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Phenomenology of Minimal SU(5) Unification with Dynamical Supersymmetry Breaking *

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Abstract

We consider the constraints from proton decay and b - τ unification in the minimal supersymmetric SU(5) grand unified theory with a 'visible' dynamical supersymmetry breaking sector. We show how the presence of vector-like messenger fields and the constrained superparticle mass spectrum affect the phenomenology of the model. We include the messenger fields in our renormalization group analysis between the messenger scale (~ 100 TeV) and the GUT scale. We show that the simplest model of this type, a minimal SU(5) GUT with an additional $\mathbf{5} + \bar{\mathbf{5}}$ of messenger fields is excluded by the constraints from proton decay and b - τ unification.

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1 Introduction

Supersymmetric grand unification is one of the viable possibilities for the physics that lies beyond the standard model [1]. Interest in supersymmetric grand unified theories (SUSY GUTs) has been motivated primarily by two observations: (i) supersymmetry eliminates the quadratic divergences in radiative corrections to the Higgs mass, which can destabilize the hierarchy between the GUT and electroweak scales [2], and (ii) gauge coupling unification can be achieved to considerable accuracy provided that superparticle masses are $\lesssim 1$ TeV [3]. While this picture may turn out to be correct, it provides no explanation for why the supersymmetry breaking scale is so low (for example, in comparison to the Planck scale). In addition, it gives no explanation for the near degeneracy of the squark masses, necessary to avoid large flavor-changing neutral current (FCNC) effects. While supersymmetry can resolve (i) and (ii) above, the origin, scale and pattern of supersymmetry breaking masses required to produce a viable phenomenology remains a separate and important puzzle.

Recently proposed models of dynamical supersymmetry breaking (DSB) provide possible solutions to these remaining problems [4, 5]. If supersymmetry is broken by nonperturbative dynamics triggered when some asymptotically-free gauge coupling $g(\Lambda)$ becomes large, then we would expect

$$\Lambda \approx m_0 \exp \frac{8\pi^2}{bg^2(m_0)} \quad (1)$$

where $b < 0$ is a beta function, and m_0 is some high scale, like M_{Planck} . The exponential suppression can account for a hierarchy between the SUSY-breaking scale and the Planck scale. Furthermore, the models that have been proposed so far employ a mechanism [7] by which supersymmetry breaking is transmitted to the ordinary particles through loop diagrams involving vector-like “messenger” fields, that carry electroweak quantum numbers, and ordinary gauge interactions. When the messenger fields feel SUSY breaking originating from the dynamical SUSY breaking sector of the theory, they transmit it to the ordinary squarks via these diagrams, which are flavor independent [4, 5]. As a result, the squarks of different generations remain degenerate, and the FCNC problem is naturally avoided.

In this paper, we will comment on the GUT phenomenology of models of this type. In the present context, GUT refers to the unification of the ordinary gauge groups of the standard model, but not to the unification of these groups with the additional gauge groups responsible for dynamical supersymmetry breaking. We restrict our discussion to the minimal SU(5) grand unified model [6] for definiteness. We will first argue that the existence of vector-like multiplets with electroweak quantum numbers at a scale ~ 100 TeV is generic to any workable model. We will catalog the possible particle content of the messenger sector that is allowed by the requirement of perturbative gauge coupling unification, and we state the predicted squark and gaugino mass relations that follow in each case. Using this information, and including the messenger fields in our renormalization group analysis, we determine the bounds from the nonobservation of proton-decay, and from the requirement of b - τ Yukawa unification. We conclude that a minimal SU(5) GUT with the simplest messenger sector possible is excluded by the lower bounds on the proton half-life.

2 Messenger Sector

The part of the messenger sector that is relevant to our analysis involves those fields which carry electroweak quantum numbers. These fields transmit SUSY breaking to the ordinary sector via loop diagrams involving electroweak gauge interactions. Since these diagrams have appeared in a number of places in the literature [4, 5, 7], we do not display them again here. In the model of Ref. [5], the relevant part of the messenger sector superpotential is

$$W_m = \lambda_D S \bar{D} D + \lambda_l S \bar{l} l \quad (2)$$

where the fields have the quantum numbers $D \sim (3, 1)_{-1/3}$, $\bar{D} \sim (\bar{3}, 1)_{1/3}$, $l \sim (1, 2)_{-1/2}$, and $\bar{l} \sim (1, 2)_{1/2}$ under the standard model gauge group, and S is a singlet chiral superfield. This particle content forms full SU(5) multiplets, $\mathbf{5} + \bar{\mathbf{5}}$, so that the apparent gauge unification at $\sim 2 \times 10^{16}$ GeV is preserved. The vacuum expectation value (vev) of the scalar component of S , $\langle S \rangle$, determines the SU(3) \times SU(2) \times U(1) invariant masses in (2), while the vev of the F component of S , $\langle F_S \rangle$, parametrizes the degree of SUSY breaking. In Ref. [5] the 3-2 model [8] is assumed as the source of DSB, and

the authors show that the remaining portion of the messenger superpotential can be constructed so that DSB in the 3-2 sector generates vevs for both S and F_S .

We first would like to argue that the portion of the messenger superpotential given in (2) is generic to a wide variety of realistic models in which SUSY breaking is transmitted to the ordinary sector via gauge interactions. If the messenger quarks and leptons are vector-like under the standard model gauge group, and we allow no dimensionful couplings in the superpotential, then the couplings in (2) will be present. The latter requirement is a philosophical one, namely, that all mass scales in the theory be generated via dimensional transmutation. One might imagine constructing a model in which the messenger fields are chiral rather than vector-like under the standard model gauge group. The general problem of a model of this type is that radiatively-generated gaugino masses are too small. The one-loop diagram responsible for generating a gaugino mass necessarily involves chirality flips on the fermion and scalar lines. These chirality flips are proportional to the messenger fermion masses, m_f , and therefore the gaugino masses are of order $(\alpha/4\pi)m_f^2/m_{SUSY}$. Since m_f is of order the weak scale, the gauginos in this scenario are unacceptably light.[†]

One might also imagine that the messenger quarks and leptons are vector-like under the standard model gauge group, but carry nonstandard quantum numbers as well. In this case, it is likely that the additional gauge couplings would be perturbative. The only gauge group that is nonperturbative is the one in the DSB sector, and introducing additional particles that transform under it could lead to disaster in two ways: the vacuum structure of the theory may change so that supersymmetry is restored, or the multiplicity of messenger quarks and leptons may be too large to retain perturbative unification of the ordinary gauge couplings [4]. We know of no workable model in which the messenger quarks and leptons couple directly to a strongly interacting group from the DSB sector. If the messenger quarks and leptons couple to a nonstandard gauge group that is perturbative (like SU(3) in Ref. [4]), one might still worry that the conclusions of the two-loop proton decay analysis that we will present in the next section could be altered. While

[†]We do not consider the possibility of light gluinos in this paper [9].

the standard model gauge couplings will run differently in this case, one can show that the quantities relevant to our analysis (e.g., the mismatch of gauge couplings at the GUT scale, mass ratios of particles at the messenger scale, etc.) will remain unaltered and our conclusions will remain the same.

In what follows we assume minimal $SU(5)$ unification, so that the gauge structure of the theory is $SU(5) \times G_{\text{DSB}}$. In G_{DSB} we include any nonstandard gauge groups that may be necessary for communicating SUSY breaking to the full messenger sector superpotential (e.g., the messenger hypercharge group discussed in [5]). $SU(5)$ gauge invariance implies that the messenger quarks and leptons form complete $SU(5)$ representations. In the minimal case of a $\mathbf{5} + \overline{\mathbf{5}}$ in the messenger sector, the messenger superpotential at the GUT scale has the form

$$W_m = \lambda S \overline{\mathbf{5}} \mathbf{5} . \quad (3)$$

However, below the GUT scale, $SU(5)$ is broken, and we recover the superpotential given in (2). The assumption of unification allows us to compute λ_D and λ_l in terms of λ , by running these couplings down to the messenger scale. Once the messenger scale has been specified, threshold corrections at this scale are calculable, and can be included in our renormalization group analysis without introducing any additional uncertainty. This is true for representations larger than $\mathbf{5} + \overline{\mathbf{5}}$ as well.

Next, we must specify what other $SU(5)$ representations are allowed in the messenger sector. Introducing additional $SU(5)$ multiplets preserves gauge unification, but the gauge coupling at the GUT scale $\alpha_5(m_{GUT})$ increases as we add additional multiplets. If we require that $\alpha_5(m_{GUT})$ remains perturbative, then we may add 1, 2, 3 or 4 $(\mathbf{5} + \overline{\mathbf{5}})$ pairs, or a single $(\mathbf{10} + \overline{\mathbf{10}})$ pair, or $(\mathbf{5} + \overline{\mathbf{5}}) + (\mathbf{10} + \overline{\mathbf{10}})$ to the particle content of the minimal $SU(5)$ GUT. Additional $\mathbf{5}$ s or $\mathbf{10}$ s, or larger $SU(5)$ representations will render $\alpha_5(m_{GUT})$ nonperturbative [10].

The messenger sector fields not only affect the renormalization group analysis between the messenger scale Λ and the GUT scale (including the calculable threshold corrections at the scale Λ) but also constrain the threshold corrections at the weak scale. As presented in Ref. [5], the radiatively-generated gaugino and squark masses, m_i and \tilde{m} , assuming a $\mathbf{5} + \overline{\mathbf{5}}$ in the

messenger sector, are given by

$$m_i = \frac{g_i^2 \langle F_S \rangle}{16\pi^2 \langle S \rangle}, \quad (4)$$

$$\tilde{m}^2 = \sum_a 2C_F^{(a)} \left(\frac{g^{(a)2}}{16\pi^2} \right)^2 \frac{\langle F_S \rangle^2}{\langle S \rangle^2}, \quad (5)$$

where C_F is $3/4$ for $SU(2)$ doublets, $4/3$ for $SU(3)$ triplets, and $(3/5)Y^2$ for $U(1)$. The quantity $\langle F_S \rangle / \langle S \rangle \equiv m_0$ has dimensions of mass, and is of the same order as the messenger scale Λ . In a specific model, m_0 is calculable by minimizing the messenger sector potential. Once m_0 is fixed, the particle content of the messenger sector completely determines the gaugino and squark masses at the scale Λ ; these can subsequently be run down to the weak scale. We explain how we fix the precise value of m_0 in the following section. If the messenger sector consists of n_5 $\mathbf{5} + \bar{\mathbf{5}}$ pairs, then (4) and (5) both scale as n_5 , while the ratio m_i / \tilde{m} scales as $\sqrt{n_5}$. In the case of a $\mathbf{10} + \bar{\mathbf{10}}$ pair, we obtain the same result for the gaugino and squark masses as the case where $n_5 = 3$. These observation will be useful in our discussion of the proton decay bound in the following section.

3 Proton Decay Analysis

Our analysis of the proton decay constraints is similar in spirit to that of Refs. [11, 12]. By including threshold corrections at the GUT scale, we can determine the largest color-triplet Higgs mass that is consistent with gauge coupling unification. We can then constrain the remaining free parameters involved in the proton decay matrix element using the current lower bounds on the proton half-life. Our algorithm is as follows:

i. We fix n_5 or n_{10} , the number of $\mathbf{5} + \bar{\mathbf{5}}$ or $\mathbf{10} + \bar{\mathbf{10}}$ pairs in the messenger sector. The messenger sector scale is set at 100 TeV, to insure that the superparticle masses are of order the weak scale (recall equations (4) and (5) above).

ii. We define the GUT scale M_{GUT} as the scale where the $SU(2)$ and $U(1)$ gauge couplings unify. We use the input values $\alpha_1^{-1}(m_Z) = 58.96 \pm 0.05$ and $\alpha_2^{-1}(m_Z) = 29.63 \pm 0.05$, that follow from Ref. [13]. We first run them up to $m_{top} = 176$ GeV using the standard model renormalization group

equations (RGEs). We then numerically solve the two-loop RGEs with the supersymmetric particle content, taking into account the change in the one- and two-loop beta functions as we cross the messenger scale. The RGEs and beta functions are provided in the appendix.

iii. Given an input value of $\alpha_3(m_Z)$, we determine the mismatch between $\alpha_3^{-1}(M_{GUT})$, and $\alpha_5^{-1}(M_{GUT})$, the GUT coupling determined in step *ii*. We ascribe this mismatch to the sum effects of threshold corrections at the weak scale, the messenger scale, and the GUT scale. Using the one-loop expressions for threshold corrections given in Ref. [11], we find that the mismatch $\Delta\alpha_3^{-1} \equiv \alpha_3^{-1}(M_{GUT}) - \alpha_5^{-1}(M_{GUT})$ is given by

$$\Delta\alpha_3^{-1} = -\frac{1}{4\pi} \left[\frac{12}{5} \ln \frac{M_{H_c}}{M_{GUT}} + 4 \ln \frac{m_{\tilde{g}}}{m_{\tilde{w}}} - \frac{8}{5} \ln \frac{m_{\tilde{h}}}{m_{top}} + \frac{12}{5} n_5 \ln \frac{m_D}{m_L} \right]. \quad (6)$$

The first term gives the threshold correction at the GUT scale as a function of the color-triplet Higgs mass M_{H_c} . The next two terms give the largest [11] threshold corrections at the weak scale, depending on the gluino, wino, and higgsino masses. We checked the threshold corrections from scalars can be safely neglected in this analysis. The third term gives the threshold correction at the messenger scale, from the splitting of the original n_5 $\mathbf{5} + \overline{\mathbf{5}}$ pairs. In the case of a $\mathbf{10} + \overline{\mathbf{10}}$ pair, we have a different particle content at the messenger scale, and we should make the replacements

$$\frac{12}{5} n_5 \log \frac{m_D}{m_L} \quad \longrightarrow \quad -\frac{6}{5} \log \frac{m_Q}{m_E} - \frac{18}{5} \log \frac{m_Q}{m_U} \quad (7)$$

where the new fields have the quantum number assignments $Q \sim (3, 2)_{1/6}$, $E \sim (1, 1)_{-1}$ and $U \sim (3, 1)_{-1/3}$.

Note that most of the variables in (6) and (7) can be estimated reliably enough, making it possible to place an upper bound on M_{H_c} . We take $m_{\tilde{g}}/m_{\tilde{w}} = \alpha_3/\alpha_2 \approx 3.5$, and set $m_{\tilde{h}} = 1$ TeV to maximize M_{H_c} . The ratio m_Q/m_L can be computed by running the Yukawa couplings in (2) between the GUT scale and the messenger scale. We use a one-loop estimate of this ratio which takes into account the effect of the gauge interactions on the running. We find

$$m_D/m_L \approx \left[\frac{\alpha_5^{-1}}{\alpha_1^{-1}(\Lambda)} \right]^{\frac{1}{6} \frac{1}{33+n_5}} \left[\frac{\alpha_5^{-1}}{\alpha_2^{-1}(\Lambda)} \right]^{\frac{3}{2} \frac{1}{1+n_5}} \left[\frac{\alpha_5^{-1}}{\alpha_3^{-1}(\Lambda)} \right]^{-\frac{8}{3} \frac{1}{-3+n_5}} \quad (8)$$

for $n_5 < 3$. For $n_5 = 3$ (the case where α_3 no longer runs at the one-loop level above the messenger scale), one makes the substitution

$$\left[\frac{\alpha_5^{-1}}{\alpha_3^{-1}(\Lambda)} \right]^{-\frac{8}{3} \frac{1}{-3+n_5}} \rightarrow \left[\frac{\Lambda}{M_{GUT}} \right]^{-\frac{4}{3} \frac{\alpha_3(\Lambda)}{\pi}} \quad (9)$$

in eq. (8). Finally, the ratios that are relevant in the $\mathbf{10}+\overline{\mathbf{10}}$ case are given by

$$m_Q/m_E \approx \left[\frac{\alpha_5^{-1}}{\alpha_1^{-1}(\Lambda)} \right]^{\frac{35}{288}} \left[\frac{\alpha_5^{-1}}{\alpha_2^{-1}(\Lambda)} \right]^{-\frac{3}{8}} \left[\frac{\Lambda}{M_{GUT}} \right]^{-\frac{4}{3} \frac{\alpha_3(\Lambda)}{\pi}}, \quad (10)$$

$$m_Q/m_U \approx \left[\frac{\alpha_5^{-1}}{\alpha_1^{-1}(\Lambda)} \right]^{\frac{15}{288}} \left[\frac{\alpha_5^{-1}}{\alpha_2^{-1}(\Lambda)} \right]^{-\frac{3}{8}} \quad (11)$$

For any input value of $\alpha_3(m_Z)$, we determine the mismatch $\Delta\alpha_3^{-1}$, and the maximum M_{H_c} that follows from (6) or (7). Note that in determining this upper bound, we take into account the uncertainties in M_{GUT} and $\alpha_5(M_{GUT})$ that follow from the experimental uncertainties in $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$ at 90% confidence level. The results are shown in Figure 1. In the case where we ignore threshold corrections at the messenger scale, Figure 1a, we see that the addition of $\mathbf{5}+\overline{\mathbf{5}}$ pairs tends to weaken the bound on M_{H_c} ; in the $\mathbf{10}+\overline{\mathbf{10}}$ case the bound is strengthened. However, we see that in the final result, Figure 1b, the upper bound on M_{H_c} is not much different from the minimal case. This is due to an accidental cancellation between two different effects, namely the two-loop contribution to the gauge coupling evolution due to the additional fields, and the threshold correction at the messenger scale.

iv. Using our results shown in Figure 1, we can study the bounds from the proton life-time. Consider the mode $n \rightarrow K^0 \bar{\nu}_\mu$. The nucleon life-time is given in Ref. [11] as

$$\tau(n \rightarrow K^0 \bar{\nu}_\mu) = 3.9 \times 10^{31} \text{yrs.} \times \left| \frac{0.003 \text{GeV}^3}{\zeta} \frac{0.67}{A_S} \frac{\sin 2\beta}{1 + y^{tK}} \frac{M_{H_c}}{10^{17} \text{GeV}} \frac{\text{TeV}^{-1}}{f(u, d) + f(u, e)} \right|^2 \quad (12)$$

Here ζ parametrizes the uncertainty in the hadronic matrix element, and is estimated to be between 0.003 GeV^3 to 0.03 GeV^3 [14]. We set $\zeta = 0.003$ to be the most conservative. A_S represents the short distance renormalization of the proton-decay operators, i.e. from running the Yukawa couplings up

to the GUT scale, and then running the dimension-5 operators generated by integrating out the color-triplet Higgs, back down to the weak scale. We have checked that the effect of the messenger sector fields on the numerical value of A_S is negligible, and set $A_S \approx 0.67$. Note that this is an extremely conservative assumption. The effect of going to $n_5 = 2$, for example, reduces A_S to 0.63. However, we have not included the effect of the top quark Yukawa coupling in the running which can enhance A_S (and strengthen the resulting proton decay bound) by as much as a factor of ~ 3 [11]. We simply set $A_S = 0.67$ in our numerical analysis as a conservative estimate. f is the “triangle” function obtained by dressing the dimension-5 supersymmetric operators with winos to produce dimension-6 four-fermion operators that are responsible for the decay [15, 16]. It is given by

$$f(u, d) \equiv \frac{m_{\tilde{w}}}{m_{\tilde{u}}^2 - m_{\tilde{d}}^2} \left(\frac{m_{\tilde{u}}}{m_{\tilde{u}}^2 - m_{\tilde{w}}^2} \ln \frac{m_{\tilde{u}}^2}{m_{\tilde{w}}^2} - \frac{m_{\tilde{d}}}{m_{\tilde{d}}^2 - m_{\tilde{w}}^2} \ln \frac{m_{\tilde{d}}^2}{m_{\tilde{w}}^2} \right). \quad (13)$$

The squark, slepton and wino masses fed into the triangle function are those given by the generalization of (4) and (5) to the case of n_5 $\mathbf{5} + \overline{\mathbf{5}}$ pairs or n_{10} $\mathbf{10} + \overline{\mathbf{10}}$ pairs, as we discussed earlier. The precise values that we choose depend on the value of the parameter m_0 , which is of order the messenger scale. We choose m_0 such that the squark masses are normalized 1 TeV at the weak scale, and the gaugino masses are fixed by (4). We make this choice because the proton-decay bounds are weakest for the heaviest squark masses. However, we do not allow squark masses greater than 1 TeV in the interest of naturalness. Finally, the quantity $1 + y^{tK}$ parametrizes possible interference between the diagrams involving \tilde{c} and \tilde{t} exchange [16]. While the bound from the $K\nu$ mode can be made arbitrarily weak by assuming destructive interference between diagrams (i.e. $y^{tK} = -1.0$), one cannot simultaneously weaken the bounds from the other possible decay modes [11]. For example, for $|1 + y_{tK}| < 0.4$, the mode $n \rightarrow \pi^0 \bar{\nu}_\mu$ becomes dominant, giving a comparable decay rate. Thus, we use the $K\nu$ mode with $|1 + y_{tK}| = 0.4$ to obtain a characteristic bound on the parameter space.

With all the parameters fixed as described above, we obtain a bound on the quantity $\sin 2\beta$, or alternately $\tan \beta$, the ratio of up- to down-type Higgs vevs, for each value of $\alpha_3(m_Z)$ that we input in step *ii*. The results are shown in Figures 2a through 2c. The area of the α_3 - $\tan \beta$ plane that is consistent

with the proton decay constraints is the region above the solid line labeled proton decay. The region between the horizontal dashed lines show the area allowed by the two standard deviation uncertainty in the experimental value of $\alpha_3(m_Z)$ measured at LEP 0.116 ± 0.005 [13]. It is clear that in going from $n_5 = n_{10} = 0$ to $n_5 = 1$, these regions no longer overlap. In the $n_5 = n_{10} = 0$ case, we have chosen the most conservative set of parameters possible, namely, we have set all the squark and slepton masses $\tilde{m} = 1$ TeV, and the wino mass $m_{\tilde{w}} = 45$ GeV. However, in the $n_5 = 1$ case, with squark masses normalized to 1 TeV, the slepton and gaugino masses are predicted from (4) and (5). Because we have greater predictivity in this case, we can no longer choose the most favorable parameter set, and the bound is strengthened. This effect is large enough to overwhelm the competing effect of the slightly weaker bound on the color triplet Higgs mass that we found in Figure 1. As we pointed out earlier, the ratio of gaugino to squark mass scales at $\sqrt{n_5}$, so as we go to larger values of n_5 , we obtain an even less favorable set of parameters. Our proton decay bound stays more or less the same however, due to the competition between this effect and the weaker bound on the color triplet Higgs mass. Finally, we obtain a slightly tighter bound in the $\mathbf{10}+\overline{\mathbf{10}}$ case, Figure 2c, as a result of the slightly tighter upper bound on M_{H_c} shown in Figure 1. For each case in which a messenger sector is present, the proton decay region never intersects with the allowed region for $\alpha_3(m_Z)$.

4 b - τ unification

For completeness, we have also determined the region of the α_3 - $\tan \beta$ plane that is consistent with b - τ unification. Here we work only to the one-loop level. In our proton-decay analysis, the bound on M_{H_c} followed from threshold corrections, so we needed to include all other effects of equal importance. This necessitated the two-loop analysis. In the case of b - τ unification, however, we do not need the higher level of accuracy as we will explain below, so we worked at one loop.

Our algorithm is straightforward. For a given choice of α_3 and $\tan \beta$, we determine the top, bottom, and tau Yukawa couplings. We run these Yukawa

couplings up to the scale m_{top} using the standard model renormalization group equations. Above m_{top} we run these Yukawa couplings using the one-loop supersymmetric renormalization group equations up to the GUT scale [18]. We impose two conditions to determine whether we have a valid solution with b - τ unification: (a) we require λ_b and λ_τ to be within 0.5% of each other at the GUT scale (b) we require λ_b , λ_τ and λ_{top} to be less than 2. Condition (b) is imposed so that the couplings do not blow up below 10 times the GUT scale. If the cutoff scale is lower than this, higher-dimension operators can generate corrections to the Yukawa unification at more than the 10% level [17]. The messenger sector particles alter the analysis through their effect on the running of the gauge couplings, which in turn enter into the one-loop RGEs for the Yukawa couplings. These renormalization group equations are provided in the appendix. The requirement of unification to within 0.5% at the GUT scale is somewhat arbitrary. Threshold corrections and the effects of M_{Planck} suppressed operators could in principle account for a larger mismatch between the Yukawa couplings at the GUT scale. Our results should therefore be considered qualitative. Unlike our proton decay analysis, we do not do a two-loop analysis including threshold corrections at each of the relevant scales. The presence of M_{Planck} suppressed operators can give important GUT scale threshold corrections in the small $\tan\beta$ region [17], rendering the extra accuracy of such an analysis meaningless.

Our results are shown in Figures 2a through 2c. The crescent shaped region is generated by allowing the b -quark $\overline{\text{MS}}$ mass to vary between 4.1 and 4.5 GeV, the range suggested by the QCD sum rule analysis [19]. Note that as we increase n_5 or n_{10} , the b - τ region moves towards smaller values of α_3 . While this is not inconsistent with the measured value of α_3 , the distance between the b - τ region and the area allowed by the proton decay bounds increases monotonically with n_5 or n_{10} .[‡]

[‡]Note that the low $\tan\beta$ ‘cusp’ of the crescent would extend into the proton decay region for allowed $\alpha_3(m_Z)$ in the minimal case ($n_5 = n_{10} = 0$) had we imposed the less stringent constraint $\lambda_{top} < 3.3$ [18] on the acceptable size of the Yukawa couplings at the GUT scale. Even in this case, however, the three-way overlap between proton decay, b - τ unification and $\alpha_3(m_Z)$, does not persist when the messenger fields are present. If we had chosen a smaller value of $m_{top} = 150$ GeV, on the other hand, the b - τ region would extend to smaller $\tan\beta$, but not to larger $\alpha_3(m_Z)$, and our conclusions would remain the same.

5 Conclusions

We have shown that the minimal supersymmetric SU(5) model cannot be embedded successfully within the simplest type of scenario suggested in Ref. [5]. The effect of messenger sector particles on the renormalization group analysis, as well as the predicted gaugino and squark mass ratios that follow from the messenger sector particle content lead to a conflict with the lower bound on the proton lifetime. In addition, the region of parameter space preferred for b - τ unification moves farther away from the region preferred by the proton decay bounds as the size of the messenger sector is increased. We must emphasize that this is only a mild obstacle to the type of scenario proposed in Ref. [5]. Non-minimal GUTs can be constructed which evade the proton decay bound [20]. It is nonetheless interesting that we can exclude what is perhaps the *simplest* models of this type.

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A Appendix

The two-loop supersymmetric renormalization group equation for the gauge couplings that we use in our proton decay analysis is

$$\mu \frac{\partial g_i}{\partial \mu} = \frac{1}{16\pi^2} b_i g_i^3 + \left(\frac{1}{16\pi^2} \right)^2 \sum_{j=1}^3 b_{ij} g_i^3 g_j^2 \quad (14)$$

where the beta functions are given by

$$b_i = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} n_g + \begin{bmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{bmatrix} n_h + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} n_5 + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} n_{10} + \begin{bmatrix} 0 \\ -6 \\ -9 \end{bmatrix} \quad (15)$$

$$\begin{aligned}
b_{ij} = & \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} n_g + \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_h \\
& + \begin{pmatrix} \frac{21}{45} & \frac{9}{5} & \frac{32}{15} \\ \frac{3}{5} & 7 & 0 \\ \frac{4}{15} & 0 & \frac{34}{3} \end{pmatrix} n_5 + \begin{pmatrix} \frac{23}{5} & \frac{3}{5} & \frac{48}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{6}{5} & 6 & 34 \end{pmatrix} n_{10} \\
& + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} .
\end{aligned} \tag{16}$$

Here n_g , n_h , n_5 and n_{10} are the number of generations (3), higgs doublets (2), messenger sector $\mathbf{5}+\overline{\mathbf{5}}$ and $\mathbf{10}+\overline{\mathbf{10}}$ pairs, respectively.

The one-loop renormalization group equations for the top, bottom, and tau Yukawa couplings used in our analysis of b - τ Yukawa unification are given by [18]

$$\mu \frac{\partial \lambda_{top}}{\partial \mu} = \frac{1}{16\pi^2} \left(-\sum_i c_i g_i^2 + \lambda_b^2 + 6\lambda_{top}^2 \right) \lambda_{top} , \tag{17}$$

$$\mu \frac{\partial \lambda_b}{\partial \mu} = \frac{1}{16\pi^2} \left(-\sum_i c'_i g_i^2 + \lambda_\tau^2 + 6\lambda_b^2 + \lambda_{top}^2 \right) \lambda_b , \tag{18}$$

$$\mu \frac{\partial \lambda_\tau}{\partial \mu} = \frac{1}{16\pi^2} \left(-\sum_i c''_i g_i^2 + 4\lambda_\tau^2 + 3\lambda_b^2 \right) \lambda_\tau , \tag{19}$$

where $c_i = (\frac{13}{15}, 3, \frac{16}{3})$, $c'_i = (\frac{7}{15}, 3, \frac{16}{3})$, and $c''_i = (\frac{9}{5}, 3, 0)$.

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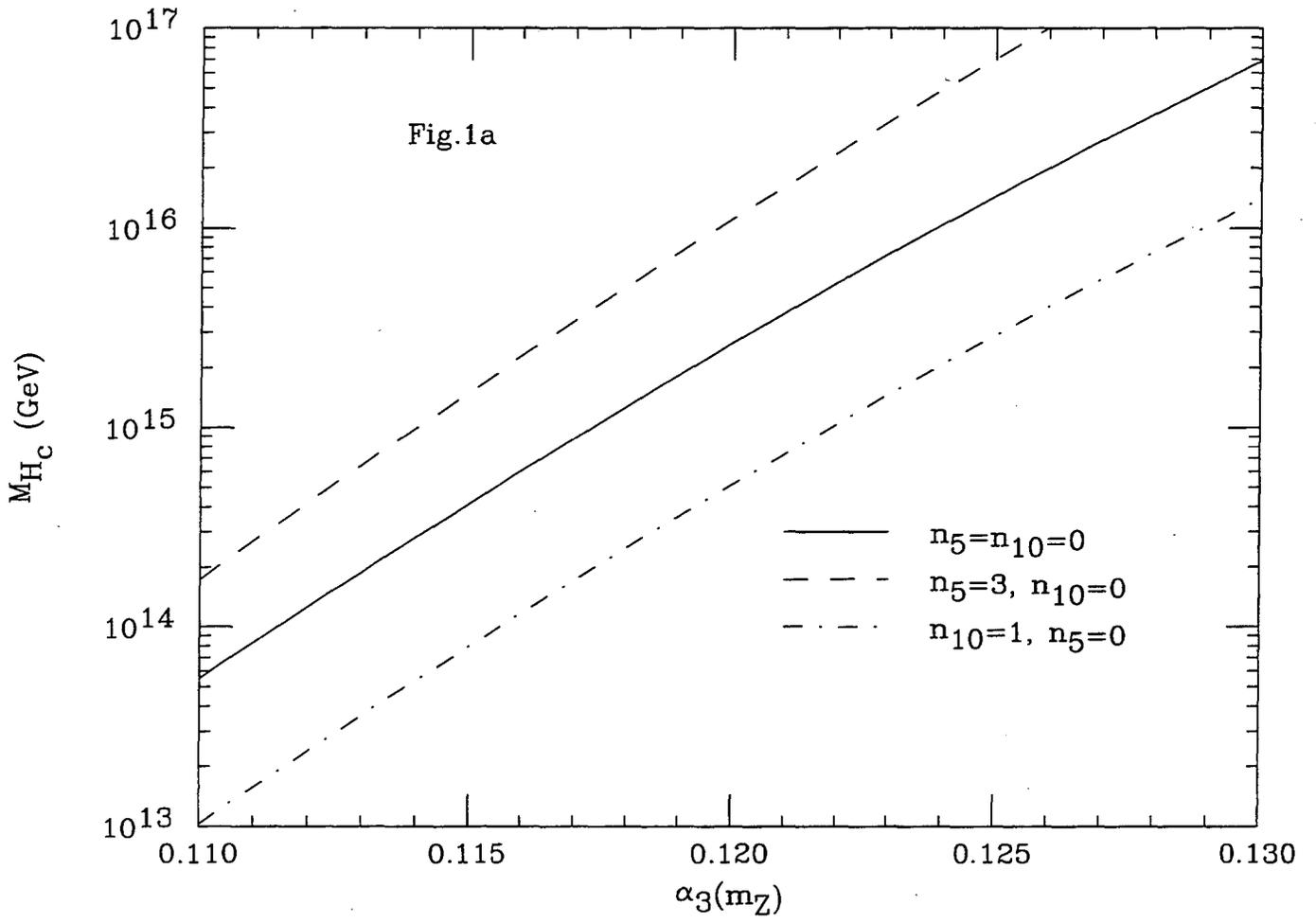
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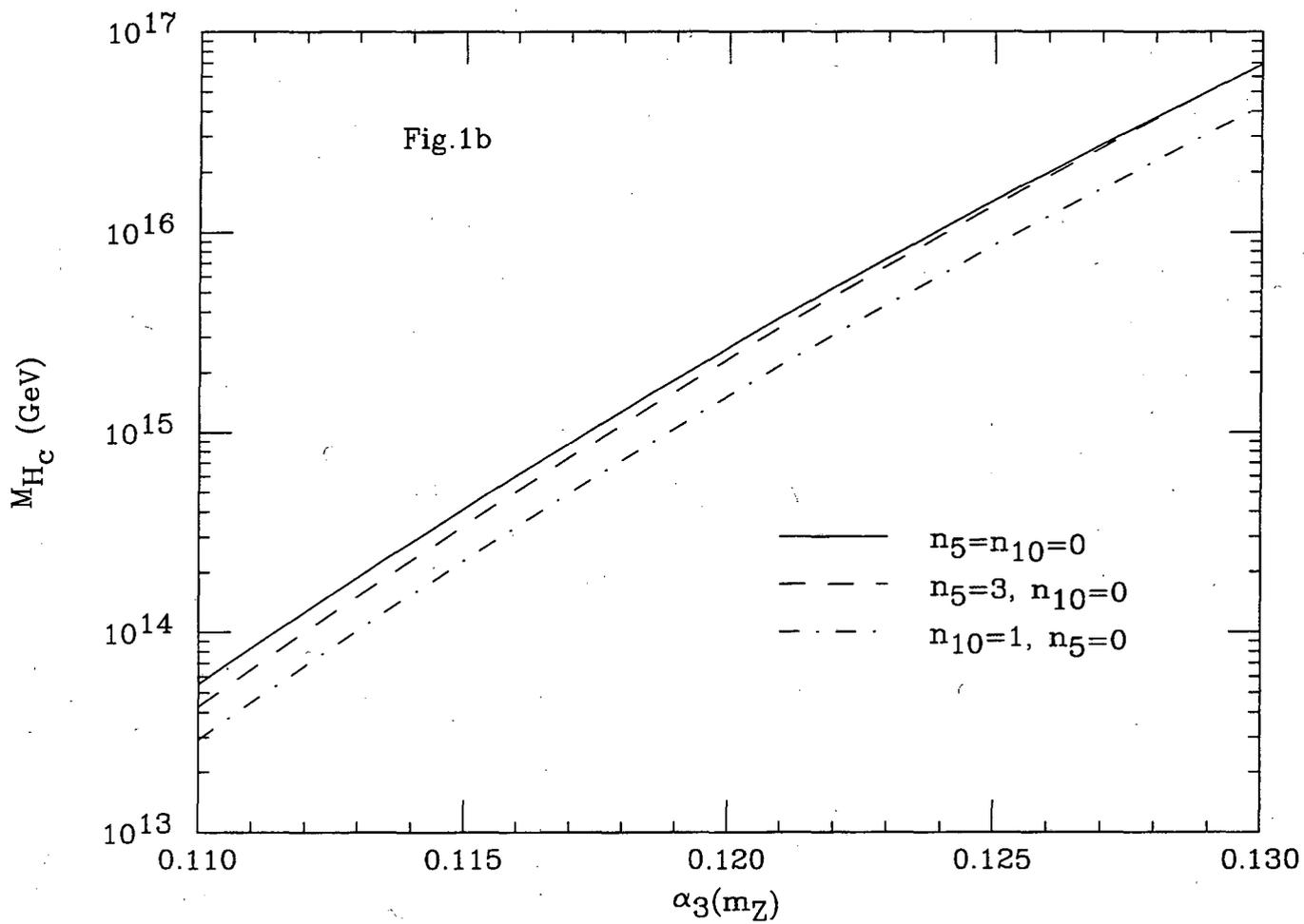
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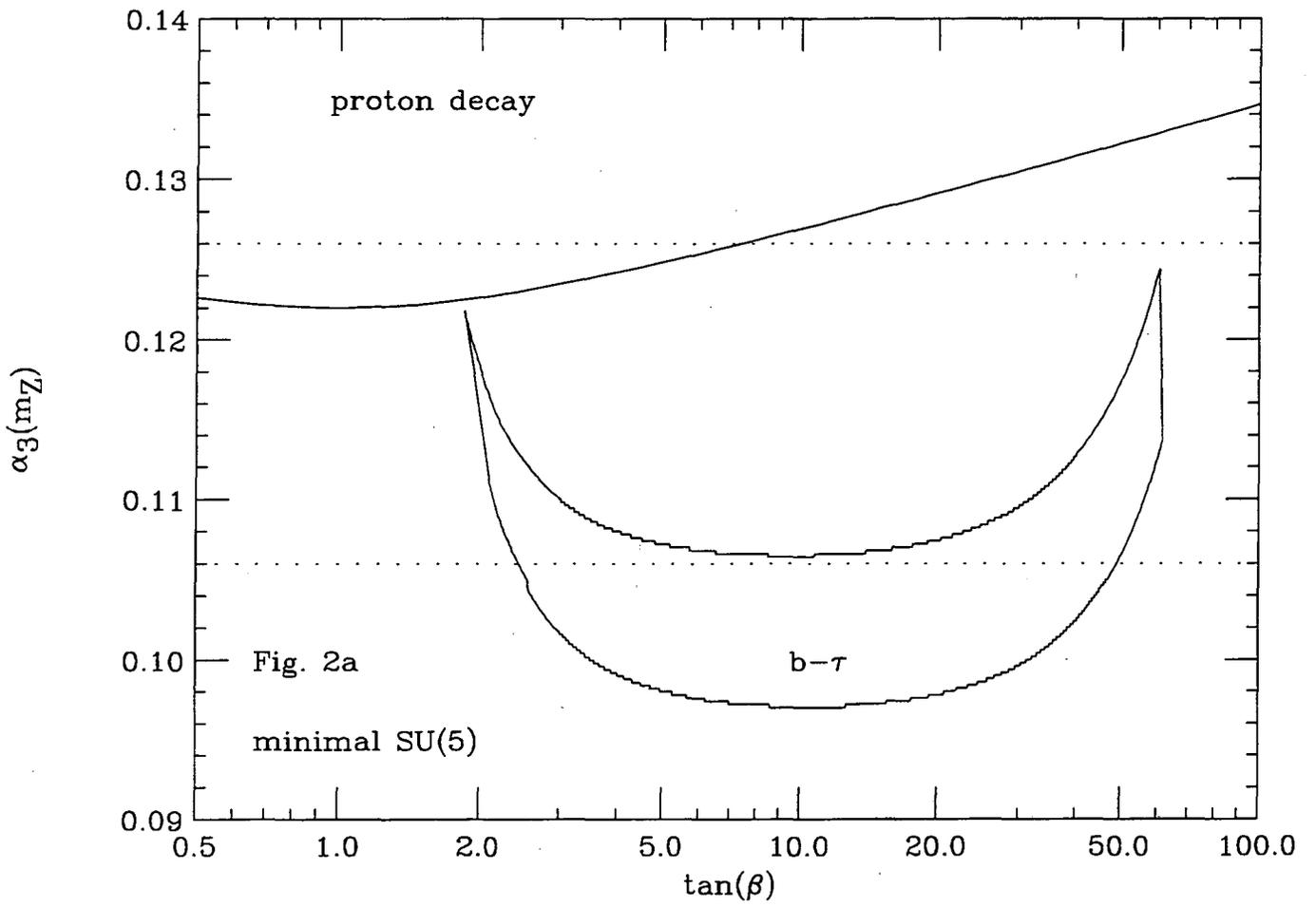
Figure Captions

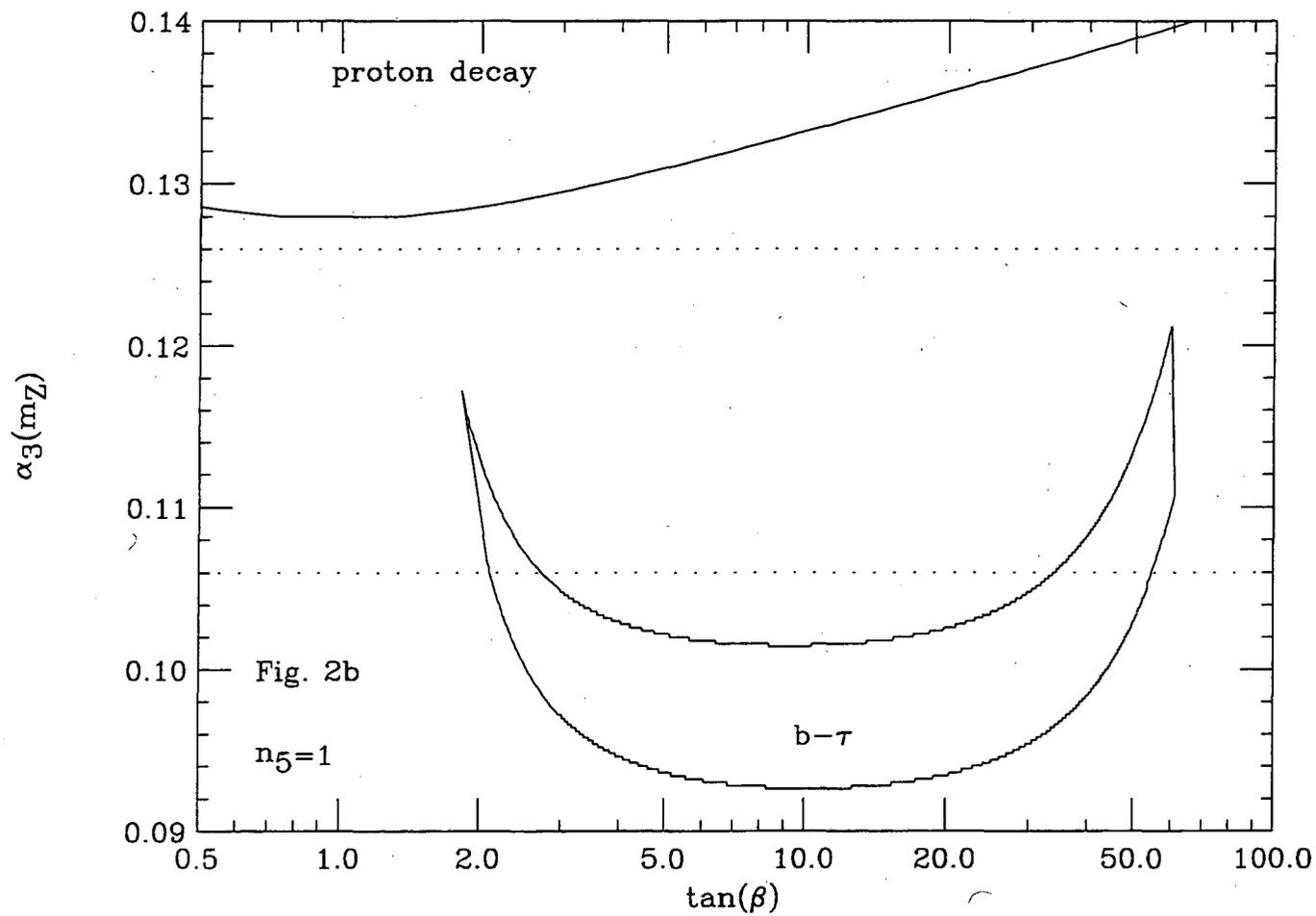
Fig. 1 (a) Upper bound on the color-triplet Higgs mass M_{H_C} as a function of $\alpha_3(m_Z)$, ignoring threshold corrections (i.e. mass splittings) at the messenger scale. The solid line is the minimal SU(5) result, the dashed line is the case of three $5+\bar{5}$ pairs in the messenger sector, and the dotted line is the case of one $10+\bar{10}$ pair. (b) The complete result, including threshold corrections at the messenger scale.

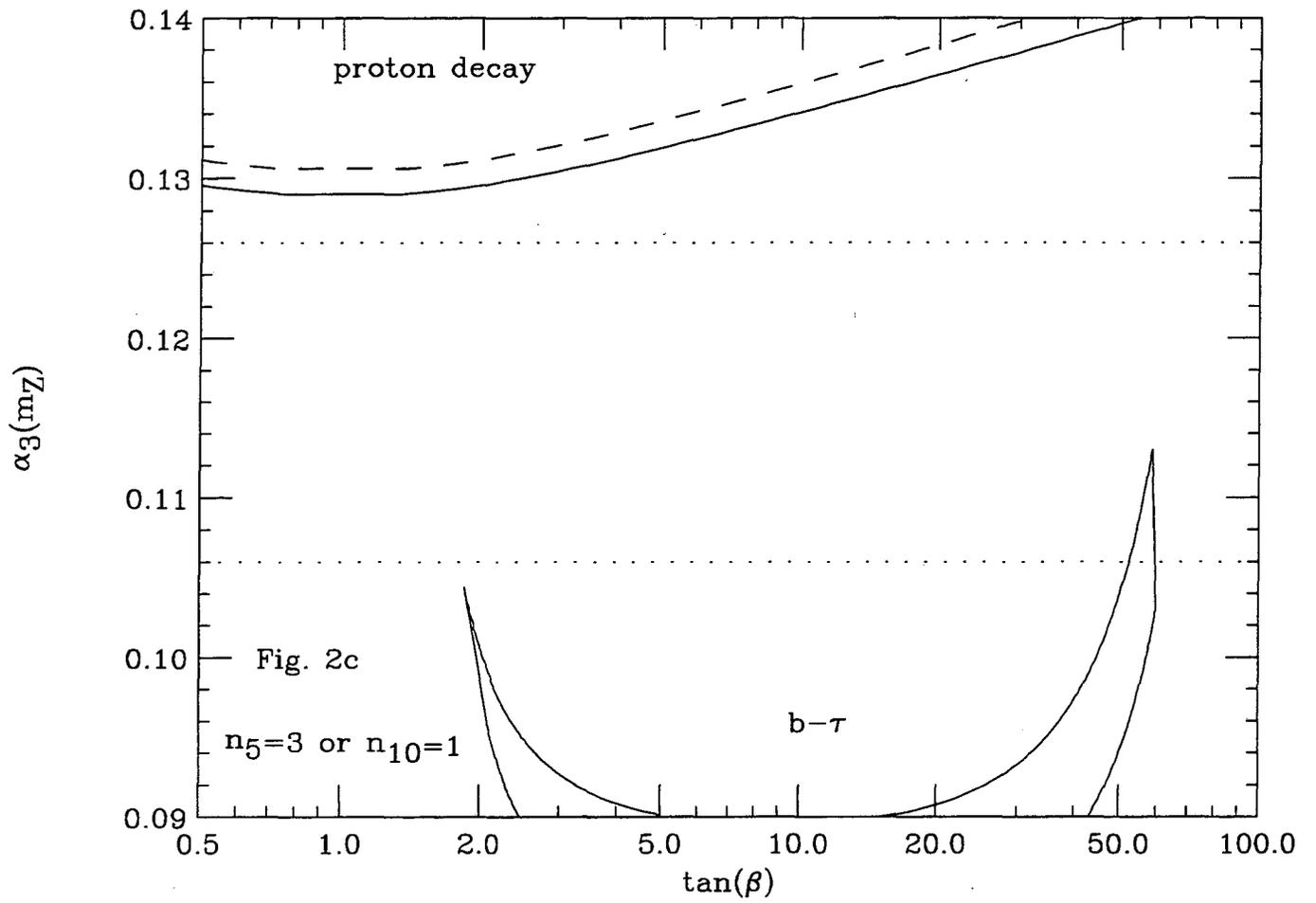
Fig. 2(a) Preferred regions of the $\tan(\beta)$ - $\alpha_3(m_Z)$ plane, in minimal SU(5) unification, for $m_{top} = 176$ GeV. The region above the proton-decay line is allowed by the lower bound on the proton lifetime (see the text) at 90% confidence level, while the region within the crescent shaped contour is preferred by the constraint of b - τ Yukawa unification with $m_b(m_b) = 4.1$ - 4.5 GeV. The horizontal band shows the experimentally allowed range of $\alpha_3(m_Z) = 0.116 \pm 0.005$ at two standard deviations. (b) Same as (a) for $n_5 = 1$, (c) Same as (a) for $n_5 = 3$. The dashed line shows the result for $n_{10} = 1$ and $n_5 = 0$; the b - τ region remains the same for $n_5 = 3$ or $n_{10} = 1$.











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