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THE DISCRETE AMBIGUITY RESOLUTION AND BARYON-RESONANCE PARAMETER DETERMINATION

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THE DISCRETE AMBIGUITY RESOLUTION AND BARYON-RESONANCE PARAMETER DETERMINATION*

D. M. Chew† and M. Urban††

(April 1978)

ABSTRACT

We have performed a partial-wave analysis on elastic π^+p data between 1400 and 2200 MeV, using principles of analyticity (to select and amalgamate data), causality and unitarity together with Barrelet zeros. We examine here in detail the resonating waves between 1500 and 1800 MeV and show how a new resolution of the discrete ambiguity gives, for the S31 and D33 resonances, different parameters than found in an earlier resolution using less accurate information. In either case, we observe mass degeneracy of these resonances in agreement with general considerations regarding smooth zero trajectories.

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Introduction

Baryon-resonance parameters are usually obtained from partial wave analysis, but the reliability of the results is difficult to evaluate -- depending on many different factors. One is data amalgamation and smoothing which is very dependent on the quality of the data and on the method used. High-statistics data containing systematic error can be given excessive weight in a method of smoothing which uses only minimal χ^2 and ignores the difference between well and wrongly measured points. We have proposed a method,[1] based on work by Barrelet,[2] which allows identification of measured points with systematic error larger than statistical. Discrimination between data with equally high statistics then becomes possible, improving their overall quality.[3,4] Analyticity of scattering amplitudes is the general principle employed in our selection criteria; analyticity also allows us to renormalize scattering data so that the forward differential cross-section value agrees with the value of the tables calculated with dispersion relations and total cross-section measurements.[5]

A further weakness present in most partial wave analysis is inadequate attention to the discrete ambiguity -- arising from the fact that more than one amplitude reproduces the same scattering and polarization data, the various amplitudes being interrelated by transforming the complex variable $z_i = \cos\theta_i$ (or $w_i = e^{i\theta_i}$) into $\bar{z}_i$ (or $\bar{w}_i^{-1}$). For an amplitude approximated by a polynomial of order N there exist 2^N such amplitude approximations to the same data.[2,6] Therefore some supplementary information beyond a minimum χ^2 fit is needed in order to "lift

the ambiguity". Experimentally, in the case of a $1/2 + 0$ reaction, an extra measurement (in addition to scattering and polarization) that can bring the needed information is the spin-rotation parameter.[2] However, no such measurement exists in the energy range where we have analyzed elastic π^+p data, and therefore one must try to determine, for each of the roots of the polynomial which approximates the data, which choice (z_i or z_i^{-1} , w_i or $\bar{w}_i^{-1}$) is most likely to correspond to the actual amplitude.

The foregoing has been accomplished using causality principles in the case of π^-p charge-exchange[1] and elastic π^+p data,[7,8] in a way which was considered preliminary in 1976 at the Oxford Conference[7] because of the sparcity of data at that time, especially in the $\Delta(1600)$ region. Following the analysis of a more complete set of elastic π^+p data in Ref. [4], it became possible to check the table of critical points and the resolution of discrete ambiguity presented in Ref. [7]. The recent results of Ref. [8] first help specify the energy at which the zero trajectories cross the physical region (where the polarization reaches ± 1 at some value of $\cos\theta$). But they also show two new critical points on two trajectories (the ones designated (E) and (F) in Ref. [7] and [8]), implying an amplitude different from the one we would have determined from the critical points specified in the table of Ref. [7]. We wish to contrast the "new" and "old" amplitudes in order to show in the particular case of the $\Delta(1600)$ mass region how two different resolutions of the discrete ambiguity substantially change the resonance parameters.

Of course, an additional reason for variation of resonance parameters is the "parametrization-dependent ambiguity", as referred to by the

Particle Data Group.[9] We will be careful to use the same parametrisization for both "new" and "old" amplitudes, presenting our results in the last section of this paper. Let us review first the zero partial wave analysis (ZPWA) and the determination of the polynomial approximation to the amplitude that we use to get the partial waves.[10]

I. Zero partial wave analysis

Our amplitude determination is based on the fact that for a $1/2 + 0$ reaction, according to Barrelet,[2] some of the zeros (in the variable $w = e^{i\theta}$) which can be determined at each s, for the experimental quantity

$$\sum^{\pm}(s,w) = \frac{d\sigma}{d\Omega}(1 \pm P) \tag{1}$$

(where $\frac{d\sigma}{d\Omega}$ is the scattering differential cross section and P the polarization), are also zeros of the amplitude $F(s,w)$, as:

$$\sum^{\pm}(s,w) = F(s,w) \cdot \overline{F(s,\overline{w}^{-1})} \tag{2}$$

This last formula shows that an amplitude with a zero at w_i and an amplitude with a zero at $\overline{w}_i^{-1}$ will both give the same set of data $\Sigma^{\pm}(s,w)$ (and therefore $\frac{d\sigma}{d\Omega} = \frac{1}{2}(\Sigma^+ + \Sigma^-)$ and $P\frac{d\sigma}{d\Omega} = \frac{1}{2}(\Sigma^+ - \Sigma^-)$). If N zeros are close enough to the physical region to be determined (see Section II), the data -- for a $1/2 + 0$ reaction -- are approximated by:

$$\sum_{\pm}(w,s) = \frac{d\sigma}{d\Omega}(1 \pm P) \simeq \sum(0) \prod_{i=1}^{N} \frac{(w - w_i)\overline{(w^{1} - \overline{w}_i^{-1})}}{(1 - w_i)(1 - w_i^{-1})} \tag{3}$$

and the amplitude is proportional to:

$$F(s,w) \propto F(0) \prod_{i=1}^{N} \frac{\left(w - \begin{matrix} w_i \\ \text{or} \\ \overline{w}_i^{-1} \end{matrix}\right)}{\left(1 - \begin{matrix} w_i \\ \text{or} \\ \overline{w}_i^{-1} \end{matrix}\right)} \tag{4}$$

In order to resolve the discrete ambiguity (i.e., choose for each of the N zeros between w_i or $\overline{w}_i^{-1}$) for the amplitude, we use the principle of causality at energies close to the mass of a dominating resonance (such as the $\Delta(1900)$ with $J = \frac{7}{2}$ and $N = 6$). As a resonance of spin J and naturality ε is approached, $2J-1$ zeros should move in a clockwise sense about the zeros of $R_{J,\varepsilon}(w)$. (This condition is referred to as either the Wigner condition or causality.)

Another unknown factor -- which cannot be determined from the data -- is the phase of $F(s,w)$ and first of all the power n of a factor $1/w^n$ which can always multiply (4) without changing the data approximated in (3). Barrelet and Urban[11] have shown from unitarity that only two values of n are possible: either $\frac{N}{2}$ or $\frac{N}{2} + 1$, N being the (even) number of zeros which can be determined from the available data. It follows from the empirical study of Ref. [7] that only $\frac{N}{2}$ is possible (at $\sqrt{s} \simeq 1900$ MeV , $\frac{N}{2} + 1$ would give a resonance in the G37 partial wave instead of the F37 and -- as with the Minami ambiguity -- even suppress

completely the F37 wave!).

The last phase factor -- which is necessarily independent of w in order for the approximation to remain polynomial in w -- is a normalization, which we choose to be in agreement with dispersion relations and the total cross-section measurements. We use the value of the forward-amplitude phase as tabulated in Ref. [5], the value of the data in the forward direction being already in agreement with $(d\sigma/d\Omega)_{\theta=0}$ from the same tables.[8]

The amplitude is thus approximated -- as in Ref. [7] -- according to:

$$F(s,w) \sim \sqrt{\left(\frac{d\sigma}{d\Omega}\right)_{\theta=0}} \cdot \frac{e^{i\theta_0}}{w^{N/2}} \cdot \prod_{i=1}^{N} \left(\frac{w - w_i}{1 - w_i}\right) \tag{5}$$

each w_i being a root (inside or outside the unit circle) as determined from the resolution of the discrete ambiguity.

The partial-wave analysis, as in Ref. [7] follows from the projection of this amplitude onto the orthogonal set of polynomials $R_{J,\varepsilon}$:

$$F(s,w) = \sum_{\substack{\varepsilon=\pm 1 \\ J=1/2}}^{J=\infty} T_{J,\varepsilon}(s) \cdot R_{J,\varepsilon}(w) \tag{6}$$

with the inverse:

$$T_{J,\varepsilon} = \frac{-1}{2(2J + 1)} \int_{\gamma} F(w) \cdot R_{J,\varepsilon}(\overline{w}) \, |dz| \tag{7}$$

(γ) being the unit circle representing the physical region in the w-plane. Knowledge of the N closest zeros of the amplitude $F(s,w)$ translates

into an approximate determination of the waves with $J \leqslant \frac{N+1}{2}$.

Statistical errors of the data may be straight-forwardly converted into errors of partial wave amplitudes.[7,12] How do we handle the possibility that unselected data may have systematic errors larger than statistical errors? We explain now how the Barrelet moments[2,13] can be used to establish a "selective amalgamation".[4]

II. Determination of Barrelet zeros through selective amalgamation of the data

The zeros of the experimental data which can be determined are the zeros of the polynomial approximation to the analytic function $\Sigma^{\pm}(s,w)$ (square of the amplitude). An exact polynomial expansion of the exact analytic function converges within a domain determined by the nearest t singularity (in the πN case, this in effect is the ρ pole in the t-channel); in the w-plane, this domain of convergence is a ring whose two radii, R_ρ and R_ρ^{-1} are given by the relation between t and $\cos\theta$. The coefficients of the polynomial approximation should then also behave like those of a convergent series, which is asymptotically:

$$\lim_{k \to \infty} |a_k| \sim \left(\frac{1}{R}\right)^k = e^{-k \ln R} \tag{8}$$

A method to calculate successive terms of the series as objectively as possible is Barrelet's method of moments.[2,13] The coefficients of the polynomial approximation are calculated independently one from another (never the case in a least-square fit), as averages on the

experimental distribution ($\frac{d\sigma}{d\Omega}$ and $P\frac{d\sigma}{d\Omega}$) of pseudo polynomials, orthogonal on the $\cos\theta$ interval of these distributions:

$$A_\ell = \int_a^b \frac{d\sigma}{d\Omega}(\cos\theta) \cdot p_\ell(\cos\theta)\, n_p(\cos\theta)\, d(\cos\theta) \ , \tag{9}$$

the pseudo polynomials $p_\ell(\cos\theta)$ being defined with respect to the norm n_p so that:

$$\int_a^b n_p(\cos\theta) \cdot p_\ell(\cos\theta) \cdot p_{\ell'}(\cos\theta)\, d(\cos\theta) = \delta_{\ell\ell'} \ . \tag{10}$$

The approximations to the data are then the series:

$$\frac{d\sigma}{d\Omega}(\cos\theta) = \sum_{\ell=0}^{\ell=N_1} A_\ell p_\ell(\cos\theta) \tag{11}$$

and

$$P\frac{d\sigma}{d\Omega}(\cos\theta) = \sum_{\ell=0}^{\ell=N_2} B_\ell q_\ell(\cos\theta) \tag{12}$$

the coefficients B_ℓ being calculated in a similar way as the average of pseudo polynomials q_ℓ over the experimental distribution of $P\frac{d\sigma}{d\Omega}$.

For data free from systematic error, the coefficients plotted vs. their order exhibit the exponential decrease (8). This means that, after a certain order N_1, the coefficients become compatible with zero within their errors (calculated according to the statistical errors of the experimental points); it is meaningless to add more terms to the polynomial development beyond N_1.

A way to exhibit this lack of meaning of adding more terms is to

calculate the polynomial approximations for a succession of N_1 (between, say, 6 and 14) and to calculate for each N_1 the chi-square of such an approximation with respect to the experimental distribution given the statistical errors of each point therein. A useful quantity to quantify the goodness of fit is the chi-square-per-point -- first used in Ref. [1] -- which, when errors are only statistical, we expect to decrease to the order of 1 for N_1 sufficiently large. The plot of this goodness-of-fit measure versus N_1 should show how the introduction of higher terms helps up to a point, beyond which the goodness of fit remains roughly constant.[4]

Now suppose we observe a rise of the chi-square/point with increasing N_1* after a plateau (i.e., after the coefficients have become compatible with zero within errors), what can we say?

We cannot put the blame on the method which is based on the analyticity of the amplitude. Therefore we can only put the blame on the data themselves which must be revealing what are called by experimenters "systematic errors", a terminology that groups all non-statistical errors which have not been adequately represented in the stated error and are perhaps impossible to correct.

With our method of analysis in cases of high statistics it sometimes is easy to pinpoint the experimental data points which have a systematic error larger than their statistical one (their individual chi-square at a reasonable order of N_1 is very high) and therefore to choose either to ignore the point or replace it by a correct value from another experiment. Alternatively we may ignore the whole experiment when the

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*We tacitly assume that N_1 is small compared to the total number of data points and that all data points have comparable statistical errors.

individual culprit points cannot be isolated or when too many such points exist! Fig. 1 illustrates such selection with data of Ref. 14(a).

Such evaluation has been carried out for all the elastic π^+p scattering sets of measurements with more than 20 data points per energy[4] in the range $0.6 < p_{lab} < 4.0$ GeV/c . We can complete many sets of data by mixing different experiments in order to achieve a $\cos\theta$ interval close to [-1, +1]; such an amalgamation is used only if the corresponding plot of χ^2/pt vs. N_1 does not indicate systematic error.

In the particular case of the $\Delta(1600)$ mass region that we present here, we use two independent sets of scattering data[14a] which we believe have been rendered free from systematic errors significantly larger than the stated errors (statistical)[4] . We check that -- with the same set of polarization data,[14b] -- they give results in good agreement one with the other at neighboring energies.* The selected scattering data have about 100 points/energy. Errors on polarization data (about 40 points/energy) are so much larger that we have not felt worthwhile an effort to "clean" them, even though they show signs of some systematic errors larger than the overall stated error.

III. The results of the ZPWA in the $\Delta(1600)$ mass region

In Ref. [10], we give the final results of this zero partial-wave analysis carried out between 1150 and 2400 MeV, new results from our selective amalgamation of the data[4] being included from 1400 MeV on. The resolution of the discrete ambiguity justified in detail in Ref. [8],

*An example of such case is provided in Fig. 2(a) where the D33 wave is observed with the adjustement either of the mixture of results from all the scattering data of Ref.14(a), or of the results from one set of experiments only (see Table II(a)).

is reproduced partially in Table I(a) for the mass region between 1450 and 1800 MeV. In this interval, where we have selected 26 independent sets of measurements, we find that two waves are certainly resonating: the S31 and D33.

In order to determine the parameters of these resonances, we have parametrized the resonant component by a simple Breit-Wigner formula:

$$T_{J,\varepsilon} \sim \frac{\frac{1}{2}\Gamma_e}{(M-M_R) - i\left(\frac{\Gamma}{2}\right)} \tag{13}$$

-- where Γ_e and Γ are the elastic and total width related by the elasticity $x_e = \Gamma_e/\Gamma$ -- the more elaborate forms described in detail in ref. [15] seeming to us inappropriate to the available data. Also, we use a variety of backgrounds and mass intervals for the fit by a minimun chl2 method [16] to both S_{31} and D_{33} amplitudes and obtain the results of Table II(a); both resonances have mass above 1700 and are nearly degenerate -- within an interval of 10 MeV for the parabolic background. Furthermore, the widths of these resonances are of the same order of magnitude, (between 150 and 300 MeV) whatever the parametrization of the background (linear or parabolic, for both ReT and ImT), so the approximate degeneracy holds for both real and imaginary parts of the mass*. Fig. 2(a) shows these partial waves in the Argand plot and their projections on ReT and ImT axis vs. $\sqrt{s}$ for some of these fits.

In order to show the influence of the discrete ambiguity on the parameters of the resonance, we have compared it to the ambiguity resolution proposed in ref. [7], reproduced in Table I(b), the main difference from the previous

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* However the fit to the selected data from only one of the two sets of experiments of ref. [14(a)], (B76), leads to the narrowest value of the widths, inplying that better measurements might reveal narrower resonances.

analysis, besides the energy location of the critical points, is the existence for trajectories (E) and (F) of two additional critical points. The reliability of these critical points is characterized by one star only (see refs [1,2,4,7,8] and therefore it is not surprising that they were missed in the previous analysis based on less complete data.

We observe in fig. 2(b) that with the old resolution of the discrete ambiguity, and the new selected data of ref. [14], the resonances S31 and D33 are indeed present more clearly than with less accurate data[7]. However as listed in Table II(b) the mass of both these resonances is shifted to values below 1700 MeV -- different from what they are in Table II(a) for our current resolution of the discrete ambiguity -- this fact being true independent of our parametrization of the bakcground. Also inside each of these resolution of the discrete ambiguity, the masses of the D33 and S31 resonance are found remarkably close -- within 10 MeV of each other, again with the parabolic background--leading to the suspicion that high resolution data will show degeneracy to be a rather general phenomenon -- as foreseen by Barrelet[2] and observed by ourselves in the direct analysis of zero trajectories.[17]

In conclusion, zero partial wave analysis (ZPWA) finds degeneracy of the resonances S31 and D33 whatever the resolution of the discrete ambiguity, a result anticipated from general zero-trajectory considerations. The two solutions found in the different resolutions of the discrete ambiguity give different results for the mass (found to be around 1730 MeV with our latest resolution, versus 1680 MeV with the "old" one [7]) although both sets give similar widths (between $\sim$ 150 and 300 MeV). Therefore it is understandable that conventional partial wave analysis (CPWA) finds these two resonances whatever their (accidental) resolution of the discrete ambiguity: their different masses reflect only the lack of smoothness of the zero trajectories found in CPWA[7].

It is smooth behavior of certain zero trajectories that implies resonance degeneracy. In order to establish such smoothness one requires a high density

of data (in energy and angle) free from systematic error. We believe that, by identifying and eliminating erroneous data points through Barrelet-moment analysis based on analyticity, we have been able to combine sufficient data as to make a strong case for smoothness of two zero trajectories. Conventional partial-wave analysis has no such capacity yet.

There nevertheless remain many aspects of the data that need augmentation. Energy gaps occur where there are no accurate measurements of the polarization. It is up to experimenters now to measure the spin rotation parameters and to show whether Table I(a) remains a viable solution or/and how it can be improved. It will then be possible to discriminate[18] between solutions for baryonic parameters given by different partial wave analyses -- according to their resolution of the discrete ambiguity.

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TABLE CAPTIONS

I. Resolution of the discrete ambiguity according to
   (a) the recent analysis of new available data[8]
   (b) an earlier analysis[7]

II. Results of the Breit-Wigner fit of the partial waves D33 and S31 resulting from the two different resolutions of the discrete ambiguity as listed in Table I.

FIGURE CAPTIONS

1. Data illustrating the methods used in selecting points with statistical error larger than systematic one. The eliminated points (with systematic error larger than their statistical one) are circled:

   (i) at 0.665 GeV/c, the two points in the forward direction have been eliminated before any attempt at smoothing because of their value being larger than given for the forward direction from total cross section measurements and dispersion relations.[5]

   (ii) at 0.665 and 2.037 GeV/c, the chi2/point vs N_1 rises for large values of N_1 after reaching a minimum value[4] (for $6 < N_1 < 10$), indicating that some point(s) have systematic error(s) larger than statistical.[1] Removal of the points located respectively at $\cos\theta = 0.29$ and 0.15 leaves a smoothly decreasing chi2/point[4], these two points being the only ones with a chi2 larger than 9.0 at each energy (in fact, respectively, $\chi^2 = 18.$ and 26.).

   At $P_{lab} = 1.475$ GeV/c, one sees[4] no objectionable behavior of the global chi2/point vs N_1 in agreement with the absence of data point with a $\chi^2 > 9.0$.

2. (a) and (b) illustration of the results of some of the fits of Table II (a) and (b) respectively: Waves D33 and S31 obtained with the resolution of the discrete ambiguity of respectively Ref. [8] (in Table Ia) and Ref. [7]

(in Table Ib). The darker points of S33 in (a) are from the experiment by Hughes[14a]. Though we observe a good agreement between the two fits of D33, a better χ^2 and a smaller width are obtained from the partial wave analysis of the smoothed data of Bardsley et al. alone.

TABLE I

Resolution of the discrete ambiguity from elastic π^+p data

P_{lab} (GeV/c) of $\frac{d\sigma}{d\Omega}$	s (MeV)	(a)[8] Critical points Traj.	(a) Critical points Conf. level*	(a) Zero trajectories A	B	C	D	E	F	(b)[7] Critical points Traj.	(b) Critical points Conf. level*	(b) Zero trajectories A	B	C	D	E	F
				0	0	I	I	0	I			0	0	I	I	I	0
0.753		C	****							C	****						
				0	0	0	I	0	I			0	0	0	I	I	0
0.776		E	**							E	**						
				0	0	0	I	I	I			0	0	0	I	0	0
0.788		F	**														
				0	0	0	I	I	0								
0.849		F	**							F	**						
				0	0	0	I	I	I			0	0	0	I	0	I
0.881		C	*							C	*						
				0	0	I	I	I	I			0	0	I	I	0	I
1.105		E	***							E	***						
				0	0	I	I	0	I			0	0	I	I	I	I
1.235		E	*														
				0	0	I	I	I	I								
1.364		B, A	****							B, A	****						
				I	I	I	I	I	I			I	I	I	I	I	I

TABLE II

Resonance-parameter dependence on the resolution of the discrete ambiguity

			(a)						(b)					
L_{212J}	Mass interval (Number of data points)	Type of Background	M	Γ	Γ_e	x_e	χ^2/ND	Fig	M	Γ	Γ_e	x_e	χ^2/ND	Fig.
D_{33}	1550-1800 (21)	linear	1737 ± 4	163 ± 25	22 ± 5	.13	5.2	–	1691 ± 9	151 ± 16	10 ± 1	.07	8.5	–
	1600-1800 (18)	linear	1731 ± 4	204 ± 15	31 ± 2	.15	4.4	2a	1670 ± 7	182 ± 33	18 ± 4	.10	6.3	2b
	1600-1800 (11 of B76[14])	linear	1723 ± 5	123 ± 34	15 ± 6	.12	3.2	2a						
	1600-1800 (18)	parabolic	1733 ± 7	184 ± 23	20 ± 5	.11	4.8	–	1671 ± 10	150 ± 18	10 ± 2	.07	6.1	–
S_{31}	1600-1800 (18)	linear	1760 ± 44	300 ± 156	43 ± 18	.14	0.9	–	1691 ± 11	103 ± 21	10 ± 2	.10	1.0	–
		linear + M and Γ fixed	(1731)	(204)	19 ± 5	.09	0.9	2a*	(1670)	(182)	19 ± 2	.10	1.0	2b*
	1550-1850 (23)	parabolic	1720 ± 15	267 ± 34	39 ± 6	.15	1.4	2a	1682 ± 18	294 ± 19	61 ± 15	.21	1.0	2b

* Chosen because of the lack of sensitivity of χ^2/ND to the fixed values of M and Γ (from D_{33}).

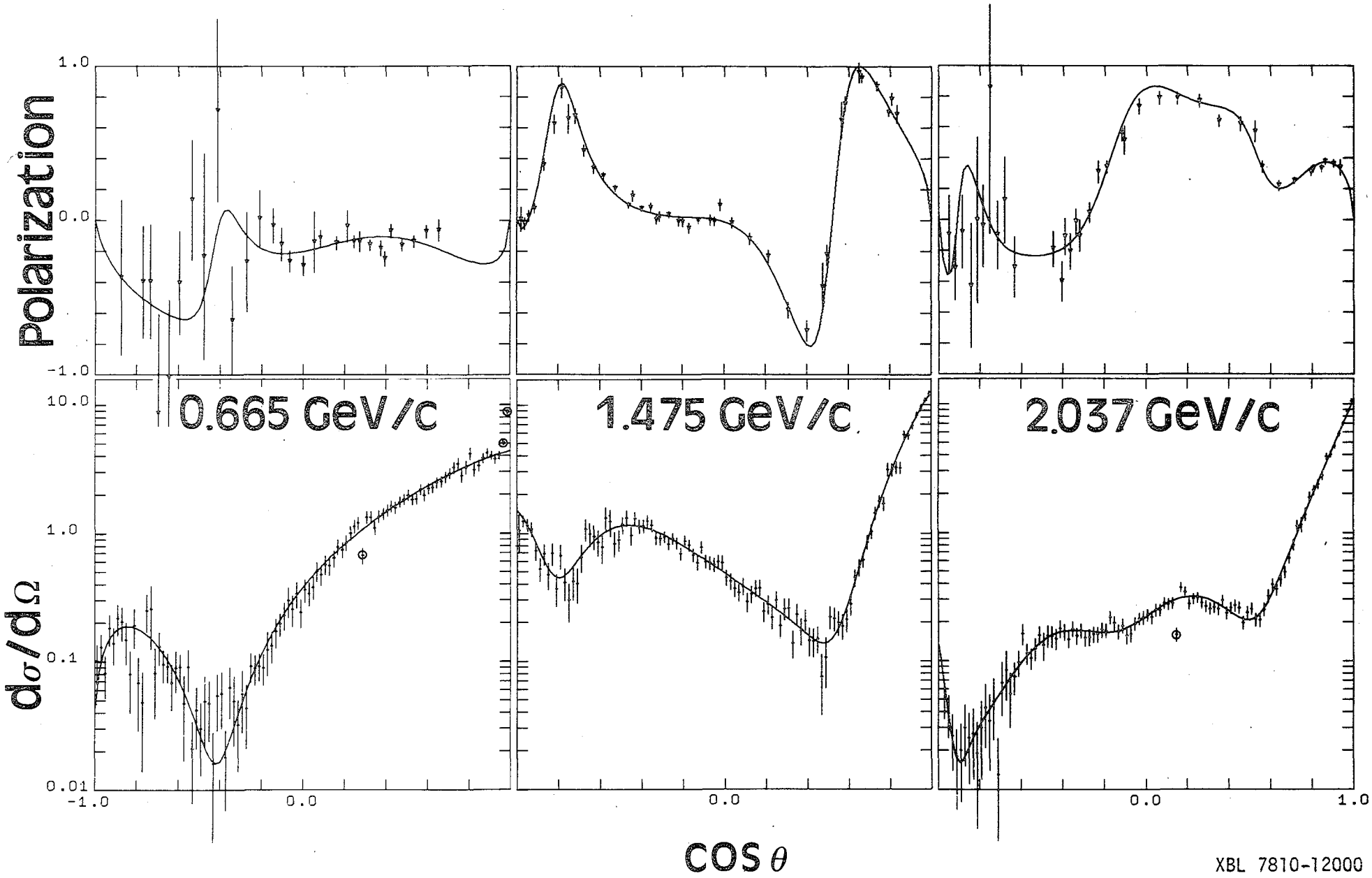
Polarization
1.0
0.0
-1.0
0.665 GeV/c
1.475 GeV/c
2.037 GeV/c
dσ/dΩ
10.0
1.0
0.1
0.01
-1.0
0.0
0.0
0.0
1.0
COS θ
XBL 7810-12000

Figure 1

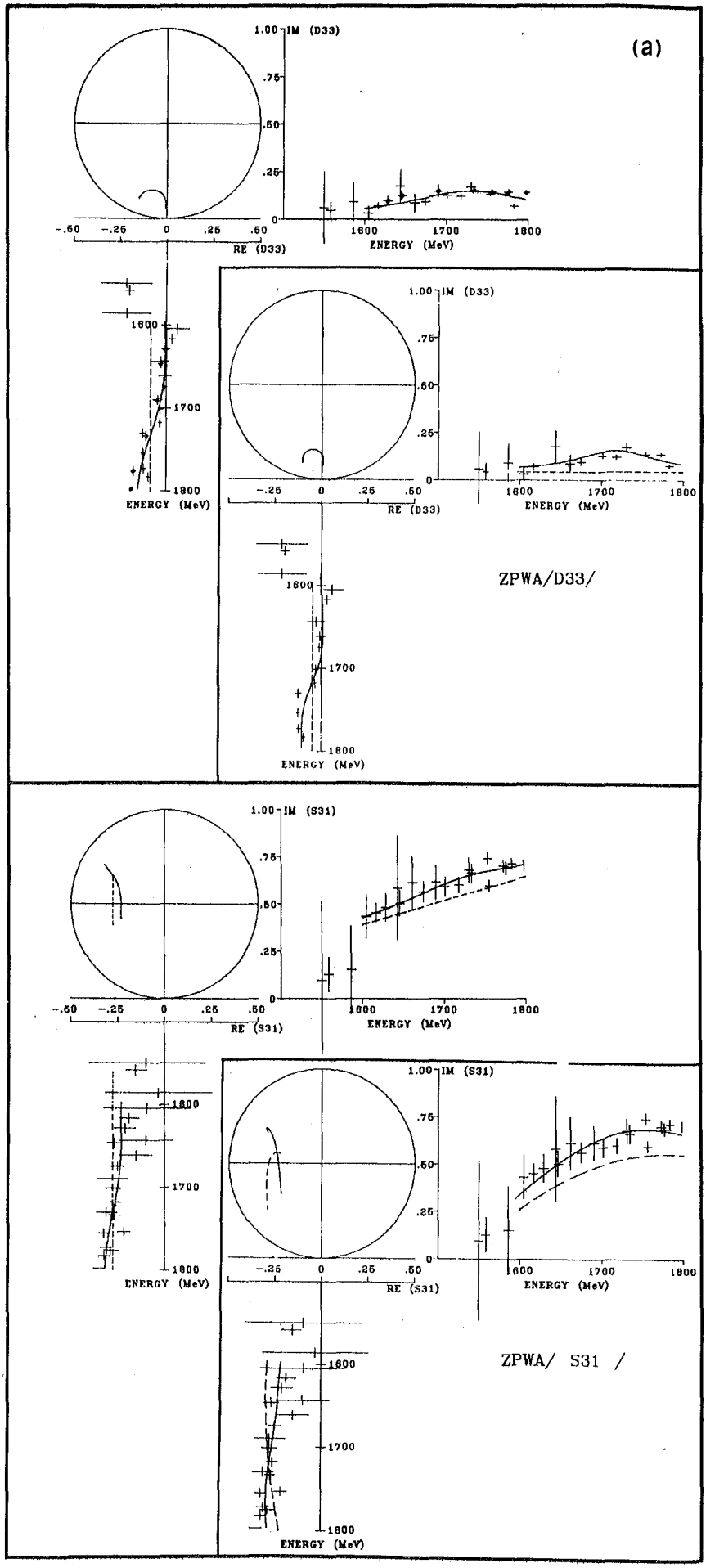

Figure 2(a)

XBL 7810-12007

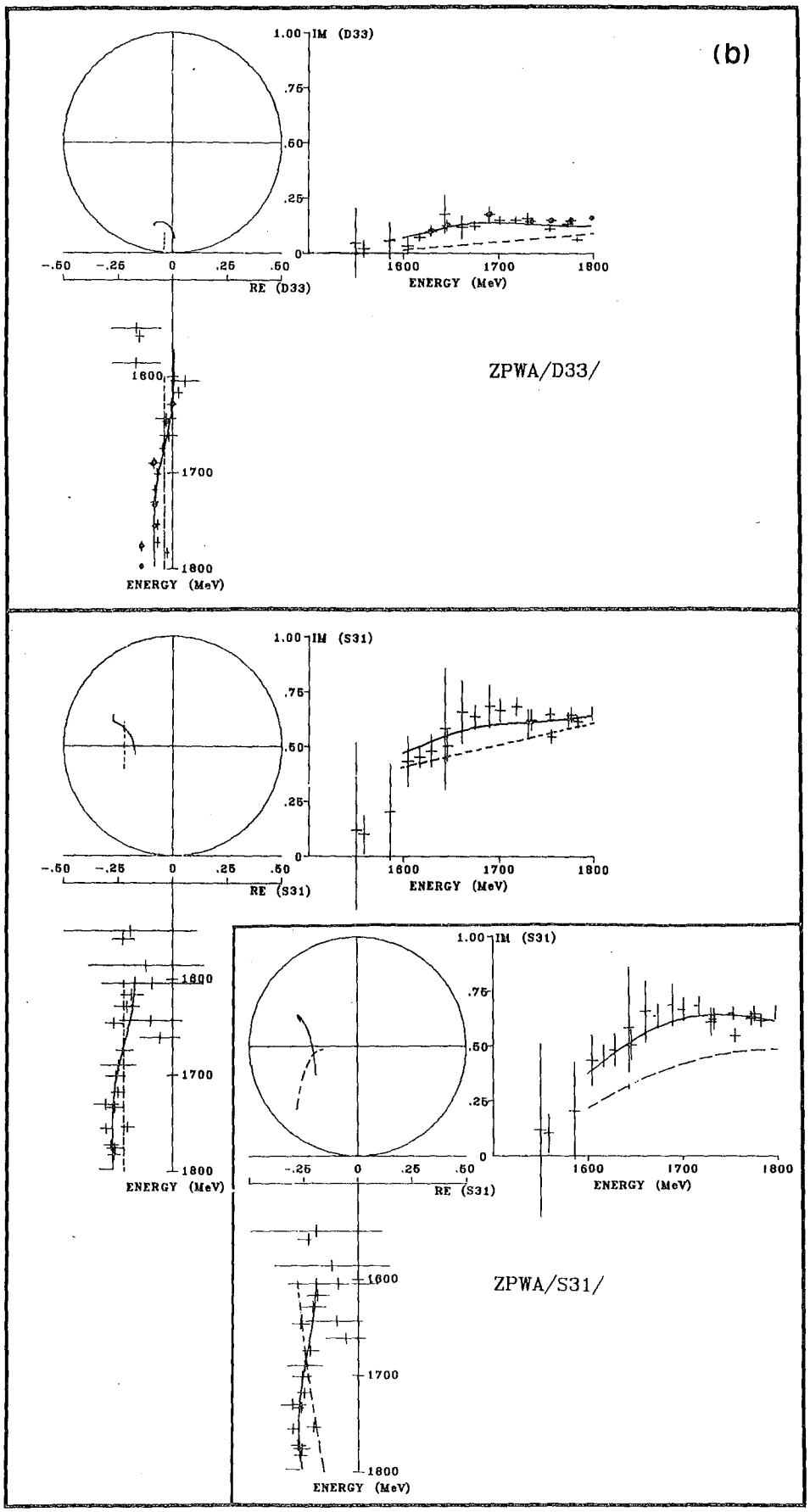

Figure 2(b)

XBL 7810-12002

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