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FINITE RANGE EVALUATION OF (p-d, d-t) WITH MOMENTUM SPACE TECHNIQUES*

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ABSTRACT

A momentum space methodology is presented for calculating second order D.W.B.A. matrix elements without the use of the zero-range approximation. This methodology is applied to the sequential transfer term in (p,t) reactions. Examples are presented for $^{208}Pb(p,t)^{206}Pb$ which show that; 1) finite-range effects can be large 2) the shape of the calculated differential cross section can be strongly dependent on the contribution from a sequential transfer mechanism and 3) the post-prior interchange usually used is probably the best realistic approximation.

I. INTRODUCTION

Recently a great deal of work has been done to investigate the contribution of a sequential transfer mechanism in (p,t) reactions.[1-5] This is usually done by adding a simultaneous term, calculated using D.W.B.A., and a sequential term using "second order" D.W.B.A. Both terms are almost always evaluated using the zero-range approximation. It is the purpose of this paper to present a methodology, using momentum space techniques, for calculating the sequential transfer term without using the zero range approximation. Examples will be presented which were calculated by using a full finite range formalism for both the simultaneous and sequential terms.

II. FORMAL THEORY

Two potential theory[6,7] allows the two-nucleon transfer T-matrix element to be written as

$$T_{tp} = \left\langle \chi_t^{(-)} | \hat{V}_t + \hat{V}_t \, G^{(+)} \hat{V}_p | \chi_p^{(+)} \right\rangle \tag{2.1}$$

where the notation is apropriate for a (p,t) reaction,

$$|\chi_p^{(+)}\rangle = \chi^{(+)}(\underset{\sim}{k}_p, \underset{\sim}{r}_p) | \Phi_p \rangle, \tag{2.2}$$

$$\hat{V}_p = V_p - U_p, \tag{2.3}$$

and

$$G^+ = (E^+ - H)^{-1}. \tag{2.4}$$

In the above $\chi(\underset{\sim}{k},\underset{\sim}{r})$ is the usual distorted wave (second order inelastic effects are neglected), Φ the appropriate bound state, U the optical potential used to generate $\chi(\underset{\sim}{k},\underset{\sim}{r})$, V the exact potential acting between the two fragments, and G^+ an outgoing wave Greens Function. The symbol χ without an argument shall mean both the distorted wave and the appropriate bound state. The Hamiltonian H can be written

$$H = H_p + V_p = H_t + V_t = H_d + V_d \tag{2.5}$$

with d representing a channel which contains fragments which differ by

one nucleon from those in channels p and t. The Greens function of Eq. (2.4) can be written

$$G^{(+)} = G_d^{(+)} + G_d^{(+)}\hat{V}_d G^{(+)} \tag{2.6}$$

with

$$G_d^{(+)} = (E^+ - H_d - U_d)^{-1} . \tag{2.7}$$

Keeping only the first term in Eq. (2.6) allows Eq. (2.1) to become

$$T_{tp} = T_{tp}^{sim} + T_{tp}^{seq} \tag{2.8}$$

where

$$T_{tp}^{sim} = \langle \chi_t^{(-)} | \hat{V}_t | \chi_p^{(+)} \rangle \tag{2.9}$$

is the usual D.W.B.A. term, and

$$T_{tp}^{seq} = \langle \chi_t^{(-)} | \hat{V}_t \, G_d^{(+)} \, \hat{V}_p | \chi_p^{(+)} \rangle \tag{2.10}$$

transfers the nucleons one at a time. This procedure gives the T-matrix element to second order in the $\hat{V}$'s. However, the channel d could have been chosen to be any channel that conserved the number of nucleons. The particular choice made is justified by the reaction mechanism to be studied (i.e., sequential transfer). The two terms in Eq. (2.8) came from a consideration of the formal scattering theory involved and thus represent two separate mechanisms for transferring the same pair of nucleons. That is, the second term results from allowing a third channel (in addition to the initial and final fragmentations) and does not depend explicitly on the single particle or cluster properties of the two nucleons involved.

The evaluation of Eq. (2.10) requires the insertion of a complete set of states between the two transfer potentials. This, in principle, means a sum (integral) over all bound (continuum) internal states for the two fragments and an integral over the relative momentum between them. In practice, only a few internal states are kept. Using Φ_d^i to denote each pair of internal states allows Eq. (2.10) to become

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$$T_{tp}^{seq} = \sum_i \left\langle \chi_t^{(-)} | \hat{V}_t | \Phi_d^i \right\rangle \mathcal{G}_d^{(+)} \left\langle \Phi_d^i | \hat{V}_p | \chi_p^{(+)} \right\rangle \tag{2.11}$$

where

$$\mathcal{G}_d^{(+)} = (E_i^+ - H_d^0)^{-1}. \tag{2.12}$$

H_d^0 is the optical model Hamiltonian and E_i^+ is the relative energy between the two fragments with the plus still requiring an outgoing wave Greens function.

III. EVALUATION OF THE GREENS FUNCTION WITH MOMENTUM SPACE TECHNIQUES

The Greens function of Eq. (2.12) can be evaluated in several ways. In Ref. 8 a closed form is used which, in coordinate space and for each partial wave, is

$$g_L^{(+)}(r,r') = -(2mk/\hbar^2)\, u_L^{(+)}(r_>)\, u_L^{(R)}(r_<) \tag{3.1}$$

where the $r_>$ ($r_<$) denotes the larger (smaller) of r and r'. Calculationally this form, in effect, couples the two one step terms since the value of both coordinates must be fixed in order to use the proper form. That is, if r and r' appeared instead of $r_>$ and $r_<$ the two one step terms would be separable.

Another way of evaluating Eq. (2.12) is that of Ref. 7 where a set of biorthogonal distorted wave states are introduced. Since this method would require the numerical evaluation of distorted waves at many energies it isn't calculationally useful and, in fact, is used only formally in the above reference.

A third method will be used here and was suggested, in a different context, in Ref. 9. It allows the Greens function to be written in a separable form and, for this reason, could be more efficient than Eq.(3.1) even in standard coordinate space calculations. It is first desired to consider a unit operator over a region of space r = 0 to r = R which is given by

$$\delta^R(\underset{\sim}{r}-\underset{\sim}{r}') = \sum_{LM} \sum_{n,n_2=1}^{N} |LM\rangle |Ln_1\rangle N^L_{n_1n_2} \langle Ln_2| \langle LM| \tag{3.2}$$

where

$$|LM\rangle = Y^M_L(\hat{\underset{\sim}{r}}) \tag{3.3}$$

and

$$|Ln\rangle = j_L(k_n r). \tag{3.4}$$

The matrix N orthonormalizes the states $|Ln\rangle$ and would become a diagonal matrix if the k_n's were chosen to give orthogonal states. Calculationally, it is given by

$$N^L_{n_1n_2} = (\underset{\sim}{O}^{L^{-1}})_{n_1n_2} \tag{3.5}$$

where

$$(\underset{\sim}{O}^L)_{n_1n_2} = \langle n_1\ L \mid n_2\ L\rangle \tag{3.6}$$

and $\underset{\sim}{O}^L$ is a matrix.

The superscript R implies that δ^R acts over a sphere of radius R. The following properties can be rather easily shown:

$$1)\quad \lim_{\substack{N\to\infty \\ R\to\infty}} \delta^R(\underset{\sim}{r} - \underset{\sim}{r}') = \delta(\underset{\sim}{r} - \underset{\sim}{r}'), \tag{3.7}$$

$$2)\quad \delta^R(\underset{\sim}{r} - \underset{\sim}{r}'')\ \delta^R(\underset{\sim}{r}'' - \underset{\sim}{r}') = \delta^R(\underset{\sim}{r} - \underset{\sim}{r}') \tag{3.8}$$

and

$$3)\quad \int d\hat{\underset{\sim}{r}}' \int_0^R r'^2 dr'\ \delta^R(\underset{\sim}{r} - \underset{\sim}{r}')\ f(\underset{\sim}{r}') = f(\underset{\sim}{r})\ . \tag{3.9}$$

The symbol on the right-hand-side of Eq.(3.7) is a Dirac Delta function and Eq.(3.9) is true if the function f can be represented over 0 to R by the N Bessel functions of Eq. (3.2). Thus δ^R is a unit operator over 0 to R in the same sense that a Dirac Delta function is over 0 to ∞, and the

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two are in fact identical in the $\lim_{N,R\to\infty}$. Placing δ^R on either side of the Greens function, Eq. (2.12), gives

$$\delta^R(\underset{\sim}{r}'_d-\underset{\sim}{r}_d)\ (E_i^+-H_d^0)^{-1}\ \delta^R(\underset{\sim}{r}_d-\underset{\sim}{r}''_d)$$

$$= \sum_{LM}\sum_{n_1,n_2=1}^{N}\sum_{n_1',n_2'=1}^{N} |LM\rangle\ |Ln_1\rangle\ N^L_{n_1,n_2}$$

$$\left\langle Ln_2\right| \left[E_i^+ + \frac{\hbar^2}{2\mu_d r_d^2}\left\{\frac{\partial}{\partial r_d}\left(r_d^2\ \frac{\partial}{\partial r_d}\right)\right.\right.$$

$$\left.\left. - L(L+1)\right\} - U_d(r_d)\right]^{-1}\left|\ Ln_2'\right\rangle$$

$$N^L_{n_1',n_2'}\ \left\langle Ln_1'\right|\ \left\langle LM\right| \tag{3.10}$$

It is now necessary to evaluate the matrix elements of the inverse operator. A general method for evaluating such an operator is as follows. By definition an operator P satisfies

$$P\ P^{-1} = 1. \tag{3.11}$$

Inserting the radial part of the unit operator (Eq. (3.2)) and taking a matrix element gives

$$\sum_{n_2'n_2=1}^{N} \left\langle Ln_1'\ |\ P|Ln_2'\right\rangle N^L_{n'_2n_2}\left\langle Ln_2|P^{-1}|Ln_1\right\rangle = \left\langle Ln_1'|Ln_1\right\rangle \tag{3.12}$$

Or in matrix form

$$\underset{\sim}{P}^L\ \underset{\sim}{N}^L\ \underset{\sim}{\tilde{P}}^L = \underset{\sim}{O}^L \tag{3.13}$$

where $\underset{\sim}{\tilde{P}}^L$ contains the matrix elements of P^{-1}. Solving gives

$$\underset{\sim}{\tilde{P}}^L = \underset{\sim}{N}^{L^{-1}}\ \underset{\sim}{P}^{L^{-1}}\ \underset{\sim}{N}^{L^{-1}}. \tag{3.14}$$

The Greens function of Eq. (3.10) thus may be written as

$$\delta^R(\underset{\sim}{r}_d'-\underset{\sim}{r}_d)\ (E_i^+-H_d^0)^{-1}\ \delta^R(\underset{\sim}{r}_d-\underset{\sim}{r}_d'') = \sum_{LM}\ \sum_{n,n'=1} |LM\rangle|\ Ln\rangle\ g^L_{nn'}\langle Ln'|\ \langle LM|\ . \tag{3.15}$$

where the matrix $g^{L^{-1}}$ has elements

$$(\underset{\sim}{g}^{L^{-1}})_{nn'} = \left\langle Ln\left|\left[E_i^+ + \frac{\hbar^2}{2\mu_d r_d^2}\left\{\frac{\partial}{\partial r_d}\left(r_d^2\frac{\partial}{\partial r_d}\right) - L(L+1)\right\} - U_d(r_d)\right]\right|Ln'\right\rangle \tag{3.16}$$

$$= \left\langle Ln\left|\left[E_i^+ - \frac{\hbar^2 k_{n'}^2}{2\mu_d} - U_d(r_d)\right]\right| Ln'\right\rangle\ . \tag{3.17}$$

To this point, nothing has been said calculationally about the outgoing wave boundry condition on the Greens function. This is probably best included in the above by use of a Bloch operator formalism.[9,10] An appropriate boundary condition can be imposed by replacing E_i^+ with

$$E_i^+ \to E_i - \frac{\hbar^2}{2\mu_d}\ \delta(r_d - R)\left(\frac{\partial}{\partial r_d} - \frac{b-1}{R}\right) \tag{3.18}$$

where

$$b = \left(\frac{r_d}{B}\ \frac{\partial B}{\partial r_d}\right)\Bigg|_{r_d = R}$$

and B is the functional form of the Greens function in the external ($r_d > R$) region. Specifically for an outgoing wave

$$B = G_L(\eta, k_d r_d) + i\ F_L(\eta, k_d r_d) \tag{3.18a}$$

where η is the Sommerfeld parameter, k_d is the on-shell (energy conserving) wave number and $F_L(G_L)$ is a regular (irregular) Coulomb function.

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Using Eq. (3.15) and

$$j_L(k_n r)\, Y_L^M(\hat{\underset{\sim}{r}}) = (i^{-L}/4\pi) \times \int d\hat{\underset{\sim}{k}}_n\, Y_L^M(\hat{\underset{\sim}{k}}_n)\, e^{i\underset{\sim}{k}_n \cdot \underset{\sim}{r}} \tag{3.19}$$

gives the form for the Greens function to be used in the next section. Namely,

$$G^{(+)}(\underset{\sim}{r},\underset{\sim}{r}') = (4\pi)^{-2} \sum_{LM} \sum_{n,n'=1}^{N} \left[\int d\hat{\underset{\sim}{k}}_n\, Y_L^M(\hat{\underset{\sim}{k}}_n)\, e^{i\hat{\underset{\sim}{k}}_n \cdot \underset{\sim}{r}}\right]$$
$$\times \left[\int d\hat{\underset{\sim}{k}}_{n'}\, Y_L^{M*}(\hat{\underset{\sim}{k}}_{n'})\, e^{-i\hat{\underset{\sim}{k}}_{n'} \cdot \underset{\sim}{r}'}\right]. \tag{3.20}$$

The above expression is a sum of terms separable in $\underset{\sim}{r}$ and $\underset{\sim}{r}'$. Thus, each of the two steps ((p,d) and (d,t)) of Eq. (2.10) can be calculated independently as a function of n(or n') and L. The results of these two calculations can then be combined with the g^L of Eq. (3.20). It should also be noted that $\underset{\sim}{r}$ and $\underset{\sim}{r}'$ appear only in the plane wave exponentials. Thus, if it is necessary in calculating one of the steps to use coordinates $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ related to $\underset{\sim}{r}$ by

$$\underset{\sim}{r} = a\underset{\sim}{r}_1 + b\underset{\sim}{r}_2, \tag{3.21}$$

the Greens function is further separable in $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$.

Two additional points need to be considered. First, how does one choose the k_n and second, how many are needed (i.e. what is N in Eq. (3.20))? For the results presented later, it was required that one of the k_n's be the on-shell wave number. Others were added above and below, in steps of π/R, until convergence was achieved. The on-shell wave number is chosen since the $g^L_{nn'}$ tend to be larger when n or n' denote the physical wave number and tend to be largest when both do. The interval π/R is suggested by the zeros of $j_0(kr)$, but in practice represents a convenient (not a necessary) choice. The convergence, for a calculated sequential cross section, is shown in Fig. 1. As may be seen, the use of about twenty momentum states gives the cross section to high accuracy.

It is concluded from the discussion of this section that the plane wave methodology is a viable and highly convenient way to handle an optical potential Green's function.

IV. EVALUATION OF THE SEQUENTIAL TRANSFER TERM

The techniques used in evaluating the simultaneous transfer term (Eq. (2.9)) were discussed in detail in Ref. 11 and will not be presented here. The methodology used for each of the two steps of the sequential transfer term is essentially identical to that of Refs. 12 and 13. Thus, the discusssion of this section will be limited to relating the Greens function of Eq. (3.20) to the momentum space techniques of these references except where additional discussion is needed. The sequential term can be written, using Eqs. (2.11) and (3.20) as

$$T_{tp}^{seq} = \sum_{i} \sum_{L_d M_d} \sum_{n_d, n'_d=1}^{N} \left\langle \chi_t^{(-)} | \hat{V}_t | \Phi_d^i \, L_d M_d n_d \right\rangle$$

$$\times \, g_{n_d n'_d}^{L_d} \left\langle \Phi_d^i \, L_d \, M_d \, n'_d \, | \hat{V}_p | \, \chi_p^{(+)} \right\rangle \tag{4.1}$$

where

$$| L_d M_d n_d \rangle = (4\pi)^{-1} \int d\hat{k}_{\underset{\sim}{n}_d} \; Y_{L_d}^{M_d} (\hat{k}_{\underset{\sim}{n}_d}) \times e^{i \underset{\sim}{k}_{n_d} \cdot \underset{\sim}{r}_d} . \tag{4.2}$$

To this point no approximations have been made on the transfer potentials ($\hat{V}_t$ and $\hat{V}_p$). The usual approximation may be made on $\hat{V}_p$. Namely,

$$\hat{V}_p = \sum_{i=1}^{A} v_{pi} - U_p \tag{4.3}$$

$$= v_{p1} + \left[\sum_{i=2}^{A} v_{pi} - U_p \right] \tag{4.4}$$

$$\approx V_{p1} \tag{4.5}$$

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where the nucleus in channel p has A nucleons, v_{pi} is the nucleon-nucleon potential, U_p is the optical potential between p and A, and V_{p1} is the potential which binds the neutron and proton to form the intermediate deuteron. The above is not exact since it assumes that the second and third terms on the right hand side of Eq. (4.4) cancel. They can not since 1) the optical potential has an imaginary part and the nucleon-nucleon potential does not and 2) the second term contains the interaction between the proton and A-1 nucleons while the third term represents the interaction between the proton and A nucleons. This approximation, however, has proven extremely useful for (p,d) reactions and thus its use is partially justified here. The other transfer potential ($\hat{V}_t$) is more worrisome. In Refs. 2,4 and 5 a "post-prior" interchange is made which replaces $\hat{V}_t$ with $\hat{V}_d$. This is done primarily so that one can follow the same procedure as for $\hat{V}_p$ and use the zero range approximation. However, the "post-prior" interchange is valid only on-shell.[14] In the results presented later, approximate forms for both $\hat{V}_t$ and $\hat{V}_d$ will be used. The form taken for $\hat{V}_t$ will be

$$\hat{V}_t = \sum_{i=1}^{3} \sum_{j=1}^{B} v_{ij} - U_t \tag{4.6}$$

$$= \sum_{j=1}^{B} v_{1j} + \left[\sum_{i=2}^{3} \sum_{j=1}^{B} - U_t \right] \tag{4.7}$$

$$\approx V_{1B} \tag{4.8}$$

where V_{1B} is the potential which binds the second transfered neutron to the intermediate nucleus. The transfer potential $\hat{V}_d$, will be approximated as

$$\hat{V}_d = \sum_{i=1}^{2} \sum_{j=1}^{C} v_{ij} - U_d \tag{4.9}$$

$$= \sum_{i=1}^{2} v_{i1} + \left[\sum_{i=1}^{2} \sum_{j=2}^{C} v_{ij} - U_d \right] \tag{4.10}$$

$$\approx V_{nd} \tag{4.11}$$

with V_{nd} the potential acting between the second transferred neutron and the deuteron.

In the D.W.B.A. methodology of Ref. 12 the distorted waves are expanded as

$$\chi^{(\pm)}(\underset{\sim}{k},\underset{\sim}{r}) = \sum_{LMn} a_{nL}^{(\pm)} Y_L^{M^*}(\hat{\underset{\sim}{k}}) \int d\hat{\underset{\sim}{k}}_n e^{i\underset{\sim}{k}_n \cdot \underset{\sim}{r}} Y_L^M(\underset{\sim}{k}_n). \tag{4.12}$$

It may thus be seen, by referring to Eqs. (4.1) and (4.2) that the sequential transfer term may be evaluated by using the D.W.B.A. methods of the above reference twice but replacing the a_{nL}'s in the final state of the (p,d) reaction and the initial state of (d,t) reaction by $g_{n_d n'_d}^{Ld}$ of Eq. (4.1). Thus the sequential calculation is done by constructing a D.W.B.A. matrix for the (p,d) step, another for the (d,t) step, a third for the propagator and multiplying them.

V. RESULTS AND DISCUSSION

Results will be shown in this section using the methodology discussed previously. The reaction $^{208}Pb(p,t)^{206}Pb$ is considered. This particular reaction was chosen because the study of Ref.11 found that a D.W.B.A. formalism gave reasonable agreement with experiment. Since using only a simultaneous transfer mechanism gave fair results, it might be expected that the remaining descrepancy (between theory and experiment) would be removed by considering a single additional mechanism.

Results shown consider simultaneous transfer, sequential transfer and a coherent addition of the two. Both terms were evaluated with full finite range calculations. The deuteron was described by a Hulthen form with parameters given in Ref.15, and the triton wave function used

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was identical to that in Ref. 11. The relative motion between the deuteron and the second transferred neutron was described by taking an overlap of the above deuteron with the triton. This procedure is, of course, required for consistency between the simultaneous and sequential terms.

The transferred neutrons were bound to the appropriate nucleus by a Woods-Saxon potential with parameters as given in Ref. 11. The optical model parameters referred to below are shown in Table I.

In Fig. 2 the cross section for $^{208}Pb(p,t)^{206}Pb_{3+}$ is shown. (This result was also presented in a preliminary report of this work in Ref. 15.) The unnatural parity nature of the final state makes this transition D.W.B.A. (simultaneous) forbidden. The transfer would have a cross section that is zero in zero-range D.W.B.A. and very small in finite range D.W.B.A. Thus the calculated curve represents a purely sequential process. The state was taken to be pure $(3p^{1/2})^{-1}$ $(2f^{5/2})^{-1}$. This required that a coherent sum over transitions through the ^{207}Pb ground state $(1/2^-)$ and 0.57 MeV state $(5/2^-)$ be taken. Both a zero range and a finite range result are shown. As may be seen the differences are consistent with the 20% one would usually like to see. Thus, finite range effects would be needed only for a detailed study. This would be expected, however, since finite range effects should be small when the D.W.B.A. term is small.[16] This result confirms the claim of Ref. 2 that this transition can be understood as a sequential process.

In Fig. 3 a result for the ground state transition is shown. Optical parameter set 2 was used. The use of only a simultaneous mechanism gives an excellent fit to the shape of the cross section in the forward direction but gives a progressively worse fit as the scattering angle is increased. As shown in the figure, the calculated curve was multiplied by 5.2 before plotting. A large part of the additional strength can be gained by using a more complete wave function. As reported in Ref. 11, the theory can be brought within about 40% of the data by including all but two components from the major shell below ^{208}Pb. Including these components does not alter the shape. Adding the sequential term increases the strength by about 40%. It would thus be expected that the

use of a more complete wave function in both terms would give about the right strength. However, the shape of the cross section, when both terms are included, is not as well described. Since one must match the shape of the cross section in order to convincingly claim additional strength, it is felt that the simultaneous plus sequential result in Fig. 3 is probably no more than a parametrization which happens to give about the right normalization to match the data. The D.W.B.A. result is also probably a parametrization of the shape since adding an additional physical mechanism destroys the reasonable agreement in the structure of the cross section. These results also suggest that D.W.B.A. fits which closely match data do not imply a complete understanding of the reaction. In view of the prior adjustments needed in the strengths of the simultaneous and sequential terms and in their relative phase in a zero-range study,[2] it should be noted that both the strength and relative phase used here are those given by the finite-range theory.

In Fig. 4 results for the ground state transition are also shown, but a different optical parameter set was used. The simultaneous transfer calculation gives a reasonable fit at back angles but the calculation drops in the extreme forward direction while the data rises. Adding the sequential term, however, gives a fit superior to the others shown. The data clearly selects optical parameter set 1 and requires the addition of the sequential term. As noted in the figure, the addition of the two mechanisms does not increase the strength of the calculated cross section. It is thus concluded that the examples shown here suggest a strong shape signature for the sequential transfer mechanism but do not indicate a solution to the traditional problems with the normalization.

In all the examples shown previously the approximate form for $\hat{V}_d$ discussed in Section IV has been used to transfer the second neutron. The effect of using $\hat{V}_t$ instead is shown in Fig. 5. As may be seen the use of $\hat{V}_d$ gives a superior fit. As discussed in Section IV, the formal theory requires the use of $\hat{V}_t$. The interchange of $\hat{V}_t$ and $\hat{V}_d$ is, however, perfectly correct on-shell. Thus, deviations from the theory occur only off-shell. Also the approximate form used for $\hat{V}_t$ (see Eq. (4.8) assumed

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the potential between a deuteron and the final nucleus was the same as that between a triton and the final nucleus. Since a triton optical potential has a real strength of typically 150 MeV and a deuteron optical potential has a real strength of about 100 MeV, the approximate form used for $\hat{V}_t$ is probably not as good as that used for $\hat{V}_d$. Equation (4.11), for $\hat{V}_d$, assumed that the potential between a deuteron and the intermediate nucleus was the same as the potential between a deuteron and a nucleus with one less nucleon. This assumption is quite commonly made in using various optical parameter sets and thus would seem valid. The results of Fig. 5 would thus suggest that the post-prior interchange introduces a smaller error than the use of the obvious simple form for $\hat{V}_t$.

In Fig. 6, a comparison is shown between a finite-range and a zero-range sequential calculation. The curves have no physical significance since the D.W.B.A. term has not been included. As may be seen, they give very different shapes. The zero-range result is about a factor of three smaller in the extreme forward direction. Thus, when the simultaneous term is small (Fig. 2), finite range effects are small but when the D.W.B.A. term is large finite range effects are large as claimed in Ref. 16.

Three conclusions are drawn from the work in this paper. First, the methodology presented for calculating second order D.W.B.A. matrix elements is both viable and efficient. Second, the examples presented show that a sequential transfer mechanism can drastically effect the shape of a differential cross section. This suggests that matching the shape of a two-nucleon transfer cross section with D.W.B.A. can be no more than a parametrization of the data. Reasonable agreement with the data for $^{208}Pb(p,t)^{206}Pb_{g.s}$ was found by using one set of optical parameters and D.W.B.A. However, the addition of a different physical mechanism (sequential transfer) destroyed the agreement. The use of another parameter set gave poor agreement when only simultaneous transfer was allowed but when both mechanisms were allowed the theory was in excellent agreement with the shape of the cross section. Thus, the data required optical parameter set 1 and the coherent addition of the simultaneous and sequential terms. The third conclusion is also drawn from the examples. Namely,

the use of a simple form for the potential which transfers the second neutron is a poorer approximation than using an incorrect form off-shell for the second transfer potential. That is, using the form for $\hat{V}_t$ of Eq. (4.8) is not as good an approximation as performing a post-prior interchange and using $\hat{V}_d$ of Eq. (4.11).

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FOOTNOTES AND REFERENCES

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Table I. Optical Parameters

Set	Channel	V	W	W_D	r_c	r_v	a_v	r_w	a_w	Ref.
1	p	47.9	0	10.0	1.25	1.25	0.65	1.25	0.76	17
	d	97.8	0	14.0	1.25	1.25	0.68	1.25	0.78	17
	t	149.8	12.0	0	1.24	1.24	0.68	1.43	0.87	18
2	P	53.4	5.0	5.6	1.17	1.17	0.75	1.32	0.66	19
	d	103.9	1.2	16.1	1.17	1.17	0.75	1.32	0.66	2
	t	167.0	10.0	0	1.16	1.16	0.75	1.50	0.82	18

0 0 0 4 0 4 4 2 2 0

FIGURE CAPTIONS

Fig. 1. Dependence of the sequential cross section on the number of momentum states (N_d) used.

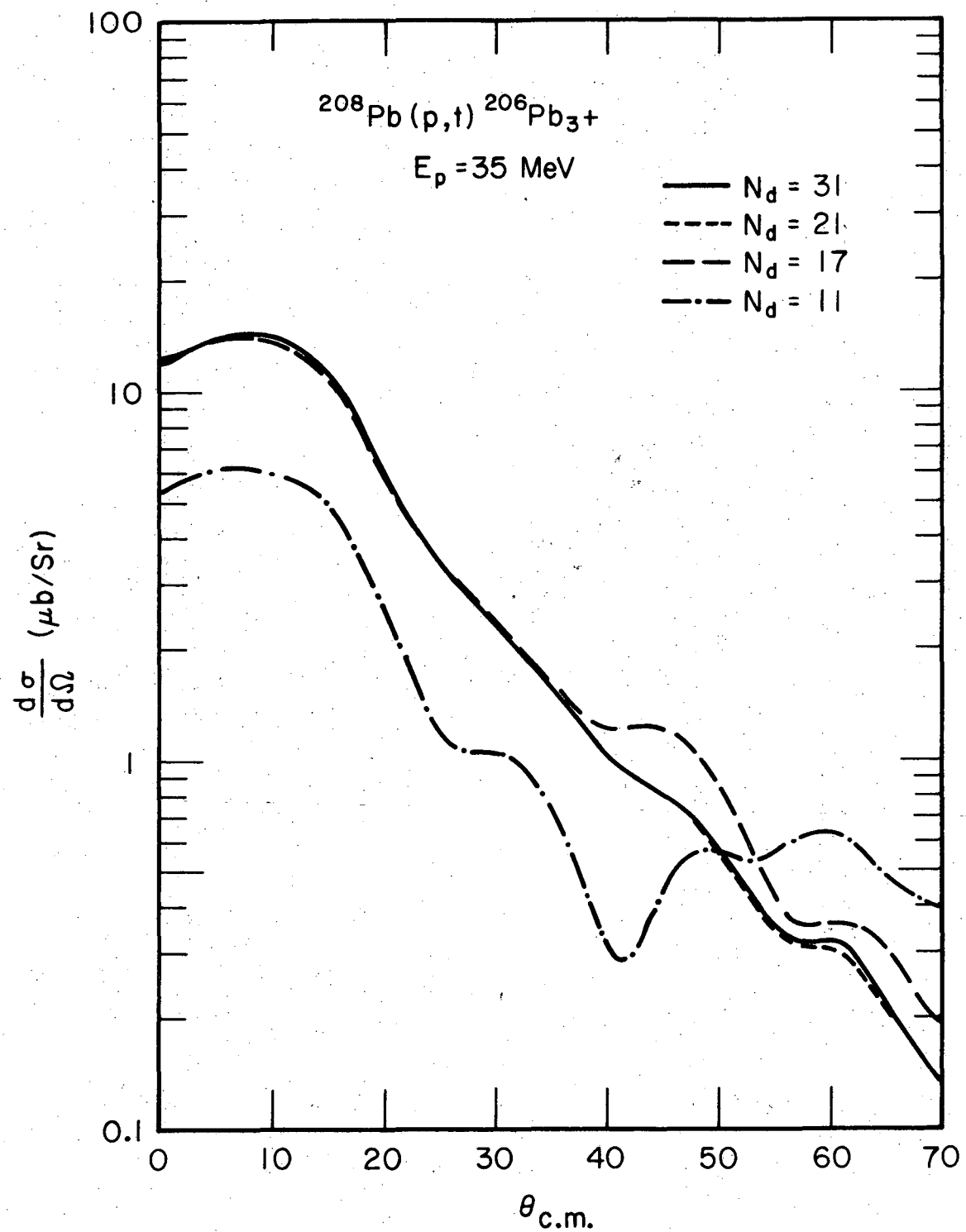
Fig. 2. Differential cross section for the reaction $^{108}Pb(p,t)^{206}Pb_{3+}$. The transfer potential $\hat{V}_d$ has been used (see Eq. (4.11)). The normalization of the calculated curves is as shown.

Fig. 3. Differential cross section for $^{208}Pb(p,t)^{206}Pb_{g.s}$. The calculated curves have been multiplied by the factor N. Optical parameter set 2 was used. Sim. denotes simultaneous and Seq. sequential. See Fig. 2.

Fig. 4. Differential cross section for $^{208}Pb(p,t)^{206}Pb_{g.s.}$. Optical parameter set 1 has been used. See Figs. 2 and 3.

Fig. 5. Differential cross section for $^{208}Pb(p,t)^{206}Pb_{g.s.}$. The use of $\hat{V}_d$ (see Eq. (4.11)) is compared to the use of $\hat{V}_t$ (see Eq. (4.8)). The calculated curves have been multiplied by the factor N.

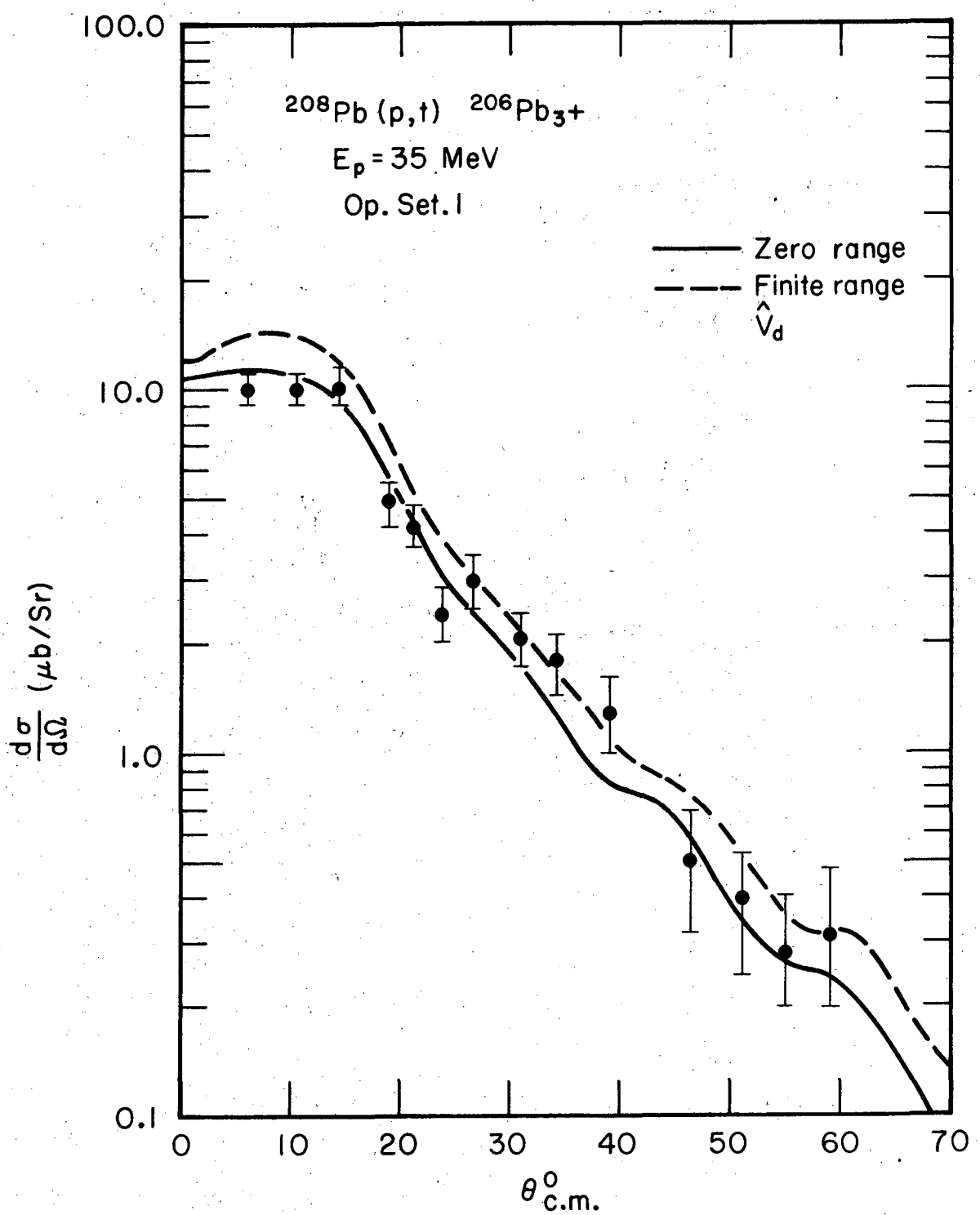
Fig. 6. Finite-range and zero-range calculations for $^{208}Pb(p,t)^{206}Pb_{g.s}$ are shown. Only the sequential term was included. Although the absolute normalization is arbitrary the relative normalization between the two curves is as shown.



XBL 762-2195

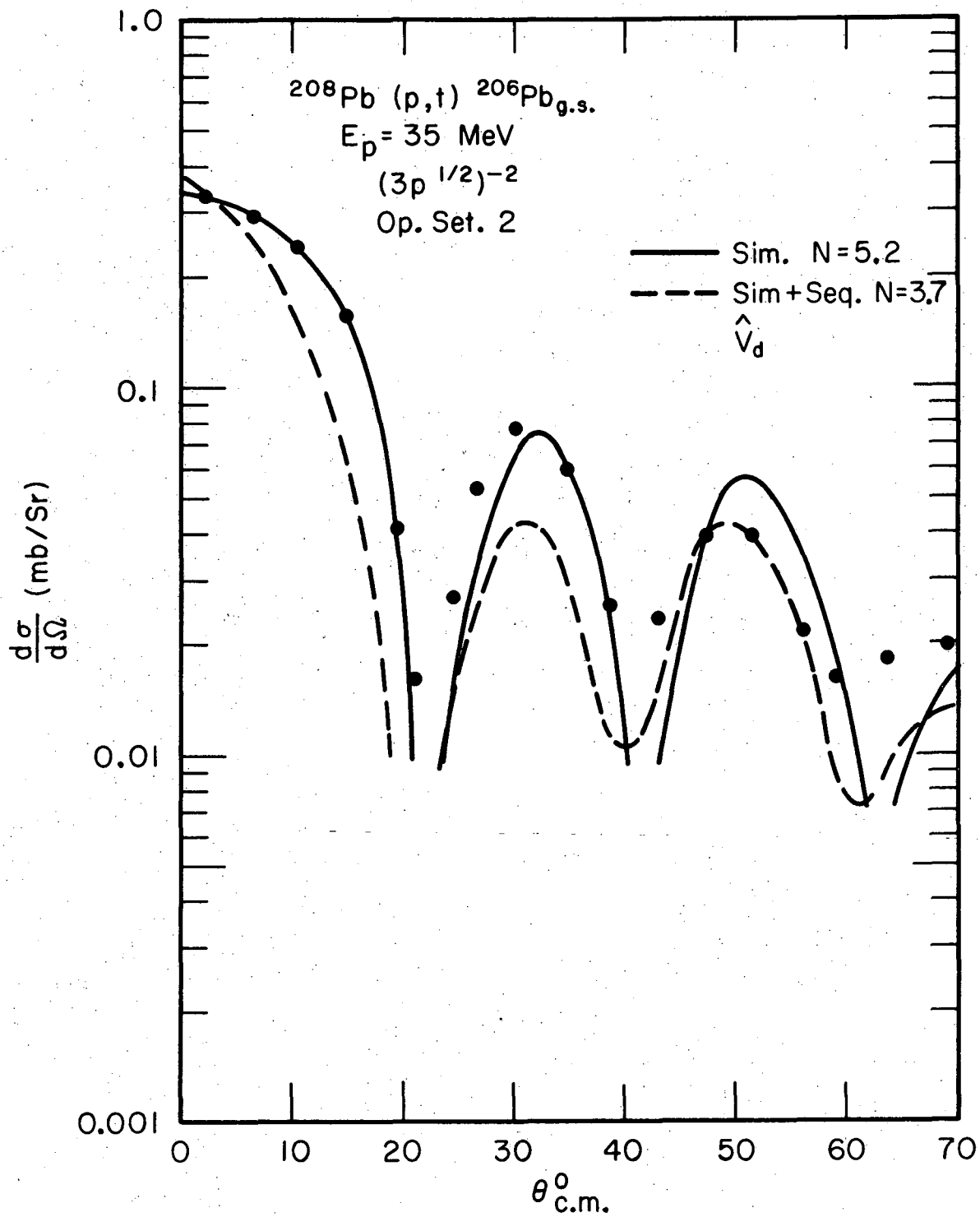
Fig. 1

0 0 0 0 4 4 0 4 2 1 9



XBL 762-2194

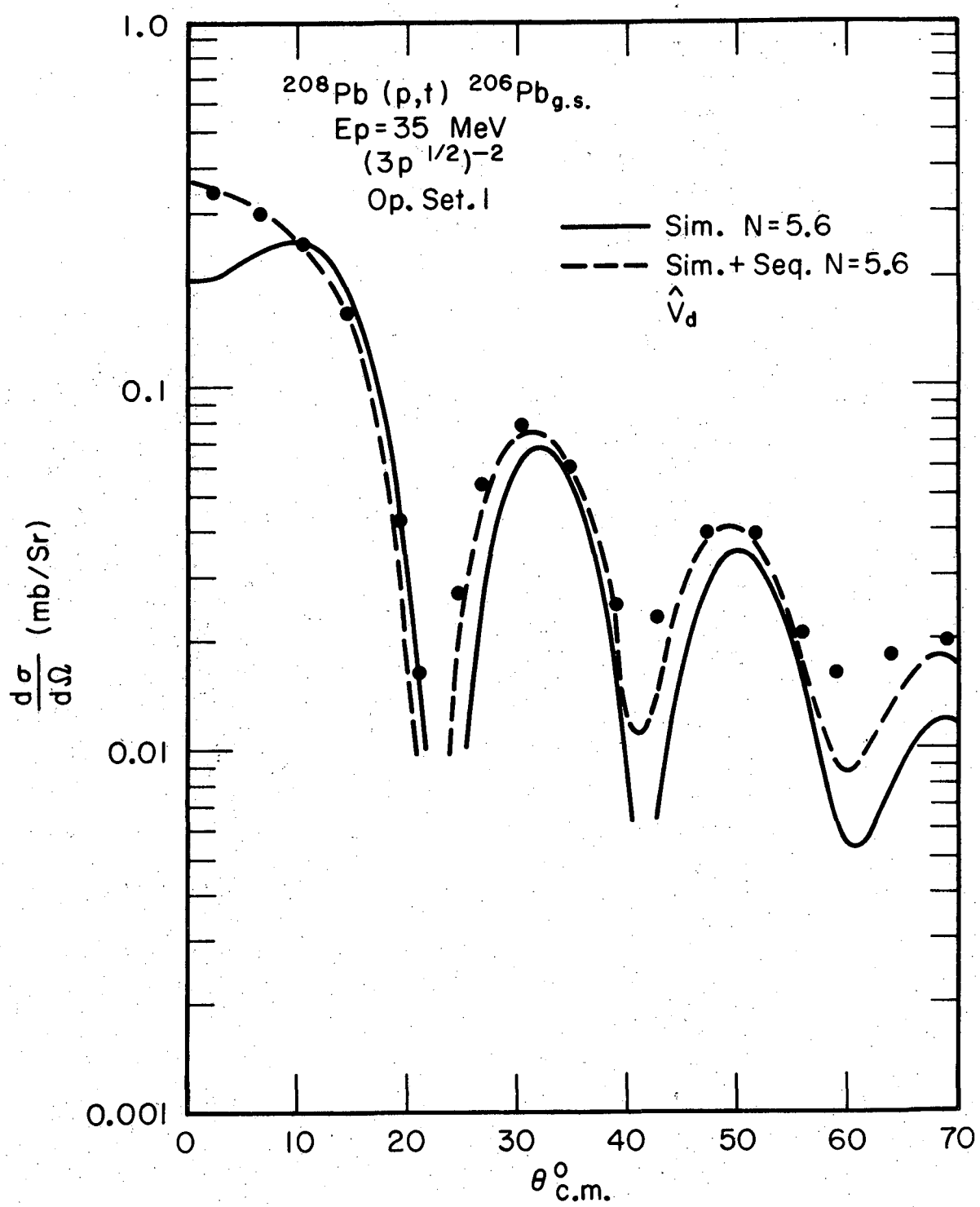
Fig. 2



XBL 762-2193

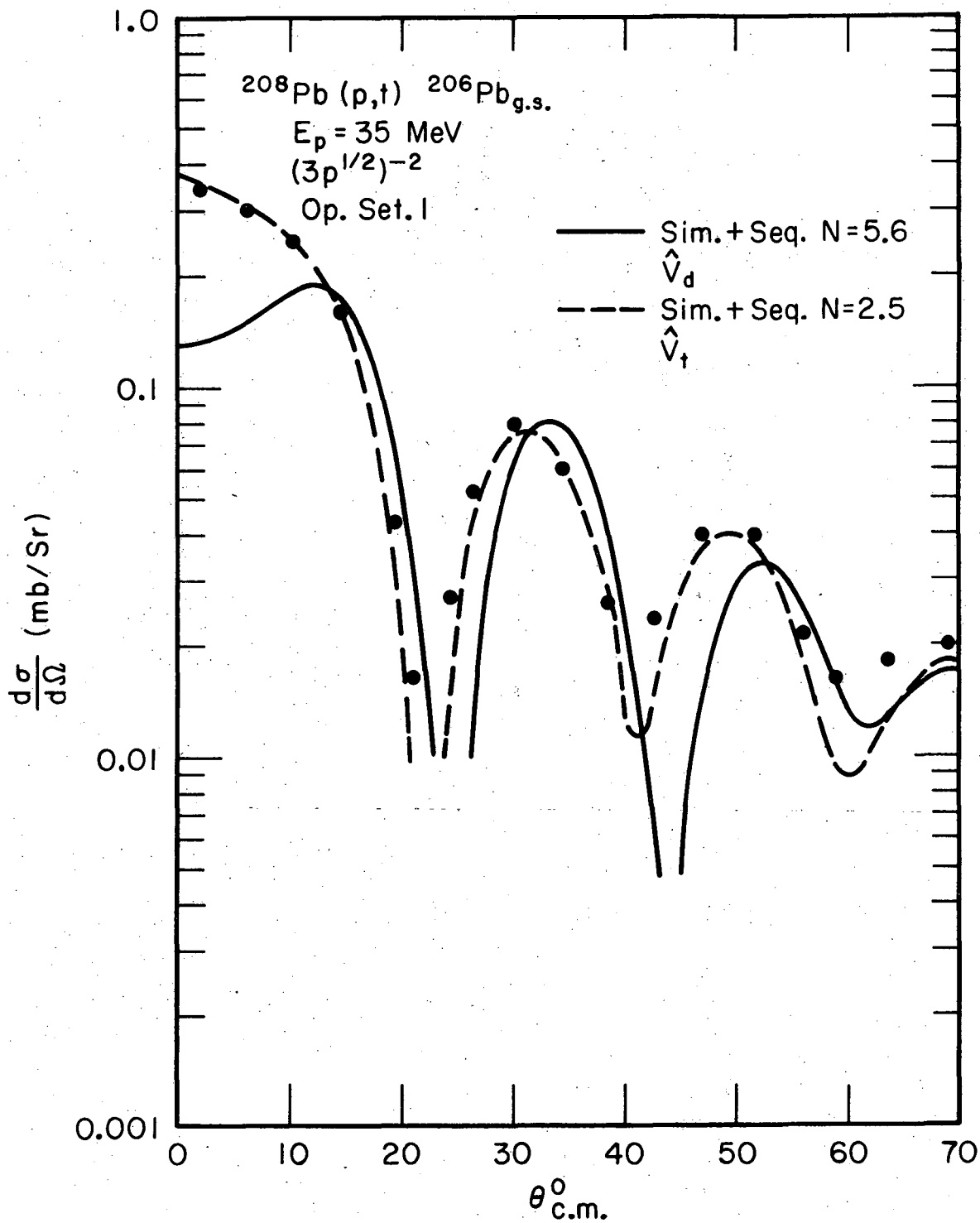
Fig. 3

0 0 0 0 4 4 0 4 2 1 8



XBL 762-2192

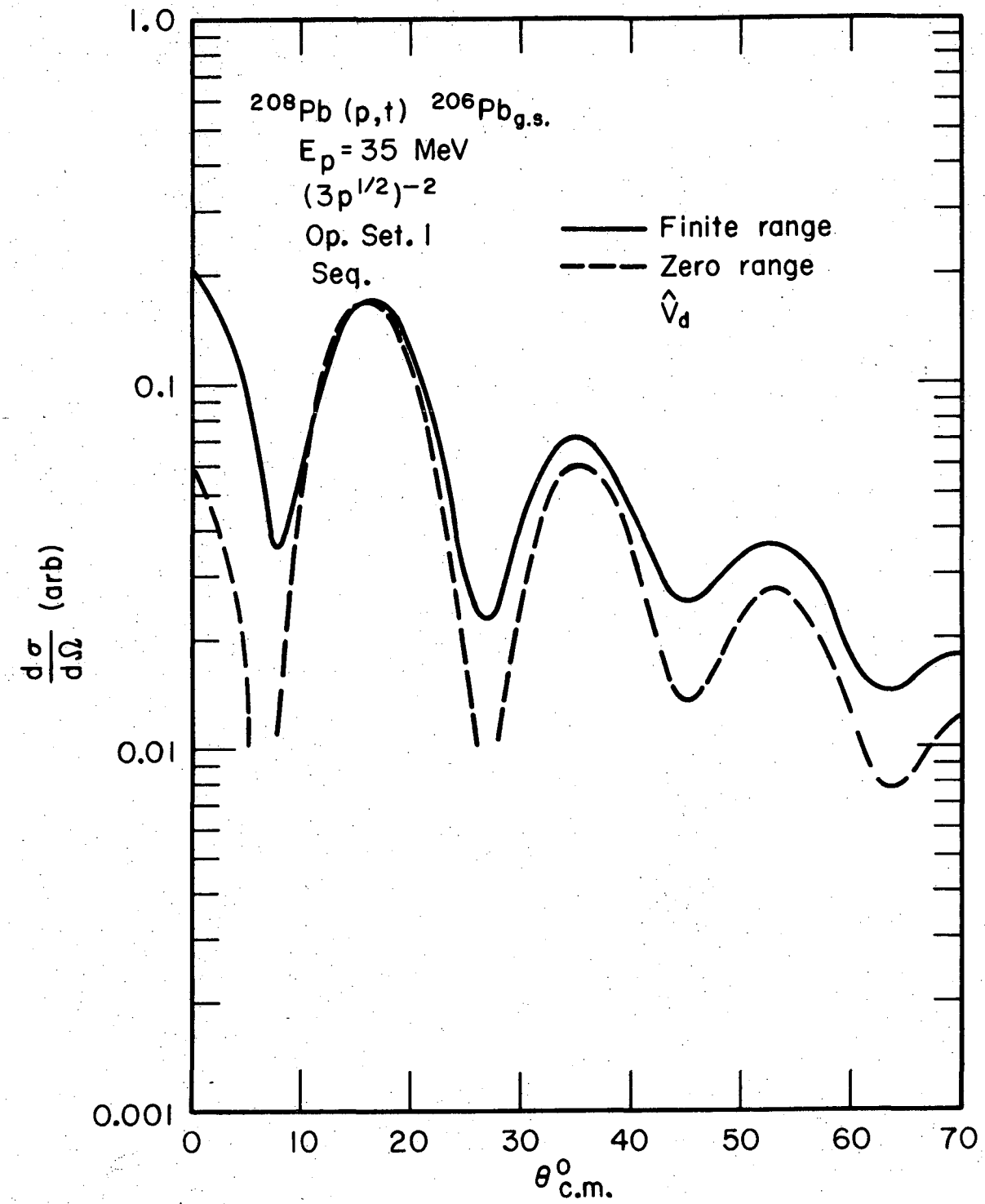
Fig. 4



XBL 762-2191

Fig. 5

0 0 0 0 4 4 0 4 2 1 7



XBL 762-2190

Fig. 6

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