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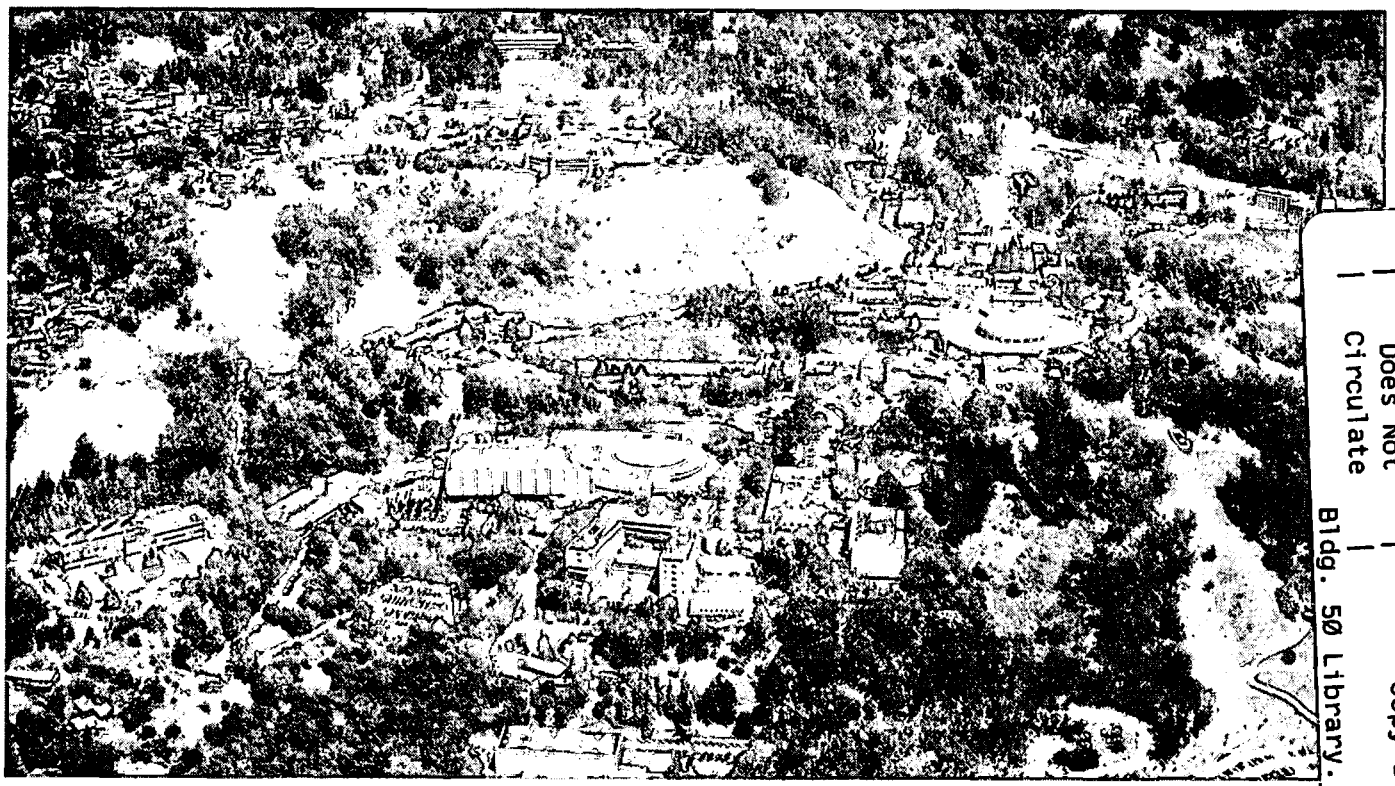

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Conserved "Angular Momentum" in Standpoint Cosmology; A Global Meaning for Light *

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Abstract

Conservation of a quantity analogous to angular momentum in standpoint cosmology segregates unphysical ("tachyonic") particle trajectories and provides a definition of lightlike trajectory. By-product is a definition of "Riemannian cold matter".

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I. Introduction

At first glance the non-Riemannian gravitational action of standpoint cosmology[1] seems to forego distinction between unphysical and particle trajectories, a distinction provided by the sign of ds^2 in standard theory where negative ds^2 is sometimes described as "tachyonic". The apparent deficiency is displayed by the standpoint-spacetime expression for the square of the increment of action,

$$ds^2 = \frac{1}{2}\left\{[G(dt^2 - dr^2) - gr^2 d\vec{n}^2]^2 + [G'(dt^2 - dr^2) - g'r^2 d\vec{n}^2]^2\right\}^{1/2}, \qquad (I.1)$$

where the coefficients G, g, G', g' are prescribed Hubble-scale functions of r and t (see Appendix) and $d\vec{n}^2 = d\theta^2 + \sin^2\theta d\varphi^2$. Positivity of the quartic form inside the curly bracket of (I.1) precludes any physical-unphysical distinction achieved simply through the sign of ds^2. This paper presents an alternative Hubble-scale definition of "particle trajectory" based on a dynamical constant of motion that we designate by the symbol J because of its relation to angular momentum. Close to standpoint on Hubble scale, where a Riemannian approximation to (I.1) becomes accurate, our definition accords with that of general relativity. Lightlike Hubble-scale trajectories in standpoint cosmology are characterized, not by zero ds^2, but by zero J.

Complementary to our definition of lightlike trajectory is a definition of "cold matter". It will be found that for cold matter everywhere there is an approximately-Riemannian metric that is equivalent to gravitational action..

II. Action and Angular Momentum

The standpoint-spacetime homogeneous-universe gravitational action $\int ds$ we write as $\int L dt$, where corresponding to (I.1),

$$L(t,r,\dot{r},\dot{\varphi}) \equiv \frac{ds}{dt} = \frac{1}{\sqrt{2}}\{[G(1-\dot{r}^2)-gr^2\dot{\varphi}^2]^2+[G'(1-\dot{r}^2)-g'r^2\dot{\varphi}^2]^2\}^{1/4}. \quad (II.1)$$

The fourth root in (II.1) brings a 4-fold ambiguity $(\pm 1, \pm i)$ whose resolution will be occupying our attention. Even with adoption of a real positive value for L at low velocities ($\dot{r}^2 << 1$, $r^2\dot{\varphi}^2 << 1$), and thus a real positive value everywhere (so long as $g/G \neq g'/G'$), the presence in L of branch points in the complex-velocity plane will relate to our distinction between particle-like and "tachyonic" trajectories and to our definition of "lightlike" trajectory.

In (II.1) we have exploited the rotational invariance of the standpoint's homogeneous universe to suppress the variables θ and $\dot{\theta}$. Only those ($\dot{\theta} = 0$) geodesics will be considered that lie in the plane belonging to $\theta = \pi/2$. Absence of L dependence on φ is a further important aspect of rotational invariance.

It is sometimes convenient to replace the velocities $\dot{r}$ and $\dot{\varphi}$ by the alternative variables x and z defined by

$$\begin{aligned} x &\equiv 1-\dot{r}^2, \\ z &\equiv \frac{r^2\dot{\varphi}^2}{1-\dot{r}^2} = \left(1+\frac{1-v^2}{v_{tr}^2}\right)^{-1}. \end{aligned} \quad (II.2)$$

where $v_{tr} \equiv r\dot{\varphi}$ and $v^2 \equiv \dot{r}^2 + v_{tr}^2$, and to write

$$L = \frac{1}{\sqrt{2}}\left\{(G-gz)^2+(G'-g'z)^2\right\}^{1/4} x^{1/2}. \quad (II.3)$$

It will turn out that "angular momentum" is determined by the 3 variables z, r, t independently of x. Regarded as an analytic function of x and z, L is seen in (II.3) to have a square-root x branch point at the x origin and a pair of fourth-root z branch points at the complex-conjugate locations,

$$z_b^{\pm}(r,t) = \frac{G \mp iG'}{g \mp ig'}. \quad (II.4)$$

For future use we calculate

$$z_m(r,t) \equiv \mathrm{Re}\, z_b^{\pm}(r,t) = \frac{gG+g'G'}{g^2+g'^2}, \quad (II.5)$$

$$Im\ z_b^{\pm}(r,t) = \pm\frac{g'G - gG'}{g^2 + g'^2}. \qquad (II.6)$$

According to the explicit functions G, g, G', g' given in the appendix, $z_m(r,t) \geq 1$ throughout the (compact) standpoint spacetime.

"Angular momentum" J we define in terms of $L(t, r, \dot{r}, \dot{\varphi})$ as

$$J \equiv -\frac{\partial L}{\partial \dot{\varphi}} \text{ (at fixed } t, r, \dot{r}), \qquad (II.7)$$

thereby guaranteeing J to be constant along a trajectory, since one of the "geodesic" equations is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = \frac{\partial L}{\partial \varphi} = 0. \qquad (II.8)$$

The dimension (length) of the "J" defined by (II.7) symptomizes an important difference from the usual meaning of angular momentum. The difference will receive attention in what follows. By explicit calculation from (II.7) and (II.1), one finds

$$\frac{J^2}{r^2} = \frac{z}{2}\frac{[g(G-gz) + g'(G'-g'z)]^2}{[(G-gz)^2 + (G'-g'z)^2]^{3/2}} \qquad (II.9)$$

$$= \frac{(g^2+g'^2)^{1/2}}{2}\frac{z(z_m - z)^2}{[(z_m - z)^2 + (\mathrm{Im}\, z_b^{\pm})^2]^{3/2}}. \qquad (II.10)$$

It may be verified that, when L is regarded as a function of z at fixed t, r, x as in Formula (II.3), the *minimum* of L occurs at $z = z_m(r,t)$. Below we shall find $z_m(r,t)$ to be the *maximum* value of z (and of v_{tr}^2) that can be reached at (r,t) on any particle geodesic. Thus the subscript "m" may be understood as denoting both "maximum" and "minimum".

Let us check that near standpoint ($r = 0$) the unfamiliar expression (II.10) reduces to a recognizable form. At $r = 0$, according to the appendix, $g = G$ and $g' = G'$, so $z_m = 1$ and $\mathrm{Im}\, z_b^{\pm} = 0$. Because the functions g, G, g', G' are controlled by a single parameter of order Hubble length (see Appendix), the foregoing values of z_m and $\mathrm{Im}\, z_b^{\pm}$ remain good approximations at all spatial distances from standpoint that are small on Hubble scale. Thus in standpoint spatial neighborhood (where the metric (I.1) becomes Minkowskian to the extent that $g = G$ and $g' = G'$ may be approximated by constants) we have

$$\frac{J^2}{r^2} \approx \frac{(G^2+G'^2)^{1/2}}{2}\frac{z}{1-z}$$

$$= \frac{(G^2 + G'^2)^{1/2}}{2} \frac{v_{tr}^2}{1 - v^2}. \qquad (II.11)$$

Taking the square root of (II.11) reveals J as there proportional to $p_{tr}r$ where p_{tr}, apart from a missing rest-mass factor, is the transverse component of particle momentum. Up to a factor constant along a trajectory, (II.11) thus accords with the familiar flat-space meaning of angular momentum.

It is of course because the dynamics here considered ignores non-gravitational forces that rest mass fails to make appearance. The here-defined dimension-length "J" is a label on a homogeneous-universe "geodesic" that may be followed by *any* particle, independent of rest mass. Evidently, the value of J carried by a particle trajectory is standpoint dependent; both the age and the spatial location of standpoint need specification.

III. Barrier Between Unphysical and Particle Trajectories

Because J is conserved along a trajectory, a trajectory with nonvanishing J can never, in the 5-dimensional (t, r, φ, x, z) space, pass through a point where z is equal either to zero or to $z_m(r,t)$; according to (II.10), J vanishes at such a point. Thus any $J \neq 0$ trajectory must fall into *one* of 3 categories, being characterized along its entirety *either* by $0 < z < z_m(r,t)$, by $z > z_m(r,t)$, or by $z < 0$. (A point where $r = 0$ and $\operatorname{Im} z_b^{\pm}(r,t) = 0$ might seem to require special attention but cannot be reached if $J \neq 0$.) Because in standpoint spatial neighborhood, where $z_m(r,t) \approx 1$, the former category associates with particles, we postulate $0 \leq z < z_m(r,t)$ to be a general requirement of a particle trajectory. A lightlike trajectory is characterized by $z = z_m(r,t)$ (and thus by zero J!). Notice that massive particles of zero J have $z = 0$ whereas photons never have $z = 0$ but always have $J = 0$. Notice also that the physical requirement $z \geq 0$ demands $\dot{r}^2 \leq 1$.

It is illuminating to consider at fixed t, r the complex $-z$ plane for the function $L(z)$, which has a pair of complex-conjugate branch points as shown in Figure 1. At $r = 0$ (any t), where $g = G$ and $g' = G'$, these two fourth-root branch points collapse to a single square-root branch point at $z_m = 1$. From (II.2) one recognizes $z_m = 1$ in the Minkowskian standpoint neighborhood as corresponding to the familiar flat-space boundary at $v^2 = 1$ that separates particle-like from unphysical geodesics.

Amusingly, Figure 1 allows a generalization of the usual imaginary-L tachyonic region to be identified through analytic continuation in a complex plane cut by branch lines connecting infinity to the two branch points at $z_b^+(r,t)$ and $z_b^-(r,t)$. On the real-z line reached from the physical region by a path passing *above* z_b^+ or *below* z_b^- (arrows in Figure 1), L is pure imaginary.

The real-L, real $-z$ nonphysical region- -where $z > z_m$- - has no analogue in general relativity. Although the standpoint-cosmology Lagrangian L lacks singularity at $z = z_m$ or $z = 0$ (except when $r = 0$), "angular-momentum" conservation insulates the real-L unphysical region by a "barrier" analogous to the familiar "centrifugal barrier" that prevents $J \neq 0$ particle trajectories from reaching the

spatial origin.* Positive$-ds^2$ unphysical geodesics ($z > z_m$ or $z < 0$) may consistently be ignored. Exclusion of the region $z < 0$ as "unphysical" corresponds to exclusion of $\dot{r}^2 > 1$ and conforms to the boundary of (compact)standpoint spacetime. Everywhere in this spacetime, $r \leq t$.

*Quantum-mechanical penetration of the z_m barrier is probably not meaningful. Standpoint spacetime is a classical *approximate* concept requiring hugeness of Hubble length on particle scale. For a standpoint sufficiently "young" (sufficiently-small Hubble length) to allow barrier penetration, meaning for spacetime is absent from standpoint cosmology.[1]

IV. Riemannian Cold-Matter

Throughout any standpoint's universe the condition $z << z_m$ correlates with "cold" matter- -matter moving slowly in the coordinates of its *own* standpoint. A small value of v/c in such coordinates guarantees, from *any* standpoint, $z \lesssim v^2/c^2$. Rewriting the non-Riemannian (I.1) in the form

$$ds^2 = \frac{(g^2+g'^2)^{1/2}}{2}\left\{(z_m - z)^2 + (Imz_b^+)^2\right\}^{1/2}(dt^2 - dr^2), \qquad (IV.1)$$

and invoking the relation,

$$z_m^2 + (Imz_b^+)^2 = \frac{G^2+G'^2}{g^2+g'^2}, \qquad (IV.2)$$

one recognizes that when $z << z_m$ the Riemannian small-z expansion of (IV.1) to first order namely

$$ds^2 \approx \frac{1}{2}\left\{(G^2+G'^2)^{1/2}(dt^2 - dr^2) - \frac{gG+g'G'}{(G^2+G'^2)^{1/2}}r^2 d\vec{n}^2\right\}, \qquad (IV.3)$$

becomes accurate- -non-Riemannian corrections thereto being of second order in z. The considerations of Reference (2) guarantee that *in the standpoint's spatial neighborhood*, where $g \approx G$ and $g' \approx G'$, Formula (IV.3) is compatible with a Riemannian approximation valid for *arbitrary* spacetime displacements (including "hot" as well as "cold" matter). The approximate "metric" (IV.3), on the other hand, is accurate for *cold* matter *anywhere* (as a representation of gravity).

To the extent that cold matter underlies *physical* meaning for spacetime- -providing measuring rods and clocks- -standpoint cosmology might be described as "physically" Riemannian and correspondingly "objective". In particular, cold-matter trajectories in matter-standpoint spacetime (over distances small on the local Hubble scale) may be approximately mapped onto the spacetime of a distant standpoint. By contrast, "hot-matter" trajectories (such as light rays) are only known to be mappable over Hubble-scale distances if they nearly intersect the standpoints being compared[(2)]. ("Nearly" means impact parameter small on Hubble scale.)

Notice that for $z << z_m$, the general formula (II.10) for J reduces to

$$\frac{J^2}{r^2} \approx \frac{(gG+g'G')^2}{2(G^2+G'^2)^{3/2}}z, \qquad (IV.4)$$

a cold-matter approximation compatible with (II.11) in standpoint spatial neighborhood but valid everywhere. Thus "coldness" generally implies low J^2/r^2 although the converse is not true.

V. General Nonradial Trajectories

The general (nonperturbative) trajectory is obtainable by solving the second-order nonlinear radial equation

$$\frac{d}{dt}\frac{\partial L(t,r,\dot{r},\dot{\varphi})}{\partial \dot{r}} = \frac{\partial L}{\partial r}, \tag{V.1}$$

with $\dot{\varphi}$ and $\ddot{\varphi}$ expressed in terms of $J, t, r, \dot{r}$. When $\dot{\varphi} = 0$, a general 2-parameter closed-form solution has been given in Reference (2). For $\dot{\varphi} \neq 0$ solutions of (V.1) remain to be found. Expecially interesting will be the general lightlike trajectory- -obeying $r^2\dot{\varphi}^2 = (1 - \dot{r}^2)z_m(r,t)$.

What is the meaning of vanishing J for Hubble-scale photon trajectories even when $\dot{\varphi} \neq 0$? Where the action is non-Riemannian, vanishing J replaces vanishing photon mass (or vanishing ds^2). The meaning of light is then not local but derives from the angular isotropy of the large-scale universe as perceived from any standpoint in the frame of that standpoint. The quantity J should not be called "angular momentum" when applied to light.

Appendix

For ease of reference the action-determining coefficients G, g, G', g' are here given throughout that double-cone interior- -a region where $0 \leq t \pm r \leq 2R$- -which constitutes the spacetime of a standpoint whose age parameter is R (Reference (2) deduces Hubble length $2(\sqrt{2}-1)R$). The coefficients are conveniently expressed through a pair of angles, $-\frac{\pi}{2} \leq \varphi_\pm \leq \frac{\pi}{2}$, such that

$$\frac{t \pm r}{R} = 1 + \sin\varphi_\pm. \tag{A.1}$$

The standpoint locates at $\varphi_\pm = 0$, i.e., at $r = 0, t = R$. Defining

$$\begin{aligned} h_\pm &\equiv 1 + 2\sin\varphi_\pm, & f_\pm &\equiv (1 - 2\sin\varphi_\pm)\frac{1+\sin\varphi_\pm}{\cos\varphi_\pm}, \\ u &\equiv 1 + \sin\varphi_+ + \sin\varphi_-, & v &\equiv \frac{(1+\sin\varphi_+)\cos\varphi_+ - (1+\sin\varphi_-)\cos\varphi_-}{\sin\varphi_+ - \sin\varphi_-}, \end{aligned} \tag{A.2}$$

the action coefficients are

$$\begin{aligned} G &= h_+ f_- + f_+ h_-, & g &= 2uv, \\ G' &= h_+ h_- - f_+ f_-, & g' &= u^2 - v^2. \end{aligned} \tag{A.3}$$

Note that when $\varphi_+ = \varphi_-(r = 0), u = h_+ = h_-, v = f_+ = f_-$ and $g = G, g' = G'$.

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2. G.F. Chew, Lawrence Berkeley National Laboratory preprint LBL-37854 (1995), submitted to *Physical Review D.*

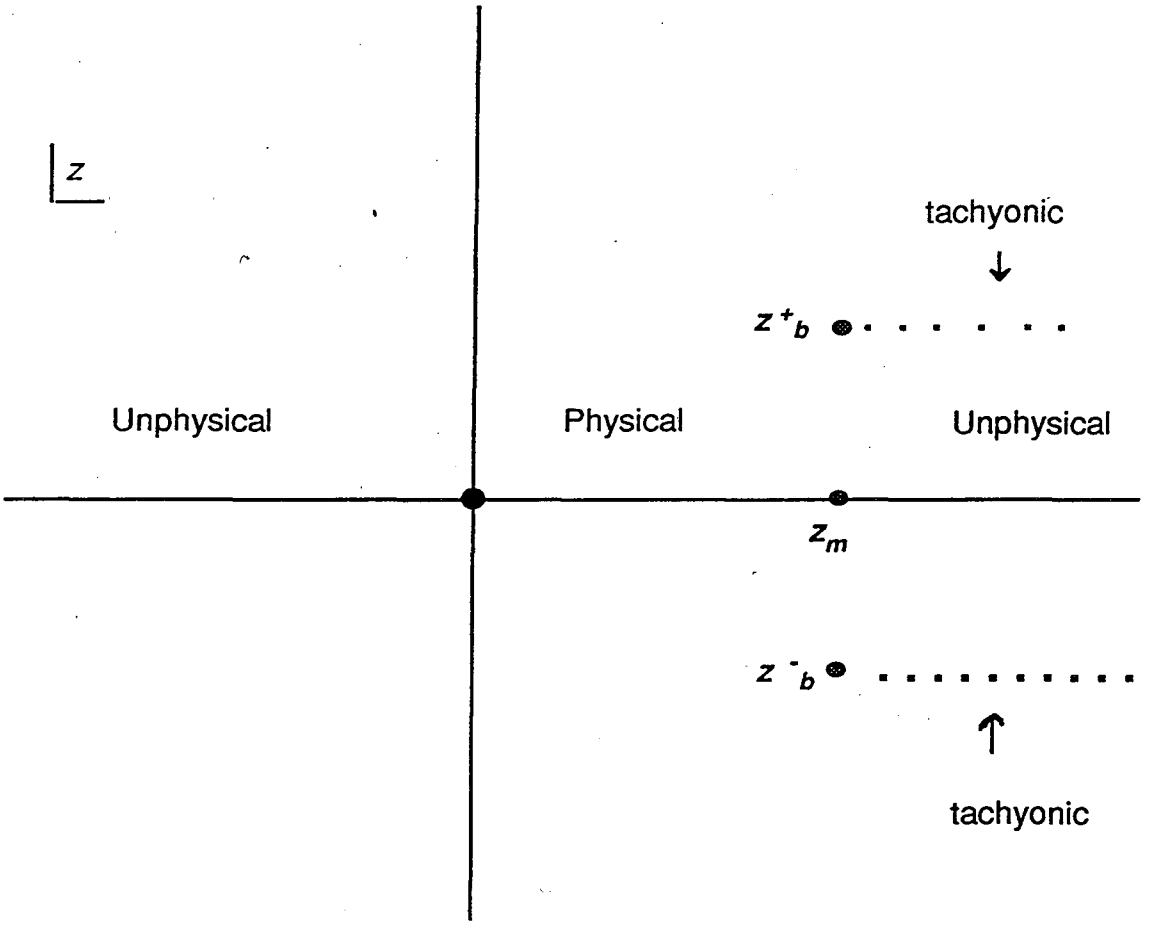

Figure 1

The complex-z plane, showing location of branch points in $L(z)$ at $z = z_b^{\pm}$.

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