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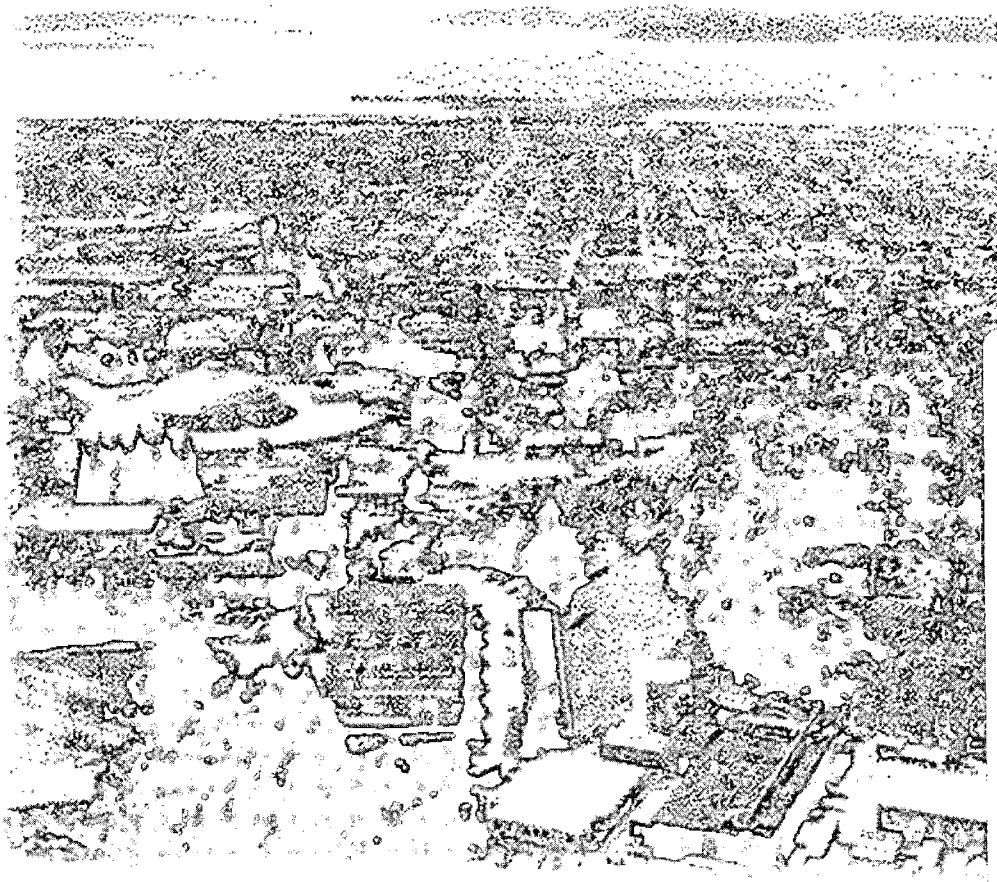

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PURE PERMANENT MAGNET HARMONICS CORRECTOR RING*

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Pure Permanent Magnet Harmonics Corrector Ring

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Abstract—**A concept for creating any desired harmonics mix in a pure permanent magnet (PM) corrector ring is presented. Useful for nulling various harmonics simultaneously, such a device is versatile for many accelerator applications. The harmonic mix can be changed without redesign or replacement by a new ring or parts and, if desired, can be accomplished in-situ via remote control of rotor motors. Harmonics suppression of greater than a factor of 100 or even 1000 are possible; exact functional dependencies of harmonics suppresion capability versus magnet geometry are given. Sensitivity to positioning and corrector ring PM errors are given, and shown to be themselves nullable.**

I. Introduction

Presently, much effort is put into designing magnets with tight harmonics specifications. In the case of electromagnets this entails laborious attention to iron/coil design geometry and often tedious and costly experimentation of end chamfers and designs. In the case of permanent magnets, block quality, sorting, and, positioning must be carefully controlled. Furthermore, in both instances these factors limit the attainable level of field quality achievable.

The theory of pure PM design in two dimensions has been described thoroughly [1,2]. Here we present a concept utilizing a PM corrector ring, insertable at any desired location in the beam path, capable of providing any desired harmonic mix. The present application is to null the harmonics of the Q2 septum quadrupole for SLAC's B-factory. Herein, the PM material with $\mu = 1$ is represented by magnetic charge sheets on surfaces [3,4].

II. Field from Homogeneously Magnetized PM Cylinders

In 2-D, the field at location $z = x + iy$ due to a cylinder of permanent magnet material of radius r_c centered at

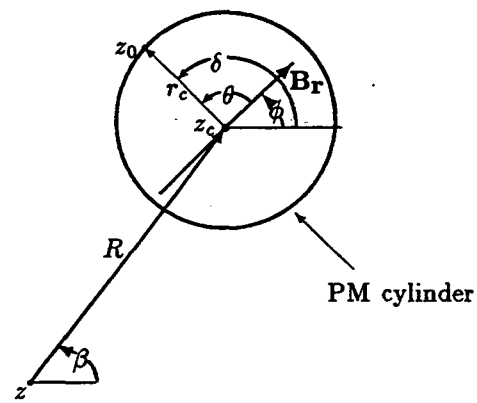

Fig. 1. 2-D geometry for a arbitrarily positioned and oriented PM cylinder (beam axis is into paper, ϕ is magnetization direction w.r.t. horizontal, δ is the integration angle w.r.t. horizontal, and $\theta \equiv \delta - \phi$)

$z_c = z + Re^{i\beta}$ with uniform magnetization $\mathbf{B_r} = B_r e^{i\phi}$ is given by [5] (See Figure 1):

$$B^*(z) = \frac{B_r}{2\pi}\int_0^{2\pi} \frac{r_c \cos\theta}{z - z_0(\delta)} d\delta = \frac{B_r e^{i\phi}}{2}\left(\frac{r_c}{z_c - z}\right)^2, \quad (1)$$

where $B_r \cos\theta(\delta)$ is the equivalent magnetic charge density at the point on the cylinder surface $z_0(\delta) = z_c + r_c e^{i\delta}$, $0 \le \delta < 2\pi$. The derivation is given in the appendix.

The integrated field, $I^*(z) = \int_{-\infty}^{\infty} B^*(z) dZ$, along an axis ($Z$) parallel to that of a lone cylinder of length L is

$$I^*(z) \equiv \sum_{n=1}^{\infty} b_n z^{n-1} = \frac{B_r L r_c^2}{2}\sum_{n=1}^{\infty} \frac{n z^{n-1}}{z_c^{n+1}} e^{i\phi}, \quad (2)$$

where the infinite series on the right is the multipole expansion of Eq. (1) about $z = 0$.

Using two independently rotatable cylinders each of length $L/2$ placed end-to-end, arbitrary net magnetization orientation and strength are achievable by rotation and counter-rotation, respectively, of cylinders in a pair. The effective magnetization orientation of the cylinder pair is adjustable by rotating both cylinders an angle ϕ.

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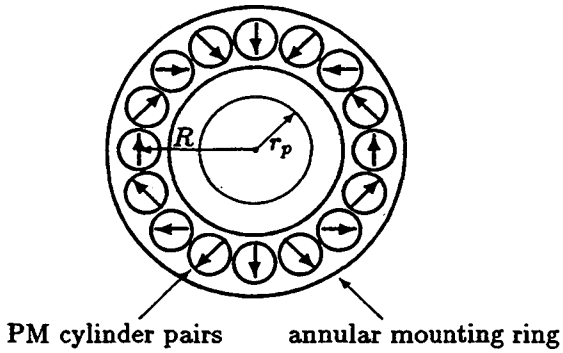

Fig. 2. Pure PM harmonics corrector ring (beam axis at $z = 0$ is into paper at center of ring)

The effective strength $\epsilon B_r L$ of the cylinder pair is adjustable by subsequently counter-rotating cylinders an incremental angle $\pm\eta$:

$$\epsilon B_r L = \cos\eta B_r L \tag{3}$$

For M cylinder pairs spaced uniformly in azimuth (see Fig. 2) $\beta_m = m2\pi/M$, $0 \le m \le M-1$, with net magnetization directions ϕ_m and strengths ϵ_m, the multipole expansion coefficients about $z = 0$ of the integrated field become

$$b_n = k_n \sum_{m=0}^{M-1} e^{-i(n+1)\beta_m} p_m; \tag{4}$$

$$\text{where } k_n \equiv \frac{B_r L n r_c^2}{2R^{n+1}}; \quad p_m \equiv \epsilon_m e^{i\phi_m}.$$

Equivalently, in matrix form:

$$\mathbf{b} = [K][T]\mathbf{p}, \tag{5}$$

where $[K]$ is a diagonal matrix consisting of the elements k_n, $[T]$ is a matrix with elements $T_{n,m} = e^{-i(n+1)\beta_m}$, and $\mathbf{p}$ is an M-element vector quantifying the net orientation and strength of each of the M cylinder pairs.

III. CREATING AN ARBITRARY MULTIPOLE MIX FOR FIELD CORRECTION

To produce a given $\mathbf{b}$, the required $\mathbf{p}$ is given by

$$\mathbf{p} = [T]^{-1}[K]^{-1}\mathbf{b}. \tag{6}$$

where the elements of $[T]^{-1}$ and $[K]^{-1}$ are

$$T^{-1}_{m,n} = e^{i(n+1)\beta_m}/M; \quad K^{-1}_{n,l} = \begin{cases} 1/k_n & \text{if } n = l \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Thus required orientation and strength of the m cylinder pairs are given by

$$p_m \equiv \epsilon_m e^{i\phi_m} = \sum_n e^{i(n+1)\beta_m} b_n / M k_n. \tag{8}$$

Let error fields in a magnet be characterized by the multipole expansion a la POISSON Code[6] format:

$$I^*(z) = i \sum_{N=N_1}^{N=N_{max}} \frac{N c_N}{r_p} \left(\frac{z}{r_p}\right)^{N-1}, \tag{9}$$

where the complex c_N [G-cm^2] $\equiv s_N e^{i\alpha_N}$. Setting $b_{n=N}$ of Eq. (8) equal to the negative of the N error coefficient $i s_N e^{i\alpha_N}/r_p^N$ of Eq. (9), the N-pole error term of any accelerator magnet may be nulled in an adjacent coaxial harmonics corrector ring consisting of M cylinder pairs of effective length $\epsilon_N L$ and magnetization orientations defined by ϕ_{m_N}, given by

$$\epsilon_N L = \frac{N s_N}{r_p} \frac{2}{M N B_r} \left(\frac{R}{r_c}\right)^2 \left(\frac{R}{r_p}\right)^{N-1}, \text{ and} \tag{10}$$

$$\phi_{m_N} = (N+1)\beta_m + \alpha_N - \frac{\pi}{2} \equiv (N+1)\beta_m + \phi_{0_N}.$$

To null a single multipole term N_1, the effective length $\epsilon_{N_1} L$ of all M cylinder pairs in the corrector ring are identical. Nulling of an arbitrary number of multipole terms is accomplished by a vectorial superposition, per Eq. (8). Effective lengths $\epsilon_m L$ of the M cylinder pairs are then different, but none are larger than $L \sum_{N=N_1}^{N=N_{max}} \epsilon_N$:

$$L\epsilon_m e^{i\phi_m} = L \sum_{N=N_1}^{N=N_{max}} \epsilon_N e^{i\phi_{m_N}}. \tag{11}$$

where the "N" are the mulipole error terms to be nulled.

Nulling an N-pole term in the corrector ring will in turn introduce higher order multipole errors, per Eqs. (10) and (4):

$$I^*(z) = \frac{B_r \epsilon_N L M r_c^2}{2} \sum_{n=N+\nu M}^{\infty} \frac{n e^{i\phi_{0_N}} r_p^{n-1}}{R^{n+1}} \left(\frac{z}{r_p}\right)^{n-1} =$$

$$\frac{-iNc_N}{r_p} \sum_{n=N+\nu M}^{\infty} \frac{n}{N} \left(\frac{r_p}{R}\right)^{n-N} \left(\frac{z}{r_p}\right)^{n-1}, \tag{12}$$

where $\nu = 0, 1, 2, ..., \infty$. The first $N_1, .., N_{max}$ terms of Eq. (12) (i.e., with $\nu = 0$) are the negative of those that were to be nulled from Eq. (9). The corrector ring should consist of more cylinders than the highest harmonic component to be nulled, i.e., $N_{max} < M$, otherwise nulling the highest terms would introduce lower harmonics. Field contributions from the newly introduced error terms start with the $n = N + M$ term and are relatively small compared with the original N-pole error term (i.e., where $\nu = 0$) that was nulled. For each of the nulled error terms N, at $|z| = r_p$ the ratio f of newly introduced error terms to the corresponding nulled term is:

$$f = \sum_{\nu=1}^{\infty} \left(\frac{N+\nu M}{N}\right) \left(\frac{r_p}{R}\right)^{\nu M}. \quad (13)$$

The largest term is where $N = 1$, and occurs when $\nu = 1$ if $r_p/R < ([1+M]/[1+2M])^{1/M}$ ($\simeq .96$ for $M = 16$), yielding a reduction factor

$$f = (1+M)(r_p/R)^M. \quad (14)$$

IV. PRACTICAL DESIGN ISSUES

A. *What level of harmonics reduction is possible?*

The number and radial placement of cylinders necessary to achieve a desired level of harmonics reduction follows from Eq. (14).

For the Q2 magnet, assume original B field quality at the normalization radius $r_p = 4.5$ cm is good to 10^{-2} in all harmonics, and that we desire to make it 10^{-4}. For specified stay clear radii $r_1 = 5.0$ cm and $r_2 = 6.4$ cm, choosing for scenario (*i*) $M = 16$, $R = 5.7$ cm, and $r_c = 0.7$ cm, appropriately orienting cylinders would null the N=1 harmonic (as well as others) and would introduce a new $N = 17$ term that contributes a field equal to 39% of that of the original $N = 1$ term at r_p. For scenario (*ii*) let $M = 16$, $R = 5.9$ cm and $r_c = 0.5$ cm, yielding $f = 22\%$. Correction strength capability goes as $r_c^2 r_p^{N-1}/R^{N+1}$; thus the $N = 1$ term strength correction capability of scenario (*ii*) is only 48% of that of scenario (*i*) and marginally less for higher harmonics. Scenario (*ii*) yields a harmonics reduction factor of 5, still far short of the factor of 100 sought.

More effective in the radially restricted Q2 case is increasing the number of rotors. For scenario (*iii*) let $M = 32$, $R = 5.7$ cm, and $r_c = 0.7$ cm, resulting in $f = 1/58$. For scenario (*iv*) letting $M = 32$, $R = 5.9$ cm, and $r_c = 0.5$ cm, gives $f = 1/176$. For the latter design, which more than meets the harmonics reduction criteria, cylinder packing factor $2r_cM/2\pi R = 0.86$, leading to a 1.6 mm spacing between cylinders.

For some instances of quadrupole magnet correction, $N = 1$ and $N = 2$ terms need not be nulled, in which case minimum harmonic reduction is $\sim$ three times better than that of the above scenarios (See Eq. (13).) In cases that are not so radially restricted, much greater harmonics rejection factors are attainable via decreasing r_p/R (e.g., for $M = 16$ and $r_p/R = 0.5$, $f \simeq 1/4000$), though at the expense of harmonic strength nulling capability.

B. *What magnitude of harmonic can be nulled?*

The length of cylinder pairs to achieve a desired harmonic strength nulling capability follows from Eqs. (10) and (11). Assume a level of $q_N\%$ N^{th}-harmonic at r_p must be nulled, i.e., $Ns_N/r_p = 0.01 q_N N_f s_{N_f}/r_p$, where N_f is the fundamental harmonic and the N are the harmonics to be nulled. We have

$$\epsilon_N L = \frac{0.01 q_N N_f s_{N_f}}{r_p} \frac{2}{MNB_r} \left(\frac{R}{r_c}\right)^2 \left(\frac{R}{r_p}\right)^{N-1}. \quad (15)$$

For the Q2 magnet, $2s_2/r_p = 55800 r_p$ G-cm. Assume for instance that harmonics $N = 1, 3, \& 4$ with magnitudes $q_N = 1\%, 0.5\%$, and 0.25%, respectively at $r_p = 4.5$ cm, must be nulled and other harmonics are negligible. For the parameters of scenario (*iv*) and with $B_r = 10,000$ G, required lengths from Eq. (15) are: $\epsilon_{N=1}L = 2.19$ cm, $\epsilon_{N=3}L = 0.573 q_3 \epsilon_{N=1} L$, and $\epsilon_{N=4}L = 0.564 q_4 \epsilon_{N=1} L$. From Eq. (11), if the regular/skew mixes were such that all three ϕ_{m_N} were identical for some m, the required effective length of that longest cylinder pair would be 3.2 cm.

It is not feasible to use the harmonics corrector ring to null the high order allowed harmonics occuring in a PM device (e.g. the $N = 18$ harmonic of a 16-block PM Q2 quadrupole); these can be nulled or made negligible rather by spacing of the PM blocks [1] (an 11% space penalty), by employing finer block segmentation (e.g. 24 blocks in the Q2 magnet itself), and/or by reducing the ratio r_p/r_1.

It remains then, to be sure to conservatively estimate the magnitude of uncorrected harmonics (i.e., $q\%$ of the fundamental) so that the capacity, i.e., the length L of the designed corrector ring to null them is sufficient. In general, for a given corrector length L, there is a tradeoff between attainable harmonics reduction factor f (Eq. 13) and nullable harmonic magnitude Ns_N/r_p (Eq. 10). Larger M and R/r_p lead to a better harmonics reduction ratio f, but lower the maximum nullable harmonic magnitude (assuming Mr_c is constant). Nonetheless, as illustrated above, both impressive rejection ratios and large absolute magnitudes of nullable harmonics are simultaneously attainable.

C. *What if the harmonic mix to be nulled changes?*

If the mix of harmonics to be nulled is known and expected to remain invariant, a corrector ring can be designed using cylinders of different lengths per Eqs. (11) and (10). Alternatively, inverting the same matrix of Eq. (5), cylinders of different radii squared $r_{c_m}^2$ (or B_r, were they available) could likewise be employed. Inverting a different matrix, variable radial position R_m could also be employed to null harmonics.

However, the beauty of the counter-rotating cylinder pairs scheme is its flexibility; arbitrary cylinder pair net magnetization strength and orientation allow changing the mix of harmonics to be nulled without resorting to ring or parts replacement. Alternatively, several corrector rings of a standard cylinder pair design can be utilized to null a different error harmonics mix in a series of nominally identical magnets. Furthermore, this robust scheme

provides for self-correction as shown in the following section. The counter-rotation scheme can also be employed with blocks having other shapes, e.g. with square PM cross-sections inside a machined cylindrical sleeve to facilitate rotation, and/or with a different geometrical arrangement of tuning blocks.

A uniform temperature excursion will not alter the harmonic mix of either an accelerator magnet or its companion corrector ring, though it causes a field magnitude change in both of equal percentage. Thus it will not affect harmonic corrector ring performance.

D. Would corrector ring shielding affect performance?

Corrector ring shielding creates additional (virtual) field sources as images of the originals. A dipole or PM cylinder of strength and orientation $p_m = \epsilon_m e^{i\phi_m}$ at location $Re^{i\beta_m}$ in a device with an infinitely permeable annular shield of radius R_s centered at $z = 0$ will produce an image source of strength and orientation $t_m = \epsilon_m (R_s/R)^2 e^{i(2\beta_m - \phi_m)}$ at $R_t e^{i\beta_m}$, where $R_t = R_s^2/R$ and where it is assumed that $r_c/R \leq 0.1$.

Nulling an N-pole term in the shielded corrector ring will introduce further error terms, in addition to those given by Eq. (12). Using the ϕ_{m_N} given in Eq. (10), Eq.(8) gives the coefficient for a single nulled multipole in terms of the uniform effective length ϵ_N and reference orientation ϕ_{0_N}:

$$b_N = M k_N \epsilon_N e^{i\phi_{0_N}}. \tag{16}$$

Error terms created due to image sources, from Eq. (4) are:

$$b_{n=\nu M-N} = M k_n \epsilon_N e^{-i\phi_{0_N}} \left(\frac{R}{R_s}\right)^{2(n-1)}, \tag{17}$$

where $\nu = 0, 1, 2, ..., \infty$. Field contributions from the image source related error terms start with the $n = M - N$ term and are relatively small compared with the original N-pole error term that was nulled. For each of the nulled error terms N, at $|z| = r_p$ the ratio f of image source induced error terms to the corresponding nulled term is:

$$f \equiv \sum_{n=\nu M-N}^{\infty} (nc_n/r_p)/(Nc_N/r_p) = \tag{18}$$

$$e^{-i2\phi_{0_N}} \sum_{\nu=1}^{\infty} \left(\frac{\nu M - N}{N}\right) \left(\frac{R}{R_s}\right)^{2(\nu M-N-1)} \left(\frac{r_p}{R}\right)^{\nu M-2N}.$$

The largest terms are those for which $\nu = 1$ and they increase with N. Thus one must choose M and radii R, R_s, and r_p such that the rejection ratio f is sufficiently small for the highest order terms to be nulled, as is the case for scenario *iv* above.

V. POSITIONING SENSITIVITES AND IMPLICATIONS FOR HARMONICS

Harmonic mix sensitivities due to a perturbation δP in
(1) radial position R_m of the mth rotor pair,
(2) azimuthal position β_m of the mth rotor pair,
(3) angle ϕ_m of magnetization of the mth rotor pair,
(4) length L_m of the mth rotor pair,
(5) cosine of separation angle η_m between magnetization directions of the two cylinders comprising the mth rotor pair,
(6) cylinder radius squared r_c^2 of the mth rotor pair, or
(7) remanent field strength B_r of the mth rotor pair,
is given by (using Eq. (2)):

$$\delta P \frac{dI^*(z)}{dP} = \sum_{n=1}^{\infty} \left\{ g_n \left(\frac{r_p}{R_m}\right)^{n-1} B_{r_m} L_m \left(\frac{r_{c_m}}{R_m}\right)^2 e^{i(\phi_m - (n+1)\beta_m)} \right\} \left(\frac{z}{r_p}\right)^{n-1}, \tag{19}$$

where for

$$P \equiv \begin{cases} R_m \\ \beta_m \\ \phi_m \\ L_m, \\ \cos\eta_m \\ r_{c_m}^2 \\ B_{r_m} \end{cases} \quad g_n \equiv \begin{cases} (-n[n+1]/2)(\delta R_m/R_m) \\ (-n[n+1]/2)\delta\beta_m \\ (in/2)\delta\phi_m \\ (n/2)(\delta L_m/L_m) \\ (n/2)\delta\cos\eta_m \\ (n/2)(\delta r_{c_m}^2/r_{c_m}^2) \\ (n/2)(\delta B_{r_m}/B_{r_m}). \end{cases}$$

The kernel of Eq. (17) in brackets {} are merely new inc_n/r_p terms which themselves are nullable via new $\epsilon_N L, \phi_{m_N}$ contributions, calculated from Eq. (10), which when added to the previous summation in Eq. (11) yield new L_m and ϕ_m. Thus, the corrector ring is capable of self-correction!

The magnitude of the corrector ring perturbation-induced harmonics are directly calculable. For the parameters of scenario (*iv*) and with $B_r = 10,000$ G, $L_m = 5$ cm, and a perturbation $\delta P/P = 1\%$ (or $\delta P = 0.01$ rad when P represents an angle or $\delta P = 0.01$ when P represents $\cos\eta_m$), the contribution at $|z| = r_p$ of the new $N = 1$ term s_1/r_p normalized to the Q2 fundamental $N_f = 2$ term is $3.59/251000 = 0.14 \cdot 10^{-4}$ for P representing R_m or β_m, and half that amount for the other perturbation parameters.

The largest multipole contribution at $|z| = r_p$ for P representing R_m or β_m, which occurs at the integer nearest the harmonic

$$n = \frac{2\hat{r}_p}{1-\hat{r}_p}, \text{ is } \frac{(ns)_{max}}{s_1} = \frac{(\hat{r}_p+1)}{(1-\hat{r}_p)^2} \hat{r}_p^{(2\hat{r}_p/(1-\hat{r}_p))} \tag{20}$$

times as large as the $N = 1$ term, where $\hat{r}_p \equiv r_p/R$. For scenario (*iv*) $r_p/R = 4.5/5.9$ and $(ns)_{max}/1s_1 = 5.5$, thus

the largest harmonic contribution at $|z| = r_p$ normalized to the Q2 fundamental is $0.77 \cdot 10^{-4}$ for $\delta R_m/R_m$ or $\delta\beta_m = 0.01$. Actual deviations of these parameters should be much smaller than 0.01 and thus self-correction of these harmonics is most likely unnecessary.

For the other perturbation parameters the largest multipole contribution, which occurs at the integer nearest

$$n = \frac{\hat{r}_p}{1-\hat{r}_p}, \text{ is } \frac{\hat{r}_p^{(\hat{r}_p/(1-\hat{r}_p))}}{1-\hat{r}_p} \tag{21}$$

times as large as the $N = 1$ term. For scenario (iv) $r_p/R = 4.5/5.9$ and $(ns)_{max}/1s_1 = 1.8$, thus the largest harmonic contribution at $|z| = r_p$ normalized to the Q2 fundamental is $0.13 \cdot 10^{-4}$ for $\delta\phi_m$, $\delta L_m/L_m$, $\delta B_{r_m}/B_{r_m}$, $\delta r_{c_m}^2/r_{c_m}^2$, or $\delta\cos\eta_m = 0.01$. These contributions are negligible compared with the target harmonics level of $1 \cdot 10^{-4}$ and need not be corrected if actual perturbation parameters $\delta P/P$ or $\delta P \simeq 0.01$.

VI. SUMMARY

The pure PM harmonics corrector ring described herein enables nulling of an arbitrary harmonic mix in an accelerator magnet. For the B-factory's Q2 septum quadrupole, relatively high harmonic magnitudes ($\sim 1\%$ @ r_p) can be nulled with a compact ($\sim$ 5 cm long) corrector. For Q2, high harmonics rejection factors (> 100) are attainable with reasonable device design complexity (32 PM cylinders). For other magnets, harmonics rejection factors of over 10^3 are possible, limited only by the corresponding absolute strength nulling capability of a specified corrector length. Flexibility for infrequent in-situ harmonics mix changing is easily incorporated in the design and is accomplished by manual rotation and counter-rotation of cylinder pairs. Likewise, several corrector rings of a standard cylinder pair design can be utilized to null a different error harmonics mix in a series of nominally identical magnets. Frequent in-situ harmonics mix changing is possible via remote control of rotor rotation. Harmonics introduced by positioning and magnetization errors are themselves nullable in this robust device. Finally, the need to get the requisite field quality directly from shimming, shaping, or positioning the companion accelerator magnet is obviated, simplifying fringe field design compensation, parts tolerancing, PM quality isssues, etc.

This concept of an independent arbitrary harmonic generating/nulling device embodied in the inexpensive, flexible, robust, high-strength PM design provides a powerful new tool for wide application in accelerator design, tuning, and harmonics suppression.

VII. APPENDIX A: INTEGRAL DERIVATIONS

Eq. (1) becomes, with $\theta \equiv \delta - \phi$; $z_0 - z = Re^{i\beta} + r_c e^{i\delta}$:

$$B^*(z) = \frac{B_r}{2\pi}\int_0^{2\pi} \frac{r_c\cos\theta}{z - z_0(\delta)}d\delta$$

$$= \frac{-B_r}{2\pi}\int_0^{2\pi} \frac{r_c(\cos\delta\cos\phi + \sin\delta\sin\phi)}{Re^{i\beta} + r_c e^{i\delta}}d\delta. \tag{22}$$

Defining $Z \equiv r_c e^{i\delta}$ it follows that $dZ = iZd\delta$, $2\cos\delta = Z/r_c + r_c/Z$, $2i\sin\delta = Z/r_c - r_c/Z$, and

$$B^*(z) = \frac{B_r i}{4\pi}\oint \frac{\cos\phi(Z^2 + r_c^2) - i\sin\phi(Z^2 - r_c^2)}{Z^2(Z + Re^{i\beta})}dZ$$

$$= \frac{B_r i}{4\pi}\oint \frac{e^{-i\phi}}{Z + Re^{i\beta}} + \frac{e^{i\phi}r_c^2}{Z^2(Z + Re^{i\beta})}dZ. \tag{23}$$

The pole at $-Re^{i\beta}$ and the double pole at 0 lie outside and inside, respectively, the circle $Z = r_c e^{i\delta}$. Thus from Cauchy's integral formulas it follows directly that

$$B^*(z) = \frac{B_r i}{4\pi}\left(0 + 2\pi i \frac{-r_c^2 e^{i\phi}}{(Z + Re^{i\beta})^2}\bigg|_{Z=0}\right)$$

$$= \frac{B_r r_c^2 e^{i\phi}}{2(Re^{i\beta})^2} = \frac{B_r e^{i\phi}}{2}\left(\frac{r_c}{z_c - z}\right)^2. \tag{24}$$

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REFERENCES

[1] K. Halbach, "Design of permanent multipole magnets with oriented rare earth cobalt material", *Nucl. Instr. Meth.* vol. 169, p. 1, 1980.

[2] K. Halbach, "Physical and optical properties of rare earth cobalt magnets", *Nucl. Instr. Meth.* vol. 187, p. 109, 1981.

[3] K. Halbach, "Insertion device design", LBL Report V8811-1.1-16, 1989.

[4] R.D. Schlueter, "Field errors in hybrid insertion devices", *Halbach Symposium on Magnet Technology* vol. 1, LBL Report 36839, p. 55, 1995.

[5] K. Halbach, "Perturbation effects in segmented rare earth cobalt multipole magnets", *Nucl. Instr. Meth.* vol. 198, p. 213, 1982.

[6] *Reference Manual for the Poisson/Superfish Group of Codes*, LA-UR-87-126, Los Alamos National Laboratory, Los Alamos, NM 1987.

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