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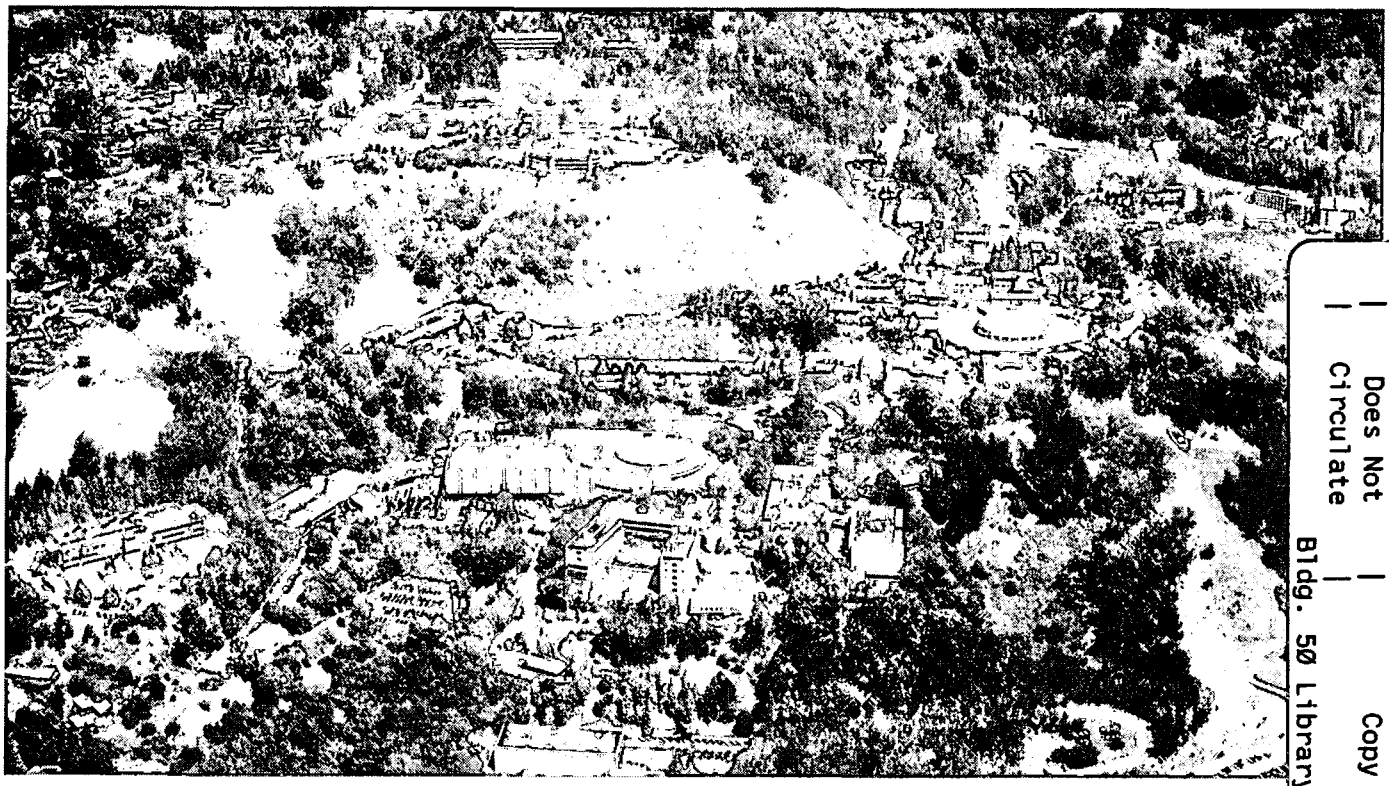

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A Supersymmetric Theory of Flavor with Radiative Fermion Masses *

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Abstract

Supersymmetric theories involving a spontaneously broken flavor symmetry can lead to fermion masses which vanish at tree level but are generated by radiative corrections. In the context of supersymmetric theories with minimal low energy field content we discuss which fermion masses and mixings may be obtained radiatively, and find that constraints from flavor changing phenomenology imply that only the first generation fermion masses and some (but not all) CKM mixings can naturally come

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from radiative corrections. We also consider general conditions on theories of flavor which guarantee the existence of tree level massless fermions while having non-trivial CKM matrix elements at tree level. Two complete models of flavor are presented. In the first model, all first generation fermion masses are radiatively generated. In the second model, the electron and up quark mass are due to radiative corrections whereas the down mass appears at tree level, as does a successful prediction for the Cabibbo angle $\sin\theta_c = \sqrt{m_d/m_s}$.

1 Introduction

A complete supersymmetric theory of flavor must address both the fermion mass problem and the flavor changing problem [1]. An early proposal to address the flavor changing problem by invoking a $U(N)$ flavor symmetry of the Kahler potential in supergravity [2] was very incomplete; it did not address how the symmetry could be broken to get the fermion mass interactions of the superpotential. By studying broken flavor symmetries, one can study both issues simultaneously [3], opening the door to a new field of flavor model building. Although there is considerable freedom in the choice of the flavor symmetry group and the pattern of symmetry breaking, the enterprise is nevertheless constrained by the direct link between the flavor changing and fermion mass problems. Many candidate theories of fermion masses are excluded by flavor changing phenomenology. In this paper we study the possibility that some fermion masses arise radiatively, which requires large flavor changing interactions of the squarks or sleptons. Hence theories of flavor, based on spontaneously broken flavor symmetries, which involve radiative fermion masses, are very highly constrained by flavor changing phenomenology.

Flavor symmetries should forbid Yukawa couplings of the light fermions. After the flavor symmetries are broken, the light generation fermions should acquire small Yukawa couplings. Many models of fermion masses use the Froggatt-Nielsen mechanism [4] to generate small Yukawa couplings: assuming a flavor symmetry is broken by the VEV of some fields $\langle\phi\rangle$, and after integrating out heavy states of mass M, one can get light generation Yukawa couplings suppressed by $\frac{\langle\phi\rangle}{M}$. This mechanism can naturally generate second generation Yukawa couplings, but in order to ensure small enough first generation Yukawa couplings one usually has to assume contrived representations of the flavor group and/or contrived patterns of flavor breaking. There is, however, another possibility for generating small Yukawa couplings: if generated radiatively, they are suppressed by the loop factor $\frac{1}{16\pi^2}$. This intriguing possibility has been extensively studied in the literature[5]. A universal feature of all models must be that an "accidental" chiral symmetry is present in the Yukawa sector to force a zero Yukawa coupling at tree level, while this symmetry must be broken in another sector of the theory in order for the Yukawa coupling to be radiatively generated.

As we pointed out in [6], supersymmetric theories can provide a natural way for this to happen: the constraints of holomorphy can force the superpotential to have accidental symmetries not shared by the D-terms. Given that the supersymmetric extension of the standard model is of interest for other reasons, we are naturally led to explore the idea of radiative fermion masses in supersymmetric models. To be specific, we consider supersymmetric $SU(3) \times SU(2) \times U(1)$ theories with minimal low energy field content, i.e. we do not consider extra Higgses or extra families etc. We will find that, with this assumption, the set of possibilities for radiative fermion masses is highly constrained, and yields robust experimental predictions.

The outline of this paper is as follows. In section 2 we consider general possibilities for radiative fermion masses in supersymmetric theories with minimal low-energy field content, and conclude that, quite generally, only the lightest generation can be obtained radiatively. In section 3 we discuss phenomenological constraints and consequences which follow from generating the lightest generation radiatively. In the subsequent sections, we consider issues related to building models which naturally implement radiative fermion Yukawa couplings for the first generation: In section 4, we discuss some general properties such models should have; and in section 5 we extend the lepton model presented in [6] to the quark sector. Our conclusions are drawn in section 6.

2 General possibilities for radiative fermion masses

We now consider general possibilities for radiatively generated Yukawa couplings in supersymmetric theories with minimal low energy field content. We know that, in the limit of exact supersymmetry, a Yukawa coupling which is zero at tree level will never be generated radiatively. Thus, in order to have radiative Yukawa couplings, we need soft supersymetry breaking operators which, further, must explicitly break the chiral symmetries associated with the zero Yukawa couplings of the superpotential. Also, the particles in the radiative loop must be at the weak scale: since the generated Yukawa coupling λ is dimensionless and vanishes in the limit m_S (the supersymmetry breaking scale) goes to zero, we must have $\lambda \sim \frac{1}{16\pi^2}\frac{m_S}{M}$, where M is a typical mass for the particles in the loop. Thus, M must be near the weak scale (rather than the GUT or Planck

scale) in order to generate large enough Yukawa couplings.

Thus, we see that the breaking of the flavor symmetries associated with the zero Yukawa couplings must lie in the weak scale soft supersymmetry breaking operators: the trilinear scalar A terms and the soft scalar masses. In this paper we make the plausible assumptions that the flavor symmetry is not an R symmetry and that supersymmetry breaking fields are flavor singlets. Then, the A terms must respect the same flavor symmetries as the the Yukawa couplings, since any flavor symmetry forbidding $\int d^2\theta f(\phi)$ (where $f(\phi)$ is some function of the superfields ϕ in the theory) will also forbid $\int d^2\theta\theta^2 f(\phi)$. Hence, all the flavor symmetry breaking responsible for generating radiative fermion masses resides in the scalar mass matrices. (However, in appendix A, we repeat the analysis without this assumption. Requiring our vacuum to be the global minimum of the potential and using constraints from flavor-changing neutral currents (FCNC), the A terms are such that the conclusions of this section are not greatly altered.)

For simplicity, let us work in the lepton sector, and consider the possibility of radiatively generating K lepton masses for $K = 3,2,1$ in turn.

***K*=3.** In this case, we have a vanishing tree level Yukawa matrix which has a large $U(3)_\ell \times U(3)_e$ symmetry. By our assumption that the flavor symmetry is not an R symmetry and that supersymmetry breaking fields do not carry flavor, the A terms must also vanish. But then, all the soft scalar mass matrices can be simultaneously diagonalized, leaving an independent, unbroken $U(1)$ symmetry acting on every superfield, preventing the radiative generation of any Yukawa couplings.

***K*=2.** Here, we only have the third generation Yukawa coupling at tree level. This case is more interesting. We shall find that, although it is possible to generate two Yukawa eigenvalues radiatively, strong constraints from FCNC force the ratio of the (radiatively generated) first to second generation Yukawa couplings to a value too small to be compatible with experiment.

Let us work in a basis where the Yukawa matrix $\boldsymbol{\lambda}_E$ is diagonal,

$$\boldsymbol{\lambda}_E = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda \end{pmatrix}. \tag{2.1}$$

Since $\boldsymbol{\lambda}_E$ is invariant under independent rotations of the first two generation left

and right handed lepton superfields, we can make these rotations on the left and right handed scalar masses $\mathbf{m}^2_{L(R)}$,

$$\mathbf{m}^2_{L(R)} \to U_{L(R)}\mathbf{m}^2_{L(R)}U^\dagger_{L(R)}, \tag{2.2}$$

where the $U_{L(R)}$ are unitary rotations in the upper 2×2 block,

$$U_{L(R)} = \begin{pmatrix} u_{L(R)} & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.3}$$

If we write

$$\mathbf{m}^2_L = \begin{pmatrix} m^2_{2\times 2} & m^2_{2\times 1} \\ m^{2\dagger}_{2\times 1} & m^2_{33} \end{pmatrix}, \tag{2.4}$$

then under U_L we have

$$\mathbf{m}^2_L \to \begin{pmatrix} u_L m^2_{2\times 2} u^\dagger_L & u_L m^2_{2\times 1} \\ {u_L}^\dagger m^{2\dagger}_{2\times 1} & m^2_{33} \end{pmatrix}, \tag{2.5}$$

and we can choose u_L so that

$$u_L m^2_{2\times 1} = \begin{pmatrix} 0 \\ m^2_{23} \end{pmatrix}. \tag{2.6}$$

Thus, we can choose a basis where the 1-3 and 3-1 entries of $\mathbf{m}^2_L$ are 0, and similarly for $\mathbf{m}^2_R$; the scalar masses have the form

$$\mathbf{m}^2_{L(R)} = \begin{pmatrix} m^2_1 & \delta m^2_{12} & 0 \\ \delta m^{2*}_{12} & m^2_2 & \Delta m^2_{23} \\ 0 & \Delta m^{2*}_{23} & m^2_3 \end{pmatrix}_{L(R)}. \tag{2.7}$$

The 1-2 entries, δm^2_{12}, are constrained to be very small compared to m^2_1 and m^2_2 from FCNC considerations. Suppose we put just one of the δm^2_{12}, say δm^2_{12L}, equal to zero. Then, we have a $U(1)$ symmetry acting on the left-handed lepton superfield of the first generation, which will prevent the generation of any Yukawa coupling for the first generation. Hence, the radiatively generated first generation Yukawa coupling will be suppressed relative to the second generation one by roughly

$$\frac{\lambda_1}{\lambda_2} \sim \frac{\delta m^2_{12L}}{m^2_L}\frac{\delta m^2_{12R}}{m^2_R}, \tag{2.8}$$

where the $m^2_{L,R}$ are typical scalar masses for the first two generations.

Let us make a more careful estimate for the size of this suppression. For simplicity, we work in the mass insertion approximation where m_1^2, m_2^2, m_3^2 are taken to be degenerate and equal to m^2. We find the radiatively generated Yukawa matrix for the upper 2×2 block is

$$\lambda_{2\times 2} = \begin{pmatrix} \frac{\delta m^2_{12L}}{m^2}\frac{\delta m^2_{12R}}{m^2} f(7)x & \frac{\delta m^2_{12L}}{m^2} f(6)x \\ \frac{\delta m^2_{12R}}{m^2} f(6)x & f(5)x \end{pmatrix}, \tag{2.9}$$

where

$$f(n) = m^{2n-4} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m^2)^{n-1}(k^2-M^2)}, \quad x = \text{const} \times M \frac{\Delta m^2_{23L}}{m^2}\frac{\Delta m^2_{23R}}{m^2},$$

and M is the gaugino mass. Since $f(n)$ is only logarithmically sensitive to the ratio $\frac{M^2}{m^2}$, we put $M^2 = m^2$. Then, $f(n) = \frac{1}{(n-2)(n-1)}$ and we have

$$\lambda_{2\times 2} = \begin{pmatrix} \frac{1}{30}\frac{\delta m^2_{12L}}{m^2}\frac{\delta m^2_{12R}}{m^2} x & \frac{1}{20}\frac{\delta m^2_{12L}}{m^2} x \\ \frac{1}{20}\frac{\delta m^2_{12R}}{m^2} x & \frac{1}{12} x \end{pmatrix}. \tag{2.10}$$

Diagonalizing the above matrix, we find the ratio of the first to second generation eigenvalues to be

$$\frac{\lambda_1}{\lambda_2} \sim \frac{1}{25}\frac{\delta m^2_{12L}}{m^2}\frac{\delta m^2_{12R}}{m^2}. \tag{2.11}$$

We see that it is impossible to generate large enough first generation Yukawa couplings consistent with FCNC constraints (unless the scalars are taken to be unacceptably heavy), which require (for 300 GeV sleptons and 500 GeV squarks)

$$\begin{aligned} &\frac{1}{25}\frac{\delta m^2_{12\ell}}{m^2}\frac{\delta m^2_{12e}}{m^2} < 2\times 10^{-4} \ (\mu \to e\gamma) \\ &\frac{1}{25}\frac{\delta m^2_{12q}}{m^2}\frac{\delta m^2_{12d}}{m^2} < 1\times 10^{-6} \ (K_1 - K_2 \text{ mixing}) \\ &\frac{1}{25}\frac{\delta m^2_{12q}}{m^2}\frac{\delta m^2_{12d}}{m^2} < 6\times 10^{-5} \ (D_1 - D_2 \text{ mixing}). \end{aligned} \tag{2.12}$$

We are left with the case K=1, where Yukawa couplings for two generations occur at tree level, while the remaining Yukawa couplings, which necessarily correspond to the lightest generation, are radiatively generated. In the next section, we study the phenomenological constraints on this scenario in detail.

3 Phenomenological constraints

In this section, we discuss the phenomenology of obtaining the first generation Yukawa coupling radiatively. Recall that we are relying on the scalar mass matrices to break the chiral symmetries associated with the Yukawa matrices; in particular, then, the scalar mass matrices cannot be diagonalized in the same basis as the Yukawa matrices. Thus, if we work in the mass eigenstate basis for all fields, we will have non-trivial mixing matrices at the gaugino vertices. Let us set the relevant notation here, following [7]. The superpotential contains

$$W \supset Q^T \boldsymbol{\lambda}_U U^c h_u + Q^T \boldsymbol{\lambda}_D D^c h_d + L^T \boldsymbol{\lambda}_E E^c h_d \tag{3.1}$$

where $\boldsymbol{\lambda}_U$, $\boldsymbol{\lambda}_D$, $\boldsymbol{\lambda}_E$ are the Yukawa matrices, and are diagonalized by

$$\begin{aligned} \boldsymbol{\lambda}_U &= V^*_{U_L} \overline{\boldsymbol{\lambda}_U} V^\dagger_{U_R} \\ \boldsymbol{\lambda}_D &= V^*_{D_L} \overline{\boldsymbol{\lambda}_D} V^\dagger_{D_R} \\ \boldsymbol{\lambda}_E &= V^*_{E_L} \overline{\boldsymbol{\lambda}_E} V^\dagger_{E_R}. \end{aligned} \tag{3.2}$$

The soft supersymmetry breaking masses matrices are contained in

$$\tilde{Q}^\dagger \mathbf{m}^{2*}_Q \tilde{Q} \; + \tilde{U}^{c\dagger} \mathbf{m}^2_U \tilde{U}^c + \widetilde{D}^{c\dagger} \mathbf{m}^2_D \widetilde{D}^c + \tilde{L}^\dagger \mathbf{m}^{2*}_L \tilde{L} + \tilde{E}^{c\dagger} \mathbf{m}^2_E \tilde{E}^c$$

and are diagonalized by

$$\begin{aligned} \mathbf{m}^{2*}_Q &= U_Q \overline{\mathbf{m}}^{2*}_Q U^\dagger_Q, \;\; \mathbf{m}^2_U = U_U \overline{\mathbf{m}}^2_U U^\dagger_U, \;\; \mathbf{m}^2_D = U_D \overline{\mathbf{m}}^2_D U^\dagger_D, \\ \mathbf{m}^{2*}_L &= U_L \overline{\mathbf{m}}^{2*}_L U^\dagger_L, \;\; \mathbf{m}^2_E = U_E \overline{\mathbf{m}}^2_E U^\dagger_E, \end{aligned} \tag{3.4}$$

In the mass eigenstate basis, the rotation matrices V, U appear in the gaugino couplings,

$$\begin{aligned} \mathcal{L}_g \; &= \sqrt{2} g' \sum_{\pi=1}^{4} \Big[-\frac{1}{2} \overline{e}_L W^\dagger_{E_L} \tilde{e}_L N_n (H_{n\widetilde{B}} + \cot\theta_W H_{n\widetilde{w}_3}) + \overline{e}^c_L W^\dagger_{E_R} \tilde{e}_R N_n H_{n\widetilde{B}} \\ &+ \frac{1}{2} \cot\theta_W \overline{\nu}_L \tilde{\nu}_L N_n H_{n\widetilde{w}_3} \\ &+ \overline{u}_L W^\dagger_{U_L} \tilde{u}_L N_n (\frac{1}{6} H_{n\widetilde{B}} + \frac{1}{2} \cot\theta_W H_{n\widetilde{w}_3}) + \overline{d}_L W^\dagger_{D_L} \tilde{d}_L N_n (\frac{1}{6} H_{n\widetilde{B}} - \frac{1}{2} \cot\theta_W H_{n\widetilde{w}_3}) \\ &- \frac{2}{3} \overline{u}^c_L W^\dagger_{U_R} \tilde{u}_R N_n H_{n\widetilde{B}} + \frac{1}{3} \overline{d}^c_L W^\dagger_{D_R} \tilde{d}_R N_n H_{n\widetilde{B}} + h.c. \Big] \end{aligned}$$

$$+ g\sum_{c=1}^{2}[\bar{e}_L W^\dagger_{E_L}\tilde{\nu}_L(\chi_c K_{c\tilde{w}}) + \bar{\nu}_L\tilde{e}_L(\chi^\dagger_c K^*_{c\tilde{w}})$$
$$+ \bar{d}_L W^\dagger_{D_L}\tilde{u}_L(\chi_c K_{c\tilde{w}}) + \bar{u}_L W^\dagger_{U_L}\tilde{d}_L(\chi^\dagger_c K^*_{c\tilde{w}}) + h.c.]$$
$$+ \sqrt{2}g_3[\bar{u}_L W^\dagger_{U_L}\tilde{u}_L\tilde{g} + \bar{d}_L W^\dagger_{D_L}\tilde{d}_L\tilde{g} + \bar{u}^c_L W^\dagger_{U_R}\tilde{u}_R\tilde{g} + \bar{d}^c_L W^\dagger_{D_R}\tilde{d}_R\tilde{g} + h.c.], \qquad (3.5)$$

here[1] the neutralino and chargino mass eigenstates are related to the gauge eigenstates by e.g. $\tilde{B} = \sum_{n=1}^{4} H_{n\tilde{B}} N_n$, $\tilde{w}_3 = \sum_{n=1}^{4} H_{n\tilde{w}_3} N_n$, $\tilde{w}^+ = \sum_{c=1}^{2} K_{c\tilde{w}}\chi_c$, and

$$W_{E_L} = U^\dagger_L V_{E_L},\ W_{E_R} = U^\dagger_E V_{E_R},\ W_{U_L} = U^\dagger_Q V_{U_L},\ W_{D_L} = U^\dagger_Q V_{D_L},$$
$$W_{U_R} = U^\dagger_U V_{U_R},\ W_{D_R} = U^\dagger_D V_{D_R}. \qquad (3.6)$$

Having defined our notation, we now consider the dominant radiative contributions to the lepton, up and down mass matrices given in Fig. 1. In the following, we assume that the first two generation scalars are degenerate, since we know from the previous section that the contribution to the mass matrix from the non-degeneracy between the first two generations is negligible. Evaluating the diagrams, we find (keeping only the contribution from the third generation tree-level mass) [8] :

$$\Delta \mathbf{m}_{\mathbf{e}\,\alpha\beta} = \sum_{n=1}^{4}\frac{H_{n\tilde{B}}}{M_n}(H_{n\tilde{B}} + \cot\theta_W H_{n\tilde{w}_3})$$
$$\times \frac{\alpha m_\tau}{4\pi\cos^2\theta_W}(A + \mu\tan\beta) \times \{W_{E_L 3\alpha}W^*_{E_L 33}\, W_{E_R 3\beta}W^*_{E_R 33}$$
$$[h(x_{3L_n}, x_{3R_n}) - h(x_{3L_n}, x_{1R_n}) - h(x_{1L_n}, x_{3R_n}) + h(x_{1L_n}, x_{1R_n})]$$
$$+ W_{E_L 3\alpha}W^*_{E_L 33}\,\delta_{3\beta}[h(x_{3L_n}, x_{1R_n}) - h(x_{1L_n}, x_{1R_n})]$$
$$+ \delta_{\alpha 3}\, W_{E_R 3\beta}W^*_{E_R 33}[h(x_{1L_n}, x_{3R_n}) - h(x_{1L_n}, x_{1R_n})]$$
$$+ \delta_{\alpha 3}\delta_{3\beta}h(x_{1L_n}, x_{1R_n})\}, \qquad (3.7)$$

$$\Delta \mathbf{m}_{\mathbf{u}\,\alpha\beta} = \frac{8}{3}\frac{\alpha_s m_t}{4\pi}\left(\frac{A + \mu\cot\beta}{M_{\tilde{g}}}\right) \times \{W_{U_L 3\alpha}W^*_{U_L 33}\, W_{U_R 3\beta}W^*_{U_R 33}$$
$$[h(x_{3L}, x_{3R}) - h(x_{3L}, x_{1R}) - h(x_{1L}, x_{3R}) + h(x_{1L}, x_{1R})]$$

[1]Neutrino masses are not discussed here.

$$+ W_{E_L 3\alpha} W^*_{E_L 33} \delta_{3\beta} [h(x_{3L}, x_{1R}) - h(x_{1L}, x_{1R})]$$
$$+ \delta_{\alpha 3}\, W_{E_R 3\beta} W^*_{E_R 33} [h(x_{1L}, x_{3R}) - h(x_{1L}, x_{1R})]$$
$$+ \delta_{\alpha 3}\, \delta_{3\beta}\, h(x_{1L}, x_{1R})\}, \tag{3.8}$$

where $x_{3L(R)_n} = \frac{m^2_{\tilde{\tau}_{L(R)}}}{M_n^2}$, $x_{1L(R)_n} = \frac{m^2_{\tilde{e}_{L(R)}}}{M_n^2}$ in the lepton sector, $x_{3L(R)} = \frac{m^2_{\tilde{t}_{L(R)}}}{M_{\tilde{g}}^2}$, $x_{1L(R)} = \frac{m^2_{\tilde{u}_{L(R)}}}{M_{\tilde{g}}^2}$ and $\Delta \mathbf{m}_{\mathbf{d}\,\alpha\beta}$ is the same as $\Delta \mathbf{m}_{\mathbf{u}\,\alpha\beta}$ with the replacements $\cot\beta \to \tan\beta, m_t \to m_b$ and $\tilde{t}, \tilde{u} \to \tilde{b}, \tilde{d}$, and where

$$h(x,y) = \frac{f(x) - f(y)}{x - y},$$
$$f(x) = \frac{x \ln x}{1 - x}. \tag{3.9}$$

Let us begin our phenomenological discussion with the lepton sector. The above expression for the radiative contribution to the lepton mass matrix is rather unwieldy; while we can use it for numerical work, in order to get an approximate feeling for the size of the radiative electron mass, we simply look at the 11 entry of the radiative correction matrix $m_e \approx \Delta \mathbf{m}_{e11}$. For simplicity, we assume that one of the neutralinos is pure bino, that the scalar tau's are degenerate with mass m and much lighter than the selectrons. Then we find as in [6]

$$m_e = \frac{\alpha m_\tau}{4\pi \cos^2\theta_W} \frac{(A + \mu \tan\beta)}{M_1} \times W_{E_L 31} W_{E_R 31} h(x_3, x_3), \tag{3.10}$$

where M_1 is the bino mass, $h(1,1) = 1/2$, and we have assumed $W_{E33} \simeq 1$. As explained in [6], we must work in the large $\tan\beta$ regime, and so we can neglect the A term contribution above. If we set $\tan\beta = 60$ and $\mu = M_1 = m$, equation (3.10) reproduces the electron mass if the product $W_{E_R 31} W_{E_L 31} \simeq 0.01$. This is roughly speaking a lower bound for this product. In this calculation we have taken the selectron to be much heavier than the stau so that the super-GIM cancellation in the loop can be ignored. In fact, however, for selectrons moderately heavier than the staus, there will be a super-GIM cancellation and $W_{E_R 31} W_{E_L 31}$ will be correspondingly larger. In Fig. 2, we give a plot for the relevant super-GIM suppression factor. Assuming left and right handed scalars degenerate, scalars of the first two generations degenerate, and the third generation scalar degenerate with the gaugino, we plot the super-GIM factor against the ratio

of first two generation to the third generation scalar masses. This implies that each of $W_{E_{R31}}, W_{E_{L31}}$ should be at least 0.1. In the following we will explore the consequences of having such large mixing angles.

$-\mu \to e\gamma$: One immediate observation is that, if in the diagram of Fig. 1(a) we replace one of the external electrons with a muon and attach a photon to the graph, we get a potentially dangerous contribution to the rare process $\mu \to e\gamma$. How dangerous is this effect? In appendix B, we present the FCNC constraints on the elements of the mixing matrices W. Requiring the $\mu \to e\gamma$ rate to be smaller than current experimental bound constrains $W_{E_{L(R)32}} W_{E_{R(L)31}}$ to be smaller than $\sim 10^{-4}$. Since we know that we need $W_{E_{L(R)31}} \sim 0.1$ in order to generate the electron mass radiatively, we must have that $W_{E_{L(R)32}} \lesssim 10^{-3}$ in order to avoid a dangerous $\mu \to e\gamma$ rate. It may seem strange that $W_{E_{L(R)}31}$ and $W_{E_{L(R)}32}$ have such disparate sizes; any theory of lepton flavor with radiatively generated electron mass must naturally explain why $W_{E_{L(R)}32}$ is so much smaller than $W_{E_{L(R)}31}$. Speaking more loosely, if the electron mass is radiative, muon number must be very nearly conserved.

$-\tau \to e\gamma$: What about the decay $\tau \to e\gamma$? Since it is a 3-1 transition, it is directly related to $W_{E_{L(R)31}}$. Under the same set of assumptions that went into the simplified equation (3.10), the amplitude for $\tau_{L(R)}$ decay is

$$F_{L(R)} = \frac{\alpha m_\tau}{4\pi \cos^2\theta_W} \frac{(A + \mu \tan\beta)}{M_1^3} \times W_{E_{L(R)}31} g(x_3, x_3), \tag{3.11}$$

where

$$\begin{aligned} g(x,y) &= \frac{f'(x) - f'(y)}{x - y}, \\ f'(x) &= \frac{x^2 - 2x\ln x - 1}{2(x-1)^3}, \end{aligned} \tag{3.12}$$

and $g(1,1) = \frac{1}{12}$. The branching ratio for $\tau \to e\gamma$ is proportional to $|W_{E_{L31}}|^2 + |W_{E_{R31}}|^2 \geq 2|W_{E_{L31}} W_{E_{R31}}|$, which is the product constrained by the requirement of obtaining radiative electron mass. Putting $\mu = M_1 = m = 300$ GeV gives $B(\tau \to e\gamma) \approx 10^{-6}$, a factor of 100 beneath the current bound. We make a more careful analysis as follows. Assuming that the left and right scalars, as well as the scalars of the first two generations are degenerate, both the radiatively generated m_e and the $\tau \to e\gamma$ rate depend on the following parameters (other

than the mixing angles) in the large $\tan\beta$ regime: $(\mu, M_1, M_2, m_{\tilde{\tau}}^2, m_{\tilde{e}}^2, \tan\beta)$ Putting $\tan\beta = 60$ and assuming the grand unification relation $M_2 \sim 2M_1$, the dependence is reduced to only $(\mu, M_1, m_{\tilde{\tau}}^2, m_{\tilde{e}}^2)$. Specifying these parameters determines what the product $|W_{E_L 31} W_{E_R 31}|$ should be to obtain the correct electron mass, and this in turn provides us with a lower bound on $B(\tau \to e\gamma)$. In Fig. 3, we give a representative contour plot for this lower bound on $B(\tau \to e\gamma)$. Over a significant portion of the parameter space, the rate is only 10-100 times smaller than the current bound $B(\tau \to e\gamma) \lesssim 1.2 \times 10^{-4}$ [9].

$-d_e$: If there are CP violating phases in the theory, we have further considerations. First, we note that if there is no mixing with the second generation (as seems to be required for avoiding dangerous $\mu \to e\gamma$), then we can choose a basis where the mixing matrices $W_{E_{L(R)}}$ are real: the only potentially complex coupling is $(\tilde{e}_L^* m_{13L}^2 \tilde{\tau}_L + h.c. + L \to R)$. Since the tree level electron Yukawa coupling is zero, we can independently rephase the superfields $e_{L(R)}$ to make $m_{13L(R)}^2$ real. Thus, the only sources of CP violation are the phases in the A and μ parameters. Ordinarily, (when no fermion masss are generated radiatively), the phases of A and μ are constrained to be small, since arbitrary phases lead to large electric dipole moments via diagrams proportional to the tree level first generation Yukawa couplings. Does the situation change when we generate the lightest generation Yukawa coupling radiatively? To answer this question, let us look at the lepton mass matrix and dipole moment matrix in the 2 dimensional space of the first and third generation (the second generation has no mixing and is thus irrelevant). For simplicity, we again consider taking the first two generation scalars much heavier than those of the third generation so that they are decoupled, and we set $\mu = M_1 = m$. Then, we have

$$\frac{\mathbf{m_e}}{m_\tau} \simeq \begin{pmatrix} .02\, W_{E_L 31} W_{E_R 31} e^{i\theta} & .02\, W_{E_L 31} e^{i\theta} \\ .02\, W_{E_R 31} e^{i\theta} & 1 \end{pmatrix},$$

$$\frac{\mathbf{d_e}}{e} \simeq 1.5 \times 10^{-21} cm \times \left(\frac{300\text{GeV}}{M_1}\right)^2 \times \begin{pmatrix} W_{E_L 31} W_{E_R 31} & W_{E_L 31} \\ W_{E_R 31} & 1 \end{pmatrix} e^{i\theta}, \tag{3.13}$$

where θ is the phase of $A + \mu \tan\beta$. We can approximately diagonalize the lepton

mass matrix as follows

$$\frac{\mathbf{m_e}}{m_\tau} \simeq V_{E_L}^* \begin{pmatrix} .02\, W_{E_L 31} W_{E_R 31} & 0 \\ 0 & 1 \end{pmatrix} V_{E_R}^\dagger,$$

$$V_{E_{L(R)}} \simeq \begin{pmatrix} e^{-i\theta/2} & .02\, W_{E_{L(R)}31} e^{-i\theta} \\ -.02\, W_{E_{L(R)}31} e^{i\theta/2} & 1 \end{pmatrix}. \tag{3.14}$$

In the basis where the lepton mass matrix is diagonal with real eigenvalues, the electric dipole moment matrix is $\mathbf{d}'_e = V_{E_L}^T \mathbf{d}_e V_{E_R}$, and the electric dipole moment of the electron is $d_e = Im(\mathbf{d}'_{e11})$. We find with $M_1 = 300$ GeV and $W_{E_L 31} W_{E_R 31} \sim .01$ (as required to generate the electron mass),

$$\frac{d_e}{e} = 6 \times 10^{-24}\text{cm} \times \sin\theta. \tag{3.15}$$

Thus, $\sin\theta$ must be smaller than $\sim 7\times 10^{-4}$ for $\frac{d_e}{e}$ not to exceed the experimental limit of 4×10^{-27} cm. So, we have not made any progress on the supersymmetric CP problem. However, as we have already mentioned, if we assume that $\sin\theta$ is sufficiently suppressed, there are no other CP violating contributions when muon number is conserved.

What if the electron mass is not all radiative in origin and has some small tree level contribution? If there is an $O(1)$ phase mismatch between the tree and radiative parts of the electron mass, there will be a phase in the electron electric dipole moment of order $\frac{m_e^{tree}}{m_e}$ even if A and μ are taken to be real. This would again give too large a dipole moment unless $\frac{m_e^{tree}}{m_e} \lesssim 10^{-3}$. (Of course, in deriving this result,we assume that most of the electron mass is radiative, otherwise there is no reason for the $W_{E_{L(R)}31}$ to be big enough to cause trouble with the dipole moment). We conclude that if there are large CP violating phase differences in the theory, the electron mass must either be nearly all radiative or nearly all tree level.

In the quark sector, in addition to the first generation quark masses, we are also interested in the possibility of generating CKM mixing angles by finite radiative corrections. Table 1 shows the relevant ratios of quark masses and mixing angles.

The constraints on SUSY FCNC have been studied in [10, 11], and the results are given in terms of $\delta_{ij} = \frac{\delta\tilde{m}_{ij}^2}{M_{\tilde{q}}^2}$, where $\delta\tilde{m}_{ij}^2$ is the off diagonal squark mass

$\frac{m_t}{m_t}$	1	$\frac{m_b}{m_t}$	1.6×10^{-2}	$\frac{m_b}{m_b}$	1
$\frac{m_c}{m_t}$	3.6×10^{-3}	$\frac{m_s}{m_t}$	4.5×10^{-4}	$\frac{m_s}{m_b}$	2.7×10^{-2}
$\frac{m_u}{m_t}$	1×10^{-5}	$\frac{m_d}{m_t}$	2×10^{-5}	$\frac{m_d}{m_b}$	1.3×10^{-3}
$\frac{\sin\theta_c m_c}{m_t}$	8×10^{-4}	$\frac{\sin\theta_c m_s}{m_t}$	1×10^{-4}	$\frac{\sin\theta_c m_s}{m_b}$	6×10^{-3}
$\frac{V_{cb} m_t}{m_t}$	4×10^{-2}	$\frac{V_{cb} m_b}{m_t}$	6.4×10^{-4}	$\frac{V_{cb} m_b}{m_b}$	4×10^{-2}
$\frac{V_{ub} m_t}{m_t}$	4×10^{-3}	$\frac{V_{ub} m_b}{m_t}$	6.4×10^{-5}	$\frac{V_{ub} m_b}{m_b}$	4×10^{-3}
$\frac{V_{td} m_t}{m_t}$	1×10^{-2}	$\frac{V_{td} m_b}{m_t}$	1.6×10^{-4}	$\frac{V_{td} m_b}{m_b}$	1×10^{-2}

Table 1: The relevant ratios of quark masses and mixing angles with all quantities taken at the scale of top quark mass. The values of quark masses, mixing angles, and the RG mass enhancement factors η_i are taken as follows: $m_t(m_t) = 168\,\mathrm{GeV}$, $m_b(m_b) = 4.15\,\mathrm{GeV}$, $m_c(m_c) = 1.27\,\mathrm{GeV}$, $m_s(1\,\mathrm{GeV}) = 180\,\mathrm{MeV}$, $m_d(1\,\mathrm{GeV}) = 8\,\mathrm{MeV}$, $m_u(1\,\mathrm{GeV}) = 4\,\mathrm{MeV}$, $\eta_b = 1.5$, $\eta_c = 2.1$, $\eta_{u,d,s} = 2.4$, $\sin\theta_c = 0.22$, $V_{cb} = 4 \times 10^{-2}$, $V_{ub} = 4 \times 10^{-3}$, $V_{td} = 1 \times 10^{-2}$.

in the super-KM basis and $M_{\tilde{q}}$ is the "universal squark mass". However, in order to generate the light generation quark masses entirely by radiative corrections, the splitting between scalar masses of the first two and the third generations must be quite large so that the super-GIM cancellation is not effective. As we can see from Fig. 2, this typically requires $\frac{\tilde{m}_1}{\tilde{m}_3} \gtrsim 3$. Then it is not clear which scalar mass should be used for $M_{\tilde{q}}$. In appendix B, we translate thes results obtained in [10, 11] into constraints directly on the mixing matrix elements, which are more suitable for our dicussions.

When $\tan\beta$ is large, some of the one-loop diagrams for the down type quark Yukawa couplings are enhanced by $\tan\beta$ (Figs. 1(c), 4(a)(b)). They can give significant corrections to the down type quark masses and CKM matrix elements[12]. Here we are interested in the possibility that some of the light generation quark masses and mixing angles are entirely generated by these loop corrections. Because of the large $\tan\beta$ enhancement, it is easier to generate CKM mixing angles in the down sector than in the up sector. In fact, we can see from Table 1 that it is impossible to generate V_{cb} in the up sector, while generating V_{ub} and θ_c requires $W_{U_L 31}$ to be greater than about 0.4 and 0.2 respectively. W_{U_L} is linked to W_{D_L} by the CKM matrix: $V_{CKM} = {W_{U_L}}^\dagger W_{D_L}$. To get the correct V_{ub}, $W_{U_L 31}$ has to be canceled by the mixing angles of the

same size in W_{D_L}, which will violate the FCNC constraints listed in Table B1. Therefore, we will only consider generating CKM mixing angles in the down sector.

The flavor diagonal gluino diagram could give large corrections to the down quark masses if the corresponding Yukawa couplings already exist at tree level. It does not generate fermion masses if they are absent at tree level, but gives large uncertainties in the tree level bottom Yukawa coupling λ_b^0, which appears in these gluino diagrams. The flavor-changing gluino diagram (through $m_b^0 \mu \tan\beta$) can give sizable down quark mass matrix elements involving light generations and therefore generate m_d and CKM mixing angles. The first chargino diagram (Fig. 4(a)) only gives significant contributions when one of the external leg is b_R, i. e., it contributes to λ_{D13}, λ_{D23}, λ_{D33}. With some unification assumptions at high scales, one usually finds the chargino contribution to the bottom quark mass is smaller than and opposite to the gluino contribution [13, 14]. Here we do not make assumptions about physics at high scales so both contributions lead to uncertainties in the tree level λ_b^0. The contributions to λ_{D13} and λ_{D23} are proportional to V_{td} and V_{ts} respectively, so they can only give corrections to the already existing mixing angles but not generate them entirely. The second chargino diagram (Fig. 4(b)) is supressed by the weak coupling constant compared with other diagrams and will be ignored. In the following we will concentrate on the possibilities that the light fermion masses and mixing angles are generated by the flavor-changing gluino diagram.

–m_u: The possibility that m_u comes from radiative corrections by mixing with the third generation has been pointed out in [7]. We can see from Fig. 2 and $\frac{m_u}{m_t}$ in Table 1 that if $W_{U_{L31}} W_{U_{R31}} \sim 10^{-3}$, m_u can be generated entirely from radiative corrections. There is no direct constraint on the 1-3 mixing. The induced splitting between the first two generation left-handed squark masses could contribute to $K - \bar{K}$ mixing. However, this constraint is easily satisfied, so it is possible that m_u is entirely radiative.

–m_d: From Fig. 2 and Table 1. we can see that to generate m_d requires $W_{D_{L31}} W_{D_{R31}} \sim 2 \times 10^{-3}$. Compared with the constraints derived from $B - \bar{B}$ mixing in Table B1(a), this requires the sfermion masses to be in the TeV range, which is somewhat uncomfortably large. In addition, if m_d does get its mass from radiative corrections, we also generate the 1-3 entry for the down Yukawa

matrix. Their ratio is:

$$\frac{\Delta\lambda_{D11}}{\Delta\lambda_{D13}} = \frac{W_{D_{L31}}W_{D_{R31}}\widetilde{\widetilde{H}}}{W_{D_{L31}}W_{D_{R33}}\widetilde{H}} < \frac{W_{D_{R31}}}{W_{D_{R33}}} \lesssim 0.1, \tag{3.16}$$

for $m_{\tilde{b}} \sim 1\text{TeV}$, assuming $W_{D_{R33}} \simeq 1$, where $\widetilde{\widetilde{H}} = h(x_{3L}, x_{3R}) - h(x_{3L}, x_{1R}) - h(x_{1L}, x_{3R}) + h(x_{1L}, x_{1R})$, $\widetilde{H} = h(x_{3L}, x_{3R}) - h(x_{1L}, x_{3R})$, and h, $x_{1(3)L(R)}$ are defined in (3.8), (3.9). On the other hand, $\frac{m_d}{V_{ub}m_b} \simeq 0.3$. We see that the generated $\Delta\lambda_{D13}$ gives a too big contribution to V_{ub} which has to be canceled by a tree level λ_{D13}.

We now discuss the possibilities for radiative generation of CKM elements. We take the independent parameters of the CKM matrix to be V_{us}, V_{ub}, V_{cb} and the CP violating phase.

–θ_c: To generate θ_c we need $W_{D_{L31}}W_{D_{R32}} \sim 10^{-2}$, assuming $W_{D_{L(R)33}} \simeq 1$. From $B - \bar{B}$ mixing and $b \to s\gamma$ decay, or $K - \bar{K}$ mixing alone, the sfermion masses are also required to be $\gtrsim 1\text{TeV}$ in order to satisfy these constraints. Furthermore the phase of $W_{D_{L31}}W_{D_{R32}}$ has to be small ($< 10^{-1}$) from the ϵ parameter of CP violation. Similar to the case of m_d, generating θ_c radiatively may also give a too big contribution to V_{ub}. If we try to generate m_d, θ_c, and V_{ub} all by radiative corrections, ignoring the difference between $\widetilde{H}$ and $\widetilde{\widetilde{H}}$, we obtain the following ratio for the mixing matrix elements from Table 1:

$$W_{D_{R33}} : W_{D_{R32}} : W_{D_{R31}} \simeq V_{ub}m_b : \sin\theta_c m_s : m_d \simeq 4 : 6 : 1.3. \tag{3.17}$$

By unitarity we obtain

$$W_{D_{R33}} \simeq 0.55,\ W_{D_{R32}} \simeq 0.82,\ W_{D_{R31}} \simeq 0.18. \tag{3.18}$$

(Taking into account that $\widetilde{H} > \widetilde{\widetilde{H}}$ gives larger $W_{D_{R32}}$, $W_{D_{R31}}$.) From Table B1, we can see that $m_{\tilde{b}}$ has to be pushed above 2 TeV (even higher for the first two generations) to satisfy the constraints from both ΔM_K and $b \to s\gamma$. If there are O(1) phases in these W's, the ϵ constraints raise the lower limit of the squark masses to ~ 20 TeV, which is unacceptably large. Furthermore, it is unnatural for models to have such a large $W_{D_{R32}}$ mixing. Therefore, it is unlikely that all CKM matrix elements can be generated by radiative corrections.

–V_{ub}: To generate V_{ub} we need $W_{D_{L31}} \sim 5 \times 10^{-3}$, which easily satisfies the $B - \bar{B}$ mixing constraints. Hence V_{ub} can be generated radiatively, but as we

learned from above, V_{ub} and θ_c cannot both come from radiative corrections, and neither can V_{ub} and m_d.

–V_{cb}: Attaching a photon to the diagram which generates Δm_{D23} gives a diagram contributing to the decay $b \to s\gamma$. Hence one can write down the following simple relation between gluino diagram contributions to V_{cb} and to the Wilson coefficient $c_7(M_W)$ [15] for $b \to s\gamma$,

$$\Delta c_7(M_W) = q_D \frac{4\pi}{\alpha} \sin^2\theta_W \frac{M_W^2}{m_{\tilde{g}}^2} \frac{\tilde{G}}{\widetilde{H}} \frac{\Delta m_{D23}}{V_{cb} m_b}, \tag{3.19}$$

$$\Rightarrow \frac{\eta^{16/23}\Delta c_7(M_W)}{c_7(m_b)_{\rm SM}} \simeq (\frac{8m_W}{m_{\tilde{g}}})^2 (\frac{5\tilde{G}}{\widetilde{H}}) (\frac{\Delta m_{D23}}{V_{cb} m_b}). \tag{3.20}$$

where $\tilde{G} = g(x_{3L}, x_{3R}) - g(x_{1L}, x_{3R})$, and g is defined in (3.12). The gluino diagram contribution to $b \to s\gamma$ interferes constructively with the Standard Model contribution if V_{cb} is generated by the similar gluino diagram. Therefore, generating V_{cb} radiatively requires heavy gluino and squark masses ($\gtrsim 1$ TeV) or cancellation between the chargino diagram contributions to $b \to s\gamma$ and other contributions.

–CP-violating phases: From the above discussion we found that it is very difficult to generate all CKM mixing matrix elements by radiative corrections. This means that a non-trivial CKM matrix should occur at tree level. There is one physical CP-violating phase in V_{CKM}, and several more in the quark-squark-gaugino mixing matrices. The number of CP-violating phases in the quark sector (not including the possible phases of the parameters A and μ) is counted as in the following. There are four unitary mixing matrices W_{U_L}, W_{U_R}, W_{D_L}, W_{D_R}, ($V_{CKM} = {W_{U_L}}^\dagger W_{D_L}$ is not independent,) connecting 7 species of quark and squark fields u_L, d_L, u_R, d_R, $\tilde{Q}$, $\tilde{U}$, $\widetilde{D}$. Among the phases of these fields, 6 are fixed by the 6 eigenvalues of the Yukawa matrices $\boldsymbol{\lambda}_U$ and $\boldsymbol{\lambda}_D$ (if there are no zero eigenvalues), one overall phase is irrelevant, so we can remove 14 of the 24 phases in the W's by phase redefinition of the quark and squark fields. Each massless quark removes one more phase by allowing independent phase rotations on the left and right quark fields. Each pair of degenerate quarks or squarks of the same species removes one phase as well. Assuming m_u and m_d massless at the tree level, and degeneracies between the first two generation squarks, we can remove 5 more phases and there are still 5 independent phases left. One of

them cannot be moved to the W_U's and it can give significant contributions to the CP violation effects in the K and B systems.

4 Guidelines for model building

In the introduction and in [6] we indicated some general features effective theories of flavor should have in order to generate radiative fermion masses. In particular, we pointed out that, in supersymmetric theories, an accidental superpotential symmetry is needed to ensure that the first generation is massless at tree-level, while this symmetry must be broken by D terms in order to obtain radiative masses. For instance, in the effective lepton models considered in [6], all holomorphic and flavor symmetric operators possess an accidental $U(1)_{\ell_1} \times U(1)_{e_1}$ which is violated by the D-terms. From the point of view of an effective theory, then, it is representation content and holomorphy which are responsible for accidental symmetries for every possible superpotential operator, thereby forbidding some Yukawa couplings. However, this is by no means a necessary condition for the existence of tree level massless fermions: We do not always generate every operator consistent with symmetries when we integrate out heavy states. Thus, the condition that every effective operator in the superpotential possess an accidental symmetry is clearly too strong; we only need an accidental symmetry to exist for those operators induced by integrating out heavy states. For this reason, it seems that a deeper understanding of the accidental symmetries lies in examining the full theory, including superheavy states. This is our purpose in this section. We will find simple, sufficient conditions for guaranteeing the existence of tree level massless states after integrating out heavy states. We will also describe (in view of later application to the quark sector) the structure of the tree-level CKM matrix. These conditions will serve as convenient guides for the explicit models we construct in the next section.

We begin by considering sufficient conditions for the existence of tree level massless states. Consider the lepton sector for simplicity. In Froggatt-Nielsen schemes, we have fields ℓ_α, e_α ($\alpha = 1, 2, 3$) which would be the 3 low energy left and right handed lepton fields in the flavor symmetric limit. However, there are also superheavy states with which ℓ and e mix after flavor symmetry breaking. In general, we have vector-like superheavy states $(L_i \oplus \bar{L}_i)$ and $(E_a \oplus \bar{E}_a)$, $(i =$

$4, ..., n+3$, $a = 4, ..., m+3$), with L, E having the same gauge quantum numbers as ℓ, e respectively, and with the barred fields having conjugate gauge quantum numbers. We also have a set of gauge singlet fields ϕ with VEV's $\langle\phi\rangle$ breaking the flavor group G_f. In the superpotential, we have bare mass terms for the $(L, \bar{L})$ and the $(E, \bar{E})$ fields, as well as trilinear couplings mixing ϕ's with light and superheavy states. We also have a large Yukawa matrix $\boldsymbol{\Lambda}_{IA}$ ($I = 1, ..., n+3$, $A = 1, ..., m+3$), connecting the down-type Higgs h_d to the (ℓ_α, L_i) and (e_α, E_a),

$$W \supset \begin{pmatrix} \ell & L \end{pmatrix} \boldsymbol{\Lambda} \begin{pmatrix} e \\ E \end{pmatrix} h_d. \tag{4.1}$$

Once the fields ϕ develop VEV's, we will have mass terms like, $\ell\,\langle\phi\rangle\,\bar{L}$ mixing light and heavy states. In order to diagonalize the bare mass matrix and go from the flavor basis to the mass basis (where "light" and "heavy" are correctly identified), we must make appropriate $\langle\phi\rangle$ dependent unitary rotations on the fields:

$$\begin{pmatrix} \ell' \\ L' \end{pmatrix} = U_L(\langle\phi\rangle) \begin{pmatrix} \ell \\ L \end{pmatrix}, \; \bar{L}' = U_{\bar{L}}(\langle\phi\rangle)\bar{L},$$

$$\begin{pmatrix} e' \\ E' \end{pmatrix} = U_E(\langle\phi\rangle) \begin{pmatrix} e \\ E \end{pmatrix}, \; \bar{E}' = U_{\bar{E}}(\langle\phi\rangle)\bar{E}. \tag{4.2}$$

In this basis, the mass terms are $\sum_{i=4}^{n+3} M_i \bar{L}'_i L'_i + \sum_{a=4}^{m+3} M_a \bar{E}'_a E'_a$, and the Yukawa matrix becomes

$$\boldsymbol{\Lambda}'_{IA} = U_L^*(\langle\phi\rangle)_{IJ} \boldsymbol{\Lambda}_{JB} U_E^\dagger(\langle\phi\rangle)_{BA}, \tag{4.3}$$

where summation over J and B is understood. In order to integrate out the (now correctly identified) heavy states at tree level, we simply throw out any coupling involving them. The Yukawa matrix $\boldsymbol{\lambda}$ for the three low energy generation leptons is then

$$\boldsymbol{\lambda}_{\alpha\beta} = U_L^*(\langle\phi\rangle)_{\alpha J} \boldsymbol{\Lambda}_{JB} U_E^\dagger(\langle\phi\rangle)_{B\beta}, \; (\alpha, \beta = 1, 2, 3). \tag{4.4}$$

We would now like to understand circumstances under which we can guarantee a certain number of zero eigenvalues for $\boldsymbol{\lambda}$. For $\boldsymbol{\lambda}$ to have $k \leq 3$ zero eigenvalues, its rank must be $3 - k$. To see when this is possible, we make the simple observation that each row (or alternatively each column) of $\boldsymbol{\Lambda}$ contributes

one rank to $\boldsymbol{\lambda}$. Consider for instance the contribution to $\boldsymbol{\lambda}$ from the J_0'th row of $\boldsymbol{\Lambda}$. Defining

$$x_\alpha = U^*_{L_{\alpha J_0}},\ y_\beta = \boldsymbol{\Lambda}_{J_0 B} U^\dagger_{E_{B\beta}},$$

we have

$$\boldsymbol{\lambda}^{\text{from row } J_0}_{\alpha\beta} = x_\alpha y_\beta, \tag{4.5}$$

which is manifestly rank 1. Define a non-zero row (column) of $\boldsymbol{\Lambda}$ to be a row (column) with at least one non-zero entry. Then, it is clear that a *sufficient* condition for $\boldsymbol{\lambda}$ to have rank $3-k$ is that the number of non-zero rows (or the number of non-zero columns) of $\boldsymbol{\Lambda}$, up to rotations, equal $3-k$, i. e., $\boldsymbol{\Lambda}$ also has rank $3-k$; since in this case $\boldsymbol{\lambda}$ is of the form

$$\boldsymbol{\lambda}_{\alpha\beta} = \sum_{J=1}^{3-k} x^J_\alpha y^J_\beta, \tag{4.6}$$

which is manifestly rank $3-k$ (the case of interest to us is $k=1$). We will make use of this criterion in the following section.

We next turn to examining the tree-level CKM matrix in the quark sector. In analogy to the lepton sector, we have Yukawa matrices $\boldsymbol{\Lambda}_D$ and $\boldsymbol{\Lambda}_U$,

$$W \supset \begin{pmatrix} q & Q \end{pmatrix} \boldsymbol{\Lambda}_U \begin{pmatrix} d \\ D \end{pmatrix} h_d + \begin{pmatrix} q & Q \end{pmatrix} \boldsymbol{\Lambda}_D \begin{pmatrix} u \\ U \end{pmatrix} h_u, \tag{4.7}$$

where all new fields are in obvious analogy with the lepton sector. Let us assume that the general condition stated above, ensuring the existence of a massless eigenvalue for $\boldsymbol{\lambda}_D$ and $\boldsymbol{\lambda}_U$, is realized by $\boldsymbol{\Lambda}_D$ and $\boldsymbol{\Lambda}_U$. Then, we can write

$$\boldsymbol{\lambda}_{D\alpha\beta} = x^1_\alpha y_\beta + x^2_\alpha z_\beta,\ \boldsymbol{\lambda}_{U\alpha\beta} = x'^1_\alpha y'_\beta + x'^2_\alpha z'_\beta, \tag{4.8}$$

Suppose in particular that $\boldsymbol{\Lambda}_D$ and $\boldsymbol{\Lambda}_U$ have nontrivial entries in the same two rows, in which case we can choose $x^i_\alpha = x'^i_\alpha$, $i = 1, 2$. Then, the resulting CKM matrix has non-zero entries only in the 2-3 sector. The reason is that, since the first generation is massless, we can always choose a basis where the first generation quark doublet has no component of superheavy quark doublets with Yukawa couplings, and so both $\boldsymbol{\lambda}_D$ and $\boldsymbol{\lambda}_U$ are only non-zero in the lower 2×2 block. We can see this more explicitly as follows. First note that we can make a rotation on the left handed quarks to make x^1_α point in the 3 direction, and

make independent rotations on the right-handed up and down quarks to make y_β and y'_β also point in the 3 direction. In this basis, we have

$$\boldsymbol{\lambda}_{D\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \eta \end{pmatrix}_{\alpha\beta} + x_\alpha^2 z_\beta, \; \boldsymbol{\lambda}_{U\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \eta' \end{pmatrix}_{\alpha\beta} + x_\alpha^2 z'_\beta \tag{4.9}$$

However, we can always make rotations on the upper 2×2 block so that x^2, z, z' have 0 entries in the first component. Using equation (4.9), we easily see that both $\boldsymbol{\lambda}_D$ and $\boldsymbol{\lambda}_U$ are only non-zero in the lower 2×2 block, and CKM mixing only occurs in the 23 sector, as claimed. Thus, in order to have, for example, a tree level θ_c or V_{ub} (as is necessary from our discussion in section 3), we must ensure that $\boldsymbol{\Lambda}_D$ and $\boldsymbol{\Lambda}_U$ do not have entries in the same two rows. Other than this case, we expect generically that all elements of the CKM matrix exist at tree level.

In this section we have shown that if the Higgs couples in only 2 rows or 2 columns of the full Yukawa matrix to matter, then there will be a light generation which is massless at tree level. The required sparseness of Higgs couplings is due to G_f and holomorphy.

5 Realistic models for radiative fermion masses

In [6], we gave explicit lepton models of flavor with radiative electron mass, which naturally fulfilled the phenomenological requirements of Sec. 3; namely, the electron is massless at tree level, the muon picks up a tree level mass upon integrating out heavy states, muon number is conserved, and D terms yiels $e-\tau$ mixing which generates a radiative electron mass. In this section, our purpose is to give an extension to the quark sector. We begin by reviewing the lepton model most readily extended to the quark sector, the full model with flavor group $G_f = SU(2)_\ell \times SU(2)_e \times U(1)_A$. The fields are categorized as light/heavy and matter/Higgs in Table 2.

We require the theory to be invariant under matter parity (*Matter* $\rightarrow$ $-$*Matter*) and heavy parity (*Heavy* $\rightarrow$ $-$*Heavy*). Here, matter parity is crucial to avoid dangerous R-parity violating couplings, but the heavy parity is imposed

	Light	Heavy
Matter	$\ell_3(0), \ell_I(+1)$	$L(+2), L_I(+1), \bar{L}(-2), \bar{L}^I(-1)$
	$e_3(0), e_i(-1)$	$E(-2), E_i(-1), \bar{E}(+2), \bar{E}^i(+1)$
Higgs	$h(0)$	$\phi_{\ell I}(+1), \phi_{ei}(-1), S(0)$

Table 2: Field content and G_f transformation properties for the lepton model. I, i are $SU(2)_\ell$ and $SU(2)_e$ indices respectively, the numbers in brackets are the $U(1)_A$ charges.

only for simplicity.[2] Requiring these discrete symmetries and G_f invariance gives us the following renormalizable superpotential (where all dimensionless couplings are $O(1)$)

$$\begin{aligned} W = \lambda_3 l_3 e_3 h + \lambda_4 L E h \\ + f_1 l_3 \bar{L}^I \phi_{\ell I} + f_2 \ell_I \bar{L}^I S + f_3 \ell_I \epsilon^{IJ} \phi_{\ell J} \bar{L} \\ + f_1' e_3 \bar{E}^i \phi_{ei} + f_2' e_i \bar{E}^i S + f_3' e_i \epsilon^{ij} \phi_{ej} \bar{E} \\ + M_L \bar{L} L + M_{L_I} \bar{L}^I L_I + M_E \bar{E} E + M_{E_i} \bar{E}^i E_i. \end{aligned} \tag{5.1}$$

Note that this superpotential has only two Yukawa couplings λ_3 (for the τ) and λ_4 (for the superheavy L, E). Therefore, using the results of the last section, we are guaranteed to have a tree-level massless state after we integrate out the heavy fields;[3] we identify this state with the electron.

The fields ϕ_ℓ, ϕ_e and S take VEV's which break the flavor symmetries. We can assume without loss of generality that $\langle \phi_\ell \rangle = (v_\ell, 0), \langle \phi_e \rangle = (v_e, 0)$. As described generally in the previous section, these VEV's mix the light and heavy states and we must rotate to the mass basis where "light" and "heavy" are properly identified. An approximation to the resulting rotation on the Yukawa

[2]However, both of these parities are automatic in the $SU(3)_\ell \times SU(3)_e$ models considered in [6]. The $U(1)_A$ factor in G_f also finds a natural explanation in these theories. We do not use the $SU(3)$ theories here as a starting point here because the requisite modifications to go to the quark sector are more difficult to see than in the $SU(2)_\ell \times SU(2)_e \times U(1)_A$ model we are considering.

[3]Actually, in this theory the existence of a massless state can already be seen in the effective theory as described in [6].

matrix is shown in Fig. 5, and we generate the following superpotential term for the light fields:

$$\begin{aligned}\Delta W &= (\frac{f_3 \ell_I \epsilon^{IJ} \langle \phi_{\ell_J} \rangle}{M_L}) \lambda_4 h (\frac{f_3' e_i \epsilon^{ij} \left\langle \phi_{e_j} \right\rangle}{M_E}) \\ &= \lambda_4 (\frac{f_3 v_\ell}{M_L})(\frac{f_3' v_e}{M_E}) \ell_2 e_2 h, \end{aligned} \tag{5.2}$$

so, we can identify (ℓ_2, e_2) with the muon and (ℓ_1, e_1) with the electron.

Let us look at the above rotation more directly [16]. Setting ϕ_ℓ, ϕ_e, S to their VEV's gives the follwing mass terms in the superpotential:

$$W_{mass} = M_L \bar{L}(L + \epsilon_\ell \ell_2) + M_{L_I} \bar{L}^1 (L_1 + \epsilon_\ell' \ell_3 + \epsilon_\ell'' \ell_1) + M_{L_I} \bar{L}^2 (L_2 + \epsilon_\ell'' \ell_2), \tag{5.3}$$

plus similar terms for the E's, where $\epsilon_\ell = -\frac{f_3 v_\ell}{M_L}$, $\epsilon_\ell' = \frac{f_1 v_\ell}{M_{L_I}}$, $\epsilon_\ell'' = \frac{f_2 \langle S \rangle}{M_{L_I}}$. Thus, the mass basis is related to the flavor basis via $\ell' = U_\ell \ell$, where $\ell^{(\prime)T} = (\ell_1, \ell_2, \ell_3, L, L_1, L_2)^{(\prime)}$. To a first approximation, we have

$$U_\ell = \begin{pmatrix} 1 & 0 & 0 & 0 & -\epsilon_\ell''^* & 0 \\ 0 & 1 & 0 & -\epsilon_\ell^* & 0 & -\epsilon_\ell''^* \\ 0 & 0 & 1 & 0 & -\epsilon_\ell'^* & 0 \\ 0 & \epsilon_\ell & 0 & 1 & 0 & 0 \\ \epsilon_\ell'' & 0 & \epsilon_\ell' & 0 & 1 & 0 \\ 0 & \epsilon_\ell'' & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5.4}$$

Completely similar statements hold for the e's. Now, in the original flavor basis, the Yukawa matrix $\boldsymbol{\Lambda}$ is

$$\boldsymbol{\Lambda} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{5.5}$$

After rotating to the mass basis, we have

$$\boldsymbol{\Lambda}' = U_\ell^* \boldsymbol{\Lambda} U_e^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon_\ell \epsilon_e \lambda_4 & 0 & -\epsilon_\ell \lambda_4 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & \epsilon_e' \lambda_3 & 0 \\ 0 & -\epsilon_e \lambda_4 & 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & \epsilon_\ell' \lambda_3 & 0 & \epsilon_\ell' \epsilon_e' \lambda_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{5.6}$$

Dropping all couplings to the heavy states, we obtain the low energy Yukawa matrix $\boldsymbol{\lambda}$,

$$\boldsymbol{\lambda} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_\ell \epsilon_e \lambda_2 & 0 \\ 0 & 0 & \lambda_1 \end{pmatrix}, \tag{5.7}$$

just as we found earlier.

Note that the VEV's $\langle \phi_\ell \rangle$ and $\langle \phi_e \rangle$ do not completely break G_f; the generator

$$T_\mu = T_{U(1)_A} - 2(T_\ell^3 - T_e^3) \tag{5.8}$$

annihilates both $\langle \phi_\ell \rangle$ and $\langle \phi_e \rangle$, and corresponds to the muon number:[4]

$$e^{i\theta T_\mu} \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix} = \begin{pmatrix} \ell_1 \\ e^{2i\theta} \ell_2 \end{pmatrix}, \; e^{i\theta T_\mu} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} e_1 \\ e^{-2i\theta} e_2 \end{pmatrix}. \tag{5.9}$$

We now have most of what we want; we need only show that the required mixing between the τ and e is generated in the scalar mass matrix. We can generate D term mixings upon integrating out heavy states [16]. The one in Fig. 6 gives

$$\frac{f_2 \ell_I \langle S \rangle}{M_{L_I}} \frac{f_1^* \ell_3^\dagger \langle \phi_{\ell 1} \rangle}{M_{L_I}} = \frac{f_2 \langle S \rangle}{M_{L_I}} \frac{f_1^* v_\ell^*}{M_{L_I}} \ell_1 \ell_3^\dagger. \tag{5.10}$$

Note that this term explicitly breaks the $U(1)_{\ell_1}$ chiral symmetry associated with the zero tree-level Yukawa coupling of the electron, so we expect the required mixing between $\tilde{\tau}$ and $\tilde{e}$ to occur. Let us check it more explicitly. The D-term

[4]The $U(1)_A$ factor in G_f can be replaced with its Z_4 subgroup and still avoid dangerous muon number violating processes; after the VEV's are taken there is a symmetry under $(\ell_2, e_2) \to (-\ell_2, -e_2)$ which still forbids mixing between the scalar μ and τ, e, therefore avoiding the dangerous $\mu \to e\gamma$ decay.

part of the lagrangian is $\int d^4\theta(\phi^\dagger\phi+\theta^2\bar{\theta}^2\phi^\dagger m^2\phi)$, where ϕ is a collection of all the fields and m^2 is the soft supersymmetry breaking scalar mass matrix. When we rotate to the mass basis, we send $\phi \to U\phi$. Under this rotation, $\phi^\dagger\phi$ is invariant, but $m^2 \to Um^2U^\dagger$.[5]. In our example, the scalar mass term for the left-handed lepton fields is $\ell^\dagger m_\ell^2 \ell$, with $m_\ell^2 = \mathrm{diag}(m_{\ell_I}^2, m_{\ell_I}^2, m_{\ell_3}^2, m_L^2, m_{L_I}^2, m_{L_I}^2)$. The scalar mass matrix for the three low energy generations is then

$$\begin{aligned}
\mathbf{m}_{\ell_{\alpha\beta}}^{2\,(3\times3)} &= (U_\ell m_\ell^2 U_\ell^\dagger)_{\alpha\beta} \\
&= \begin{pmatrix} m_{\ell_3}^2 + |\epsilon_\ell'|^2 m_{L_I}^2 & 0 & \epsilon_\ell'^* \epsilon_\ell'' m_{L_I}^2 \\ 0 & m_{\ell_I}^2 + |\epsilon_\ell|^2 m_L^2 + |\epsilon_\ell''|^2 m_{L_I}^2 & 0 \\ \epsilon_\ell' \epsilon_\ell''^* m_{L_I}^2 & 0 & m_{\ell_I}^2 + |\epsilon_\ell''|^2 m_{L_I}^2 \end{pmatrix}.
\end{aligned} \tag{5.11}$$

The zero entries in the above matrix are a consequence of the unbroken $U(1)_\mu$ symmetry of the theory. We can explicitly see the 1-3 entry generated in the scalar mass matrix, which, together with the corrseponding 1-3 entry in the the right-handed scalar mass matrix, is responsible for generating the radiative electron mass.

There are two difficulties when we try to extend the lepton model for radiative electron mass to the quark sector. First, the radiative down quark mass is severely constrained by $B - \bar{B}$ mixing as we showed in Sec. 3. This can be resolved if the SUSY-breaking masses are heavy enough ($\gtrsim$ 1 TeV). The other problem is that in addition to the quark masses, we also have to get the correct CKM mixing matrix. As we have shown in Sec. 3, it is very difficult to generate all CKM mixing matrix elements: squark masses have to be pushed up to unacceptably high scales and unnatural flavor mixing gaugino interactions are needed. Excluding that possibility, one has to put in some mixing angles at tree level. In subsection 5.1 we present a model in which all first generation fermion masses come from radiative corrections. In subsection 5.2 we construct a model

[5]This is not strictly speaking correct, since supersymmetry breaking can affect the rotation to the mass basis. For instance, in Fig. 6, we could attach spurions θ^2 and $\bar{\theta}^2$ to the superpotential vertices, obtaining a direct contribution to the scalar mass matrix of order $|A|^2$, where A is the trilinear soft term associated with the superpotential vertex. Put another way, we can have spurions θ^2 in the rotation matrix U, and get contributions to the scalar masses from rotating $\phi^\dagger\phi$. These contributions are of the same order as the ones we are discussing, but do not affect any of our results.

in which m_e and m_u come from radiative corrections while m_d and θ_c appear at tree level with the prediction $\sin\theta_c = \sqrt{m_d/m_s}$. We show that this model can be naturally embedded in the flipped $SU(5)$ grand unified theory.

5.1 A complete model for radiative first generation fermion masses

The complete model for quarks and leptons is based on the same flavor group $G_f = SU(2)_l \times SU(2)_r \times U(1)_A$ as in the lepton model. However, a minimal direct extension of the lepton model to the quark sector does not give tree level CKM mixing angles. Following the guidelines to generate tree level θ_c and V_{ub} in Sec. 4, we need to introduce two heavy left-handed $SU(2)_l$ singlet quarks Q, Q' (and their conjugates $\bar{Q}$, $\bar{Q}'$).[6] Their $U(1)_A$ charges are assigned such that Q only couples to the up-type Higgs but not the down-type Higgs and vice versa for Q'. In addition, there cannot be an unbroken $U(1)$ left in the quark sector, so we introduce a second $SU(2)_l$ doublet ϕ_l', and a second $SU(2)_r$ doublet ϕ_r', whose VEV's are in different directions from the directions of ϕ_l and ϕ_r VEV's, breaking G_f completely. The field content and G_f transformation properties of the quark sector are shown in Table 3. We also impose matter-parity and heavy parity. The VEV's of ϕ, ϕ' and S are assumed to take the most general form:[7]

	Light	**Heavy**		
	$u_3(0)$, $u_i(-1)$	$U(-2)$, $\bar{U}(+2)$,		$U_i(-1)$, $\bar{U}^i(+1)$
Matter	$q_3(0)$, $q_I(+1)$	$Q(+2)$, $\bar{Q}(-2)$,	$Q'(0)$, $\bar{Q}'(0)$,	$Q_I(+1)$, $\bar{Q}^I(-1)$
	$d_3(0)$, $d_i(+1)$		$D(0)$, $\bar{D}(0)$,	$D_i(+1)$, $\bar{D}^i(-1)$
Higgs	$h_u(0)$, $h_d(0)$	$\phi_{lI}(+1)$, $\phi_{ri}(-1)$,	$\phi_{lI}'(-1)$, $\phi_{ri}'(+1)$,	$S(0)$

Table 3: Field content and G_f transformation properties of the quark sector. I and i are $SU(2)_l$ and $SU(2)_r$ doublet indices and the numbers in brackets are $U(1)_A$ charges.

[6]Second pairs of heavy U', $\bar{U}'$ and D', $\bar{D}'$ are not included in our discussion. They can be added as long as their $U(1)_A$ charge assignments forbid their Yukawa interactions with the Q's and Higgses.

$$\langle \phi_{lI} \rangle = \begin{pmatrix} v_{l0} \\ 0 \end{pmatrix}, \ \langle \phi_{ri} \rangle = \begin{pmatrix} v_{r0} \\ 0 \end{pmatrix},$$

$$\langle \phi'_{lI} \rangle = \begin{pmatrix} v_{l1} \\ v_{l2} \end{pmatrix}, \ \langle \phi'_{ri} \rangle = \begin{pmatrix} v_{r1} \\ v_{r2} \end{pmatrix}, \ \langle S \rangle = v_s. \tag{5.12}$$

Because we are dealing with a full theory, we restrict ourselves to renormalizable interactions only and all possible renormalizable interactions consistent with the symmetries are included. Nonrenormalizable interactions are assumed to be absent or suppressed enough so that they can be ignored. The G_f transformation properties of the up sector are identical to those of the lepton model so the analysis is exactly the same as in the lepton model. The superpotential for the up sector is

$$\begin{aligned} W_u = & \lambda_{u3} q_3 h_u u_3 + \lambda_{u4} Q h_u U \\ & + f_{q1} q_3 \bar{Q}^I \phi_{lI} + f_{q2} q_I \bar{Q}^I S + f_{q3} \epsilon^{IJ} q_I \bar{Q} \phi_{lJ} \\ & + f_{u1} u_3 \bar{U}^i \phi_{ri} + f_{u2} u_i \bar{U}^i S + f_{u3} \epsilon^{ij} u_i \bar{U} \phi_{rj} \\ & + M_U \bar{U} U + M_{U_i} \bar{U}^i U_i + M_Q \bar{Q} Q + M_{Q_I} \bar{Q}^I Q_I. \end{aligned} \tag{5.13}$$

Note that although we introduce another pair of G_f breaking fields ϕ'_{lI} and ϕ'_{ri}, they do not have renormalizable interactions with the up sector and the lepton sector. The only such G_f invariant interactions

$$L\bar{L}^I \phi'_{lI}, \ E\bar{E}^i \phi'_{ri}, \ Q\bar{Q}^I \phi'_{lI}, \ U\bar{U}^i \phi'_{ri} \tag{5.14}$$

are forbidden by heavy parity. Therefore, we do not generate muon number violating operators even though G_f is completely broken.

The superpotential of the down sector is given by

$$\begin{aligned} W_d = & \lambda_{d3} q_3 h_d d_3 + \lambda_{d4} Q' h_d D \\ & + f'_{q3} \epsilon^{IJ} q_I \bar{Q}' \phi'_{lJ} + f'_{q4} q_3 \bar{Q}' S \\ & + f_{d1} d_3 \bar{D}^i \phi'_{ri} + f_{d2} d_i \bar{D}^i S + f_{d3} \epsilon^{ij} d_i \bar{D} \phi_{rj} + f_{d4} d_3 \bar{D} S \\ & + M_D \bar{D} D + M_{D_i} \bar{D}^i D_i + M_{Q'} \bar{Q}' Q'. \end{aligned} \tag{5.15}$$

[7] ϕ_{lI}, ϕ_{ri} can be put in this form by $SU(2)_l$ and $SU(2)_r$ rotations, then ϕ'_{lI}, ϕ'_{ri} VEV's will take the general directions if there are no alignments between ϕ'_{lI}, ϕ'_{ri} and ϕ_{lI}, ϕ_{ri}. In this paper we do not specify the origin of these VEV's.

The f_{d1} and f_{d2} couplings are responsible for the D-term mixing between d_3 and d_i, $i = 1,2$ (with intermediate $\bar{D}^i$). f_{d3}, f_{d4} mix d_2, d_3 with D, f'_{q3}, f'_{q4} mix q_1, q_2, q_3 with Q' and they are responsible for generating tree level Yukawa couplings among d_2, d_3, and q_1, q_2, q_3 with h_d. After integrating out the heavy states, we obtain the following tree level Yukawa matrices for the up quarks and down quarks:

$$\boldsymbol{\lambda}_U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_{q2}\epsilon_{u2}\lambda_{u4} & 0 \\ 0 & 0 & \lambda_{u3} \end{pmatrix}, \boldsymbol{\lambda}_D = \begin{pmatrix} 0 & \epsilon'_{q1}\epsilon_{d2}\lambda_{d4} & \epsilon'_{q1}\epsilon_{d3}\lambda_{d4} \\ 0 & \epsilon'_{q2}\epsilon_{d2}\lambda_{d4} & \epsilon'_{q2}\epsilon_{d3}\lambda_{d4} \\ 0 & \epsilon'_{q3}\epsilon_{d2}\lambda_{d4} & \epsilon'_{q3}\epsilon_{d3}\lambda_{d4} + \lambda_{d3} \end{pmatrix}, \quad (5.16)$$

where,

$$\begin{aligned} \epsilon_{q2} &= \frac{f_{q3}v_{l0}}{M_Q}, \ \epsilon_{u2} = \frac{f_{u3}v_{r0}}{M_U}, \\ \epsilon'_{q1} &= -\frac{f'_{q3}v_{l2}}{M_{Q'}}, \ \epsilon'_{q2} = \frac{f'_{q3}v_{l1}}{M_{Q'}}, \ \epsilon'_{q3} = \frac{f'_{q4}v_s}{M_{Q'}}, \\ \epsilon_{d2} &= \frac{f_{d3}v_{r0}}{M_D}, \ \epsilon_{d3} = \frac{f_{d4}v_s}{M_D}. \end{aligned}$$

Both matrices are of rank 2, as guaranteed by the theorem of Sec. 4, (although this cannot be seen from the effective theory point of view). Now we have a massless state in each of the up and down sectors and all mixing angles are generated at tree level. m_u and m_d are then generated radiatively by the mixings between the first and the third generations induced by f_{q1}, f_{q2}, f_{u1}, f_{u2}, and f_{d1}, f_{d2} with intermediate $\bar{Q}'$, $\bar{U}$, and $\bar{D}$ states. f_{d3}, f_{d4}, f'_{q3}, f'_{q4} also induce the D term mixings among generations with intermediate $\bar{D}$ and $\bar{Q}'$ states. For example, the mixing between q_3 and q_2 is $\sim \epsilon'_{q3}\epsilon'_{q2}$, which is about the same size as the corresponding CKM mixing angle. For large $\tan\beta$ they can give sizable corrections [$\mathcal{O}(50\%)$] to the CKM matrix elements. Since we do not know the exact size and the sign of these corrections, if we just take m_s, $\sin\theta_c$ and V_{cb} to be approximately equal to the tree level results, then we have [within $\mathcal{O}(50\%)$ accuracy]

$$\begin{aligned} V_{cb} &\simeq \epsilon'_{q2}\epsilon_{d3}\frac{\lambda_{d4}}{\lambda_{d3}} \simeq 4 \times 10^{-2}, \\ \frac{m_s}{m_b} &\simeq \epsilon'_{q2}\epsilon_{d2}\frac{\lambda_{d4}}{\lambda_{d3}} \simeq 2.7 \times 10^{-2}, \end{aligned}$$

$$\sin\theta_c \simeq \frac{\epsilon'_{q1}\epsilon_{d2}}{\epsilon'_{q2}\epsilon_{d2}} \simeq 0.22 \tag{5.17}$$

Combining the above relations, we obtain the approximate tree level V_{ub}

$$V_{ub}^{\text{tree}} \simeq \epsilon'_{q1}\epsilon_{d3}\frac{\lambda_{d4}}{\lambda_{d3}} \simeq \sin\theta_c V_{cb} \simeq 9\times 10^{-3}, \tag{5.18}$$

which is about a factor of 2 bigger than the accepted value. However, as we found in Sec. 3, when we generate m_d by radiative corrections, we also generate V_{ub}^{rad} bigger than the accepted value by about a factor of 3, which has to be cancelled by the tree level V_{ub}^{tree}. If the sign is right, (5.18) is just in the range which can cancel against the radiative contribution to produce the correct V_{ub}. Therefore, correct values for all quark masses and CKM mixing angles can be obtained. Naively, one might expect that it is difficult to have massless first generation quarks at tree level because of the Cabibbo angle. Here we showed, with the help of the theorem of Sec. 4 for the rank of the Yukawa matrices, that one can naturally get massless up and down quarks at tree level, while having nonzero $\sin\theta_c$.

5.2 A model of radiative m_u, m_e, and tree level m_d

As we have mentioned, a radiative m_d is only barely consistent with $B-\bar{B}$ mixing with very heavy SUSY-breaking masses. In this subsection, we present a model in which m_d is nonzero at tree level, while m_u and m_e arise purely from radiative effects. The flavor group is $G_f = SU(2)_T \times SU(2)_F \times Z_4$. The reason for the subscripts of the $SU(2)$ groups will be clear later. $U(1)_A$ is replace by its subgroup Z_4. Matter-parity and the heavy parity are imposed as well. The field content is shown in Table 4, where I, i are $SU(2)_F$ and $SU(2)_T$ indices respectively, and the numbers in brackets are the Z_4 charges with n and (n mod 4) identified. ϕ_{Ti}, ϕ_{FI}, S and X have nonzero VEV's:

$$\langle\phi_{Ti}\rangle = \begin{pmatrix} v_T \\ 0 \end{pmatrix}, \ \langle\phi_{FI}\rangle = \begin{pmatrix} v_F \\ 0 \end{pmatrix}, \ \langle S\rangle = v_s, \ \langle X\rangle = v_x, \tag{5.19}$$

which break G_f completely. In this model there is only one pair of $SU(2)_{T,F}$ breaking fields ϕ_{Ti}, ϕ_{FI}. The tree level massless electron and up quark can be easily seen in an effective theory point of view[6], because the only $SU(2)_{T,F}$

	Light	Heavy		
	$e_3(0)$, $e_i(-1)$	$E(-2)$, $\bar{E}(+2)$	$E_i(-1)$, $\bar{E}^i(+1)$	
	$\ell_3(0)$, $\ell_I(+1)$	$L(+2)$, $\bar{L}(-2)$	$L_I(+1)$, $\bar{L}^I(-1)$	
Matter	$u_3(0)$, $u_I(+1)$	$U(+2)$, $\bar{U}(-2)$,	$U_I(+1)$, $\bar{U}^I(-1)$	
	$q_3(0)$, $q_i(-1)$	$Q(-2)$, $\bar{Q}(+2)$,	$Q_i(-1)$, $\bar{Q}^i(+1)$,	$Q'_i(+1)$, $\bar{Q}'^i(-1)$
	$d_3(0)$, $d_i(-1)$	$D(-2)$, $\bar{D}(+2)$,	$D_i(-1)$, $\bar{D}^i(+1)$,	$D'_i(+1)$, $\bar{D}'^i(-1)$
Higgs	$h_u(0)$, $h_d(0)$	$\phi_{Ti}(-1)$, $\phi_{FI}(+1)$,	$S(0)$, $X(2)$	

Table 4: Field content and G_f transformation properties of the model with radiative m_u, m_e, and tree level m_d.

invariant holomorphic combinations of the two light generations and fields with nonzero VEV's for the lepton and the up quark sectors are $\epsilon^{ij}e_i\phi_{Tj}$, $\epsilon^{IJ}\ell_I\phi_{FJ}$, $\epsilon^{IJ}u_I\phi_{FJ}$, and $\epsilon^{ij}q_i\phi_{Tj}$, which cannot give Yukawa couplings to both light generations with h_u and h_d. In the down sector, q's and d's have the same G_f transformation properties. One can write down the effective operator

$$\epsilon^{ij}q_ih_dd_jXS, \tag{5.20}$$

which generates the 12 and 21 entries of the down Yukawa matrix with equal size and opposite signs. Hence we can obtain both θ_c and m_d at tree level with the experimentally successful relation $\sin\theta_c \simeq \sqrt{m_d/m_s}$.

Compared with the lepton model discussed earlier in this section, the extra X field is required to break the left over "second generation parity" in order to generate V_{cb} and V_{us} but it may also induce a too big $\mu \to e\gamma$ rate, which will be discussed later. The Q'_i, $\bar{Q}'^i$, D'_i, $\bar{D}'^i$ are responsible for generating the operator (5.20). They can be omitted if nonrenormalizable operators are allowed and are sufficiently large. In fact, because this model can be analyzed in the effective theory point of view, including nonrenormalizable interactions will not affect our results. However, for simplicity and completeness, we will analyze the full theory and restrict ourselves to renormalizable interactions.

The lepton sector and the up quark sector are similar to the previous models. We will not repeat the detailed analysis. The only difference is that with the additional X field, we can have the following extra interactions:

$$f_{e5}Xe_3\bar{E},\ f_{\ell 5}X\ell_3\bar{L},\ f_{u5}Xu_3\bar{U},\ f_{q5}Xq_3\bar{Q}. \tag{5.21}$$

They mix the third generation with the heavy $SU(2)_{T(F)}$ singlet generation. In combination with $\epsilon^{ij}\phi_{Ti}e_j\bar{E}$, $\epsilon^{IJ}\phi_{FI}\ell_J\bar{L}$, $\epsilon^{IJ}\phi_{FI}u_J\bar{U}$, and $\epsilon^{ij}\phi_{Ti}q_j\bar{Q}$, they generate the 23 and 32 entries of the Yukawa matrices and also the D term mixing between the second and the third generations. For the up quark sector, the $D-\bar{D}$ mixing constraints are very weak and hence easily satisfied. However, for the lepton sector the constraint from the $\mu \to e\gamma$ rate requires the 2-3 mixing to be no bigger than $\mathcal{O}(10^{-3})$, while the naive expectation of 2-3 mixing in this model is of the order V_{cb}. Therefore, one has to assume that the couplings of the X field to the lepton sector are small, or prevented by some extra symmetries. We will see that this is possible to achieve later.

In the down quark sector, in addition to the usual interactions,

$$\begin{aligned} W_d = {} & \lambda_{d3}q_3h_dd_3 + \lambda_{d4}Qh_dD \\ & + f_{q1}q_3\bar{Q}^i\phi_{Ti} + f_{q2}q_i\bar{Q}^iS + f_{q3}\epsilon^{ij}q_i\bar{Q}\phi_{Tj} \\ & + f_{d1}d_3\bar{D}^i\phi_{Ti} + f_{d2}d_i\bar{D}^iS + f_{d3}\epsilon^{ij}d_i\bar{D}\phi_{Tj} \\ & + M_D\bar{D}D + M_{D_i}\bar{D}^iD_i + M_Q\bar{Q}Q + M_{Q_i}\bar{Q}^iQ_i, \end{aligned} \tag{5.22}$$

which give the tree level b and s quark masses and 1-3 D term mixing, we have the following interactions as well,

$$\begin{aligned} W_d' = {} & f_{q5}q_3\bar{Q}X + f_{d5}d_3\bar{D}X \\ & + f_{q6}q_i\bar{Q}'^i + f_{d6}d_i\bar{D}'^iX \\ & + \lambda_{d5}\epsilon^{ij}Q_i'h_dD_j + \lambda_{d6}\epsilon^{ij}Q_ih_dD_j'. \end{aligned} \tag{5.23}$$

As we have discussed before, the f_{q5}, f_{d5} couplings induce the 23 and 32 entries of the Yukawa matrix and the 2-3 D term mixing, so that V_{cb} can be generated. f_{q6}, f_{d6}, λ_{d5}, λ_{d6} together with f_{q2}, f_{d2} couplings generate the operator (5.20), which gives θ_c and m_d, and the successful relation $\sin\theta_c = \sqrt{m_d/m_s}$. The tree level down quark mass matrix takes the following form,

$$\begin{pmatrix} 0 & C & 0 \\ -C & E & B \\ 0 & B' & A \end{pmatrix}, \tag{5.24}$$

while the tree level up quark and lepton mass matrices have nonzero entries in the lower 2×2 block. In addition to m_u and m_e, V_{ub} is also generated by

radiative corrections from the 3-1 mixing $W_{D_{L31}}$. The required size of $W_{D_{L31}}$ is much smaller than that required for generating m_d radiatively, so the phenomenological constraints are easier to satisfy as we have discussed in Sec. 3.

Looking at the G_f transformation properties of the fields, one can see that this model can be embedded into the flipped $SU(5)$ grand unified theory[17]: q and d (and the not discussed right-handed neutrino n) belong to the **10** representation of flipped $SU(5)$, u and ℓ belong to the $\bar{\mathbf{5}}$ and e is a singlet **1** under flipped $SU(5)$. $SU(2)_T$ is a flavor group for the **10**'s and $SU(2)_F$ is a flavor group for the $\bar{\mathbf{5}}$'s. In Table 4, the e's are assigned to transform under $SU(2)_T$. Here one can either have them transform under a different $SU(2)_S$, or simply identify $SU(2)_S$ with $SU(2)_T$.

One nice feature of embedding this model into flipped $SU(5)$ is that the X field can be assigned to the **75** of $SU(5)$. Because only the $\mathbf{10} \times \overline{\mathbf{10}}$ contains **75** and the $\mathbf{5} \times \bar{\mathbf{5}}$, $\mathbf{1} \times \mathbf{1}$ do not, the X field can only couple to q and d but not the lepton sector. Then the μ-τ mixing and hence the troublesome $\mu \to e\gamma$ decay rate can be removed.

After flipped $SU(5)$ is broken, we do not expect the couplings and the mixings to be the same for fields belonging to the same representations of the flipped $SU(5)$.[8] But if we assume that they are of the same order, the radiative m_e, m_u and V_{ub} are also consistent: radiative V_{ub} does not need a big $W_{D_{L31}}$ ($\sim 10^{-2}$), then $W_{U_{R31}}$ has to be quite big ($\gtrsim 10^{-1}$) for generating m_u; but so is its flipped $SU(5)$ partner $W_{E_{L31}}$ for generating m_e. On the other hand, λ_U, λ_D, and λ_E are independent in flipped $SU(5)$ models. They can take suitable values so that all the tree level quantities come out correctly.

It is possible to extend the $SU(2)$ flavor groups to $SU(3)$ for these quark models as we did for the lepton model in [6]. However, here we do not gain much by paying the price that the third generation Yukawa couplings arise at the nonrenormalizable level. More heavy fields have to be introduced and more complicated stages of flavor symmetry breakings are involved. Therefore, we will not pursue this direction further in this paper.

[8] If flipped $SU(5)$ were not broken, the tree level 12 and 21 entries of the down quark mass matrix would not be generated, because $\epsilon^{ij}\mathbf{10}_i\mathbf{10}_j h_d XS$ vanishes. However, since the flipped $SU(5)$ is broken, q's and d's can have different mixings so that $\epsilon^{ij} q_i d_j h_d XS$ can be nonzero.

6 Conclusions

In this paper, we have considered the possibility of generating some of the light fermion masses through radiative corrections. Any theory of radiative fermion masses must have an accidental symmetry for the Yukawa sector guaranteeing the absence tree level masses, while this symmetry must be broken elsewhere in the theory for any mass to be generated radiatively. In our discussion, supersymmetry has been crucial in naturally implementing this scenario: supersymmetric theories automatically have two sectors (the superpotential and D terms) which need not have the same symmetries; because of holomorphy the superpotential may have accidental symmetries not shared by the D terms. Furthermore, the particles in the radiative loop generating the fermion masses are just the superpartners of known particles, and must be near the weak scale if supersymmetry is to solve the hierarchy problem. Thus, supersymmetric theories of radiative fermion masses can lead to testable predictions. Working with supersymmetric theories with minimal low energy field content, we found (with the plausible assumption that the accidental flavor symmetries of the tree level Yukawa matrix are only broken by soft scalar masses) that FCNC constraints allow only the first generation fermion masses to have a radiative origin.

In the lepton sector, a rather large mixing between the selectron and stau is needed in order to generate the electron mass. This implies that mixing with the smuon must be highly suppressed in order to avoid too large a rate for $\mu \to e\gamma$. The large selectron-stau mixing also gives rise to a significant rate for $\tau \to e\gamma$ which is only a factor 10-100 lower than the current experimental limit.

In the quark sector, in addition to the quark masses, the CKM mixing matrix must also be obtained. The FCNC constraints strongly limit the possibilities of generating light quark masses and mixing angles. We found that m_u and V_{ub} can be generated by radiative corrections, while radiatively generating any of m_d, θ_c, and V_{cb} requires heavy scalar masses ($\sim$ 1TeV). Further, it is very difficult to generate m_d, θ_c, and V_{ub} together radiatively unless the scalar masses are between 2 and 20 TeV, which we view as unacceptably high. These constraints cause the principle difficulties in constructing a model of quark flavor with radiative masses.

We extended the lepton model with flavor group $SU(2)_\ell \times SU(2)_e \times U(1)_A$

in [6] to the quark sector. The lepton model has a number of nice features: the $SU(2)$ breaking ϕ VEV's are responsible for both D-term mixing between the first and the third generation and generation of the second generation mass, so the ratio between the radiatively generated first generation mass and the second generation mass is naturally of the order $1/(16\pi^2)$. Further, muon number is conserved so that the dangerous rate for $\mu \to e\gamma$ is avoided. A direct extension of this model to the quark sector cannot generate the correct CKM mixings, which requires the addition of more fields and flavor symmetry breakings to the theory.

We presented two complete models with radiative fermion masses. In the first model, all first generation fermion masses come from radiative corrections, and there are also tree level contributions to θ_c and V_{ub} as required by the FCNC constraints. First generation fermions are guaranteed to be massless at tree level by requiring the "big" Yukawa matrices of the full theory to be rank 2. Requiring a tree level θ_c and V_{ub} forces us to add another heavy left-handed quark Q' and its conjugate $\bar{Q}'$, and another pair of $SU(2)_{l,r}$ flavor symmetry breaking fields $\phi'_{l,r}$. Muon number is still conserved as a consequence of the field content and charge assignments of the theory. With these minimal extensions, we obtain a complete theory of radiative first generation fermion masses with successful values for CKM mixing angles.

In view of the fact that a radiative m_d and $B-\bar{B}$ mixing are only compatible for very heavy scalar masses, we also constructed a second model in which m_u and m_e come from radiative corrections but m_d and θ_c arise at tree level with the successful relation $\sin\theta_c = \sqrt{m_d/m_s}$. The dangerous $\mu \to e\gamma$ rate can be naturally suppressed if we embed this model into the flipped $SU(5)$ grand unified theory.

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Appendix A

In this appendix, we consider the possibility that the soft supersymmetry breaking trilinear A terms do not respect the chiral symmetries of the Yukawa matrix [18, 19]. Before beginning the discussion of radiative fermion masses in this scenario, let us consider the constraints imposed on the form of the A matrix by requiring the desired vacuum to be the global minimum of the potential. (The extent to which this is a neccesity is discussed at the end of this appendix). Consider the lepton sector for simplicity (identical arguments hold for the quark sector). Let us work in a basis where the lepton Yukawa matrix is diagonal and has K zeros. There are D-flat directions in field space where the right and left handed lepton fields and the down type Higgs are nonzero. If we restrict ourselves to the K massless generations, there are no quartic terms in the potential along the D-flat directions; all we have are the cubic A terms and the scalar masses. But, if the A terms are non-zero in the $K \times K$ block of the massless generations, there will be directions in field space where the cubic terms become indefinitely negative and cannot be stabilized by the quadratic mass terms. This can only be avoided if the A terms are zero in the $K \times K$ block of the K massless generations. This constraint is in itself quite powerful. For instance, if $K = 3$, we must have that the A matrix is zero, and the argument that one cannot generate any radiative masses goes through exactly as in section 2. Next, let us consider the case $K = 2$. In this case, the A matrix must be zero in the upper 2×2 block. Note that we can make a rotation on the first two generation scalars to make A_{i3}, A_{3i} zero for either $i = 1$ or $i = 2$. Now, the potential is no longer unbounded below, but there is still a local minimum along the D-flat directions for the first two generations where both left and right handed fields aquire VEV's, breaking electric charge. We require that the energy of this minimum is greater than that of the usual minimum, which is $-\frac{1}{4}M_Z^2 v^2$. For scalars much heavier than $(M_Z v)^{\frac{1}{2}} = 150$ GeV, we can approximate this requirement by demanding that the electric charge breaking minimum has energy greater than zero. A straightforward calculation analogous

to that in [20] then gives us the following constraint, where we assume that all relevant scalars are degenerate with mass m:

$$\frac{1}{3}(|A_{33}| + |A_{3i}| + |A_{i3}|) \lesssim \lambda_3 m. \tag{A.1}$$

There are corrections to this inequality due to the fact that the true vacuum energy is not zero but $-\frac{1}{4}M_Z^2 v^2$; still assuming $m \gtrsim 150$ GeV the correction takes the form:

$$\frac{1}{3}(|A_{33}| + |A_{3i}| + |A_{i3}|) \lesssim \lambda_3 m(1 + \frac{1}{2\lambda_3}\frac{M_Z v}{m^2}). \tag{A.2}$$

With these constraints in hand, we begin the phenomenological analysis. Suppose that the scalar masses did not break the chiral symmetries of the Yukawa matrix. Then, since one of $A_{31,13}, A_{32,23}$ can be chosen to be zero by rotations, one generation would remain massless to all orders of perturbation theory. Thus, in order to generate both generations radiatively, we must have that both the A terms and the scalar masses break the chiral symmetries of the Yukawa sector. In the following, we consider the possibility that the A terms generate one mass radiatively while the scalar masses generate the other mass. It is easy to see that this is impossible in the lepton sector: the muon mass is too big to be generated radiatively, and even if we could, we would generate too large a rate for $\tau \to \mu\gamma$. Moving on to the quark sector, we have four cases to consider:

(1) m_d from scalar masses and m_s from A terms: In the mass insertion approximation, assuming for simplicity that all scalars are degenerate with mass m, we have in the large $\tan\beta$ limit

$$\frac{m_s}{m_t} = \frac{\alpha_s}{18\pi}(\frac{\mu M_{\tilde{g}}}{m^2})\frac{(A^d_{23}v_d)}{m^2}\frac{(A^d_{32}v_d)}{m^2}. \tag{A.3}$$

From equation (A.1), however, we must have that $\frac{(A^d_{23,32}v_d)}{m^2} \lesssim \frac{m_b}{m}$, so

$$\frac{m_s}{m_t} \lesssim 2 \times 10^{-3}(\frac{\mu M_{\tilde{g}}}{m^2})\frac{m_b^2}{m^2} \tag{A.4}$$

which, even for m=100GeV, gives too small a value for m_s by a factor of ~ 100.

(2) m_d from A terms and m_s from scalar masses: The same argument as in case (1) suggests that the generated mass for m_d will be too small by a factor of $\sim$

10. Perhaps this factor can be overcome for some choice of parameters. However, the scalars are so light that the required mixing in the scalar mass matrix to generate m_s, together with the A terms responsible for m_d, give unacceptable contributions to $K_1 - K_2$ mixing, and, if there are CP violating phases, even more unacceptable contributions to ϵ.

(3) m_u from scalar masses and m_c from A terms: The general problem with the up sector is that m_c seems to be too heavy to be radiative. In the case we are considering, we find analogously to equation (A.4)

$$\frac{m_c}{m_t} \lesssim 2 \times 10^{-3} (\frac{M_{\tilde{g}}}{m})(\frac{m_t}{m})^2 \qquad (A.5)$$

and so to generate large enough m_c we must again have fairly light squarks.

(4) m_u from A terms and m_c from scalar masses: In this case again it is difficult to get a large enough mass for the charm. In analogy to equation (3.10) we have, (in the limit where we decouple the first two generations, minimizing the super-GIM cancellation and so maximizing the generated charm mass)

$$\frac{m_c}{m_t} = \frac{2\alpha_s}{3\pi} \frac{A^u_{33}}{M_{\tilde{g}}} \times W_{U_L 31} W_{U_R 31} W_{U_L 33}^* W_{U_R 33}^* I(\frac{m_{\tilde{t}}^2}{M_{\tilde{g}}^2}). \qquad (A.6)$$

The maximium value of $W_{U_L 31} W_{U_R 31} W_{U_L 33}^* W_{U_R 33}^*$ consistent with the unitarity of the W matrices is $\frac{1}{4}$. Then, we have

$$\frac{m_c}{m_t} \lesssim 5 \times 10^{-3} \frac{A^u_{33}}{M_{\tilde{g}}} I(\frac{m_{\tilde{t}}^2}{M_{\tilde{g}}^2}). \qquad (A.7)$$

Recalling that $I(1) = \frac{1}{2}$, we see that, even with maximal mixing angles, the radiative charm mass is too small or perhaps right on the edge. However, having such large mixing in the left handed up 32 sector also implies large mixing in the left handed down 32 sector, which violates the bounds from $b \to s\gamma$ unless the third generation scalars are pushed above 1 TeV. This then makes it difficult to generate a large enough up mass, since the A term contribution is suppressed by $(\frac{m_t}{m})^2$ from (A.1). We find

$$\frac{m_u}{m_t} \lesssim 2 \times 10^{-5} (\frac{M_{\tilde{g}}}{m})(1 + \frac{\mu \cot \beta}{A_{33}})(\frac{1.7\text{TeV}}{m})^2 \qquad (A.8)$$

which is also on the edge. Another difficulty with having such large 32 mixing is that it disturbs the degeneracy between the scalar masses of the first two

generations for both left handed up and down squarks, and this could again give problems with $K_1 - K_2$ mixing and ϵ.

The above arguments certainly do not rule out the possibility of generating both light generations radiatively; there may be regions of parameter space where our rough bounds are evaded. Indeed, it may even be the case that requiring the desired vacuum to have lower energy than the charge breaking minima is not necessary, perhaps the lifetime of the false vacuum can be long enough for the universe to have stayed in it up to the present; this remains to be seen. However, these arguments, together with the fact that for the A terms not to share the same chiral symmetries as the Yukawa matrices we must entangle flavor symmetry breaking and supersymmetry breaking, provide us with sufficient motivation to restrict our detailed treatment to the scenario considered in this paper.

Appendix B

In [10, 11], the SUSY FCNC constraints are expressed in terms of the ratios of the off-diagonal scalar masses and the "universal squark or slepton masses". For example, the supersymmetric contribution to the $B-\bar{B}$ mixing is given by:[9]

$$\begin{aligned}\Delta M_B^{SUSY} = & \frac{\alpha_s^2}{216 M_{\tilde{q}}^2}\frac{2}{3} f_B^2 m_B \{(\delta_{13}^d)_{LL}^2[-66\tilde{f}_6(x) - 24x f_6(x)] \\ & + (\delta_{13}^d)_{RR}^2[-66\tilde{f}_6(x) - 24x f_6(x)] \\ & + (\delta_{13}^d)_{LL}(\delta_{13}^d)_{RR}[-12\tilde{f}_6(x) - 456x f_6(x)] \\ & + (\delta_{13}^d)_{LR}^2[132x f_6(x)] + (\delta_{13}^d)_{RL}^2[132x f_6(x)] \\ & + (\delta_{13}^d)_{LR}(\delta_{13}^d)_{RL}[228\tilde{f}_6(x)\}, \end{aligned} \qquad (B.1)$$

where,

$$\begin{aligned} f_6(x) &= \frac{1}{6(1-x)^5}(-6\ln x - 18x\ln x - x^3 + 9x^2 + 9x - 17), \\ \tilde{f}_6(x) &= \frac{1}{3(1-x)^5}(-6x^2\ln x - 6x\ln x + x^3 + 9x^2 - 9x - 1) \end{aligned} \qquad (B.2)$$

[9] We use the notation and the formula in [10], corrected by [11]

are the Feynman loop intrgrals defined in [10], and

$$x = \frac{M_{\tilde{g}}^2}{M_{\tilde{q}}^2},\ (\delta_{ij}^d)_{LL} = \frac{\delta\tilde{m}_{\bar{d}_L b_L}^2}{M_{\tilde{q}}^2},\ \text{and so on.}$$

Demanding that each term is no bigger than the experimental value of ΔM_B gives the constraints on δ_{ij}^d. However, with large splitting in scalar masses of the first two and the third generations, it is better to have constraints directly on the mixing matrix elements because of the ambiguity of what $M_{\tilde{q}}$ should be. In this appendix, we will convert the constraints on δ_{ij} into constraints on the mixing matrix elements W_{ij} directly.

We assume degeneracy between the left-handed and the right-handed scalar masses, and also the first two generation scalar masses (denoted by m_1). To reduce the number of parameters, we also assume that the relevant gaugino mass is degenerate with the third generation scalar mass (denoted by m_3). We also take the chirality-changing scalar masses much smaller than the chirality-conserving ones, so that the eigenstates and eigenvalues are not disturbed significantly. Now we can express the SUSY FCNC contributions by the mixing matrix elements and the two parameters m_3 and $y \equiv \frac{m_1^2}{m_3^2}$. For example, the first term in (B.1) becomes

$$\begin{aligned}
&\frac{\alpha_s^2}{216 M_{\tilde{q}}^2}\frac{2}{3} f_B^2 m_B \left(\frac{W_{D_{L31}}(m_1^2 - m_3^2)}{m_3^2}\right)^2 [-66\tilde{f}_6(y) + 24 f_6(y)] \\
&= \frac{\alpha_s^2}{216 M_{\tilde{q}}^2}\frac{2}{3} f_B^2 m_B (W_{D_{L31}})^2 (y-1)^2 [-66\tilde{f}_6(y) + 24 f_6(y)]. \qquad (B.3)
\end{aligned}$$

Demanding it to be smaller the ΔM_B^{EXP} gives the constraint on $W_{D_{L31}}$,

$$\sqrt{\text{Re}|W_{D_{L31}}^{\ 2}|} < \frac{18 m_3}{\alpha_s f_B}\sqrt{\frac{\Delta M_B}{m_B}}(y-1)^{-1}[-66\tilde{f}_6(y) + 24 f_6(y)]^{-\frac{1}{2}}. \qquad (B.4)$$

Similarly, we can obtain constraints on other mixing matrix elements from the other terms. The constraints from $B - \bar{B}$ mixing are shown in Table B1(a).

For $K - \bar{K}$ mixing, $\Delta m_{LL(RR)_{21}}^2$ can have two contributions. One comes from the splitting between the first two generation scalar masses, $W_{D_{L(R)21}}(m_1^2 - m_2^2)$. We can use the constraints in [10, 11] in this case because the first two generation scalar masses have to be degenerate to a high degree and there is no ambiguity in what $M_{\tilde{q}}$ is. The other comes from the large splitting of the third generation

scalar mass, $W^*_{D_{L(R)32}} W_{D_{L(R)31}}(m_1^2 - m_3^2)$. This part can be treated in the same way as in the $B - \bar{B}$ mixing described above. The terms proportional to the left-right mass insertions are a little more complicated because they involve new integrals. These terms are proportional to $[m_b\,(A + \mu \tan\beta)]^2$. For our purpose, we always work in the large $\tan\beta$ scenario. Hence the corresponding constraints scale as $\frac{m_3^3}{\mu \tan\beta}$, versus m_3 in the case of chirality-conserving terms. The results are listed in Table B1(b) for Δm_K and Table B1(c) for ϵ. The ϵ' parameter could put constraints on $|\,\mathrm{Im}\, W^*_{D_{L(R)32}} W_{D_{L(R)31}}|$ and $|\,\mathrm{Im}\, W_{D_{L(R)32}} W_{D_{R(L)31}}|$. The first one is weaker than the constraints from other places, the second one is enhanced by $\tan\beta$ and is listed in Table B1(d). The numbers are obtained by requiring its contribution to ϵ' smaller than $3 \times 10^{-3}\epsilon$.

The mixing matrix elements $W_{D_{L(R)32}}$ are constrained by the $b \to s\gamma$ decay. The $b \to s\gamma$ branching ratio has been measured to be $(2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ by CLEO [21], which is consistent with the Standard Model prediction $(2.8 \pm 0.8) \times 10^{-4}$[22]. In supersymmetric models there are many other contributions. The gluino diagram contributions depend on the mixing matrix elements $W_{D_{L(R)32}}$ so they can be used to constrain $W_{D_{L(R)32}}$. Unlike other contributions, the gluino diagrams give significant contributions to both $\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$ and $\bar{s}_R \sigma^{\mu\nu} b_L F_{\mu\nu}$ operators. The former can interfere constructively or destructively with other contributions and the latter does not. In Table B1(e) we list the constraints on $W_{D_{L32}}$ and $W_{D_{R32}}$ by requiring that each gluino diagram alone does not exceed the Standard Model contribution.

The up mixing matrices W_U's are constrained by $D - \bar{D}$ mixing, and the results are shown in Table B1(f).

In the lepton sector, the most stringent constraints come from $\mu \to e\gamma$ decay. In the large $\tan\beta$ scenario in which we are interested, the amplitude of the dominant contribution is given in Ref. [7]. Requiring that the rate does not exceed the experimental limit, $B(\mu \to e\gamma) < 4.9 \times 10^{-11}$[23] give constraints on $W_{E_{L(R)32}} W_{E_{R(L)31}}$, which are shown in Table B1(g). Because we are interested in generating m_e by radiative corrections which requires sizable mixing between the first and the third generations, $W_{E_{L(R)31}}$, the $\tau \to \mu\gamma$ decay does not give stronger constraints on $W_{E_{L(R)32}}$ than those from the $\mu \to e\gamma$ decay.

Table B1

(a) Δm_B

$\sqrt{y}$	$\sqrt{\lvert \mathrm{Re}(W_{D_{L(R)31}})^2 \rvert}$	$\sqrt{\lvert \mathrm{Re}(W_{D_{L31}}^* W_{D_{R31}}) \rvert}$
2	1.0×10^{-1}	3.1×10^{-2}
3	6.5×10^{-2}	2.4×10^{-2}
5	4.9×10^{-2}	2.0×10^{-2}

(b) Δm_K

$\sqrt{y}$	$\sqrt{\lvert \mathrm{Re}(W_{D_{L(R)32}}^* W_{D_{L(R)31}})^2 \rvert}$	$\sqrt{\lvert \mathrm{Re}(W_{D_{L32}}^* W_{D_{L31}} W_{D_{R32}} W_{D_{R31}}^*) \rvert}$	$\sqrt{\lvert \mathrm{Re}(W_{D_{L(R)32}} W_{D_{R(L)31}})^2 \rvert}^{\#}$
2	4.7×10^{-2}	5.6×10^{-3}	7.4×10^{-2}
3	3.0×10^{-2}	4.2×10^{-3}	4.7×10^{-2}
5	2.2×10^{-2}	3.6×10^{-3}	3.7×10^{-2}

(c) ϵ

$\sqrt{y}$	$\sqrt{\lvert \mathrm{Im}(W_{D_{L(R)32}}^* W_{D_{L(R)31}})^2 \rvert}$	$\sqrt{\lvert \mathrm{Im}(W_{D_{L32}}^* W_{D_{L31}} W_{D_{R32}} W_{D_{R31}}^*) \rvert}$	$\sqrt{\lvert \mathrm{Im}(W_{D_{L(R)32}} W_{D_{R(L)31}})^2 \rvert}^{\#}$
2	3.7×10^{-3}	4.6×10^{-4}	6.0×10^{-3}
3	2.4×10^{-3}	3.4×10^{-4}	3.8×10^{-3}
5	1.8×10^{-3}	2.9×10^{-4}	3.0×10^{-3}

(d) ϵ'

$\sqrt{y}$	$\lvert \mathrm{Im}(W_{D_{L(R)32}} W_{D_{R(L)31}}) \rvert^{\#}$
2	1.4×10^{-3}
3	7.7×10^{-4}
5	5.4×10^{-4}

(e) $b \to s\gamma$

$\sqrt{y}$	$\lvert W_{D_{L(R)32}} \rvert^{\#}$
2	6.9×10^{-2}
3	5.3×10^{-2}
5	4.7×10^{-2}

(f) Δm_D

$\sqrt{y}$	$\sqrt{\lvert \mathrm{Re}(W_{U_{L(R)}32}^{*} W_{U_{L(R)}31})^2 \rvert}$	$\sqrt{\lvert \mathrm{Re}(W_{U_L}{}_{32}^{*} W_{U_L 31} W_{U_R 32} W_{U_R}{}_{31}^{*}) \rvert}$	$\sqrt{\lvert \mathrm{Re}(W_{U_{L(R)}32} W_{U_{R(L)}31})^2 \rvert}^{\#}$
2	9.5×10^{-2}	3.0×10^{-2}	3.9×10^{-1}
3	6.3×10^{-2}	2.3×10^{-2}	2.5×10^{-1}
5	4.7×10^{-2}	1.9×10^{-2}	2.0×10^{-1}

(g) $\mu \longrightarrow e\gamma$

$\sqrt{y}$	$\lvert W_{E_{L(R)}}{}_{32}^{*} W_{E_{L(R)}31} \rvert^{\#}$	$\lvert W_{E_{L(R)}32} W_{E_{R(L)}31} \rvert^{\#}$
2	2.4×10^{-3}	2.2×10^{-4}
3	1.8×10^{-3}	1.3×10^{-4}
5	1.6×10^{-3}	1.0×10^{-4}

Table B1: Constraints on the fermion-sfermion flavor mixing matrix elements. The reference values are taken as: $\tilde{m}_3 = M_g = 500\,\mathrm{GeV}$, $\mu = 500\,\mathrm{GeV}$, $\tan\beta = 60$, and $\sqrt{y} \equiv \frac{\tilde{m}_1}{\tilde{m}_3}$. The ones with # scale as $(\frac{\tilde{m}_3}{500GeV})^3(\frac{500GeV}{\mu})(\frac{60}{\tan\beta})$, others scale as $\frac{\tilde{m}_3}{500GeV}$.

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Figure Captions

Fig. 1: The dominant radiative contributions to the fermion masses: (a) charged leptons, (b) up-type quarks, (c) down-type quarks.

Fig. 2: Plots of the super-GIM factor $\widetilde{\widetilde{H}} \equiv h(x_3, x_3) - h(x_3, x_1) - h(x_1, x_3) + h(x_1, x_1)$ and $\widetilde{H} \equiv h(x_3, x_3) - h(x_3, x_1)$ versus the ratio between the first two generation and the third generation scalar masses $\sqrt{y}$, $y \equiv \tilde{m_1}^2/\tilde{m_3}^2 = x_1/x_3$, with $x_3 = 1$, $(M_g = \tilde{m_3})$. $\frac{\Delta \mathbf{m}_{e\alpha\beta}}{m_\tau} = 2.4 \times 10^{-2}(\frac{\mu}{m_{\tilde{\tau}}})(\frac{\tan\beta}{60})(\frac{\widetilde{\widetilde{H}}}{0.5})\sqrt{x_3}W_{E_{L3\alpha}}W_{E_{R3\beta}}$,
$\frac{\Delta \mathbf{m}_{u\alpha\beta}}{m_t} = 1.2 \times 10^{-2}(\frac{A}{m_{\tilde{t}}})(\frac{\widetilde{\widetilde{H}}}{0.5})\sqrt{x_3}W_{U_{L3\alpha}}W_{U_{R3\beta}}$,
$\frac{\Delta \mathbf{m}_{d\alpha\beta}}{m_b} = 0.7(\frac{\mu}{m_{\tilde{b}}})(\frac{\tan\beta}{60})(\frac{\widetilde{\widetilde{H}}}{0.5})\sqrt{x_3}W_{D_{L3\alpha}}W_{D_{R3\beta}}$,
for α, $\beta = 1, 2$, and $\widetilde{\widetilde{H}}$ has to be replaced by $\widetilde{H}$ if one of the α, β is 3.

Fig. 3: Contour plot of $B(\tau \to e\gamma)$, where the mixing angles are fixed by requiring a radiative electron mass. We have put $\tan\beta = 60.$, $\mu = m_{\tilde{\tau}}$=200 GeV, and plot in the $M_1 - \sqrt{y}$ plane where M_1 is the bino mass and we have assumed the GUT relation $M_2 \sim 2M_1$; $y = \frac{m_{\tilde{e}}^2}{m_{\tilde{\tau}}^2}$. We also assume that the left and right handed mixing angles are equal, giving us a lower bound on $B(\tau \to e\gamma)$. The branching ratio scales as $\frac{\mu \tan\beta}{m_{\tilde{\tau}}^4}$.

Fig. 4: Chargino diagrams which contribute to radiative down-type quark masses and are enhanced by large $\tan\beta$.

Fig. 5: The diagram which generates the second generation masses.

Fig. 6: D term mixing between the first and the third generations.

Fig. 1

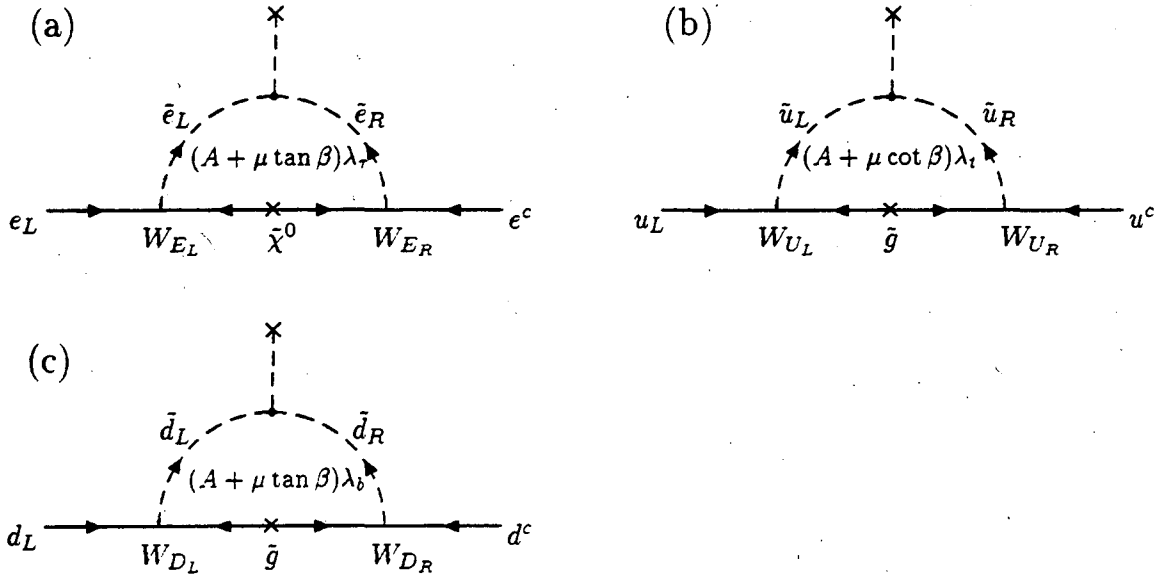


Fig. 4

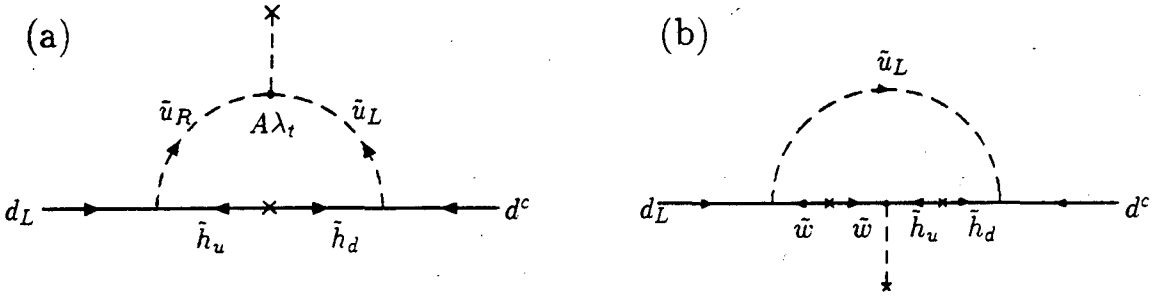


Fig. 5

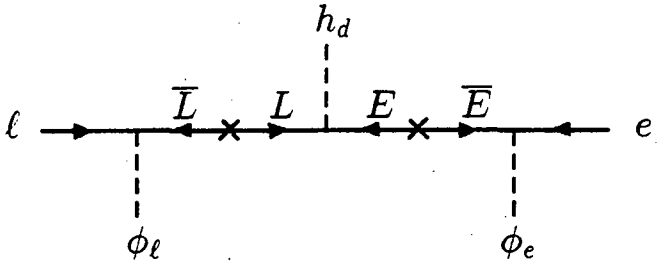


Fig. 6

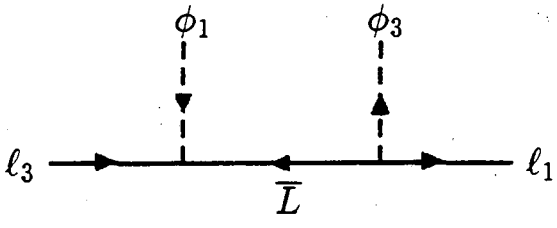


Fig. 2

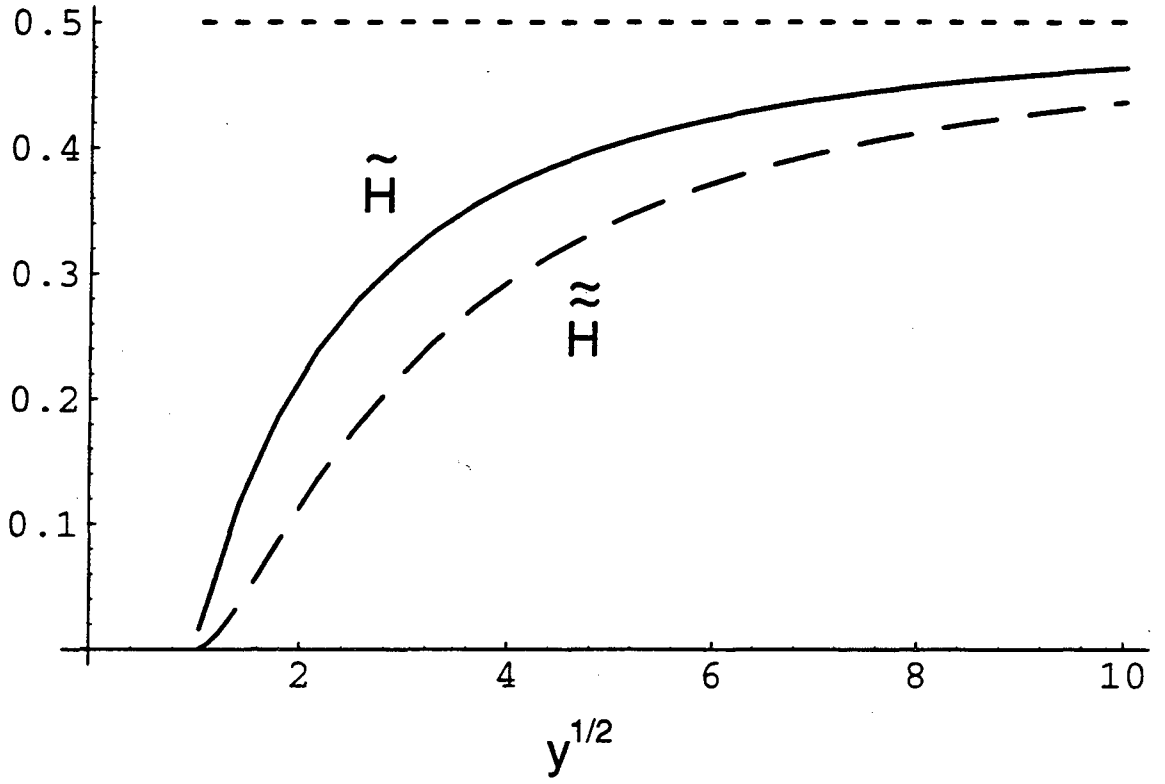
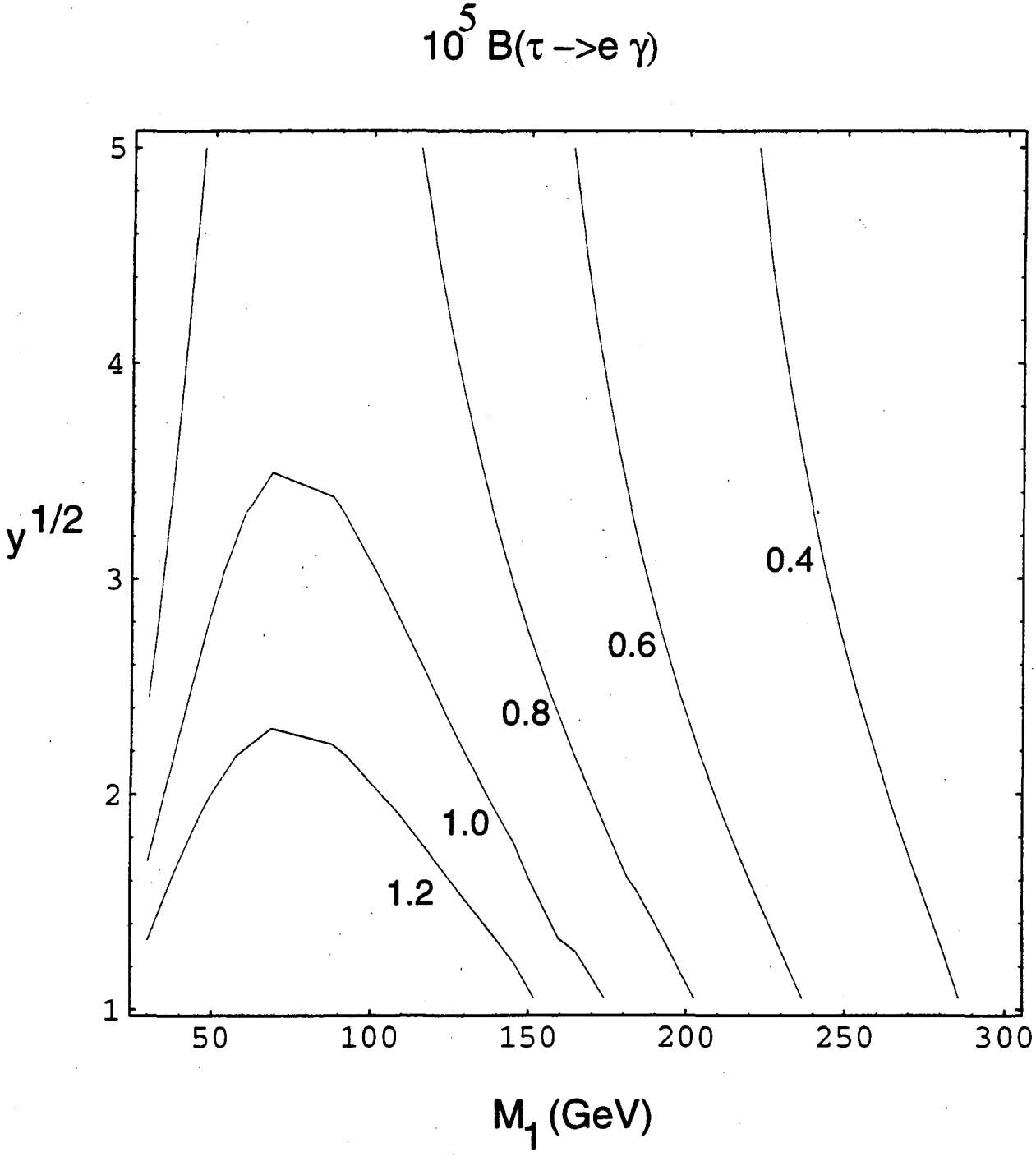


Fig. 3



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