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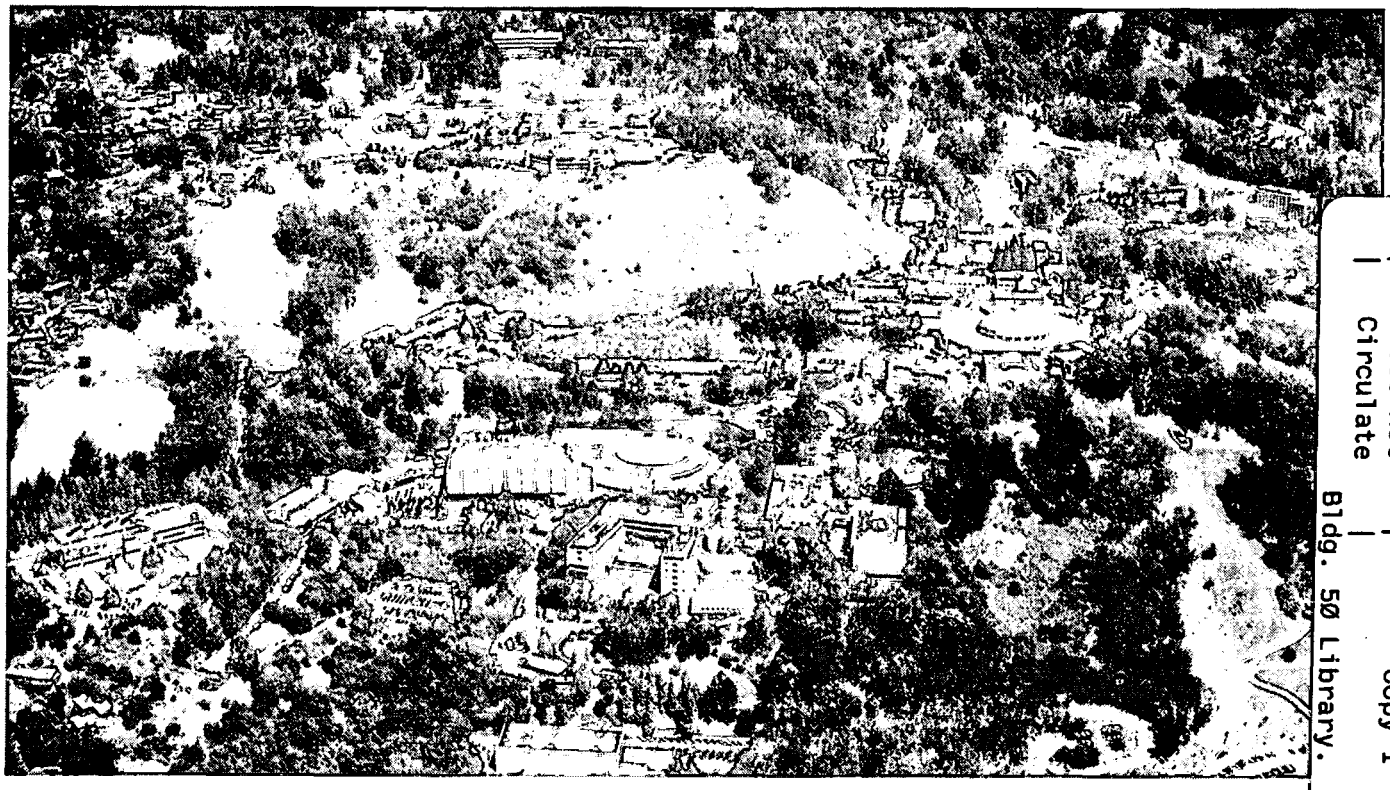

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Applications Of Classical Standpoint Cosmology*†

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Abstract

A single-parameter radical alternative to the standard cosmological model is explored; experimental tests are proposed. Among predictions of the new model are energy-density parameter $\Omega = (\frac{3}{1+\sqrt{2}})^2$, with vacuum energy comparable to matter energy in early as well as late universe. Predicted also are relations between Hubble time, standpoint age, redshift, luminosity distance and angular-diameter distance that are close to those of the standard model for zero deceleration parameter q_0. The phenomenology of early "hot" universe is qualitatively addressed.

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I. Introduction

Classical standpoint cosmology,[1] despite superficial phenomenological similarity to the "standard" cosmology of Friedmann-Robertson-Walker,[2] differs profoundly in principle. Classical standpoint cosmology is closer to Milne's "kinematic cosmology", [3] but a standpoint spacetime is compact with corresponding "curvature". Each standpoint, furthermore, has a *separate* spacetime. These separate spacetimes, metricized only in an approximate sense, relate to each other through Minkowskian approximation inside certain regions that are "small". The notion of a *single* Riemannian spacetime that underlies general relativity is seen as an approximation dependent on largeness of Hubble length. An important example of an approximately-Minkowskian region is the neighborhood of a standpoint's backward light cone - - a cone that extends back all the way to "big bang".

The Hubble-scale gravitational action of a standpoint spacetime, corresponding to "homogeneous universe", is completely specified by a single standpoint parameter of dimension length (or time, with $c = 1$) - - a parameter that for standpoint i we denote by the symbol R_i and call the "standpoint-age parameter". (The symbol R_i as used here is not to be confused with the scale parameter of standard cosmology.) In the new model the property of *age* belongs not to the entire universe as in the standard model but rather to a standpoint. It will turn out that standpoint age is $(\frac{1}{2} + \frac{1}{\sqrt{2}})^{-1} R_i$. It will also turn out that standpoint age is equal to Hubble time defined in the usual way through redshift as a function of distance.

Standard-model successes (greater than those of kinematic cosmology where there is no curvature) must eventually not only be matched but exceeded by the new model if the latter is to survive. The present paper, after reviewing conceptual novelties of standpoint cosmology, displays explicitly in standpoint-based coordinate systems not only the homogeneous-universe gravitational action and Riemannian approximations thereto but the associated radial "geodesics". These latter control relative standpoint motion. Application is then made (a) to the relation between standpoint age and redshift, (b) to energy density and pressure at standpoint in terms of Hubble and gravitational constants, (c) to relation between Hubble time and standpoint age, and (d) to luminosity distance and angular-diameter distance as function of redshift. Some of the foregoing relations

turn out the same as those of kinematic cosmology. Strikingly novel is a substantial "vacuum" component of energy density and pressure near standpoints of *all* ages.

A detailed thermodynamic approximation for use near "early" standpoints remains to be developed. Nevertheless because energy density near standpoint is of order $(GR_i^2)^{-1}$, particle mean free path becomes small compared to R_i near very young standpoints and a standpoint temperature can be defined that decreases as age advances. Age-temperature correlation dovetails with a redshift controlled entirely by ratios of standpoint ages. There is corresponding understanding of thermal cosmic background radiation and encouraging prospect for nucleosynthesis within standpoint cosmology.

The present paper does not address "inhomogeneities" - - local fluctuations of energy density on length scales small compared to R_i. We expect no difference from standard theory in describing inhomogeneities so long as errors of order $(d/R_i)^4$ are tolerable, d being the inhomogeneity scale.

II. Conceptual Novelties

The new model is economical. There is no scale parameter depending in a complicated way on universal time and no cosmological constant. Energy density in units of the standard "critical density" turns out for all ages to be $\Omega = (\frac{3}{1+\sqrt{2}})^2 \approx 1.54$. (To the extent that the present universe is "cold", $\Omega^{\text{vacuum}} = \frac{1}{3}\Omega$ and $\Omega^{\text{matter}} = \frac{2}{3}\Omega \approx 1.03$.) Paucity of parameters places the new model in immediate jeopardy of experimental falsification.

Despite the prevailing physics paradigm of covariance within a *unique* unbounded spacetime, the new model attributes to *each* standpoint a separate compact spacetime endowed with a special set of coordinates. This coordinate system is suitable for describing experiments carried out near that standpoint in the standpoint rest frame. On the scale of R_i (Hubble scale), "homogeneous universe" presents the same appearance from any standpoint when described in the coordinates belonging to that standpoint. Only a *portion* of one compact 4-dimensional standpoint spacetime generally can map onto another such spacetime. It will nevertheless be shown that familiar Poincaré symmetry (of a unique spacetime) prevails (approximately) within standpoint neighborhoods that are small on Hubble scale. The objectivity characteristic of "hard science" is encompassed by the available approximate mappings.

The separate compact spacetimes are partially tied together by invariant *radial* metric combined with common "origin" of coordinate systems in a 2-dimensional (1, 1) sense. The common "origin" associates with "big bang". A "newly-born" standpoint originates in big bang and moves "outward" in any "old-standpoint" coordinate system along a well-defined positive-timelike radial trajectory. The age of a standpoint is its invariant "distance" from big bang. Each standpoint trajectory being specified by "initial velocity" (near big bang), any standpoint is labelable by its age plus its initial velocity in the spacetime of some (other) reference standpoint.

Standpoint-spacetime is generally without metric, being defined by particle*

*Meaning for the term "particle" in this paper is classical, merely implying localization of (positive) energy within some region of 3-space. We require particle "size" to be small compared to distance from standpoint but otherwise put no upper limit on "particle diameter" and make no requirement as to "structure". A galaxy or even a cluster of galaxies may be considered a "particle".

trajectories but becomes approximately Riemannian in restricted regions of "phase-space" - - i.e., for certain special combinations of particle location and velocity. Close to standpoint in 3-space, any velocity is compatible with the Riemannian approximation; far from standpoint, Riemannian displacements must be nearly radial. "Close" means distance small compared with R_i while "far" means distance of order R_i. Close to standpoint the Riemannian approximation is sufficiently accurate that the (tiny) homogenous-universe curvature of order R_i^{-2} is physically meaningful via Einstein theory. From this curvature, mean energy density and pressure near standpoint may be inferred.

"Standpoint" represents separation between past and future - - i.e., the "present". Gravitational action for the past is different from that for the future when Hubble-scale times are considered. Only for time displacements from the present that are short compared to R_i is there approximate equivalence. Asymmetry between past and future is dramatically manifested by an impassible future boundary - - called "abyss".[1] Prediction of future based on present measurements - - i.e., measurements made near standpoint - - cannot extend beyond this boundary. The abyss limitation correlates with particle trajectories and may be regarded in the spirit of "Schwartzschild radius" of order R_i accompanying a mass of order $\rho_i R_i^3$, where the energy density ρ_i has order of magnitude $(GR_i^2)^{-1}$.

The only region within a standpoint spacetime accessible to *measurement* is the neighborhood of the standpoint's backward light cone. The remainder of a standpoint spacetime facilitates *prediction* of results from (future) measurements to be carried out near older standpoints in the same spatial neighborhood and verification of prediction based on earlier measurements made near younger standpoints. Essential to the integrity of standpoint cosmology's emphasis on measurement correlations is the "stability" of lightlike radial trajectories: a lightlike radial trajectory in one standpoint spacetime maps onto a lightlike trajectory within any other whose standpoint lies along the trajectory. Classical-measurement correlation dovetails with *S-matrix* interpretation of *quantum* standpoint cosmology.[1]

Although the present paper ignores the quantum underpinning of classical standpoint cosmology, here defining the model by gravitational action in a compact standpoint spacetime, the action displayed in Section IV was uniquely

inferred in Reference (1) from symmetry properties of a more fundamental quantum model of expanding universe. Only for standpoints whose R_i greatly exceeds $R_{min} \sim 10cm$ (age large in nanoseconds) does 3-space achieve classical significance. Quantum-model meaning for "location" within a standpoint spacetime arises in conjunction with quantum-mechanical meaning for "particles". In a region of the universe "dense" in the sense that $R_i << R_{min}$, neither quantum particles nor 3-space enjoy model meaning. According to the quantum model, "diluteness" in the sense $R_i >> R_{min}$ is essential to classical significance for 3 space.

We shall see that Milne's 1935 kinematic cosmology connects to standpoints of *infinite* age and "radially-flat" spacetime, which have past but no present and no future. It sometimes turns out calculationally convenient to invoke infinite age, but *physical* spacetime belongs to a *present* where the surrounding spacetime (in homogeneous approximation) has "curvature" of order R_i^{-2}.

Characteristic of standpoint cosmology are *two* distinct meanings for "time". There is standpoint age - - a global noncompact concept - - and there is the local time of particles within the $1 + 3$ dimensional compact spacetime attached to standpoint. Although these two times interrelate classically through the notion of trajectory for a standpoint in the spacetime of *another* standpoint, the underlying (quantum) Hilbert space achieves time doubling through a *complex* conformal group whose classical representations are 8-dimensional rather than 4-dimensional.[1] Symptomatic of this Hilbert-space doubling is a classical-particle action whose dependence on velocity is *quartic* rather than the usual (Riemannian) quadratic. Corollary is absence of complete mapping from one standpoint spacetime to another. In standpoint cosmology, objectivity is only approximate - - depending for validity on hugeness of Hubble length.

III. Special Particle Coordinates in Standpoint Spacetime

The spacetime belonging to a standpoint is compact, with boundary and well-defined center; there is an accompanying natural system of particle coordinates. The standpoint may be associated with a particle located at the center of this spacetime where it is "at rest". In such a sense, a standpoint described in a spacetime belonging to some *other* standpoint is displaced from center and generally is in motion. Any (compact) standpoint spacetime may be described as the intersection of interiors of forward and backward light cones whose vertices share the standpoint's spatial location while locating in time an interval R_i from the standpoint, one vertex in the standpoint's past and the other in its future. The 3-dimensional forward-light-cone boundary we shall associate with big bang - - an association shared with Milne's kinematic cosmology.[3]

Using the index i to designate a standpoint, and the 4 symbols $(t_i, \vec{r}_i)$ for the special attached-spacetime coordinates of a particle, restriction to the double-cone interior amounts to particle coordinates being constrained to the interval

$$0 \leq t_i \pm |\vec{r}_i| \leq 2R_i. \qquad (III.1)$$

A particle coincident with the i standpoint locates at $t_i = R_i, \vec{r}_i = 0$, i.e. at the double-cone center. It will be seen in Section V that standpoint-spacetime particle trajectories curve in conformity to (III.1) - - matter inside the double cone being unable to cross the boundary.

For many purposes, "polar" particle coordinates - - a set of 4 angles $(\varphi_0, \vec{\varphi})$ - - are appropriate for standpoint spacetime. Although these angles refer to a standpoint, in the interest of notational simplicity we shall omit the standpoint index on angles. The angular range is standpoint independent, in contrast to the range of $t_i, \vec{r}_i$. Defining a unit 3-vector $\vec{n}$ by

$$\vec{n} \equiv \frac{\vec{\varphi}}{|\vec{\varphi}|} = \frac{\vec{r}_i}{|\vec{r}_i|}, \qquad (III.2)$$

the connection between $(\varphi_0, \vec{\varphi})$ and $(t_i, \vec{r}_i)$ is

$$\begin{aligned} t_i &= R_i(1 + \sin\varphi_0 \cos\varphi), \\ \vec{r}_i &= R_i \vec{n} \sin\varphi \cos\varphi_0. \end{aligned} \qquad (III.3)$$

where $\varphi \equiv |\vec{\varphi}|$. From (III.3) one may deduce

$$\frac{t_i \pm r_i}{R_i} = 1 + \sin\varphi_{\pm}, \qquad (III.4)$$

where $r_i \equiv |\vec{r}_i|, \varphi_{\pm} \equiv \varphi_0 \pm \varphi$, and the constraint (III.1) then translates into

$$-\frac{\pi}{2} \leq \varphi_{\pm} \leq \frac{\pi}{2}. \qquad (III.5)$$

Because non-gravitational forces are ignored, the equivalence principle for homogeneous universe means *any* particle trajectory is completely determined by spatial location and velocity at a selected time. There is no dependence on particle mass; gravitational *action* may be prescribed in a form common to all (single) particles. Conversely, description of homogeneous universe is achieved by action specification for an arbitrary particle.

IV. Standpoint-Spacetime Gravitational Action

From the symmetry of underlying quantum dynamics Reference (1) has induced, as homogeneous-universe standpoint-spacetime (single-particle) gravitational action, the rotationally-symmetric mass-independent† form

$$ds_i = \frac{1}{\sqrt{2}}\left\{[G(dt_i^2 - dr_i^2) - gr_i^2 d\vec{n}^2]^2 + [G'(dt_i^2 - dr_i^2) - g'r_i^2 d\vec{n}^2]^2\right\}^{1/4}, \quad (IV.1)$$

with the four coefficients G, G', g, g' being symmetric trigonometric functions of φ_+ and φ_-.‡ Firstly,

$$G(\varphi_\pm) \equiv h_+ f_- + f_+ h_- \text{ and } G'(\varphi_\pm) = h_+ h_- - f_+ f_-, \quad (IV.2)$$

the angular functions $h_\pm$ and $f_\pm$ being

$$h_\pm = 1 + 2\sin\varphi_\pm, \; f_\pm = \frac{(1 - 2\sin\varphi_\pm)(1 + \sin\varphi_\pm)}{\cos\varphi_\pm}, \quad (IV.3)$$

while secondly,

$$g(\varphi_\pm) = 2uv \text{ and } g'(\varphi_\pm) = u^2 - v^2, \quad (IV.4)$$

the symmetric angular functions u and v being

$$u = 1 + \sin\varphi_+ + \sin\varphi_-, \; v = \frac{(1 + \sin\varphi_+)\cos\varphi_+ - (1 + \sin\varphi_-)\cos\varphi_-}{\sin\varphi_+ - \sin\varphi_-}. \quad (IV.5)$$

The fourth-root notation of (IV.1) needs elaboration. It is understood that ds_i is real and positive for low velocities, i.e., for $(dr_i/dt_i)^2 << 1$ and $r_i^2 d\vec{n}^2 << 1$. In a separate paper[6] singularities in velocity space are discussed and "physical" regions of phase space unambiguously distinguished from "unphysical" even though the distinction cannot be based simply on the sign of ds_i^2 as in general relativity.

Important to physical interpretation of (IV.1) is the spatial neighborhood of the origin at $\vec{r}_i = 0$, i.e., the neighborhood of $\vec{\varphi} = 0(\varphi_+ = \varphi_-)$ - - a point in

†Mass independence brings an unconventional dimensionality to the action (IV.1). Adding particle rest mass as an overall factor in (IV.1) yields usual action dimensionality without affecting trajectories. Our omission of mass factor not only emphasizes the equivalence principle but assists recognition of relation through "geometry" to general relativity, in certain regions of (7-dimensional) particle velocity-coordinate space.

‡The angular function $G(\varphi_\pm)$, appearing only in the present section, is not to be confused with the gravitational constant that appears elsewhere.

3-space where $g = G$ and $g' = G'$ ($h_+ = h_- = u, f_+ = f_- = v$) - - so that (IV.1) collapses to the "almost-Minkowskian" form

$$ds_i^2 = \frac{1}{2}(G^2 + G'^2)^{1/2}(dt_i^2 - d\vec{r}_i^2), \qquad (IV.6)$$

with

$$d\vec{r}_i^2 \equiv dr_i^2 + r_i^2 d\vec{n}^2 = dx_i^2 + dy_i^2 + dz_i^2. \qquad (IV.7)$$

In this *spatial* neighborhood of the standpoint we choose the *positive* value of the coefficient in (IV.6). Precisely, we take

$$(ds_i^2)_{\varphi=0} = \frac{dt_i^2 - d\vec{r}_i^2}{1 - \sin\varphi_0}, \qquad (IV.8)$$

consistent with the choices (for arbitrary φ)

$$\frac{1}{2}(G^2 + G'^2)^{1/2} = \frac{(h_+^2 + f_+^2)(h_-^2 + f_-^2)}{2} = \frac{1}{\cos\varphi - \sin\varphi_0}, \qquad (IV.9)$$

$$\frac{1}{2}(g^2 + g'^2)^{1/2} = \frac{u^2 + v^2}{2} = \frac{1 + \cos^2\varphi + 2\sin\varphi_0\cos\varphi}{2\cos^2\varphi_0}. \qquad (IV.10)$$

It may be checked that at $\varphi = 0$,

$$\begin{aligned} G &= g = 2(1 - 4\sin^2\varphi_0)\left(\frac{1 + \sin\varphi_0}{1 - \sin\varphi_0}\right)^{1/2} \\ G' &= g' = \frac{2\sin\varphi_0(3 - 4\sin^2\varphi_0)}{1 - \sin\varphi_0}, \end{aligned} \qquad (IV.11)$$

so at standpoint (where $\varphi = \varphi_0 = 0$, or $\varphi_\pm = 0$), the four coefficients become $G = g = 2, G' = g' = 0$. Here, maximally distant from boundary singularities of the action, a Riemannian approximation is optimally accurate.

Inclusion of standpoint-(spatial) neighborhood corrections up to order φ^2 is achieved by the Riemannian approximation to (IV.1) given by

$$ds_i^2 \approx \frac{1}{2}(G^2 + G'^2)^{1/2}(dt_i^2 - dr_i^2) - \frac{1}{2}(g^2 + g'^2)^{1/2} r_i^2 d\vec{n}^2 \qquad (IV.12)$$

$$= \frac{dt_i^2 - d\vec{r}_i^2}{\cos\varphi - \sin\varphi_0} + \varepsilon r_i^2 d\vec{n}^2, \qquad (IV.13)$$

where the "small" function

$$\varepsilon \equiv \frac{1}{2}(G^2 + G'^2)^{1/2} - \frac{1}{2}(g^2 + g'^2)^{1/2}$$
$$= \frac{1 - \cos\varphi}{1 - \sin\varphi_0}\left[1 + \frac{1}{\cos\varphi - \sin\varphi_0} - \frac{1}{2}\frac{1 - \cos\varphi}{1 + \sin\varphi_0}\right]. \qquad (IV.14)$$

is of order φ^2. Non-Riemannian corrections to (IV.13) are of order φ^4. The approximation (IV.13) facilitates "usual physics" (general relativity) in standpoint neighborhood - - i.e., over a spatial region small on Hubble scale ($\varphi \approx \frac{r_i}{R_i} << 1$) and within a time region distant from the spacetime boundary at $\varphi_0 \pm \varphi = \pm\pi/2$ where ε becomes large - - in particular, for time t_i close neither to zero nor to $2R_i$. From (IV.13) Section VI will calculate (Riemannian) spacetime curvature and energy density at standpoint.

A differently-based Riemannian approximation is applicable far from standpoint for "almost-radial" displacements - - where $|d\vec{n}|$ is small even though ε is not.[6] The following section deals with "exactly-radial" displacements and Section VIII with almost-radial displacements. Reference (6) relates "almost-radial" motion to "cold matter".

Despite preclusion by the non-Riemannian action (IV.1) of certain notions from general relativity, standpoint cosmology maintains the equivalence principle; gravitational mass is indistinguishable from inertial mass everywhere within a standpoint spacetime. Reference (6) gives a general procedure for finding non-Riemannian particle trajectories of homogeneous universe.

V. Radial "Geodesics"

The first of the special Riemannian situations to be contemplated here is that of *radial* displacement. Because the motion of any standpoint within the spacetime of another standpoint is radial, we shall thereby achieve understanding of relative standpoint motion; relative motion will be found to resemble that of Milne's kinematic cosmology.[3] Pursuing the dynamics of radial displacement through (IV.1) or (IV.12) with $d\vec{n} = 0$, i.e.

$$ds^2_{rad} = \frac{(dt_i^2 - dr_i^2)}{\cos\varphi - \sin\varphi_0}, \qquad (V.1)$$

the coefficient in (V.1)[§] may be reexpressed through the identity

$$\cos\varphi - \sin\varphi_0 = 2\{(1 - t_i/2R_i)^2 - (r_i/2R_i)^2\}^{1/2}. \qquad (V.2)$$

Absence of standpoint index on the ds^2_{rad} of (V.1) reflects our physics-facilitating postulate that, wherever (and to the extent that) Minkowskian approximation controls mapping between different standpoint spacetimes, "distance" along a geodesic between corresponding points is invariant under change of standpoint. Consistency of this postulate will be verified. Because relative displacement of two different standpoints admits Minkowskian description through (V.1), we shall deduce the *trajectory* along which some selected standpoint moves - - a trajectory mappable by the prescription of Appendix B between any of those standpoint spacetimes whose spatial origins are displaced from trajectory location along a *single* direction $\vec{n}$. For *infinite-R_i* spacetimes, such $1+1$ dimensional standpoint trajectories will turn out to be straight lines, as in Milne cosmology.

Notice that the radial metric (V.1) is, according to (V.2), singular along the backward-light-cone (future) spacetime boundary where $r_i^2 = (2R_i - t_i)^2$. This singularity, present also in the general action, prevents any matter from penetrating the future boundary- -which has been called "abyss".[1] The metric (V.1) implies the radial equation of motion

$$\frac{d^2 r_i}{dt_i^2} = -\frac{1}{2}\left[1 - \left(\frac{dr_i}{dt_i}\right)^2\right] \frac{r_i + (2R_i - t_i)\frac{dr_i}{dt_i}}{(2R_i - t_i)^2 - r_i^2}, \qquad (V.3)$$

[§]Nonvanishing time derivative at standpoint of this coefficient is manifested in (V.3) by "cooling" of matter in standpoint neighborhood - - a loss of kinetic energy that might be described as "cosmological friction" or "gravitational redshift".

for which explicit solutions will below be presented. Among these will be "standpoint trajectories"- -more precisely, the trajectories of matter "at rest" with respect to that standpoint which shares the matter's location. Also importantly described by (V.3) in the spacetime of an "observer" standpoint is the trajectory of a "signal", emitted from some source, that travels to the observer.

Solution of Equation (V.3) is conveniently achieved through change of variables from t_i, r_i to the alternative pair τ_i, ρ_i, $0 \leq \tau_i \pm \rho_i \leq 4R_i$, according to

$$1 - \frac{\tau_i \pm \rho_i}{4R_i} = (1 - \frac{t_i \pm r_i}{2R_i})^{\frac{1}{2}}. \qquad (V.4)$$

(Appendix B connects τ_i, ρ_i to "infinite-age" variables whose range is 0 to ∞.) In the alternative variables the "radial metric" is the $1+1$ dimensional Minkowski form

$$ds^2 = \frac{1}{2}(d\tau_i^2 - d\rho_i^2), \qquad (V.5)$$

and the equation of motion becomes

$$\frac{d^2\rho_i}{d\tau_i^2} = 0. \qquad (V.6)$$

Geodesics (i.e., trajectories) are straight lines,

$$\rho_i = \alpha + \beta\tau_i, \qquad (V.7)$$

that translate into 2-parameter (α, β) standpoint-spacetime radial trajectories through the inverse of (V.4):

$$\begin{aligned} t_i &= \tau_i - \frac{\tau_i^2 + \rho_i^2}{8R_i} \\ r_i &= \rho_i(1 - \tau_i/4R_i). \end{aligned} \qquad (V.8)$$

Notice that in the limit $R_i \to \infty, t_i \to \tau_i$ and $r_i \to \rho_i$. Nonradial trajectories are not yet generally understood, although Reference (6) provides a foundation.

A consistent postulate is that, in a *selected* standpoint spacetime, any *other* standpoint lies on a radial trajectory whose parameter α vanishes and whose β lies in the range $0 \leq \beta \leq 1$. The parameter $\beta_{ij} = \beta_{ji}$ may be described as magnitude of *initial* j-standpoint velocity in i-spacetime (or, according to Appendix B, magnitude of i-standpoint initial velocity in j spacetime). Invariant

distance of j standpoint from (big bang) origin is $s_j = \frac{1}{\sqrt{2}}(\tau_i^2 - \rho_i^2)^{\frac{1}{2}} = \frac{1}{\sqrt{2}}\tau_i(1 - \beta_{ij}^2)^{\frac{1}{2}}$ and, defining $\vec{\beta}_{ij} = \vec{n}_{ij}\beta_{ij}$ with $\vec{n}_{ij} = -\vec{n}_{ji}$ and with $\vec{\rho}_i = \vec{n}_{ij}\rho_i$ so $\vec{\rho}_i = \vec{\beta}_{ij}\tau_i$, the 4 parameters $(s_j, \vec{\beta}_{ij})$ locate the j standpoint in i-standpoint spacetime. Allowing the full range of directions $\vec{n}_{ij}$ together with $0 \leq \beta_{ij} \leq 1$ and the s_j interval $0 \leq \sqrt{2}s_j \leq 4R_i e^{-\Delta_{ij}}$, where the symbol

$$\Delta_{ij} \equiv \ln\left(\frac{1+\beta_{ij}}{1-\beta_{ij}}\right)^{\frac{1}{2}} \qquad (V.9)$$

may be described as magnitude of *initial* standpoint rapidity ($0 \leq \Delta_{ij} \leq \infty$), reveals the chosen-standpoint spacetime to be completely and non-redundantly spanned by (other) standpoint trajectories.

Standpoint invariance for the radial increment of distance ds_{rad} means that, in contrast to the parameter $\vec{\beta}_{ij}$, the parameter s_j of standpoint j is independent of the spacetime in which this standpoint is described. One may deduce from (V.4) evaluated at $t_i = R_i, r_i = 0$, that a standpoint of age-parameter R_i is at a "distance from big bang" given by

$$s_i = \frac{1}{\sqrt{2}}4R_i\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{R_i}{\frac{1}{\sqrt{2}} + \frac{1}{2}}. \qquad (V.10)$$

What time is registered by an observer who starts clock close to the origin of any "old-standpoint" spacetime and moves along a standpoint trajectory labeled by a continuously-varying R_i. The clock adds up time increments dt_i in a succession of *different* coordinate systems as R_i increases but, according to (IV.1) with $\varphi_+ = \varphi_- = 0$, the relation

$$dt_i = ds \qquad (V.11)$$

prevails continuously along the trajectory. It follows that standpoint age is its invariant distance (V.10) from origin, a result that justifies *a posteriori* calling R_i the "standpoint age parameter". Later R_i will be seen to determine spacetime curvature at standpoint. Phenomenologically, what we are calling 'standpoint age" is the quantity commonly called "age of universe". The latter terminology, which fits the standard model, seems inappropriate here and we shall avoid it.

Radial trajectories of the form (V.7) with $\alpha \neq 0$- -more general than those followed by standpoints- -will be of interest. Differentiating Formulas (V.8) and

taking the quotient leads to the following expression for matter velocity as it varies along a general radial geodesic of standpoint spacetime:

$$v_i \equiv \frac{dr_i}{dt_i} = \frac{\beta(1-\tau_i/4R_i) - \rho_i/4R_i}{(1-\tau_i/4R_i) - \beta\rho_i/4R_i}. \qquad (V.12)$$

Formula (IV.12) will be applied in Appendix A for $\beta = -1$ to gravitational redshift. Generally the parameter, $\beta \equiv \frac{d\rho_i}{d\tau_i}$, with fixed value along the radial geodesic, ranges between -1 and $+1$. Notice from (V.12) that in each of the two limits $\beta \to \pm 1, v_i$ approaches the same limit as β and that, for any allowed $\beta, |v_i| \leq 1$. It may also be verified that if $|\beta| < 1$ then, along the abyss boundary where $\tau_i + \rho_i = 4R_i$, one *always* finds $v_i = -1$- -i.e., *inward* matter motion *parallel* to boundary as required by confinement to compact standpoint spacetime.

We conclude this section with explicit formulas showing how the 4 parameters $(R_j, \vec{\Delta}_{ij})$ locate standpoint j in i spacetime. Remembering the connection (IV.10) between R_j and s_j, the j standpoint locates at

$$\begin{aligned} \tau_i &= \sqrt{2}s_j \cosh \Delta_{ij}, \\ \vec{\rho}_i &= \sqrt{2}s_j\vec{n}_{ij} \sinh \Delta_{ij}, \end{aligned} \qquad (V.13)$$

or, alternatively, at $t_i, \vec{r}_i$ through (V.8). Relation of s_j, Δ_{ij} to angle variables is

$$\begin{aligned} s_j &= 4\sqrt{2}R_i \sin\left(\frac{\varphi_+}{4} + \frac{\pi}{8}\right) \sin\left(\frac{\varphi_-}{4} + \frac{\pi}{8}\right), \\ &= 2\sqrt{2}R_i \left[\cos\frac{\varphi}{2} - \cos\left(\frac{\varphi_0}{2} + \frac{\pi}{4}\right)\right], \end{aligned} \qquad (V.14)$$

$$e^{\Delta_{ij}} = \frac{\sin(\frac{\varphi_+}{4} + \frac{\pi}{8})}{\sin(\frac{\varphi_-}{4} + \frac{\pi}{8})}. \qquad (V.15)$$

VI. Approximately-Riemannian Region Near Standpoint

Sufficiently close to the spatial origin of any standpoint spacetime the action is approximately equivalent to a Riemannian metric at almost all times. From (IV.13) one has

$$ds^2 = \frac{dt_i^2 - d\vec{r}_i^2}{\cos\varphi - \sin\varphi_0} + \varphi^2 \frac{1 - \frac{1}{2}\sin\varphi_0}{(1 - \sin\varphi_0)^2} r_i^2 d\vec{n}^2 \qquad (VI.1)$$

plus non-Riemannian corrections of order φ^4. (It is consistent to this order of approximation to replace $\cos\varphi$ in the first term of (VI.1) by $1-\varphi^2/2$ and in both terms to approximate φ by $r_i/R_i \cos\varphi_0$.) The approximation (VI.1) more than suffices to yield the Ricci curvature tensor at standpoint and, correspondingly, the energy density implied by Einstein theory. The latter will be calculated below.

An earlier unpublished version of the present paper asserted a Riemannian (in fact, Minkowskian) approximation to be valid in the neighborhood of the boundary point $t_i = 0, \vec{r}_i = 0$ - - the vertex of the "big-bang" forward light-cone boundary. Careful reexamination of (IV.1) has revealed this assertion to be incorrect. Also incorrect is a correlated earlier claim that spacetimes belonging to standpoints of infinite age are equivalent to that of Milne. As made explicit in Appendix B, correspondence of the $R \to \infty$ limit with Milne's flat spacetime is achievable if *only* radial motion is considered, but for 3+1 spacetime, standpoint action generally remains non-Riemannian in the limit of infinite standpoint age.

The approximation (VI.1) allows calculation of the Ricci tensor at standpoint. One finds

$$\mathcal{R}_{0\mu} = \mathcal{R}_{\mu 0} = 0, \qquad \mu = 0,1,2,3, \qquad (VI.2)$$

$$\mathcal{R}_{ij} = \delta_{ij}\frac{9}{2R^2}, \qquad i,j = 1,2,3. \qquad (VI.3)$$

(Note that i,j here are *not* standpoint indices.) The Einstein equation for energy-momentum tensor,

$$T_{\mu\nu} = \frac{1}{8\pi G}(\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu}), \qquad (VI.4)$$

with

$$\mathcal{R} \equiv g^{\alpha\beta}\mathcal{R}_{\alpha\beta}, \qquad (VI.5)$$

then yields (at i standpoint)

$$T_{00} = \frac{1}{8\pi G}\frac{27}{4}\frac{1}{R^2}, \qquad (VI.6)$$

$$T_{ij} = -\frac{1}{3}T_{00}\delta_{ij}. \qquad (VI.7)$$

The negative sign in (VI.7) implies a contribution to T_{00} from "vacuum energy density". Taking the relation

$$T_{ij}^{vac} = -T_{00}^{vac}\delta_{ij}, \qquad (VI.8)$$

to be a defining characteristic of "vacuum",[4] the decomposition,

$$T_{\mu\nu} = T_{\mu\nu}^{matter} + T_{\mu\nu}^{vac}, \qquad (VI.9)$$

leads from (VI.7) to

$$\begin{aligned} T_{00}^{matter} &= \left(\frac{2}{3} - \frac{\sigma_i}{6}\right)\rho_i, \quad T_{ij}^{matter} = \frac{\sigma_i}{6}\delta_{ij}\rho_i, \\ T_{00}^{vac} &= \left(\frac{1}{3} + \frac{\sigma_i}{6}\right)\rho_i, \quad T_{ij}^{vac} = -\left(\frac{1}{3} + \frac{\sigma_i}{6}\right)\delta_{ij}\rho_i, \end{aligned} \qquad (VI.10)$$

where $\rho_i = T_{00}$ is total energy density at standpoint i and σ_i is a positive parameter left undetermined by the standpoint action. Recognizing that, in an ideal fluid, $\sigma_i = 0$ for nonrelativistic (large rest-mass, "cold") matter while $\sigma_i = 1$ for zero rest-mass ("hot") matter, the parameter σ_i is restricted to the range, $\sigma_i \leq 1$, and matter energy density is determined by the standpoint Ricci tensor up to a factor varying only between 3/4 and 1. Employing as unit the standard-model critical density, the relation deduced below in Section VII between Hubble parameter and R_i leads from (VI.6) and (VI.10) to

$$\begin{aligned} \Omega &= \left(\frac{3}{1+\sqrt{2}}\right)^2 \approx 1.54, \\ \Omega_i^{matter} &= \left(\frac{2}{3} - \frac{\sigma_i}{6}\right)\Omega, \\ \Omega_i^{vac} &= \left(\frac{1}{3} + \frac{\sigma_i}{6}\right)\Omega. \end{aligned} \qquad (VI.11)$$

To the extent that the present universe is "cold", the parameter σ_i is close to zero.

VII. Hubble Redshift

The experimentally-observed redshift factor commonly denoted $1+z$ turns out equal to the ratio of observer age to source age (or R^{obs} to R^{source}). Equivalently, $1+z=e^{\Delta}$ where Δ is magnitude of source-standpoint *initial* rapidity when observer-standpoint rapidity is zero. The simplicity of this relation (shared with Milne cosmology) raises expectation of a transparent derivation. Unhappily we are not presently in possession of such. The calculation to follow combines Doppler redshift due to source motion in observer system with "propagation redshift" due to gravity experienced by photons moving from source to observer through observer-standpoint spacetime.

From (V.7) and (V.8), with $\alpha=0$ and $\beta=\tanh\Delta$, it is straightforward to calculate through (V.12) in observer system the radial rapidity Δ_s^{obs} of a source, located on the observer's backward light cone, which follows the trajectory of that standpoint whose *initial* rapidity was Δ. One finds

$$\Delta_s^{obs}=\frac{1}{2}\ln\left(\frac{e^{3\Delta}}{e^{-\Delta}+2^{3/2}\sinh\Delta}\right). \qquad (VII.1)$$

It may be verified that there is deceleration- - i.e., $\Delta_s^{obs}<\Delta$. (For small $\Delta, \Delta_s^{obs}\approx(2-\sqrt{2})\Delta$.) The Doppler redshift factor is then

$$e^{\Delta_s^{obs}}=\frac{e^{\frac{3}{2}\Delta}}{(e^{-\Delta}+2^{3/2}\sinh\Delta)^{1/2}}. \qquad (VII.2)$$

What about propagational redshift? Here we need to study geodesics along the observer's backward light cone. From a computation described in Appendix A one finds a propagational redshift factor

$$\left(1+\frac{2r_s^{obs}}{R^{obs}}\right)^{1/4}, \qquad (VII.3)$$

where r_s^{obs} is distance to source in observer system. The distance to source, from a calculation paralleling that leading to (VII.1), turns out to be

$$r_s^{obs}=e^{-2\Delta}\sinh\Delta(\cosh\Delta+\sqrt{2}\sinh\Delta)s^{obs} \qquad (VII.4)$$

and remembering (V.10), one then calculates from (VII.4) that

$$\left(1+\frac{2r_s^{obs}}{R^{obs}}\right)^{1/2}=e^{-\Delta}(e^{-\Delta}+2^{3/2}\sinh\Delta). \qquad (VII.5)$$

Thus the product of (VII.3) with (VII.2) is simply e^{Δ}.

That $e^{\Delta} = 1 + z$ gives the ratio of observer age to source age follows from the relation between Δ and location of source in observer-standpoint spacetime when expressed through the alternative variables

$$\begin{aligned} \tau_i^{source} &= \sqrt{2}\, s^{source} \cosh \Delta, \\ \rho_i^{source} &= \sqrt{2}\, s^{source} \sinh \Delta. \end{aligned} \qquad (VII.6)$$

In (VII.6) i denotes observer spacetime while s^{source} is age of source. Location of any source lying on the backward light cone of the observer satisfies

$$\tau_i^{source} + \rho_i^{source} = \sqrt{2} s^{obs}. \qquad (VII.7)$$

It follows from (VII.6) and (VI.7) that

$$e^{\Delta} s^{source} = s^{obs}. \qquad (VII.8)$$

Consider next the Hubble parameter for observer, *phenomenologically* defined by

$$H_0 \equiv \lim_{r_s^{obs} \to 0} \frac{z}{r_s^{obs}}. \qquad (VI.9)$$

Because formula (VII.4) exhibits a linear relation between r_s^{obs} and Δ for small Δ,

$$\lim_{\Delta \to 0} \frac{r_s^{obs}}{\Delta} = s^{obs}, \qquad (VII.10)$$

while in the same limit $z/\Delta \to 1$, it follows that

$$H_0^{-1} = s^{obs}. \qquad (VII.11)$$

Before closing this section we remark that, according to (VII.4), the upper limit of r_s^{obs}- - distance to source located on observer's backward light cone- -is $R^{obs}/2$, reached as $\Delta \to \infty$. In other words $R_i/2$ is "radius of i standpoint's universe".

VIII. Luminosity Distance and Angular-Diameter Distance

This section contemplates small transverse displacements so as to calculate the conventionally-defined "luminosity distance" d_L [2] and for low redshift ($z \lesssim 1$) will find

$$H_0 d_L(z) = z + \frac{z^2}{2} - \frac{3}{2}\frac{z^3}{(1+\sqrt{2})^2} + \text{ order } z^4. \qquad (VIII.1)$$

The leading deviation here from linear Hubble behavior is the same as given by the standard model for zero deceleration parameter q_0.[5] The next-order correction is numerically close to although not identical with that of the standard model when $\Omega > 1$ if $\Omega^{vac} = \frac{1}{2}\Omega^{matter}$ - - a condition implying $q_0 = 0$. (For the latter special case, the standard-model cubic correction is $-1/3z^3$).

The general standpoint-cosmology formula for luminosity distance is based on the photon impact parameter when it arrives at observer after emission from source with an angular displacement $d\vec{n}$, in source coordinates, with respect to the direction that connects observer to source. Such a transverse displacement occurs at $\varphi_- = 0$ and a fixed value of φ_+ that corresponds to distance between source and observer,

$$\frac{1}{2}R^{source}\sin\varphi_+, \qquad (VIII.2)$$

in *source-standpoint* coordinates. A source at rest in this system emits light isotropically. Although emission is anisotropic in observer system, the impact parameter (a displacement perpendicular to relative source-observer motion) is "invariant" - - having the same magnitude in both systems. Formula (IV.12) applies.

"Transverse displacement" means at fixed distance from source and at fixed time. The formula for luminosity distance is obtained by multiplying with a factor e^{Δ} the square root of the coefficient of $-d\vec{n}^2$ in (IV.12), evaluated in *source-standpoint* spacetime at $\varphi_- = 0$ - - i.e., along the source's *forward* light cone. The factor $e^{\Delta} = 1 + z$ is conventionally added in defining luminosity distance so as to accommodate decrease of light intensity due to redshift and time dilation.[2] Because $e^{\Delta}R^{source} = R^{obs}$, one finds from (IV.10) and (IV.12) with $\varphi_0 = \varphi$ so $\varphi_+ = 2\varphi$,

$$d_L(\varphi_+) = \left(\frac{1}{2} + \frac{1+\sin\varphi_+}{1+\cos\varphi_+}\right)^{1/2} \frac{R^{obs}}{2} \sin\varphi_+. \qquad (VIII.3)$$

According to (V.15), redshift along the source forward light cone relates to φ_+ by the formula

$$e^{\Delta} = 1 + z = \cos(\varphi_+/4) + (1+\sqrt{2})\sin(\varphi_+/4). \qquad (VIII.4)$$

Once the variable φ_+ in (VIII.3) has been replaced by z through (VIII.4), Formula (VIII.3) may be analytically continued to arbitrarily-large z. The approximate formula (VIII.1) gives the first 3 terms of a Taylor expansion around $z = 0$.

Also interesting is "angular-diameter distance", d_A, given by a formula analogous to (VIII.3). There, no factor of e^{Δ} is involved and the standpoint taken is that of the observer. The transverse displacement in question is "radius of source". The source resides on observer's *backward* light cone where $\varphi_+ = 0$ and φ_- is negative. Distance to source in observer system is

$$-\frac{R^{obs}}{2} \sin\varphi_-, \qquad (VIII.5)$$

and from (IV.12) one finds

$$d_A(\varphi_-) = -\left(\frac{1}{2} + \frac{1+\sin\varphi_-}{1+\cos\varphi_-}\right)^{1/2} \frac{R^{obs}}{2} \sin\varphi_-. \qquad (VIII.6)$$

Because u and v are symmetric functions of φ_- and φ_+, the functions $d_A(\varphi_-)$ and $d_L(\varphi_+)$ have the same form up to an overall minus sign. Redshift relates to φ_- on the observer's backward light cone according to

$$e^{-\Delta} = \frac{1}{1+z} = \cos(\varphi_-/4) + (1+\sqrt{2})\sin(\varphi_-/4). \qquad (VIII.7)$$

One sees from (VIII.7) that, as big-bang horizon at $\varphi_- = -\pi/2$ is approached, $z \to \infty$ since $1 + \sqrt{2} = \cot\pi/8$.

Comparison of (VIII.6) with (VIII.3) and of (VIII.7) with (VIII.4) reveals the general relation

$$d_A(z) = -d_L\left(-\frac{z}{1+z}\right). \qquad (VIII.8)$$

The Taylor expansion (VIII.1) *up to cubic terms* then agrees with the standard-model relation[5]

$$d_A(z) = \frac{d_L(z)}{(1+z)^2}. \qquad (VIII.9)$$

IX. Poincaré Symmetry in Standpoint Neighborhood

This brief section makes explicit the sense in which standpoint cosmology is compatible with "usual" classical physics inside any homogeneous-universe neighborhood that is small on Hubble scale. Consider two different standpoints i and j whose initial rapidities with respect to a (third) reference standpoint, labeled "0", are $\vec{\Delta}_{oi}$ and $\vec{\Delta}_{oj}$, respectively. Let us suppose $|R_i - R_0| << R_0$ and $|R_j - R_0| << R_0$ and further that $|\vec{\Delta}_{oi}| << 1, \left|\vec{\Delta}_{oj}\right| << 1$. The two standpoints being compared, that is to say, are close to each other on Hubble scale. "Neighborhoods" of these standpoints, defined by

$$\begin{aligned} |t_i - R_i| &<< R_0, \quad |\vec{r}_i| << R_0, \\ |t_j - R_j| &<< R_0, \quad |\vec{r}_j| << R_0, \end{aligned} \qquad (IX.1)$$

a region where all 4 angles, $\varphi^{\pm}_{i,j}$ are small, then approximately map onto each other via Minkowskian geometry even though the *full* spacetimes do *not*.

To leading order the mapping is simply

$$\begin{aligned} (t_i - R_i) - (t_j - R_j) &\approx s_j - s_i, \\ \vec{r}_i - \vec{r}_j &\approx (\vec{\Delta}_{oj} - \vec{\Delta}_{oi})s_0 \\ &\approx \vec{\Delta}_{ij} s_0. \end{aligned} \qquad (IV.2)$$

Change of standpoint is thus equivalent to a familiar Poincaré displacement. Adding the consideration that, to the foregoing order, metric is Minkowskian for both coordinate systems within the neighborhood (IX.1), one recognizes usual Poincaré covariance of a unique noncompact spacetime. For physics within this neighborhood any Poincaré transformation may be invoked, so long as errors due to finiteness of universe are to be ignored.

X. Thermodynamic Approximation Near Young Standpoints

Because energy density varies inversely with square of age, near sufficiently young standpoints particle mean free paths (in time as well as in 3-space) become small on Hubble scale, allowing a thermodynamic approximation. Isotropy of universe (as viewed from a standpoint - - i.e., using standpoint coordinates) allows an association with very young standpoints of temperature as well as energy density and pressure; in homogeneous-universe approximation, such temperature depends only on standpoint age. Accompanying energy density of order $(GR_i^2)^{-1}$, temperature is presumed to decrease monotonically with increasing standpoint age. Lack of understanding of the thermodynamic role of vacuum energy presently obstructs a more precise statement.

Assuming radiation decoupling as temperature falls below atomic ionization energies, a thermal photon ("black body") spectrum would survive to later ages with "photon temperature" decreasing inversely with age. The energy of any decoupled photon diminishes by this same factor as its age (i.e., the age of the standpoint where the photon is located) advances.

Nucleosynthesis occurs near standpoints whose temperature allows nuclear reactions but, in absence of vacuum-energy understanding, calculations have not yet been attempted. It is momentarily unknown what standpoint cosmology predicts for light-element abundances generated by primordial nucleosynthesis. Making a guess that, during thermodynamic equilibrium, energy density varies as T_i^4, the ratio $\sim 10^{22}$ is expected between age of photon decoupling and the minimum meaningful classical age, s_{min}. The latter accompanies a maximum classically-meaningful temperature on TeV scale. The Mev-scale temperatures needed for nucleosynthesis would occur near an age $\sim 10^{10} s_{min} \sim 10\,\text{sec}$.

XI. Concluding Remarks

Only partial rules for mapping between different standpoint spacetimes have been developed (Section IX and Appendix B). Nonradial physical geodesics at Hubble-scale distance from standpoint are treated in principle by Reference (6) but have not been related to mapping. Noteworthy is the failure of non-Riemannian lightlike trajectories to be characterized by zero length. Angular isotropy of the standpoint universe provides an alternative characterization of "light" through a conserved dynamical variable analogous to angular momentum.

Prime candidate for early falsifier of standpoint cosmology is the predicted redshift dependence of luminosity distance (VIII.1), in conjunction with the energy-density prediction $\Omega^{matter} = 1.03$ and the equality of standpoint age with Hubble time. Although quantitative ability of the new model to explain light element abundances is not yet established, cosmic background radiation presents no problem. A qualitative phenomenological innovation is "vacuum" energy density varying roughly as H_0^2 (inversely with square of age), the vacuum fraction of total energy lying between $\frac{1}{2}$ (hot early universe) and $1/3$ (cold late universe). It is as if the standard model's "cosmological constant" were to have not only special value but unorthodox time dependence.

Motivation behind classical standpoint cosmology has been, not addition of curvature to Milne's 1935 kinematic cosmology, but rather representation of the symmetry of an underlying quantum model. That symmetry implies for each standpoint a *compact* spacetime with a matching quartic form for gravitational action. The compactness implies a special classical "curvature": Out of quantum *symmetry* has flowed an unorthodox but explicit classical *dynamics.*

A striking set of redshift-related phenomological features from Milne's model has survived in standpoint cosmology: Beyond the equality of age with a Hubble time defined by redshift, the redshift dependence for $z \lesssim 1$ of luminosity distance and angular-diameter distance is that characterized standardly as "zero deceleration" (despite nonvanishing "curvature" of standpoint spacetime). Even though total redshift combines Doppler and gravitational shifts, photon energy varies in inverse proportion to age, just as for Milne.

We remark finally that Einstein's theory of gravitation, on which stan-

dard cosmology is based, was originally formulated without regard either for quantum principles or for the meaningless of "time before big bang". The Hubble-scale reliability of general relativity is consequently open to question. Phenomenologically-viable alternatives to the standard model deserve attention, especially if they entail fewer arbitrary parameters.

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Appendix A: Gravitational Redshift Along Standpoint Light Cone

Light arriving at standpoint from a source located on standpoint backward light cone corresponds in the radial-velocity equation (V.12) to the limiting case $v_i = \beta = -1$. Our deduction of gravitational redshift will approach this limit through the relation

$$v_i = \frac{e^{\gamma_i} - e^{-\gamma_i}}{e^{\gamma_i} + e^{-\gamma_i}} = \tanh \gamma_i \qquad (A.1)$$

between radial velocity v_i and radial *rapidity* γ_i. Even in the limits $v_i \to \pm 1$, where $\gamma_i \to \pm\infty$, there is (finite) *rapidity variation* along the trajectory- - corresponding to energy shift. For zero-mass particles, energy varies in proportion to e^{δ_i} where δ_i means *change* in the absolute value of γ_i. We may alter (V.12) to a rapidity-variation relation, applicable to incoming photons, by asymptotically expanding (A.1) for large negative rapidity

$$\begin{aligned} v_i &= -1 + 2e^{2\gamma_i} + \text{ terms of order } e^{4\gamma_i} \\ &- \gamma_i >> 1 \end{aligned} \qquad (A.2)$$

and making a corresponding expansion of (V.12) around $\beta = -1$. Writing $\beta = -1 + \epsilon, \epsilon > 0$, one finds

$$\begin{aligned} v_i = \frac{-1 + \epsilon\frac{4R_i - \tau_i}{4R_i - (\tau_i - \rho_i)}}{1 - \epsilon\frac{\rho_i}{4R_i - (\tau_i - \rho_i)}} &\underset{\epsilon << 1}{=} -1 + \epsilon\frac{4R_i - (\tau_i + \rho_i)}{4R_i - (\tau_i - \rho_i)} \\ &+ \text{ terms of order } \epsilon^2. \end{aligned} \qquad (A.3)$$

By comparing (A.2) with (A.3), it may be inferred that

$$e^{2\gamma_i} \approx \frac{\epsilon}{2}\frac{4R_i - (\tau_i + \rho_i)}{4R_i - (\tau_i - \rho_i)}. \qquad (A.4)$$

for $\epsilon << 1, -\gamma_i >> 1$. Remembering γ_i to be negative, photon energy is thus proportional to

$$\left[\frac{4R_i - (\tau_i + \rho_i)}{4R_i - (\tau_i - \rho_i)}\right]^{1/2} = \left[\frac{2R_i - (t_i + r_i)}{2R_i - (t_i - r_i)}\right]^{1/4}, \qquad (A.5)$$

the right-hand form following from (V.4). Along the standpoint backward light cone, $t_i + r_i = R_i$, while at standpoint $r_i = 0$. It follows from (A.5) that

the gravitational redshift factor for propagation between $r_i = r_s^{obs}$ (notation of Section VII) and standpoint is

$$(1 + \frac{2r_s^{obs}}{R^{obs}})^{1/4}. \tag{A.6}$$

Of interest also is gravitational redshift of light emitted from standpoint and proceeding along the standpoint's *forward* light cone where $t_i - r_i = R_i$. Repeating the foregoing calculation for the limit $v_i \to +1, \gamma_i \to +\infty$, one finds gravitational energy reduction during propagation in i-standpoint system by a factor

$$\left(1 - \frac{2r_i}{R_i}\right)^{1/4} \tag{A.7}$$

for light reaching a distance r_i from standpoint. At abyss, where $r_i = R/2$, all i-system photon energies are thus reduced to zero. On the other hand, if one considers light *absorption* by matter at and moving with some *other* standpoint located on forward cone, the motion of that standpoint produces a Doppler shift so that the net redshift in the usual sense continues to be given by the ratio of standpoint ages.

Appendix B. Radial Mapping

We here make explicit the 1 + 1 dimensional mapping between a pair of standpoint spacetimes the separation of whose spatial origins is parallel to a direction $\vec{n}$. The spatial mapping is confined to points along the separation ray. The two standpoints i and j are specified, respectively, by $(R_i, \Delta_{oi}\vec{n})$ and $(R_j, \Delta_{oj}\vec{n})$, where the index 0 designates a (third) reference standpoint whose spatial origin lies along the ray connecting i and j spatial origins. The mapping proceeds in 3 steps, in each of which the direction $\vec{n}$ is maintained:

(a) $i \to k$, where k corresponds to $(\infty, \Delta_{oi}\vec{n})$. This is the transformation (V.4), i.e.,

$$t_k \pm r_k = 4R_i[1 - (1 - \frac{t_i \pm r_i}{2R_i})^{1/2}] \qquad (B.1)$$

(b) $k \to \ell$, where ℓ corresponds to $(\infty, \Delta_{oj}\vec{n})$. This is a length-preserving boost of the 2-vector (t, r) for $R = \infty$:

$$\begin{aligned} t_\ell &= t_k \cosh \Delta_{ij} + r_k \sinh \Delta_{ij} \\ r_\ell &= t_k \sinh \Delta_{ij} + r_k \cosh \Delta_{ij}, \end{aligned} \qquad (B.2)$$

where $\Delta_{ij} \equiv \Delta_{oj} - \Delta_{oi}$.

(c) $\ell \to j$. This is the transformation (V.8), i.e.,

$$\begin{aligned} t_j &= t_\ell - \frac{t_\ell^2 + r_\ell^2}{8R_j}, \\ r_j &= r_\ell(1 - \frac{t_\ell}{4R_j}), \end{aligned} \qquad (B.3)$$

which requires $t_\ell \mp r_\ell \leq 4R_j$. If $R_i > R_j$ only a portion of the selected ray in i spacetime can be mapped onto j spacetime.

Appendix C. Distribution of Matter

We here calculate the distribution of matter throughout a standpoint's universe as a function of redshift along the standpoint's backward light cone. Three factors are to be combined:

(a) The transverse area at z per unit solid angle as observed from standpoint. This is $d_A^2(z)$, where d_A is the angular-diameter distance given by (VIII.6).

(b) The increment of radial distance in the coordinate system of that standpoint where the matter locates. This is $e^{-\Delta}H_0^{-1}d\Delta$, where $\Delta = \ln(1+z)$ so $dz = e^{\Delta}d\Delta$.

(c) Matter (energy) density in the matter-standpoint system. According to (VI.11) this is

$$\rho^{matter}(z) = \Omega^{matter}(z)\frac{3}{8\pi}\frac{e^{2\Delta}H_0^2}{G}, \tag{C.1}$$

with

$$\Omega^{matter}(z) = \frac{2}{3}\left(1 - \frac{\sigma(z)}{4}\right)\left(\frac{3}{1+\sqrt{2}}\right)^2. \tag{C.2}$$

Combining these factors and integrating over the full 4π solid angle gives

$$\frac{d(matter)}{dz} = \left(1 - \frac{1}{4}\sigma(z)\right)\left(\frac{3H_0 d_A(z)}{1+\sqrt{2}}\right)^2 \frac{1}{H_0 G}. \tag{C.3}$$

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