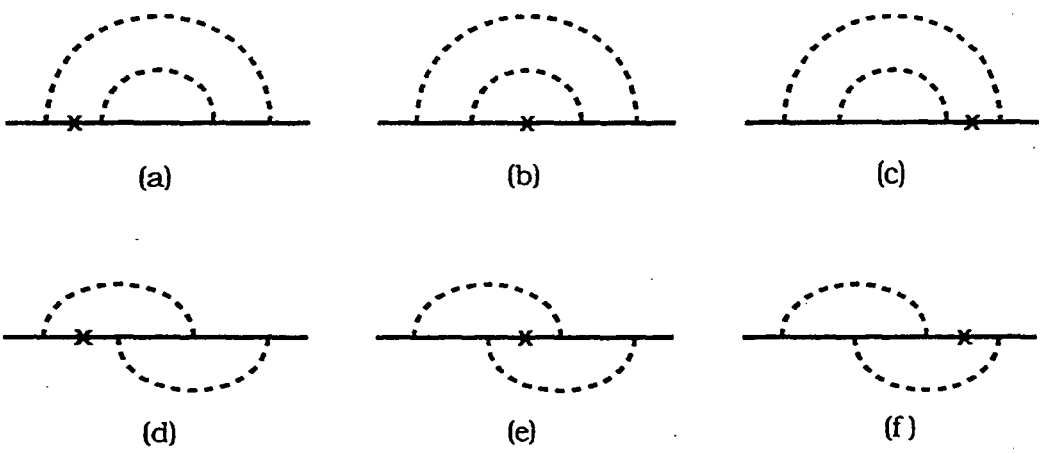

Figure 3.2: Two loop vertex renormalization diagrams.

it is found [52, 54, 58] that the leading order behaviour cancels, and the total one-loop correction is $\mathcal{O}(1)$, or $1/N$ times the tree-level value. The one-loop corrections are therefore small in the $1/N$ expansion and chiral perturbation theory seems to be valid.

As encouraging as this result is, it leaves some questions open. If the one-loop results are to be fitted to the data and believed, one should show that the two-loop contribution is small compared to the one-loop result. This is not obvious since, for example, the diagram of Fig. 3.2(a) is of order N^3 times the one–loop correction. However, once again there are several diagrams to be added together. Here we show that when all of the two–loop diagrams are taken into account, the largest terms cancel, and the result is of order $1/N$ times the one-loop contribution. Evidently, the pion loop expansion parameter turns out to be $1/N$. Although this has been suspected before [52, 54], it has not been previously demonstrated to two loops.

One result of this analysis is a demonstration that when the pion-baryon vertex is taken to leading order in $1/N$, the chiral loop corrections follow exactly the same pattern as would have been calculated in the chiral quark model [19]. This is surprising because the chiral quark model is a constituent quark model

where (to leading order) the pions interact with only one quark at a time. In the foregoing analysis, however, the pions interact coherently with all of the quarks in the baryon at once. Nevertheless, when all of the loop diagrams are taken into account, the cross terms where pions connect two or more quarks cancel exactly. All of the pions end up acting on only one quark at a time, and the chiral quark model results. It had already been noted that the constituent quark model fits the data as well as the usual baryon-pion theory [47]; the $1/N$ expansion sheds some light on why this is so.

After an introduction to the effective Lagrangian used for the calculations, the two-loop calculation is presented, followed by a discussion of the results.

3.2 The Lagrangian

The recipe for the effective Lagrangian calls for three ingredients: the large-N baryons, the Goldstone bosons, and the heavy baryon approximation.

Perturbative large-N baryon states $|B>$ can be made that have the same quantum numbers as the physical baryons $|\mathcal{B}>$[56]. These states serve the same purpose as the perturbative vacuum state $|0>$, which can be used in calculations because it is the lowest perturbative state that has the same quantum numbers as the true vacuum $|\Omega>$.

To make the baryon states, quark creation operators are used: $a^\dagger_{a\alpha}$ creates a quark with spin α and isospin a. The color can be ignored if the quark operators are bosonic. Historically, of course, color was proposed because the quarks were found to be symmetric in spin-flavor, and so there must be another quantum number with respect to which the quarks are antisymmetric if they are fermions. In a reversal of history, we ignore the color quantum numbers, and pretend that the quarks are bosons.

A particular perturbative baryon state can be written [56]

$$|B>= B^{a_1\alpha_1\ldots a_N\alpha_N} a^\dagger_{a_1\alpha_1}\ldots a^\dagger_{a_N\alpha_N}|0> \tag{3.2}$$

$|B>$ has non-relativistic normalization: $<B|B>= 1$. There is a tower of baryon states having spin $1/2,\ 3/2,\ 5/2,\ldots N/2$.

One of the main results of the large-N analysis [56] is best explained by example. (First, some notation: if $V_{a\alpha}^{b\beta}$ is a spin-flavor matrix, then define $\{V\} \equiv a_{a\alpha}^{\dagger} V_{a\alpha}^{b\beta} a_{b\beta}^{\dagger}$.) The baryons can receive a contribution to their mass from a term of the form

$$a_1 < B|\{1\}|B >$$

where a_1 is some unknown constant of order 1. The operator $\{1\} = a^{\dagger}a$ simply counts the number of quarks. Other terms that may contribute to the mass include (using the spin matrix J)

$$\frac{b_1}{N} < B|\{J^i\}\{J^i\}|B >$$

$$\frac{b_2}{N} < B|\{1\}\{1\}|B >$$

The operators $\{J^i\}\{J^i\}$ and $\{1\}\{1\}$ each contain two $a^{\dagger}$-a pairs, and so can act on two quarks at a time. Since only connected diagrams may contribute to the matrix element [56], at least one gluon must be exchanged. Since the quark-gluon vertex is $g/\sqrt{N}$, each gluon costs a factor $1/N$. Therefore an explicit $1/N$ must be included in the terms above. The generalization of this example is easy: for a term in the baryons' mass or effective action that has n pairs of a's and $a^{\dagger}$'s, $(n-1)$ factors of $1/N$ must be included.

The second ingredient needed for the Lagrangian is the set of Goldstone bosons. The chiral field [1] is

$$\Sigma = e^{2i\pi^a T^a/f}$$

where $tr(T^a T^b) = \frac{1}{2}\delta^{ab}$ and f is the pion decay constant, 93 MeV. For the N power counting, it is necessary to keep in mind that $f \propto \sqrt{N}$. Under chiral $SU(N_F) \times SU(N_F)$ transformations, the chiral field transforms as

$$\Sigma \rightarrow L\Sigma R^{\dagger}$$

We also need the field ξ, where $\xi\xi = \Sigma$. Under chiral transformations [17],

$$\xi \rightarrow L\xi U^{\dagger} = U\xi R^{\dagger}$$

[1] I have included only the goldstone boson octet. In principle, the η' should also be included, since $m_{\eta'}^2 \propto 1/N$ [60]. However, since in the real world it seems that the η' is not sufficiently close to the $N \rightarrow \infty$ limit [61], I have left it out.

where U is defined implicitly by the above transformation rule. With ξ it is possible to construct vector and axial vector fields,

$$\begin{aligned} V_\mu &= \frac{i}{2}(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi) \\ A_\mu &= \frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi) \end{aligned} \tag{3.3}$$

The third necessary ingredient is the heavy baryon approximation [49, 46]. Assume the the baryon has momentum $p_\mu = mv_\mu + k_\mu$ where v_μ is the velocity and $k_\mu/m \ll 1$. Then the non-relativistic fields ψ'_B can be constructed from the baryon fields ψ_B by removing, in the baryon rest frame, the phase factor $\exp(-im_B t)$, where m_B is the lowest mass of all the baryons in the large-N tower of states. In covariant notation, where v_μ is the four-velocity of the baryon,

$$\psi'_B = e^{im_B \not{v}(v.x)}\psi_B \tag{3.4}$$

(Note that for the spin-3/2 field, the index B implicitly contains a vector index μ; for spin-5/2, B contains two vector indices $\mu\nu$, etc. .) The free particle Lagrangian can now be written [47, 56]

$$\mathcal{L} = i\bar{\psi}'_B(v.\partial)\psi'_B - \bar{\psi}'_{B'}(\Delta m)_{B'B}\psi'_B + \mathcal{O}(1/m_B) \tag{3.5}$$

where there is an implied sum over all the large-N baryons B. $\Delta m_{B'B}$ is a diagonal matrix which takes into account the fact that the different baryons have slightly different masses. These differences are proportional to $1/N$ and/or to the flavor symmetry breaking [59], and will be ignored.

The spin of the baryon is $< B|\{\sigma^\mu\}|B >$, where

$$\sigma^\mu = (\gamma^\mu - v^\mu\not{v})\gamma_5$$

With all the ingredients assembled, we are ready to begin. The effective Lagrangian is made by contracting the indices of the various pieces in all possible ways. The Goldstone bosons are coupled to the baryons through the axial field of Eq. (3.3), as well as through a covariant derivative using the vector field of Eq. (3.3). The easiest way to keep track to the baryon indices is to use a mixed notation, using both the second–quantized baryon annihilation operator ψ'_B and

the non-relativistically normalized state $|B>$ of Eq. (3.2). (From now on, I drop the prime on ψ'_B.) For example, an interaction of the axial field with the baryons is $g\bar{\psi}_{B'} < B'|\{A.\sigma\}|B > \psi_B$ where there is a sum over baryons B and B'. The Lagrangian is [56]

$$\begin{aligned}\mathcal{L} &= \tfrac{f^2}{4} tr(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \beta tr[(\Sigma^\dagger + \Sigma) M_q] + \bar{\psi}_{B'} < B'|\{iv.D\}|B > \psi_B \\ &+ g\bar{\psi}_{B'} < B'|\{A.\sigma\}|B > \psi_B + \tfrac{h}{N}\bar{\psi}_{B'} < B'|\{A_\mu\}\{\sigma^\mu\}|B > \psi_B + \ldots \quad (3.6)\end{aligned}$$

Here M_q is the quark mass matrix; g and h are unknown constants of order 1. The last term is divided by N for the reason explained above. The ellipses indicate that the Lagrangian contains more terms of higher order in $p/m_B, p/\Lambda_\chi$, or $1/N$ in the triple heavy baryon, chiral, and large–N expansion.

One may wonder whether the three expansions get entangled. The baryon mass $m_B \propto N$, so that an expansion in $1/m_B$ is also an expansion in $1/N$. This does not affect the consistency of the above Lagrangian, but a problem may arise if the chiral symmetry breaking scale Λ_χ depends on N. In that case, the momenta running in the loops depends on N and the N-power counting is thrown off. Under a rather mild assumption, however, it is easily seen that this does not happen. The four pion vertex is $\mathcal{O}(1/N)$ in the $1/N$ expansion [4, 50], and in the chiral expansion it is schematically

$$a\frac{p^2}{f^2} + b\frac{p^2}{\Lambda_\chi^2}\frac{p^2}{f^2} + c\frac{p^4}{\Lambda_\chi^4}\frac{p^2}{f^2} + \ldots$$

If it is assumed that the four meson vertex is always $\mathcal{O}(1/N)$ independent of the kinematics, then each term above must be $\mathcal{O}(1/N)$, and since $1/f^2$ is $\mathcal{O}(1/N)$, Λ_χ must be $\mathcal{O}(1)$.

In the baryon rest frame, the spatial components of the Noether current under axial transformations is

$$\begin{aligned}< \mathcal{B}|j^i_{5a}|\mathcal{B} > &= \frac{g}{2} < B|\{(\xi T_a \xi^\dagger + \xi^\dagger T_a \xi)\sigma^i\}|B > \\ &+ \frac{h}{2N} < B|\{\xi T_a \xi^\dagger + \xi^\dagger T_a \xi\}\{\sigma^i\}|B > , \quad a = 1 \ldots N_F^2 - 1 \\ < \mathcal{B}|j^i_{50}|\mathcal{B} > &= < \mathcal{B}|\bar{q}\gamma^i\gamma_5 q|\mathcal{B} >= (g + \frac{h}{N}) < B|\{\sigma^i\}|B > \quad (3.7)\end{aligned}$$

3.3 Two–Loop Corrections

What are the meson loop corrections to the baryon axial current? The spatial components of the axial current are written

$$< \mathcal{B}'|\bar{q}\gamma^i\gamma_5 T^a q|\mathcal{B} >_{tree} = X^{ia}_{B'B}$$

where T^a is a generator of the flavor group. (The magnitude of X^{ia} is what was loosely called g_A in the introduction.) The previous section has shown that the axial current can be expressed in the $1/N$ expansion as:

$$X^{ia}_{B'B} = g < B'|G^{ia}|B > + \frac{h}{N} < B'|H^{ia}|B > + \ldots \tag{3.8}$$

where g and h are constants of order 1 and G^{ia} and H^{ia} are the operators [57, 59]

$$G^{ia} = a^\dagger_{r\alpha} T^a_{rs} \sigma^i_{\alpha\beta} a_{s\beta}$$

$$H^{ia} = (a^\dagger_{r\alpha} T^a_{rs} a_{s\alpha})(a^\dagger_{t\beta} \sigma^i_{\beta\gamma} a_{t\gamma})$$

The mesons are coupled derivatively to the baryon axial current. Some of the Feynman rules for the pion–baryon interactions are given in Fig. 3.3. The baryons are treated within the heavy fermion approximation [49, 46, 47], and the calculations are performed in the baryon's rest frame. The meson propagator uses the mass matrix m^2_{ab}, a diagonal matrix that gives the masses of the pions, kaons, and eta under flavor symmetry breaking. For the N power counting, it is important to keep in mind that the pion decay constant $f \propto \sqrt{N}$.

Now look at the vertex renormalization. The momentum integral for Fig. 3.1(a) is

$$\mathcal{I}_{ab} = i \int \frac{d^4p}{(2\pi)^4} \frac{1}{p_0^2} \frac{\frac{1}{3}\mathbf{p}^2}{p^2 - m^2_{ab}}$$

For Fig. 3.2 (a) and Fig. 3.2 (c), the integral is

$$\mathcal{J}_1^{aa'bb'} = -\int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{1}{p_0^3} \left(\frac{1}{p_0 + q_0} - \frac{1}{q_0} \right) \frac{\frac{1}{3}\mathbf{p}^2}{p^2 - m^2_{aa'}} \frac{\frac{1}{3}\mathbf{q}^2}{q^2 - m^2_{bb'}}$$

The inner loop includes a counter-term. If this term (the $1/q_0$ appearing above) is not included, the internal baryon acquires an additional mass, which must then

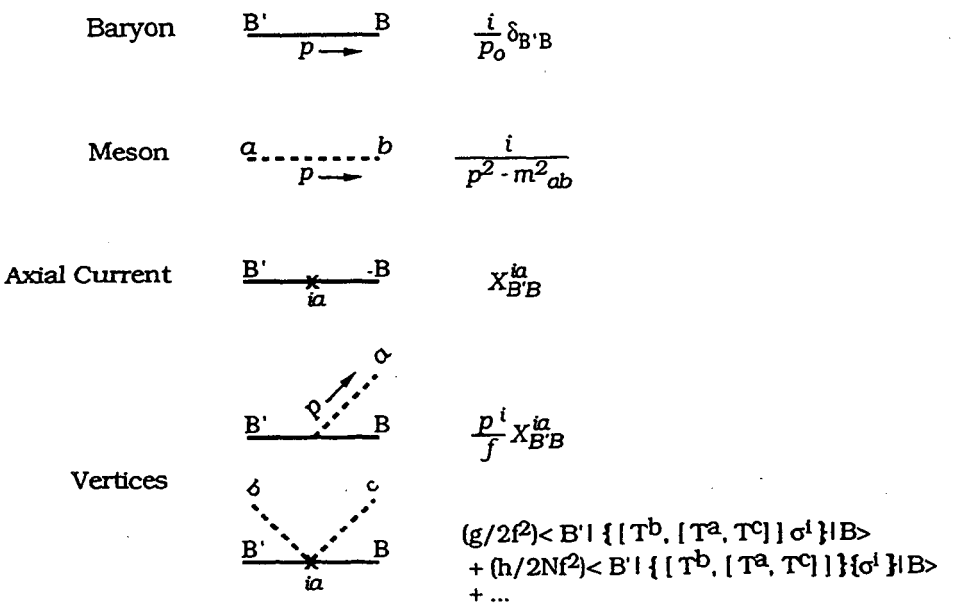

Figure 3.3: Some of the Feynman rules for baryon-meson interactions.

be transformed away by the heavy baryon transformation. It is easier simply to include the counter term explicitly. The mass differences between the various baryons are proportional to $1/N$ and/or to the flavor symmetry breaking, and will be ignored.

The integrals for Fig. 3.2 (b),(d),(e), and (f) are (respectively)

$$\mathcal{J}_2^{aa'bb'} = -\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p_0^2}\frac{1}{(p_0+q_0)^2}\frac{\frac{1}{3}\mathbf{p}^2}{p^2-m^2_{aa'}}\frac{\frac{1}{3}\mathbf{q}^2}{q^2-m^2_{bb'}}$$

$$\mathcal{K}_1^{aa'bb'} = -\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p_0^2}\frac{1}{p_0+q_0}\frac{1}{q_0}\frac{\frac{1}{3}\mathbf{p}^2}{p^2-m^2_{aa'}}\frac{\frac{1}{3}\mathbf{q}^2}{q^2-m^2_{bb'}}$$

$$\mathcal{K}_2^{aa'bb'} = -\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p_0}\frac{1}{(p_0+q_0)^2}\frac{1}{q_0}\frac{\frac{1}{3}\mathbf{p}^2}{p^2-m^2_{aa'}}\frac{\frac{1}{3}\mathbf{q}^2}{q^2-m^2_{bb'}}$$

$$\mathcal{K}_3^{aa'bb'} = -\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p_0}\frac{1}{p_0+q_0}\frac{1}{q_0^2}\frac{\frac{1}{3}\mathbf{p}^2}{p^2-m^2_{aa'}}\frac{\frac{1}{3}\mathbf{q}^2}{q^2-m^2_{bb'}} = \mathcal{K}_1^{bb'aa'}$$

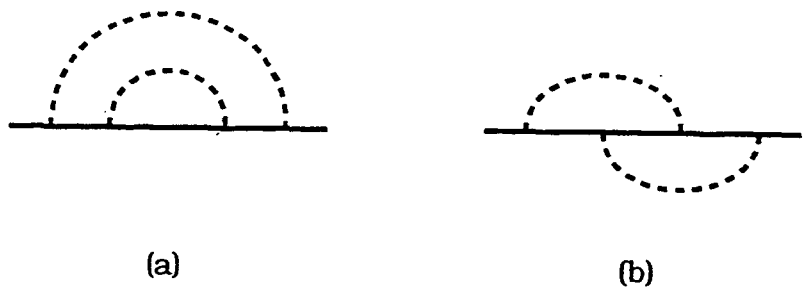

Figure 3.4: Two loop wavefunction renormalization diagrams.

The vertex renormalization to two loops can then be written:

$$
\begin{aligned}
V^{ia}_{B'B} \;=\; & \Big(X^{ia} + \frac{1}{f^2}\mathcal{I}^{bb'} X^{jb'} X^{ia} X^{jb} + \frac{1}{f^4}\mathcal{J}_1^{bb'cc'} X^{jb} X^{ia} X^{kc} X^{kc'} X^{jb'} \\
& + \frac{1}{f^4}\mathcal{J}_2^{bb'cc'} X^{jb} X^{kc} X^{ia} X^{kc'} X^{jb'} + \frac{1}{f^4}\mathcal{J}_1^{bb'cc'} X^{jb} X^{kc} X^{kc'} X^{ia} X^{jb'} \\
& + \frac{1}{f^4}\mathcal{K}_1^{bb'cc'} X^{jb} X^{ia} X^{kc} X^{jb'} X^{kc'} + \frac{1}{f^4}\mathcal{K}_2^{bb'cc'} X^{jb} X^{kc} X^{ia} X^{jb'} X^{kc'} \\
& + \frac{1}{f^4}\mathcal{K}_1^{cc'bb'} X^{jb} X^{kc} X^{jb'} X^{ia} X^{kc'} \Big)_{B'B}
\end{aligned}
\tag{3.9}
$$

The operators X^{ia} are treated as matrices with baryon indices; intermediate baryon states are summed over.

The baryon wavefunction renormalization constant can be computed from the diagrams of Fig. 3.1 (b) and Fig. 3.4 (see Appendix C):

$$
\begin{aligned}
\left(Z_2^{-1}\right)_{B'B} \;=\; & \Big(1 + \frac{1}{f^2}\mathcal{I}^{bb'} X^{jb} X^{jb'} + \frac{1}{f^4}(2\mathcal{J}_1^{bb'cc'} + \mathcal{J}_2^{bb'cc'}) X^{jb} X^{kc} X^{kc'} X^{jb'} \\
& + \frac{1}{f^4}(\mathcal{K}_1^{bb'cc'} + \mathcal{K}_2^{bb'cc'} + \mathcal{K}_1^{cc'bb'}) X^{jb} X^{kc} X^{jb'} X^{kc'} \Big)_{B'B}
\end{aligned}
\tag{3.10}
$$

Finally, the renormalized axial current is

$$
< \mathcal{B}' | \bar{q}\gamma^i\gamma_5 T^a q | \mathcal{B} > = \left(Z_2^{\frac{1}{2}}\, V^{ia}\, Z_2^{\frac{1}{2}} \right)_{B'B} \tag{3.11}
$$

When Eqs. (3.9) and (3.10) are substituted into Eq. (3.11) and the result is multiplied out to order $1/f^4$ (i.e. to two loops), the terms do not simplify in any obvious way. Some identities among the integrals must be used,

$$\mathcal{J}_2^{aa'bb'} + \mathcal{K}_2^{aa'bb'} = \mathcal{K}_1^{aa'bb'}$$

$$\mathcal{K}_1^{aa'bb'} + \mathcal{J}_1^{aa'bb'} = 0$$

$$\mathcal{K}_1^{aa'bb'} + \mathcal{K}_1^{bb'aa'} - \mathcal{I}^{aa'}\mathcal{I}^{bb'} = 0$$

as well as the identity

$$\mathcal{K}_1^{aa'bb'}[X^{ia}X^{ia'}, X^{jb}X^{jb'}] = 0$$

Then after a few pages of algebra, Eq. (3.11) becomes

$$\begin{aligned}
< \mathcal{B}'|\bar{q}\gamma^i\gamma_5 T^a q|\mathcal{B} >= X^{ia} &+ \frac{1}{2f^2}\mathcal{I}^{bb'}[X^{jb},[X^{ia},X^{jb'}]] \\
&+\frac{1}{f^4}\mathcal{K}_1^{bb'cc'}\left\{\frac{1}{4}\left[X^{jb},[[X^{kc},[X^{ia},X^{kc'}]],X^{jb'}]\right]\right. \\
&+\frac{1}{2}\left[[X^{jb},X^{ia}],[X^{kc},[X^{jb'},X^{kc'}]]\right] \\
&\left.+\frac{1}{4}\left[X^{ia},\left[X^{jb},[X^{kc},[X^{jb'},X^{kc'}]]\right]\right]\right\} \\
&+\frac{1}{4f^4}\mathcal{K}_2^{bb'cc'}\left[[X^{jb},X^{kc}],[X^{ia},[X^{jb'},X^{kc'}]]\right]
\end{aligned} \tag{3.12}$$

This is the main result of the chapter. From here it is possible to show that the one-loop corrections to the axial current are suppressed by $\mathcal{O}(1/N)$ times the tree-level, and the two-loop contribution is suppressed by $\mathcal{O}(1/N^2)$.

3.4 Interpretation

To understand the meaning of Eq.(3.12), take X^{ia} to leading order in $1/N$,

$$X^{ia} = g < B'|G^{ia}|B > \tag{3.13}$$

where $G^{ia} = a^\dagger_{r\alpha}T^a_{rs}\sigma^i_{\alpha\beta}a_{s\beta}$. Since the operator G^{ia} has one a and one $a^\dagger$, it can count the number of quarks in the baryon once, and so can be of order N when

sandwiched between states $< B'|$ and $|B >$. No accidental cancellations occur, and the tree level coupling to the axial current is

$$X^{ia} \sim \mathcal{O}(N)$$

Now using Eq. (3.13), the one-loop correction can be read off from Eq. (3.12):

$$\frac{g^3}{2f^2}\mathcal{I}^{bb'} < B'|[G^{jb},[G^{ia},G^{jb'}]]|B > \tag{3.14}$$

Each G^{ia} has one a and one $a^\dagger$, but each commutator eliminates an a–$a^\dagger$ pair due to the identity $[a^\dagger_{a\alpha}V^{b\beta}_{a\alpha}a_{b\beta}, a^\dagger_{c\gamma}W^{d\delta}_{c\gamma}a_{d\delta}] = a^\dagger_{a\alpha}[V,W]^{b\beta}_{a\alpha}a_{b\beta}$. The resultant operator in Eq. (3.14) has only one a and one $a^\dagger$, so the matrix element is at most of order N. Since $1/f^2 \sim 1/N$, the total one-loop correction is $\mathcal{O}(1)$, or $1/N$ times the tree-level value.

This result would not have followed if, for example, only the lowest lying baryon were allowed as an intermediate state between the operators X^{ia} and X^{jb} in Eq. (2.34). For a given intermediate state, the one–loop amplitude is $\mathcal{O}(N)$ times the tree–level value. However, the terms from all the various intermediate states combine in such a way that they cancel to $\mathcal{O}(1/N)$. As was mentioned in section 3.1, the decuplet terms tend to cancel the octet terms.

The quadruple commutators of Eq. (3.12), which give the two-loop corrections, also eliminate all but one a–$a^\dagger$ pair. Therefore these commutators are also of $\mathcal{O}(N)$, and when they are multiplied by a coefficient of $1/f^4$, the result is $\mathcal{O}(1/N)$. That is, the two-loop correction is $1/N^2$ times the tree-level value.

This result has the following interpretation: since the loop corrections contain only one a and one $a^\dagger$, the pion vertices and the current operator all act on the same quark; the vertex and wavefunction renormalization are carried out on each quark individually.

The reason that this happens in the one-loop case can easily be demonstrated graphically. In Fig. 3.5 (a), the ends of the pion line are attached to quark lines different from the one on which the current acts. In such a case, one of the pion vertices can be commuted past the current operator, so that the pion is emitted and absorbed before the current operator acts (see Fig. 3.5 (b)). However, this diagram has already been taken into account by the wavefunction

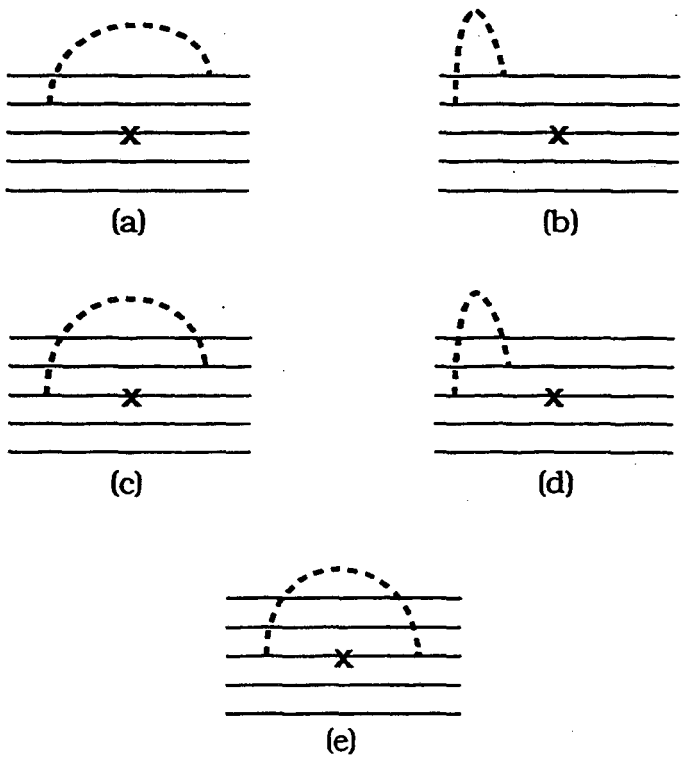

Figure 3.5: The renormalization of the baryon axial current, viewed at the quark level. Diagram (a) = diagram (b), and (c) = (d). Only (e) ends up contributing to the overall renormalization.

renormalization, and so does not contribute. Similarly, in the graph of Fig. 3.5 (c), the pion line begins on the same quark line as the one on which the current operator acts, but the pion ends on a different line. In this case also, the second vertex can be commuted past the current operator, resulting in Fig. 3.5(d). Again, this diagram has already been taken into account by the wavefunction renormalization, and so does not contribute. So we reach the conclusion stated above: the only type of graph that needs to be considered has both pion vertices acting on the same quark line as the current operator (Fig. 3.5 (e)).

This is exactly what is assumed to be true in the chiral quark model [19] as developed in Ref. [47] (see also section 1.3). In that model the leading vertex and wavefunction renormalizations act on one quark at a time, and the quark propagator is that of a (fairly heavy) constituent quark. One problem with the chiral quark model, however, is that it is difficult to understand why a free quark propagator can be used inside the proton. Why should not the bound state wavefunction be taken into account? The large-N approach offers a solution: the quark inherits the heavy fermion propagator from the baryon as a whole but manages not to interact strongly with the other quarks thanks to the cancellations that occur in the commutator structure of the loop corrections. The constituent quark emerges from the tangle of meson loops.

There are two details that might complicate the above picture, but they do not turn out to be problematic. The first is that we have left out some diagrams. Figs. 3.6 (a) - (e) also contribute [46, 47]. It turns out that these diagrams follow the same pattern as above: l-loop diagrams are suppressed by factors of $(1/N)^l$, and the result is just what would have been expected from the chiral quark model. One difference of these diagrams, however, is that they are not necessarily suppressed by powers of the coupling constant g of Eq.(3.13). For example, Fig. 3.6 (a) and Fig. 3.6 (b) are both proportional to g (rather than g^3 or g^5). This point does not affect the present discussion, but is important in the next section.

The second technicality is that Eq. (3.13) is only an approximation to the axial vertex. When the vertex is expanded to the next order in $1/N$ (as suggested by Eq. (3.8)), a new operator H^{ia} is introduced. H^{ia} acts on two

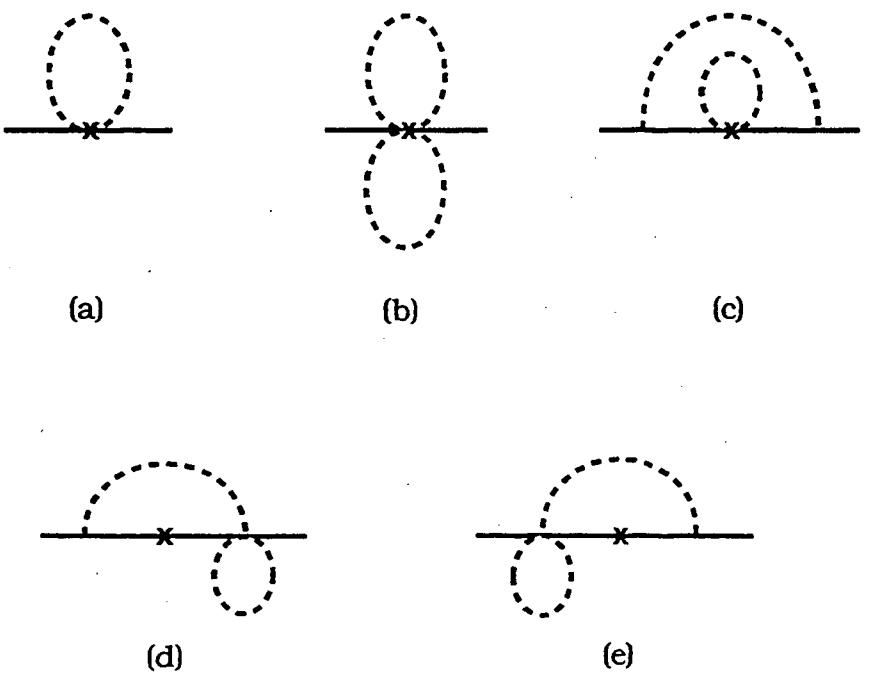

Figure 3.6: Additional one and two loop diagrams that contribute to the renormalization of the axial current.

quarks at a time, so the simplest constituent quark picture receives corrections[2]. However, the identities of Ref. [59] can be used to show the the one- and two-loop corrections are still suppressed by powers of $1/N$ and $1/N^2$ respectively.

3.5 Discussion

How does all this formalism fare in the real world? The double commutator in Eq. (3.12), which gives the one-loop correction, is of the order of the number of flavors N_F. The quadruple commutators giving the the two-loop corrections are of order N_F^2. The momentum integrals should be cut off at the chiral symmetry-breaking scale Λ, so that the one-loop effects are proportional to $\Lambda^2/16\pi^2 f^2$, and the two-loop corrections are proportional to $\Lambda^4/(16\pi^2 f^2)^2$. Let the pion decay constant f be factorized to show clearly its N-dependence:

$$f = \sqrt{N}\hat{f}$$

where $\hat{f}$ is $\mathcal{O}(1)$. As mentioned previously, some diagrams of Fig. 3.6 are not suppressed by powers of the axial coupling constants (g and h of Eq. (3.8)). Therefore the total one-loop contribution to the axial current is of order $(N_F/N)(\Lambda^2/16\pi^2\hat{f}^2)$ times the tree-level value, and the two-loop correction is of order $(N_F/N)^2(\Lambda^2/16\pi^2\hat{f}^2)^2$ times the tree value. Evidently the chiral loop expansion parameter is

$$\frac{N_F}{N}\frac{\Lambda^2}{16\pi^2\hat{f}^2}$$

This parameter is not small in any estimation [17]. However, one can adopt the following approach: start with the bare coupling g (or for example h), assume that it can be renormalized to all orders in the flavor symmetric limit ($m_\eta = m_K = m_\pi \approx 0$), resulting in the renormalized constant g_R. This new constant g_R is to be used in computations, and the effects of virtual pions can be computed loop-by-loop, keeping only those terms that *violate* $SU(N_F)$ symmetry. In this case all terms involving $\Lambda^2/16\pi^2\hat{f}^2$ are to be thrown away, since their effects have

[2]Actually such operators appear in the chiral quark model also; rather than being suppressed by $1/N$, though, they are suppressed [19] by a power of the wavefunction at the origin divided by the constituent quark mass, $|\psi(0)|^{2/3}/m_c$.

already been included in g_R. The new loop expansion parameter then becomes

$$\frac{N_F}{N}\frac{m_K^2}{16\pi^2\hat{f}^2}\log\frac{\Lambda^2}{m_K^2}$$

This procedure is equivalent to using dimensional regularization for all the integrals of Eq. (3.12).

Such a program has already been carried out by Ref. [47]. The one-loop corrections to the baryon axial current were computed in the chiral quark model using dimensional regularization. This is exactly equivalent to a leading order $1/N$ calculation. The model fits the data well; a best fit is obtained for $g_R = 0.56$.

So far we have examined the corrections to the octet axial currents. What about the singlet current, the "spin content" of the baryon? The singlet current is, to tree level and leading order in $1/N$,

$$< \mathcal{B}|\bar{q}\gamma^i\gamma_5 q|\mathcal{B} >= g < B|a^\dagger_{r\alpha}\sigma^i_{\alpha\beta}a_{r\beta}|B >$$

This current is also renormalized, resulting in a coupling $g_R^{(0)}$ that is different from the octet renormalized coupling g_R.

When computing the renormalization of the singlet current, fewer diagrams appear than for the octet current: Figs. 3.6 (a)–(c) do not exist for the singlet current. Therefore the one-loop contribution is suppressed by a factor

$$g_R^2\frac{N_F}{N}\frac{\Lambda^2}{16\pi^2\hat{f}^2}$$

compared to the tree value. The two-loop diagrams are suppressed by a factor of $g_R^2(N_F/N)(\Lambda^2/16\pi^2\hat{f}^2)$ (Figs. 3.2 and 3.4) or $(N_F/N)(m_K^2/16\pi^2\hat{f}^2)(\log\Lambda^2/m_K^2)$ (Figs. 3.6 (d) and (e)) compared to the one-loop diagrams. Therefore, the loop expansion parameter ϵ for the singlet current is

$$\epsilon = \max\left(g_R^2\frac{N_F}{N}\frac{\Lambda^2}{16\pi^2\hat{f}^2}, \frac{N_F}{N}\frac{m_K^2}{16\pi^2\hat{f}^2}\log\frac{\Lambda^2}{m_K^2}\right)$$

If ϵ is small enough, we do not have to go through the extra step of first doing the flavor-symmetric renormalization and then returning to the integrals using dimensional regularization. Cutoff regularization can be used from the outset.

Using this approach, and the leading $1/N$ baryon-pion vertex, the spin content of the proton turns out to be

$$< p \uparrow |\bar{q}\gamma^3\gamma_5 q|p \uparrow >= g\left\{1 + \frac{g_R^2}{2f^2}\mathcal{I}^{bb'} < p \uparrow |[G^{jb}, [\sigma^3, G^{jb'}]]|p \uparrow >\right\}$$

$$= g\left\{1 - g_R^2\left[\frac{32}{9}\frac{\Lambda^2}{16\pi^2 f^2} - \frac{22}{9}\frac{m_K^2}{16\pi^2 f^2}\log\frac{(2\Lambda)^2}{m_K^2}\right]\right\} \tag{3.15}$$

(Here I used $m_\pi^2 = 0$ and $m_\eta^2 = \frac{4}{3}m_K^2$.) Unfortunately, since neither g nor Λ is known, this equation has no predictive power. It is comforting, however, that a reasonable choice of the parameters gives a reasonable result. For example, for $g = 1$ and $\Lambda = 1$ GeV, Eq. (3.15) yields a spin content of 0.57, which is within $\mathcal{O}(\epsilon^2)$ of the experimental value [39],

$$< p \uparrow |\bar{q}\gamma^3\gamma_5 q|p \uparrow >= 0.27 \pm 0.11$$

In this case the expansion parameter ϵ is rather large, $\epsilon \approx 0.75$, so the effects of chiral loops are estimated to be very important.

3.6 Conclusions

In the $1/N$ expansion, one-loop chiral corrections to the baryon axial current are $\mathcal{O}(1/N)$ times the tree-level value, and two-loop corrections are $\mathcal{O}(1/N^2)$ times the tree-level. When the leading baryon–pion vertex is used, these corrections are exactly the same as would be calculated in the chiral quark model. The large-N approach therefore gives some insight into why the chiral quark model works. However, there are a number of questions that have been swept under the rug or simply not addressed at all.

First, there is some indication that the differences between baryon masses cannot be ignored [62]. This should be an order $1/N$ effect, and in order to be consistent, one should also include first order terms in the heavy baryon expansion. It is not clear how the inclusion of these additional terms will affect the goodness of fit to the data.

Second, effects of order $m_K^2 \log \Lambda^2/m_K^2$ have been computed, but those terms of order m_K^2 that appear in the chiral Lagrangian have been ignored, even though

m_K^2 is not much smaller than $m_K^2 \log \Lambda^2/m_K^2$. That is, the effects of chiral symmetry breaking have been included only in the meson propagators and not in the vertices. This is the usual approach used in chiral perturbation theory, and is based on the (rather optimistic) hope that the logarithmic terms arising from loops will turn out to be more important than the explicit terms present in the Lagrangian.

Third, in the expansion of Eq. (3.8), one would expect the constant h, which parametrizes the deviation of the axial coupling from the naive constituent quark picture, to be of $\mathcal{O}(1)$. However, a fit to available data (not including meson loops) gives [57] $h = -0.1$, much smaller than expected. In other words, even though the $1/N$ expansion goes a long way toward explaining the success of the constituent quark model, it cannot fully explain why the corrections to the model are so small.

Finally, the main result of this chapter is somewhat mysterious. Why are pion loops suppressed by powers of $1/N$? I have simply computed the diagrams by brute force and found that this suppression occurs. It would be nice to understand at a deeper level why it *has* to be that way.

Epilogue

What is to be learned from all of these investigations? Why does the constituent quark model work? Is it pure luck, or is there some physical basis for the model? If there is a physical justification, does the constituent quark exist as a quasi-particle inside the baryon, or is it merely a concept that captures some of the symmetry relations relevant for baryon properties?

According to the chiral quark model, the constituent quark is to be taken seriously as a quasi-particle that physically exists inside the baryon. However, as we have seen, the large-N expansion reproduces many of the chiral quark predictions without actually assuming the existence of the chiral quarks. This result suggests that the constituent quark is simply a formal device, and does not literally exist.

The constituent quark model also applies to heavy meson systems [18]. This might be understood in terms of the heavy quark approximation, where it is argued [63] that the heavy quark decouples from the "brown muck" which surrounds it in the B and D systems. Turning the argument around, the brown muck decouples from the heavy quark, and perhaps can be treated as a separate entity (the constituent quark). While this physical picture offers a possible justification for the constituent quark model in heavy baryon systems, it is completely different from the large-N rationale that supports the constituent quark model in baryons.

There is one further piece of evidence: the constituent quark model also works well for the light meson states (except for the Goldstone boson octet) [64]. The success of the model in this regime is not justified by the large-N, or any other, approximation.

As mentioned in section 1.3, Diakonov and Petrov [6] seem to have derived the chiral quark model from QCD, using an instanton background approach. This might explain why the constituent quark appears in a variety of quark bound states. But their resultant Lagrangian leaves out the quarks' coupling to gluons, and this is a serious weakness.

On the other hand, perhaps Diakonov and Petrov are right. Perhaps the chiral quark Lagrangian, Eq. (1.13), is the correct Lagrangian if the gluons are *not* included, but it is only to be used when sandwiched between bound states. This, after all, is just what the large-N analysis dictates for baryons. I am guessing that Diakonov and Petrov overlooked something in their derivation that would require their result to be less general than they had supposed.

If the instanton-background approach can be placed on a rigorous footing and shown to yield the chiral quark model *between bound states*, then this would be a great step forward. It would be the first time that a truly predictive low-energy theory had been extracted from QCD.

Appendix A

Matrices With Space-Time Indices

In Chapter 1 we encountered the matrix $\not{p} - \sigma - i\gamma_5 \boldsymbol{\pi}$, which was interpreted as a matrix with not only Dirac and isospace indices, but also space-time indices. How are we to understand this?

First, a function $f(x)$ can be written as a diagonal matrix:

$$(f)_{xy} = f(x)\delta(x-y)$$

The momentum operator has the following representation:

$$(p_\mu)_{xy} = i\frac{\partial}{\partial x^\mu}\delta(x-y)$$

Two matrices can be multiplied together by summing over the intermediate indices:

$$(AB)_{xz} = \int dy\, A_{xy}B_{yz}$$

Using these three definitions, it can be shown (and it is a worthwhile exercise to show!) that

$$([p_\mu, f])_{xy} = i\left(\frac{\partial f(x)}{\partial x^\mu}\right)\delta(x-y)$$

so indeed this strange matrix notation fulfills our expectations for what the momentum operator should do.

Now, how are the terms in Eq.(1.9) to be calculated? Examine, for example, the second term:

$$-\frac{1}{2}\,\mathrm{tr}\left\{\frac{1}{\not{p}-v}(s+i\gamma_5\boldsymbol{\pi})\frac{1}{\not{p}-v}(s+i\gamma_5\boldsymbol{\pi})\right\}$$

The first step is to move all the momentum operators to the left and the functions of x to the right. This can be done using the identity [65]

$$s\frac{1}{p^2-v^2}=\frac{1}{p^2-v^2}s+\frac{1}{(p^2-v^2)^2}[p^2,s]+\frac{1}{(p^2-v^2)^3}[p^2,[p^2,s]]+\dots,$$

which is to be used after "rationalizing" the Dirac denominators, as well as the identity

$$[p^2,s]=\Box s+2ip^\mu\partial_\mu s$$

After this procedure we end up with terms like

$$-\frac{1}{2}\,\mathrm{tr}\left\{\frac{\not{p}+v}{p^2-v^2}\frac{\not{p}+v}{p^2-v^2}s^2\right\}$$

Taking the Dirac and flavor traces, we get

$$-4\mathrm{tr}'\left\{\frac{p^2+v^2}{(p^2-v^2)^2}s^2\right\} \tag{A.1}$$

where the tr' represents a trace over only the spacetime indices. The representation of $1/(p^2-v^2)$ in the matrix notation is

$$\left(\frac{1}{p^2-v^2}\right)_{xy}=\frac{1}{-\Box_x^2-v^2}\delta(x-y)$$

When this expression and its analogues are substituted into Eq. (A.1), we get

$$\begin{aligned}
&-4\mathrm{tr}'\left\{\int dy\left(\frac{-\Box_x^2+v^2}{(-\Box_x^2-v^2)^2}\delta(x-y)\right)s^2(y)\delta(y-z)\right\}\\
=\;&-4\int dx\int dy\left(\frac{-\Box_x^2+v^2}{(-\Box_x^2-v^2)^2}\delta(x-y)\right)s^2(y)\delta(y-x)\\
=\;&-4\int dx\int dy\frac{-\Box_x^2+v^2}{(-\Box_x^2-v^2)^2}\int\frac{d^4p}{(2\pi)^4}e^{ip.(x-y)}s^2(y)\delta(y-x)\\
=\;&-4\int dx\,dy\int\frac{d^4p}{(2\pi)^4}\frac{p^2+v^2}{(p^2-v^2)^2}e^{ip.(x-y)}s^2(y)\delta(x-y)\\
=\;&-4\int\frac{d^4p}{(2\pi)^4}\frac{p^2+v^2}{(p^2-v^2)^2}\int dx\,s^2(x)
\end{aligned}$$

The p integral can be rewritten

$$-4\int \frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2-v^2} - 4\int \frac{d^4p}{(2\pi)^4}\,\frac{2v^2}{(p^2-v^2)^2} \tag{A.2}$$

Here the first term is equal to $i\Lambda^2/4g^2$ due to the gap equation, (1.8). Its contribution to the action is therefore (including a factor $-iN$ from Eq.(1.7))

$$\int d^4x\,\frac{N\Lambda^2}{4g^2}s^2$$

This term is exactly cancelled by a similar term in Eq.(1.7) that carries a minus sign. The leading contribution to the mass of the s-particle therefore comes from the second term of Eq.(A.2), and is logarithmically divergent. It is given in Eq. (1.10).

Appendix B

Notes on Numerical Integration

In Chapter 2 the functions F and γ were determined numerically by minimizing the classical mass of the qualiton. The classical mass is given in Ref. [20] as

$$m_{cl} = \int_0^\infty dr\, \rho(r) \tag{B.1}$$

where

$$\begin{aligned}
\rho(r) \;=\; & \frac{\alpha}{96\pi^2 r^2}[2(\pi - F) + (\gamma+1)^2 \sin 2F]^2 + \frac{1}{2r^2\alpha}[2r^2\gamma'^2 + \gamma^2(2+\gamma)^2] \\
& + 2\pi f^2[r^2 F'^2 + 2(1+\gamma)^2 \sin^2 F] \\
& + \frac{2\pi}{e^2 r^2} \sin^2 F (1+\gamma)^2 [2r^2 F'^2 + (1+\gamma)^2 \sin^2 F] \\
& + \frac{4\pi}{3}(2\nu f)^2 r^2 [2 - \cos F - \cos^2 F]
\end{aligned}$$

It is convenient to define a dimensionless variable, $\tilde{r} = 2fr$ as in the text, or alternatively $\tilde{r} = 2efr$, which corresponds more closely with the original treatment of Adkins, Nappi, and Witten [21]. The latter definition will be used in this appendix; however, to explore that region of parameter space where $1/e \to 0$, of course the former definition must be used. The differential (Euler-Lagrange) equations for F and γ that follow from the requirement that the mass m_{cl} be a minimum are:

$$
\begin{aligned}
(\tilde{r}^2 + 8\sin^2 F(1+\gamma)^2)F'' &= -2\tilde{r}F' + \sin 2F(1+\gamma)^2 - 4\sin 2F(1+\gamma)^2 F'^2 \\
&+ \frac{\alpha e^2}{24\pi^3\tilde{r}^2}[2(\pi - F) + (\gamma+1)^2 \sin 2F][\cos 2F(\gamma+1)^2 - 1] \\
&-16\sin^2 F(1+\gamma)\gamma' F' + \frac{4}{\tilde{r}^2}(1+\gamma)^4 \sin^2 F \sin 2F \\
&+ \frac{\nu^2}{3e^2}\tilde{r}^2[\sin F + \sin 2F] \qquad \text{(B.2)}
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma'' &= \frac{\alpha^2}{48\pi^2\tilde{r}^2}[2(\pi - F) + (\gamma+1)^2 \sin 2F](\gamma+1)\sin 2F \\
&+ \frac{1}{\tilde{r}^2}\gamma(\gamma+1)(\gamma+2) + \frac{\pi\alpha}{e^2}(1+\gamma)\sin^2 F + \frac{4\pi\alpha}{e^2}\sin^2 F(1+\gamma)F'^2 \\
&+ \frac{4\pi\alpha}{e^2\tilde{r}^2}\sin^4 F(1+\gamma)^3 \qquad \text{(B.3)}
\end{aligned}
$$

These differential equations are subject to the following boundary conditions:

$$
F(0) = \pi \quad ; \quad F(\infty) = 0
$$

$$
\gamma(0) = 0 \quad ; \quad \gamma(\infty) = 0
$$

More precisely, the asymptotic behaviour of the functions can be worked out (from here on I will drop the tilde on $\tilde{r}$):

$$
\begin{aligned}
\lim_{r\to 0} F(r) &= \pi - Ar \\
\lim_{r\to\infty} F(r) &= B\left[\frac{1}{r^2} + \frac{\mu}{r}\right] e^{-\mu r} \quad \text{(valid when } \alpha \to 0) \\
\lim_{r\to 0} \gamma(r) &= -\frac{1}{2}Cr^2 \\
\lim_{r\to\infty} \gamma(r) &= -D/r
\end{aligned}
$$

where A, B, C and D are unknown constants, and μ is the proportional to the mass of the Π particle, $\mu = m_\Pi/2ef = \nu/e$.

I first tried to solve the above equations by the "point and shoot" [43] method. The procedure is as follows: start with a very small value of r (say, 10^{-5}) and choose a value for A and C, above. Next, integrate Eqs. (B.2) and

(B.3) to a suitably large value of r (say 20) and compute the mass of the particle from Eq. (B.1). Then start over again with a different choice of A and C and again compute the resulting mass. Keep iterating this procedure to find those values of A and C that minimize the mass.

Fig. B.1 gives a contour plot of the result. The constant C is plotted on the x-axis, and A on the y-axis; the contours give different values of the dimensionless mass $\tilde{m}$ defined by $\tilde{m} = (e/2f)m_{cl}$. (The input parameters for this plot were $e = 5.0$, $\alpha = 1.0$, and $\nu = 1.32$. The value of e was chosen to be close to the Skyrme value, $e = 5.45$ [21]; ν was chosen to give the Π particle the same mass as the physical pion, assuming that $f = f_\pi/\sqrt{3}$, as suggested by [20].)

The terrain of Fig. B.1 is characterized by a long broad valley, and above it, a series of saddle points with many other local minima. Fig. B.2 is a close-up view of the boxed-in region near the point (0.3,1.0). For this figure, the functions were integrated to $r = 50$ instead of 20. Wild oscillations are evident, with many saddle points and many local minima.

Clearly, this method is not effective for finding the functions F and γ. The problem seems to be that the differential equations (B.2) and (B.3) are hypersensitive to the initial conditions A and C. In fact, it is well known that non-linear equations such as these are often chaotic; I would not be surprised to learn that the surfaces depicted in Figs. B.1 and B.2 are actually fractal surfaces.

All of this may be very interesting from the point of view of modern mathematics, but it gets in the way of computing the properties of the constituent quark! So what is to be done?

I learned from Ref. [23] that there are existing packages which solve differential equations with two-point boundary conditions. In fact, AT&T Bell Laboratories keeps a library of Fortran and C numerical computation programs that are open to anyone. This library can be accessed by sending an e-mail message to

`netlib@research.att.com`

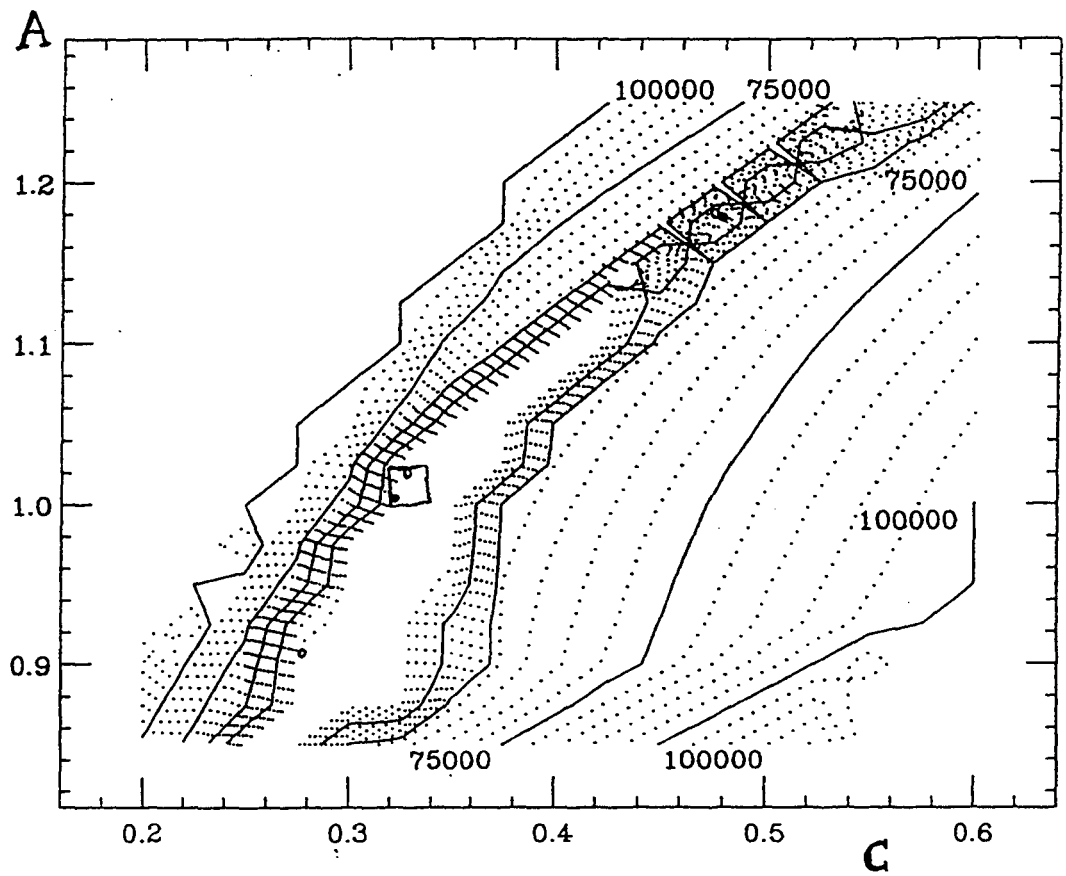

Figure B.1: A contour plot of the dimensionless mass of the qualiton for different values of C (x-axis) and A (y-axis) (see text). The inner-most solid contour line of the valley has a value 25,000; the next solid line represents 50,000. The solid circles represent local minima, found by starting at the points indicated by open circles. A detail of the area penciled in near the point (0.3,1.0) is shown in Fig. B.2.

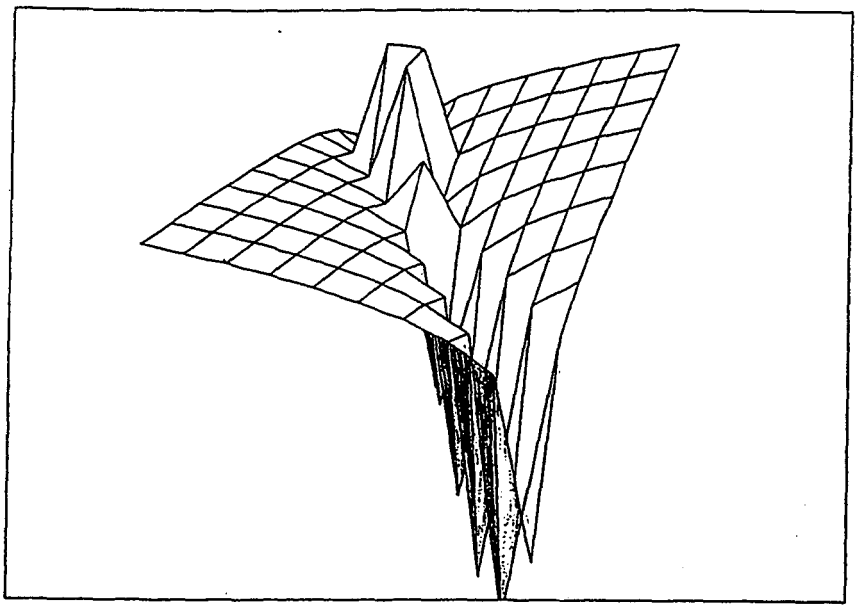

Figure B.2: Qualiton mass (proportional to the height) for different values of C and A, near (0.3,1.0).

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send index
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The program I used for the numerical solution of F and γ relies on a technique called "collocation." This program worked spectacularly well. A copy of it can be obtained by sending e-mail to the above address with the message

```
send colnew from ode
```

The program `colnew` is suitable for the problem at hand because it can find the (numerical) solution to differential equations when given boundary conditions on *both* ends of an interval.

Once the functions F and γ are known, the angular velocity functions ω_1, ω_2 and ω_3 must be determined from their characteristic differential equations [20]. These equations are most easily written in terms of new variables $w_i = r\omega_i$. (As

usual, r is dimensionless here.) Then the equation for w_3 is

$$\left\{\frac{d^2}{dr^2} - \frac{1}{2}\left(\frac{\gamma}{r}\right)^2 - \frac{\pi\alpha}{2e^2}(1-\cos F)\left(1 + F'^2 + \frac{2(1+\gamma)^2}{r^2}\sin^2 F\right)\right\} w_3 = 0$$

The solution must obey the boundary conditions [20]

$$\lim_{r\to 0} w_3(r) = A_3 r$$
$$\lim_{r\to\infty} w_3(r) = r$$

for some unknown constant A_3. Curiously, even though the collocation program worked so well before, it could not solve this problem. However, the point and shoot method is quite successful here: different values of A_3 can be tried until the asymptotic behaviour at infinity is achieved to arbitrary accuracy. To integrate the equation numerically, I used the Bulirsch-Stoer method [43]. This method is somewhat mystifying, but it is easily the best (that is, the fastest) method that I have found.

The equations that govern w_1 and w_2 are:

$$\left(\frac{d^2}{dr^2} - 2\frac{\gamma^2}{r^2}\right) w_1 + 4\frac{\gamma+1}{r^2} w_2 = 0$$
$$\left[\frac{d^2}{dr^2} - \frac{4\pi\alpha}{e^2}K_1(r) - \frac{\gamma^2+6\gamma+6}{r^2}\right] w_2 + \left[\frac{\gamma^2}{r^2} - \frac{4\pi\alpha}{e^2}K_1(r)\right] w_1 = 0$$

where

$$K_1(r) = \sin^2 F\left(\frac{1}{4} + F'^2 + \frac{(1+\gamma)^2}{r^2}\sin^2 F\right)$$

The boundary conditions are:

$$\lim_{r\to 0} w_1(r) = A_1 r \quad ; \quad \lim_{r\to 0} w_2(r) = G r^3$$
$$\lim_{r\to\infty} w_1(r) = r \quad ; \quad \lim_{r\to\infty} w_2(r) = \frac{H}{r^2}$$

for some constants A_1, G and H. Here again I used the point and shoot method: first I picked a value of A_1, then tried different values of G until I obtained a solution where w_2 at the endpoint (typically $r = 100$ or $r = 1000$) was close to zero. Unless a miracle occurred, w_1 would not have the right behaviour at large

r, so I then picked another value of A_1, and again found the optimal G. This was repeated using root finding techniques [43] until a value of A_1 was found that gave the correct asymptotic behaviour for both w_1 and w_2. This was by far the most inefficient part of the numerical work. It took about two seconds to obtain the functions F and γ, and about two minutes to obtain w_1 and w_2.

Appendix C

How To Compute the Wavefunction Renormalization

In Chapter 3 the wavefunction renormalization appears, seemingly out of nowhere, to cancel the leading vertex renormalization. To make sure that the mathematics of this cancellation does not seem too mysterious, I include this appendix, which is a tutorial on the wavefunction renormalization. (For more information, see for example [66]).

It is easiest to begin by looking at the self-energy of the baryon, which can be computed from the diagrams of Fig. 3.1 (a) (one loop) or Fig. 3.4 (two loops). For example, the one-loop self-energy is

$$\Sigma = \int \frac{d^4p}{(2\pi)^4} \frac{i}{k_0 + p_0} \frac{i}{p^2 - m_{ab}^2} \frac{p^i}{f} \frac{(-p^j)}{f} X^{ia} X^{jb}$$

The self-energy can be expanded in a power series in the external momentum k_μ. Because of the form of the baryon propagators, Σ is only a function of k_0:

$$\Sigma(k) = -i[A + Bk_0 + \ldots] \tag{C.1}$$

The constant A renormalizes the mass of the baryon. Consider, for example, the lowest baryon state: its mass was supposed to have been transformed away completely by the heavy-baryon transformation, Eq. (3.4). But after this renormalization, the new mass will have to be transformed away once again. It is

simpler, however, to adopt the following procedure (alluded to in Chapter 3): use the renormalized masses of the baryons from the beginning, and perform the heavy-baryon transformation only once. Then the Lagrangian should include a counter-term that cancels the constant A so that the masses (and mass differences Δm appearing in Eq. (3.5)) remain renormalized.

We can therefore set $A = 0$. Successive insertions of the self-energy into the propagator will then give the full propagator

$$\frac{i}{k_0} + \frac{i}{k_0}(-iBk_0)\frac{i}{k_0} + \frac{i}{k_0}(-iBk_0)\frac{i}{k_0}(-iBk_0)\frac{i}{k_0} + \ldots$$
$$= \frac{i}{k_0}(1 + B + B^2 + \ldots)$$

This geometric series can be summed:

$$\text{Full propagator} = \frac{1}{1-B}\frac{i}{k_0} \equiv \frac{iZ_2}{k_0} \tag{C.2}$$

The propagator is given by the time-ordered product of two field operators, so the above equation can be written

$$< 0|T\psi\bar{\psi}|0 >= Z_2\frac{i}{k_0}$$

where ψ is the (bare) operator that annihilates a baryon. However, the physical field ψ_{phys} must have the canonical normalization,

$$< 0|T\psi_{phys}\bar{\psi}_{phys}|0 >= \frac{i}{k_0}$$

This leads to the identification

$$\psi_{phys} = \frac{1}{\sqrt{Z_2}}\psi \tag{C.3}$$

Since the constant Z_2 evidently renormalizes the operator ψ, it is called the wavefunction renormalization constant.

The wavefunction renormalization must be included whenever matrix elements (like the baryon axial current) are evaluated because, according Eq. (C.3), whenever the *bare* field ψ annihilates a *physical* state $|\mathcal{B}>$, a factor of $\sqrt{Z_2}$ ensues:

$$< 0|\psi|\mathcal{B} >= \sqrt{Z_2}$$

This explains why the constants $\sqrt{Z_2}$ appear in Eq. (3.11).

Let us turn to the evaluation of Z_2. According to Eq. (C.2), $Z_2^{-1} = 1 - B$. Also, Eq. (C.1) implies that $B = \lim_{k\to 0} i\partial\Sigma/\partial k_0$. Therefore,

$$Z_2^{-1} = 1 - i \lim_{k\to 0} \frac{\partial\Sigma(k)}{\partial k_0}$$

For example, in the one-loop case,

$$\begin{aligned} Z_2^{-1} &= 1 - i(-1)\frac{1}{f^2}\int \frac{d^4p}{(2\pi)^4}\frac{1}{p_0^2}\frac{p^ip^j}{p^2 - m_{ab}^2}X^{ia}X^{jb} \\ &= 1 + \frac{1}{f^2}\mathcal{I}_{ab}X^{ia}X^{ib}\,, \end{aligned}$$

which agrees with Eq. (3.10).

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