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QUANTAL DESCRIPTION OF INELASTIC HEAVY-ION SCATTERING

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Abstract:

A model Schrödinger equation for scattering with energy loss is discussed. The equation is linear and is closely related to the coupled-channels approach to elastic and inelastic scattering. A possible parametrization of the model for applications to heavy-ion scattering is considered.

1. Introduction

In heavy-ion scattering with incident energies far above the Coulomb barrier, one generally observes considerable energy loss due to internal excitation of the outgoing fragments. If one focuses attention on those channels in which no or only a few units of mass and charge are transferred, one may consider these as quasi-elastic or -inelastic channels with roughly no rearrangement. In this case the system can be described in terms of the relative coordinate between the centers of mass of the scattering ions with the channel energy as an additional, non-conserved parameter. Transforming from the channel energy to its "conjugate" variable, the time, the description must then involve non-conservative interactions which depend explicitly on the time.

An example for such an approach is provided by the quantized friction theory based on the Schrödinger-Langevin equation [1-3]. Here the single-particle Schrödinger equation contains a time dependent "friction potential" proportional to the phase of the wave function, i.e. the velocity potential of the quantized motion. This term is chosen because in the classical limit, it gives rise to a friction force proportional to the particle velocity. By its construction, this friction potential makes the Schrödinger-Langevin equation nonlinear. In particular, the friction potential vanishes for the class of stationary solutions, which therefore show no damping effects at all, no matter how highly excited they are [2,4].

On the other hand, looking at the problem from the quantum-mechanical end, it is described by a system of coupled single-particle equations connecting the elastic and all inelastic channels. This system is a linear system coupling wave functions with different channel energies. We there-

fore have a linear problem with an interaction which is nondiagonal in the channel energy. In the corresponding time dependent description we are then also dealing with a linear equation with a time dependent potential. In the present paper, we discuss such a linear Schrödinger equation for scattering with energy loss.

In the next section we view energy loss as arising from current flow from the incident, elastic channel to channels with lower energy. The coupled-channels problem is reformulated in a way in which the explicit coupling goes only in a direction which is associated with loss in channel energy, while the "back-coupling" is treated implicitly via absorptive (or more precisely, "dispersive") terms in energy conserving optical potentials. Thereby the problem becomes non-hermitian and time reversal non-invariant. Going over to a continuous averaged density of channels, a Schrödinger equation results which is an integral equation in the energy variable. The corresponding time dependent approach is discussed in sect. 3. The asymmetric treatment of the coupling between channels with different energies has its analogue here in the requirement that the non-conservative time dependent potential contain only positive frequencies, thus becoming energy losing. In sect. 4 we turn to the phenomenology of the approach and consider a possible parametrization of the model. Some concluding remarks are made in sect. 5.

2. Coupled-channels approach

If rearrangement channels are disregarded (we shall amend this later), the scattering of two heavy nuclei is described by a linear system of coupled equations:

$$\left(E_0 - \varepsilon_\mu + \frac{\nabla^2}{2m} - V_{\mu\mu}(\underset{\sim}{r})\right)\psi_\mu(\underset{\sim}{r}) = \sum_{\substack{\nu=0 \\ (\nu\neq\mu)}}^{\infty} V_{\mu\nu}(\underset{\sim}{r})\,\psi_\nu(\underset{\sim}{r}) \,. \qquad (1)$$

Here the function $\psi_\mu(\underset{\sim}{r})$ is the channel wave function for channel μ with internal excitation energy ε_μ; the elastic channel corresponds to $\mu = 0$ with $\varepsilon_0 = 0$, and E_0 is the incident relative energy in the elastic channel. The terms $V_{\mu\nu}(\underset{\sim}{r})$ are the matrix elements of the nucleus-nucleus interaction taken between the internal states of the fragments in channels μ and ν, respectively, with the coordinate $\underset{\sim}{r}$ between the two centers of mass as a fixed parameter. The matrix $V_{\mu\nu}(\underset{\sim}{r})$ is real and symmetric.

In solving eq. (1) as it stands, one has to consider all channels simultaneously. Since the channels included in eq. (1) are associated with a complete set of internal states of the two nuclei, we have some channels in which the excitation energy ε_μ is discrete, while in others it is continuous; when the channel energy $E_\mu = E_0 - \varepsilon_\mu$ becomes negative, the channel is closed, and eq. (1) clearly contains an infinite number of such channels. In considering low excitations, the "close-coupling" approximation may be appropriate, where all but a few relevant open channels are discarded. But in "deep inelastic" scattering with large Q values there seems to be no natural cut-off in the number of channels to be included in a coupled-channels approach.

Guided by the consideration that the net effect of the couplings in eq. (1) is to feed current from the incident channel with energy E_0 to inelastic channels with lower energy $E_\mu < E_0$, we may approach the solution of the system (1) in a stepwise fashion, proceeding from the elastic channel in the direction of channels with ever increasing internal excitation. Ordering the channel labels in the order of increasing energies ε_ν (with degeneracies taken care of appropriately), we divide the set of channels ν into a set with $\nu = \underline{\nu} < \mu$ and a set with $\nu = \bar{\nu} > \mu$ for any given channel μ. The system (1) then separates into two coupled systems

$$(E_\mu - T - V_{\mu\mu})\psi_\mu = \sum_{\underline{\nu}=0}^{\underline{\nu}<\mu} V_{\mu\underline{\nu}}\psi_{\underline{\nu}} + \sum_{\bar{\nu}>\mu}^{\infty} V_{\mu\bar{\nu}}\psi_{\bar{\nu}}, \tag{2}$$

$$\sum_{\bar{\nu}'>\mu}^{\infty}\left[(E_\mu - (\varepsilon_{\bar{\nu}} - \varepsilon_\mu) - T)\delta_{\bar{\nu}\bar{\nu}'} - V_{\bar{\nu}\bar{\nu}'}\right]\psi_{\bar{\nu}'} = \sum_{\underline{\nu}=0}^{\underline{\nu}<\mu} V_{\bar{\nu}\underline{\nu}}\psi_{\underline{\nu}} + V_{\bar{\nu}\mu}\psi_\mu. \tag{3}$$

Inverting the matrix on the left-hand side of eq. (3) in the subspace of channels $\bar{\nu}, \bar{\nu}' > \mu$, we express the channel wave functions $\psi_{\bar{\nu}}$ in terms of the wave functions ψ_μ and $\psi_{\underline{\nu}}$ with $\underline{\nu} < \mu$, and therefore, from eq. (2),

$$(E_\mu - T - \mathcal{V}_\mu(E_\mu))\psi_\mu = \sum_{\underline{\nu}=0}^{\underline{\nu}<\mu} \mathcal{V}_{\mu\underline{\nu}}(E_\mu)\psi_{\underline{\nu}}, \tag{4}$$

with

$$\mathcal{V}_\mu(E) \equiv V_{\mu\mu} + \sum_{\bar\nu,\bar\nu'>\mu} V_{\mu\bar\nu}\left(\frac{1}{E-(\mathcal{E}-\varepsilon_\mu)+i\eta-T-V}\right)_{\bar\nu\bar\nu'} V_{\bar\nu'\mu}, \quad (5)$$

$$\mathcal{V}_{\mu\underline{\nu}}(E) \equiv V_{\mu\underline{\nu}} + \sum_{\bar\nu,\bar\nu'>\mu} V_{\mu\bar\nu}\left(\frac{1}{E-(\mathcal{E}-\varepsilon_\mu)+i\eta-T-V}\right)_{\bar\nu\bar\nu'} V_{\bar\nu'\underline{\nu}}, \quad (6)$$

where $\mathcal{E}$ stands for the diagonal matrix $\varepsilon_\nu \delta_{\nu\nu'}$.

In the system (4) the channels are coupled via a <u>triangular</u> coupling matrix. The elastic channel $\mu = 0$ is uncoupled; it is described by the Schrödinger equation

$$(E_0 - T - \mathcal{V}_0(E_0))\psi_0 = 0 . \quad (7)$$

This is the familiar optical-model equation for elastic scattering, where the optical potential is in principle given by eq. (5), i.e. Feshbach's generalized optical-model potential [5]. The first inelastic channel is explicitly coupled only to the elastic channel, but loses current to the second, third, etc. channels via the imaginary part of the optical potential $\mathcal{V}_1$. So it goes down through the channels (cf. Fig. 1) until the last open channel with $E_\mu = 0$, which is explicitly coupled to all previous channels but no longer suffers absorption into other inelastic channels, since there are none left that are open and have energy lower than E_μ. In principle one could go on and compute the closed channels with $E_\mu < 0$, but since these are only virtually excited, they are of no interest to us.

Clearly, part of the problem has been transferred to the determination of the rather complicated operators $\mathcal{V}_\mu$ and $\mathcal{V}_{\mu\underline{\nu}}$. But intuition may save us from actually calculating these quantities. It is seen from eq. (5)

that the potential $\mathcal{V}_\mu(E)$ in channel μ at energy $E_\mu = E$ is quite similar in structure to the optical potential in the elastic channel, $\mathcal{V}_0(E)$. Therefore we may simply replace it by one of the commonly used phenomenological expressions for the optical potential at the energy in question. The coupling potentials $\mathcal{V}_{\mu\underline{\nu}}(E)$ may be simplified by dropping the double sum on the right-hand side of eq. (6), since is made up of terms with non-definite sign (in contrast to the double sum in eq. (5)). Since moreover, the dispersive part of $\mathcal{V}_{\mu\underline{\nu}}$, which this double sum represents, does not play the essential physical role that it has in the optical potential, its neglect appears quite reasonable. In this fashion, we replace eq. (4) by

$$(E_\mu - T - \mathcal{V}_0(E_\mu))\psi_\mu = \sum_{\underline{\nu}=0}^{\underline{\nu}<\mu} V_{\mu\underline{\nu}}\,\psi_{\underline{\nu}} \,. \tag{8}$$

This system is to be supplemented by the boundary condition that only the channel wave function with $\mu = 0$ contains an incident plane wave, while the remainder are purely outgoing:

$$\psi_\mu(\underset{\sim}{r}) \xrightarrow[r\to\infty]{} e^{i\underset{\sim}{k}_0\cdot\underset{\sim}{r}}\,\delta_{\mu 0} + f_\mu(\hat{\underset{\sim}{k}}_\mu, E_\mu)\,\frac{e^{ik_\mu r}}{r}\,, \tag{9}$$

where $\underset{\sim}{k}_\mu^2 = 2mE_\mu$. The absolute square of the scattering amplitude $f_\mu(\hat{\underset{\sim}{k}}_\mu, E_\mu)$ determines the cross section $\sigma_\mu(\Omega, E_\mu)$ in channel μ. For the <u>in</u>elastic channels, $\mu > 0$, one may also write

$$\sigma_\mu(\Omega, E_\mu) = \frac{m}{k_0}\int(\underset{\sim}{\nabla}\cdot\underset{\sim}{J}_\mu)\,d^3r\,, \quad \mu > 0, \tag{10}$$

where

$$\underset{\sim}{J}_\mu = \frac{1}{2mi}\left(\psi_\mu^* \underset{\sim}{\nabla} \psi_\mu - c.c.\right) \tag{11}$$

is the current density in channel μ.

All channels suffer absorption due to current loss to channels with lower energy. This is over-compensated in the elastic channel by the incident flux, and in the inelastic channels by the current gain via the source terms provided by the coupling to the channels with higher energy. This can be seen in more detail: Comparing eqs. (1) and (8), we find that

$$\langle \psi_\mu, \mathrm{Im}\, U_\mu \psi_\mu \rangle = \sum_{\bar{\nu} > \mu}^{\infty} \mathrm{Im} \langle \psi_\mu, V_{\mu\bar{\nu}} \psi_{\bar{\nu}} \rangle \leq 0, \tag{12}$$

since by eq. (5), Im U_μ is non-positive. Using eq. (8), we have for the example of two inelastic channels

$$\begin{aligned}
\tfrac{1}{2}\int (\underset{\sim}{\nabla} \cdot \underset{\sim}{J}_0)\, d^3r &= \langle \psi_0, \mathrm{Im}\, U_0 \psi_0 \rangle < 0; \\
\frac{k_0}{2m}\sigma_1 = \tfrac{1}{2}\int (\underset{\sim}{\nabla} \cdot \underset{\sim}{J}_1)\, d^3r &= \langle \psi_1, \mathrm{Im}\, U_1 \psi_1 \rangle + \mathrm{Im}\langle \psi_1, V_{10} \psi_0 \rangle \\
&\equiv \langle \psi_1, \mathrm{Im}\, U_1 \psi_1 \rangle + w_{1\leftarrow 0} > 0; \\
\frac{k_0}{2m}\sigma_2 = \tfrac{1}{2}\int (\underset{\sim}{\nabla} \cdot \underset{\sim}{J}_2)\, d^3r &= \mathrm{Im}\langle \psi_2, V_{21} \psi_1 \rangle + \mathrm{Im}\langle \psi_2, V_{20} \psi_0 \rangle \\
&\equiv w_{2\leftarrow 1} + w_{2\leftarrow 0} > 0.
\end{aligned} \tag{13}$$

With the help of eq. (12), using the hermiticity of $V_{\mu\nu}$, we have

$$\begin{aligned}
\langle \psi_0, \mathrm{Im}\, U_0 \psi_0 \rangle &= -\left(\mathrm{Im}\langle \psi_1, V_{10} \psi_0 \rangle + \mathrm{Im}\langle \psi_2, V_{20} \psi_0 \rangle\right) = -w_{1\leftarrow 0} - w_{2\leftarrow 0} < 0; \\
\langle \psi_1, \mathrm{Im}\, U_1 \psi_1 \rangle &= -\mathrm{Im}\langle \psi_2, V_{21} \psi_1 \rangle = -w_{2\leftarrow 1} < 0.
\end{aligned} \tag{14}$$

We see that the current loss in channel 0 has two components, $-w_{1\leftarrow 0} - w_{2\leftarrow 0}$, which appear again as current gains $+ w_{1\leftarrow 0}$ and $+ w_{2\leftarrow 0}$ in channels 1 and 2, while the current loss in channel 1, $- w_{2\leftarrow 1}$, appears as a current gain $+ w_{2\leftarrow 1}$ in channel 2. The current gains and losses in all channels balance out, since by the hermiticity of the original system (1), the current summed over all channels is conserved,

$$\sum_{\mu=0}^{\infty} \int \underset{\sim}{\nabla} \cdot \underset{\sim}{J}_{\mu} \, d^3r = 0 . \tag{15}$$

It seems natural to interpret the quantities

$$w_{\mu\leftarrow \underline{\nu}} \equiv \mathrm{Im} \langle \psi_{\mu} , V_{\mu\underline{\nu}} \psi_{\underline{\nu}} \rangle \tag{16}$$

as "partial probabilities" for the current flow from channel $\underline{\nu}$ to channel μ, but it must be realized that the right-hand side of eq. (16) may have either sign.

Although the original equations (1) contain a real symmetric interaction and are therefore time reversal invariant, the system (8) no longer is, since it contains the complex optical potential $\mathcal{V}_0(E_\mu)$. This has come about through the incorporation in $\mathcal{V}_0(E_\mu)$, of the boundary condition that there be only outgoing waves in all channels with energy lower than E_μ (giving rise to the appearance of $+ i\eta$ in eq. (5)). The same situation occurs already in the usual optical-model equation for elastic scattering, eq. (7), where the time-reversed solutions do not satisfy the equation with an absorptive potential $\mathcal{V}_0$, but an equation with the complex conjugate, productive "optical potential".

Averaging over channels. In typical heavy-ion scattering situations at energies above the Coulomb barrier, the number of channels to be included in eq. (8) is very large. Going over to a continuous distribution of channel energies, we may write instead:

$$\left(E_0 - \varepsilon - T - U_0(E_0 - \varepsilon)\right)\psi_{\underset{\sim}{k}_0,\varepsilon} = \int_0^{\varepsilon_-} d\varepsilon' V(\varepsilon,\varepsilon')\,\bar{\rho}(\varepsilon')\,\psi_{\underset{\sim}{k}_0,\varepsilon'}, \tag{17}$$

where $V(\varepsilon, \varepsilon')$ is an average of the matrix element $V_{\mu\nu}$ (see below) and $\bar{\rho}(\varepsilon)$ is a smoothed form of the combined level distribution of the two nuclei,

$$\rho(\varepsilon) = \sum_\mu g_\mu\, \delta(\varepsilon - \varepsilon_\mu), \tag{18}$$

with g_μ representing the degeneracy of the level ε_μ. Strictly speaking, most of the levels will of course have continuous excitation energies, corresponding to particle decay of the fragments. However, one may assume that an average level density still makes sense. In line with this, one must argue that the target (or projectile), although excited above the continuum threshold, still holds together long enough during the scattering that the use of optical potentials with a range appropriate to scattering of essentially bound nuclei is justified.

The wave function $\psi_{\underset{\sim}{k}_0,\varepsilon}$ (for clarity we have indicated the incident momentum $\underset{\sim}{k}_0$) represents an average of the channel wave functions ψ_μ in the interval $\varepsilon - d\varepsilon/2 < \varepsilon_\mu < \varepsilon + d\varepsilon/2$. This will in general be an average over very many channel wave functions, and one might suspect that these have random phases and average out to zero. However, since the functions ψ_μ are expansion coefficients in the expansion of the total wave function Ψ in terms of internal fragment states Φ_μ,

$$\Psi(\underset{\sim}{r}, \xi) = \sum_{\mu} \psi_{\mu}(\underset{\sim}{r}) \Phi_{\mu}(\xi),$$

their phases depend on the (arbitrary) choice of phases for the states Φ_{μ} (cf. refs. [6,7]). By a judicious choice of the latter, the phase of ψ_{μ} can be made to vary smoothly with μ over the whole range of excitation energies, leading to a non-vanishing average function $\psi_{\underset{\sim}{k}_0, \varepsilon}$ which depends smoothly on ε (in ref. [7]) these considerations were restricted to a small interval $d\varepsilon$). We thereby transfer the possible randomness of the phases of the ψ_{μ} to the phases of the Φ_{μ}.

Introducing the "channel density wave function"

$$\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r}; E) \equiv \sqrt{\bar{\rho}(\varepsilon)}\ \psi_{\underset{\sim}{k}_0, \varepsilon}(\underset{\sim}{r}) \tag{19}$$

(where we have switched from the label ε to $E = E_0 - \varepsilon$), we obtain

$$(E - T - V_0(\underset{\sim}{r}; E))\, \phi_{\underset{\sim}{k}_0}(\underset{\sim}{r}; E) = \int_{E^+}^{E_0} dE'\, V^L(\underset{\sim}{r}; E_0 - E, E_0 - E')\, \phi_{\underset{\sim}{k}_0}(\underset{\sim}{r}; E'), \tag{20}$$

where

$$\begin{aligned} V^L(\underset{\sim}{r}; \varepsilon, \varepsilon') &\equiv \sqrt{\bar{\rho}(\varepsilon)}\ V(\underset{\sim}{r}; \varepsilon, \varepsilon')\ \sqrt{\bar{\rho}(\varepsilon')} \quad \text{for} \quad \varepsilon > \varepsilon', \\ &\equiv 0 \qquad\qquad\qquad \text{for} \quad \varepsilon \leq \varepsilon'. \end{aligned} \tag{21}$$

The quantity $V(\underset{\sim}{r}; \varepsilon, \varepsilon')$ is an average of the matrix elements $V_{\mu\nu}(\underset{\sim}{r})$ in the range $\varepsilon - d\varepsilon/2 < \varepsilon_{\mu} < \varepsilon + d\varepsilon/2$, $\varepsilon' - d\varepsilon/2 < \varepsilon_{\nu} < \varepsilon' + d\varepsilon/2$. The $V_{\mu\nu}$'s should have nearly random signs: The original $V_{\mu\nu}$'s, which are matrix elements

between basis states $\Phi_{\mu,\nu}$ chosen real by convention (and convenience for their properties under time reversal), are expected to have nearly random signs because of the many internal degrees of freedom involved. The rephasing of the Φ_μ is unlikely to remove this randomness [7]. Therefore, the average matrix element $V(\underset{\sim}{r};\, \varepsilon, \varepsilon')$ is much reduced in magnitude compared to a typical $V_{\mu\nu}(\underset{\sim}{r})$; if the average in given intervals $\varepsilon \pm d\varepsilon/2$, $\varepsilon' \pm d\varepsilon/2$ is over N and N' terms, respectively, one might expect the reduction factor to be $(NN')^{-\alpha/2}$, where $\alpha \lesssim 1$ ($\alpha = 1$ for complete randomness). In the "energy losing potential" $V^L(\underset{\sim}{r};\, \varepsilon, \varepsilon')$ this is compensated by the presence of the channel density factors $\sqrt{\bar{\rho}(\varepsilon)} \sim N^{1/2}$ and $\sqrt{\bar{\rho}(\varepsilon')} \sim N'^{1/2}$.

On the other hand, looking at the integral equation (20), where the $\phi_{\underset{\sim}{k}_0}(E)$ have smoothly varying phases over the whole range of energies E and $\mathcal{V}_0$ is a non-random, phenomenologically chosen function, the kernel V^L must be smoothly varying, too. We conclude from this that by our rephasing procedure the randomness has at least been removed from the average quantity V^L.

The boundary condition for the solutions of eq. (20) is, using eqs. (9), (18) and (19),

$$\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r}; E) = \sqrt{\bar{\rho}(E)}\, \psi_{\underset{\sim}{k}_0, \varepsilon}(\underset{\sim}{r})$$
$$\underset{r\to\infty}{\longrightarrow} \sqrt{\bar{\delta}(E_0 - E)}\, e^{ik\hat{\underset{\sim}{k}}_0 \cdot \underset{\sim}{r}} + F_{\underset{\sim}{k}_0}(\Omega, E)\, \frac{e^{ikr}}{r}, \tag{22}$$

where $\bar{\delta}(\varepsilon)$ is a smoothed-out δ-function with a peak at zero internal excitation energy $\varepsilon = \varepsilon_0 = 0$, i.e. $E = E_0$, and

$$F_{\underset{\sim}{k}_0}(\Omega, E) = \sqrt{\bar{\rho}(\varepsilon)}\, f_{\underset{\sim}{k}_0, \varepsilon}(\Omega) \tag{23}$$

is the average scattering amplitude $f_\mu(\Omega, E_\mu)$ in the interval $\varepsilon - d\varepsilon/2 < \varepsilon_\mu < \varepsilon + d\varepsilon/2$ (denoted by $f_{\underset{\sim}{k}_0,\varepsilon}(\Omega)$) multiplied by the square root of the averaged channel density. The square of the amplitude (23) determines the summed scattering cross section into all channels in the interval dE:

$$\frac{d^2\sigma}{d\Omega\, dE} = \frac{k}{k_0} \left| F_{\underset{\sim}{k}_0}(\Omega, E) \right|^2 . \tag{24}$$

For use in the integral equation (20), one may conveniently make the following replacement in the boundary condition (22):

$$\sqrt{\bar{\delta}(E_0 - E)} \longrightarrow \delta(E_0 - E) \frac{1}{\sqrt{\bar{\rho}(\varepsilon)}} ,$$

where $\bar{\rho}(\varepsilon)$ is the smooth extrapolation of the level density to $\varepsilon = 0$. In the elastic channel equation corresponding to eq. (7), the δ-function drops out, while in the inelastic equations (20) with $E < E_0$, it is folded into the integral kernel.

3. Time dependent description

The averaged coupled-channels equation (20) is an integral equation in the energy variable E. It is of interest to investigate the corresponding time dependent formulation. Without losing any essential physics, we assume for simplicity that in eq. (20), $\mathcal{V}_0$ is independent of energy, $\mathcal{V}_0(\underset{\sim}{r};E) \longrightarrow \mathcal{V}_0(\underset{\sim}{r})$, and V^L depends only on the energy difference $\varepsilon - \varepsilon' = E' - E$: $V^L(\underset{\sim}{r};\varepsilon,\varepsilon') \rightarrow V^L(\underset{\sim}{r};\varepsilon-\varepsilon') = V^L(\underset{\sim}{r};E'-E)$. Then we obtain

$$(E - T - \mathcal{V}_0(\underset{\sim}{r}))\,\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r};E) - \int_{-\infty}^{\infty} dE'\, V^L(\underset{\sim}{r};E'-E)\,\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r};E') = 0. \quad (25)$$

In going from eq. (20) to (22), we have extended the lower limit E^+ in the integral in eq. (20) to $-\infty$, using the fact that by eq. (21),

$$V^L(\underset{\sim}{r};E'-E) \equiv 0 \qquad \text{for } E' \leq E. \qquad (26)$$

Furthermore, we are allowed to replace the upper limit E_0 by $+\infty$, since the boundary condition (22) in conjunction with eq. (26), leads only to solutions of eq. (25) with $E \leq E_0$ anyway, as is best seen by a perturbation-theoretical argument.

Introducing the time dependent wave function

$$\phi(\underset{\sim}{r};t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi(\underset{\sim}{r};E)\, e^{-iEt}\, dE, \qquad (27)$$

we then obtain from eq. (25) the time dependent Schrödinger equation

$$\left(-i\frac{\partial}{\partial t} + T + \mathcal{V}_0(\underset{\sim}{r}) + V^L(\underset{\sim}{r};t)\right)\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r};t) = 0, \qquad (28)$$

where

$$V^L(\underset{\sim}{r};t) = \int_{0^+}^{\infty} V^L(\underset{\sim}{r};\omega)\, e^{i\omega t}\, d\omega \qquad (29)$$

is a time dependent potential which contains only <u>positive</u> frequencies. This corresponds to energy <u>loss</u> from the wave packet $\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r};t)$ describing the relative motion of projectile and target. We have here a situation similar to the interaction of a charged particle with the electromagnetic field, where the interaction term with positive frequencies describes creation of photons, i.e. energy loss in the particle motion. Indeed, one may view the inelastic scattering of heavy ions as being effected through the creation of excitation modes in the fragments, in a loose analogy to bremsstrahlung (where these excitation modes are free photons). Unfortunately, we are not aware of a quantal theory for the equation of motion of a charged particle undergoing bremsstrahlung (with the photon degrees of freedom eliminated); otherwise, this would have been helpful in establishing more detailed properties of the energy losing potential $V^L(\underset{\sim}{r};t)$.

The boundary condition (22) corresponds to an initial condition for the solution $\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r};t)$: for $t \to -\infty$, the solution must go over into a free normalized wave packet with mean momentum $\underset{\sim}{k}_0 = (E_0, \hat{\underset{\sim}{k}}_0)$:

$$\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r};t) \xrightarrow[t\to-\infty]{} \int \sqrt{\bar{\delta}(E_0-E)}\,\sqrt{\bar{\delta}(\hat{\underset{\sim}{k}}_0-\hat{\underset{\sim}{k}})}\,\frac{e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}}{(2\pi)^{3/2}}\, e^{-iEt}\,\sqrt{mk}\, dE\, d\hat{\underset{\sim}{k}}\,; \qquad (30)$$

here the $\bar{\delta}(\ldots)$ are smoothed-out δ-functions (usually sharply peaked Gaussians with unit area). Except for the angular part, the wave packet on the right-hand side of eq. (30) is seen to be the Fourier transform of the incident plane channel density wave function in eq. (22).

We also could start the discussion of scattering with energy loss with the time dependent description, by writing down eqs. (28) to (30) first, relating them to the coupled-channels approach later. Taking eq. (28) as starting equation, we Fourier transform it to eq. (25) using eqs. (26) and (29). Equation (25) is then interpreted as providing current flow from the incident "channel" E_0 to those with lower energies $E < E_0$, as in the previous section. Defining a current density by

$$\underset{\sim}{J}(\underset{\sim}{r}; E) = \frac{1}{2mi}\left(\phi^*(\underset{\sim}{r}; E)\underset{\sim}{\nabla}\phi(\underset{\sim}{r}; E) - c.c.\right), \tag{31}$$

we find that the incident current density

$$\underset{\sim}{J}_0(\underset{\sim}{r}; E) = \frac{\underset{\sim}{k}_0}{m}\,\bar{\delta}(E_0 - E) \tag{32}$$

is then a current density per unit energy, with the total, energy integrated incident current per unit area given by $\underset{\sim}{k}_0/m$.

The requirement that during scattering the wave may lose energy but not probability current, implies that the divergence of the total, energy integrated current density vanishes:

$$\int_0^{E_0} dE\; \underset{\sim}{\nabla}\cdot\underset{\sim}{J}(\underset{\sim}{r}; E) = 0 . \tag{33}$$

Integrating over space and using eqs. (25) and (26), we find

$$\int_0^{E_0} dE\left(\langle\phi(E), \operatorname{Im} V_0\,\phi(E)\rangle + \operatorname{Im}\langle\phi(E), \int_{E^+}^{E_0} dE'\, V^L(E'-E)\,\phi(E')\rangle\right) = 0. \tag{34}$$

The second term in eq. (34) receives contributions only from those components in the bra $\langle\phi(E)$ which have energy $E < E_0$, and which therefore contain only outgoing waves:

$$\phi(E)\rangle = G_+(E)\int_{E^+}^{E_0} dE''\, V^L(E''-E)\,\phi(E'')\rangle, \qquad (35)$$

where $G_+(E) = (E + i\eta - T - \mathcal{V}_0)^{-1}$. Equation (34) then becomes

$$\int_0^{E_0} dE\,\langle\phi(E), \mathrm{Im}\,\mathcal{V}_0\,\phi(E)\rangle = -\int_0^{E_0} dE\left(\int_{E^+}^{E_0} dE''\langle\phi(E''), V^{L\dagger}(E''-E)\,\mathrm{Im}\,G_+^{\dagger}(E)\int_{E^+}^{E_0} dE'\,V^L(E'-E)\,\phi(E')\rangle\right), \qquad (36)$$

which cannot be satisfied for a real potential $\mathcal{V}_0$, since then the left-hand side would be zero while the right-hand side is negative. The introduction of the energy losing potential $V^L(t)$ containing only positive frequencies requires the presence of absorption in the energy conserving potential $\mathcal{V}_0$,

$$\mathrm{Im}\,\mathcal{V}_0 < 0. \qquad (37)$$

That $\mathcal{V}_0$ must be complex is easily understood from the fact that the energy losing potential $V^L(\underset{\sim}{r};t)$ of eq. (29) is necessarily complex and, because of over-all current conservation, its imaginary part must be compensated in some way by the other potential appearing in eq. (28), viz. $\mathcal{V}_0$. Equation (37) is of course in accord with the interpretation, in the previous section, of $\mathcal{V}_0$ as an optical potential.

Again, the appearance of the complex potential $\mathcal{V}_0$ makes the Schrödinger equation (28) time reversal non-invariant, and thus introduces irreversibility into the problem. (Note that the presence of the energy losing potential $V^L(\underset{\sim}{r};t)$ by itself does not violate time reversal invariance, as

long as $V^L(\underset{\sim}{r};\omega)$ is real; for then $V^L(\underset{\sim}{r};-t) = [V^L(\underset{\sim}{r};t)]^*$, which is the condition for time reversal invariance.)

The condition (34) reads in the time dependent description

$$\int_{-\infty}^{\infty} dt \langle \phi(t), (\mathrm{Im}\, \mathcal{V}_0 + \mathrm{Im}\, V^L(t))\phi(t)\rangle = 0, \qquad (38)$$

which implies, using eq. (28),

$$\langle \phi(t), \phi(t)\rangle_{t=-\infty}^{t=\infty} = 0, \qquad (39)$$

i.e. probability is conserved.

Probability current conservation is seen to give rise to a relation between the absorptive part of the energy conserving "optical potential" $\mathcal{V}_0$ and the energy losing potential V^L. This relation, however, involves the wave function and an integration over channel energies (or time, in the time dependent description), and is therefore rather indirect. In physical content it corresponds to the relation between the imaginary part of the optical potential and the friction force occurring in the perturbation theory of nuclear friction [8].

We note from eq. (29) that $V^L(\underset{\sim}{r};t)$ is analytic in the upper complex t plane, so that its real and imaginary parts satisfy a dispersion relation in the time t:

$$\mathrm{Re}\; V^L(\underset{\sim}{r};t) = \frac{P}{\pi}\int_{-\infty}^{\infty} dt' \frac{\mathrm{Im}\, V^L(\underset{\sim}{r};t')}{t'-t} . \qquad (40)$$

Going back to the energy components ("channels") of the wave function, we can define the net current gain per unit volume of the "channel" E in the interval dE at the point $\underset{\sim}{r}$ by

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{J}(\underset{\sim}{r}; E)\, dE ,$$

and the net energy gain per second and unit volume at the point $\underset{\sim}{r}$ is therefore

$$W(\underset{\sim}{r}) = \int_0^{E_0} dE\; E\; \underset{\sim}{\nabla} \cdot \underset{\sim}{J}(\underset{\sim}{r}; E) , \tag{41}$$

or, using eq. (33),

$$W(\underset{\sim}{r}) = \int_0^{E_0} dE\; (E - E_0)\, \underset{\sim}{\nabla} \cdot \underset{\sim}{J}(\underset{\sim}{r}; E) . \tag{42}$$

This gain in energy density may be negative or positive, but the total, space-integrated energy gain is negative, i.e. the total energy decreases:

$$\begin{aligned} W = \int W(\underset{\sim}{r})\, d^3r &= \int_0^{E_0} dE\, (E - E_0) \int \underset{\sim}{\nabla} \cdot \underset{\sim}{J}(\underset{\sim}{r}; E)\, d^3r \\ &= \int_0^{E_0} dE\, (E - E_0) \frac{d\sigma}{dE} \frac{k_0}{m} \leq 0 . \end{aligned} \tag{43}$$

The time dependent equation (28) for scattering with energy loss corresponds to the simplified averaged coupled-channels problem (25). If we go back to the more complete coupled-channels integral equation (20), we see that eq. (28) must be amended in two ways: (i) the energy dependence of $\mathcal{V}_0(\underset{\sim}{r}; E)$ in eq. (20) introduces a time retardation in the conservative potential $\mathcal{V}_0$ in eq. (28),

$$\mathcal{V}_0(\underset{\sim}{r}) \longrightarrow \int_{-\infty}^{t} dt'\; \mathcal{V}_0(\underset{\sim}{r}; t - t')$$

(cf., e.g. ref. [9]). Furthermore, (ii) since the kernel in eq. (20) does not in general depend only on the difference $(E' - E)$, but separately on $(E_0 - E)$ and $(E_0 - E')$, the energy losing potential $V^L(t)$ in eq. (28) should be replaced by an operator which is "nonlocal" in the time and depends on the initial conditions, viz. E_0.

When the time dependent wave packet $\phi(\underset{\sim}{r};t)$ is decomposed into energy components according to eq. (27), a definite phase relation between the various "channel wave functions" $\phi(\underset{\sim}{r};E)$ is implied. This corresponds to the smooth phasing of the channel density wave functions discussed in sect. 2.

4. Application to heavy-ion scattering; phenomenology and parametrization

Nuclear scattering with excitation of a dense distribution of channels, corresponding to a continuous loss of energy from the incident wave, occurs typically in heavy-ion reactions at high energies. However, there exist competing channels involving a more or less violent rearrangement of the fragments, fusion-fission events, etc. Transfer channels in which only a few nucleons are exchanged may still approximately be regarded as quasi-elastic or -inelastic channels. All other channels in which the fragments change their composition drastically, will for simplicity be called "fusion channels". Their presence may be taken into account phenomenologically by an additional absorptive part in the energy conserving potential $\mathcal{V}_0(E)$, corresponding to a sink which removes probability current from the elastic and inelastic channels described by the channel density wave function $\phi_{\underset{\sim}{k}_0}(\underset{\sim}{r})$. In that event over-all current conservation no longer holds, and the equality sign in eq. (36) must be changed to a "smaller than" sign.

For an application of eq. (20) to fitting heavy-ion scattering data one must of course parametrize the energy conserving ($\mathcal{V}_0$) and the energy losing (V^L) potentials. As to the former, it should essentially be represented by a phenomenological optical potential for elastic scattering plus the electrostatic interaction. Currently used forms for such potentials are not precisely determined and not even unique, but at least one is somewhat familiar with them. An energy dependence in the conserving potential $\mathcal{V}_0 = \mathcal{V}_0(E)$ is very likely to be required; this would also allow for taking account of differences between the interactions in the ingoing and outgoing channels.

The energy losing potential V^L is a much less familiar quantity. Recalling eq. (21) and the subsequent discussion, we expect $V^L(\underset{\sim}{r}; \varepsilon, \varepsilon')$ to have the following behavior as a function of ε and ε': the matrix element $V(\underset{\sim}{r}; \varepsilon, \varepsilon')$ in eq. (21) is the average over the matrix elements $V_{\mu\nu}(\underset{\sim}{r})$ between internal states $\Phi_{\mu,\nu}$ with $\varepsilon_\mu \approx \varepsilon$ and $\varepsilon_\nu \approx \varepsilon' < \varepsilon$. The matrix element $V_{\mu\nu}$ will decrease as ε' is decreased downward from some given ε, since the states Φ_μ and Φ_ν will differ more and more in complexity, so that there will be ever less overlap between them. We assume here that the complexity of an internal state Φ_μ increases steadily with its excitation energy; this ignores possible doorway states, which are expected to be completely dissolved into the background of complicated states at the high excitation energies we are concerned with. The level density factor $\sqrt{\bar{\rho}(\varepsilon')}$ in eq. (21) works in the same direction, in that it decreases rapidly as ε' goes down. However, this may be offset considerably by the reduction of the magnitude of the matrix element through averaging, mentioned after eq. (21) (An analogous consideration holds for the effect of the factor $\sqrt{\bar{\rho}(\varepsilon)}$). One may thus be led, in a first attempt, to assume that the energy losing potential simply depends on the difference $\omega = \varepsilon - \varepsilon' = E' - E$ (as we have done already in eq. (25)). It is zero for $\omega \leq 0$, while for $\omega > 0$ its magnitude is rapidly decreasing as ω increases from zero. So we choose

$$V^L(\underset{\sim}{r}, \omega) = \begin{cases} 0 & \text{for } \omega \leq 0, \\ V^L(\underset{\sim}{r})\, e^{-\omega/\Delta} & \text{for } \omega > 0. \end{cases} \tag{44}$$

The energy range Δ, the interval of channel energies across which the inelastic channels couple significantly, is difficult to estimate, but from

the preceding discussion we do not expect it to be very large, perhaps of the order of several MeV. Coupling only such channels which are close in energy leads to a picture in which energy is "handed down" from one group of channels to the next lowest in a chain-like fashion, which has some intuitive appeal. Equation (44) is certainly an over-simplification, and V^L will quite possibly also depend on the channel energy E itself. We take the function $V^L(\underset{\sim}{r})$ in eq. (44) to be real and, lacking further information, assign a negative sign to it, in extrapolation from $V^L(\underset{\sim}{r};\ \omega \to 0)$, where it becomes the average nuclear interaction between the ions, which is negative (attractive). In order to gain some idea as to the magnitude of the energy losing potential, one may appeal to eq. (36), read as an inequality because of the extra absorption in $\mathcal{V}_0$ due to "fusion". Making very crude approximations in eq. (36), one obtains the estimate

$$\left(\bar{V}^L \Delta\right)^2 < -\frac{\hbar^3}{R^3 m \sqrt{m E_0}}\, \mathrm{Im}\, \bar{\mathcal{V}}_0 \approx -\frac{300}{\left(A_1^{1/3}+A_2^{1/3}\right)^3}\, \frac{1}{A_{red}^{3/2}}\, \frac{1}{\sqrt{E_0}}\, \mathrm{Im}\, \bar{\mathcal{V}}_0 \quad (\mathrm{MeV})^2,$$

where $\bar{V}^L$ and $\bar{\mathcal{V}}_0$ are the mean strengths of $V^L(\underset{\sim}{r})$ and $\mathcal{V}_0(\underset{\sim}{r})$, respectively, and R is the interaction radius. The spatial dependence of $V^L(\underset{\sim}{r})$ should perhaps be modeled on that of the imaginary part of the optical potential. If it is chosen spherically symmetric, energy is lost while angular momentum is conserved ("radial friction").

In solving the integral equation (20) numerically, one would reduce it to a discrete system of coupled equations whose number depends on the stepwidth one chooses for the energy integration in the kernel. Since the coupling matrix is triangular, no matrix inversion is required, i.e. the separate "channels" are computed in turn, starting with the elastic one at the incident energy E_0. This should provide a considerable numerical advantage, in particular, when the boundary condition of outgoing waves is to be implemented [10].

5. Concluding remarks

In this paper we have attempted to establish a single Schrödinger equation for the description of elastic and inelastic scattering, with a view to applications in heavy-ion reactions. The parameters of this model equation are related to the parameters characterizing the matrix elements and the geometry of the corresponding coupled-channels problem, as well as the densities of the channels involved. By extending this set of parameters, one may be able to generalize the formulation to one that also describes moderate mass and charge transfers, as long as recoil effects due to rearrangement in the exit channels can be neglected.

In the time dependent form the equation guarantees energy loss through the restriction to positive frequencies in the time dependent interaction $V^L(t)$. This may be contrasted with the Schrödinger-Langevin equation [2,4], which is constructed such that the time dependent potential $V^{friction}(t)$ fulfils

$$\langle \psi, \frac{\partial V^{friction}(t)}{\partial t} \psi \rangle \leq 0,$$

which also implies energy loss. The equation discussed in the present paper is linear (and can, e.g., be decomposed into partial wave equations), while the Schrödinger-Langevin equation is nonlinear.

It is intuitively clear that the physically important parts of the channel density wave function should be localized along classical paths [11]. The "downward" energy transfer through the channels would correspond to friction along such paths. It is not evident at this point, however, whether

the friction force derived in this way would be proportional to the velocity, as indicated by perturbation theory [8,12] and phenomenological fitting of data [13]. In this connection, and also for possible numerical application, one should investigate if semi-classical methods may be applied to the case of complex and nonconservative interactions we have considered here.

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Figure Caption

Fig. 1. Couplings and current flow in the energy losing Schrödinger equation

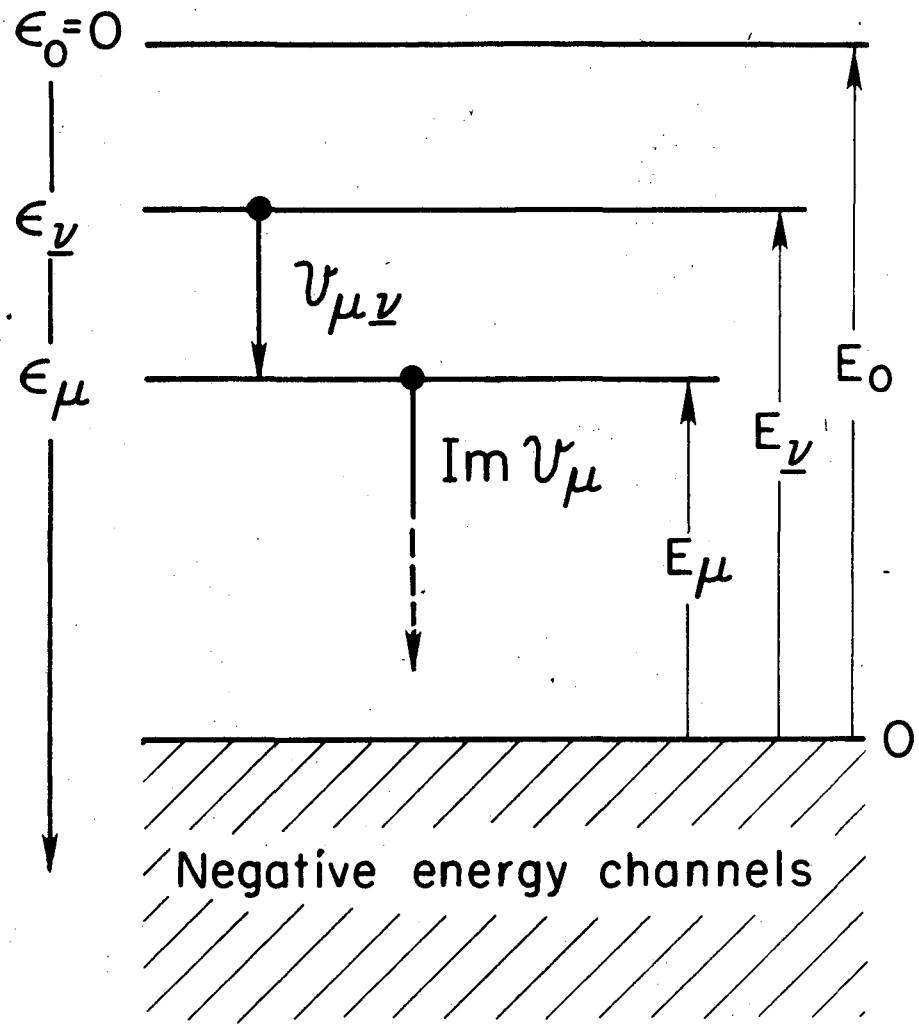

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Fig. 1

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