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FADDEEV-POPOV PROCEDURE AND GRIBOV COPIES*

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ABSTRACT

In simple examples, we find that gauges with Gribov copying are valid gauges. One need only compute the Faddeev-Popov determinant Δ exactly and procede. Δ provides the correct measure for the copies. A cut-off procedure keeping only the first copy is seen to work as well. In quantum chromodynamics, we point out a class of gauges, the field-strength gauges, for which Δ is easily computed.

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I. INTRODUCTION

In gauge theories of a generic variable A, one is interested in evaluating the generating functional

$$Z[J] = \eta \int \mathcal{D}A \, e^{-S[A]} \, e^{-JG(A)}$$
$$\eta^{-1} = \int \mathcal{D}A \, e^{-S[A]} \tag{1.1}$$

where $G(A)$ is a gauge-invariant function of A. Because of the gauge-invariance $(A \to A^{\Lambda},\ S[A^{\Lambda}] = S[A])$, the numerator and denominator contain an <u>identical</u>, <u>multiplicative</u>, infinity $(\int \mathcal{D}\Lambda)$, which cancels in $Z[J]$. In order to define a perturbation expansion, and to have a canonical formulation, however, it is a universal practice to "gauge-fix" the numerator and denominator via the procedure of Faddeev and Popov[1].

Their procedure is to choose a gauge condition $f(A) = 0$, and resolve unity via

$$1 = \Delta(A) \int \mathcal{D}\Lambda \, \delta[f(A^{\Lambda})] \tag{1.2}$$

If Δ, the Faddeev-Popov determinant, exists (see below) it is gauge-invariant. Inserting this unity into numerator and denominator of $Z[J]$, and changing variables $A' = A^{\Lambda}$, we factor out the $\int \mathcal{D}\Lambda$ (top and bottom), to reach the explicitly finite form

$$Z[J] = \bar{\eta} \int \mathcal{D}A \, \Delta(A) \, \delta[f(A)] \, e^{-S} \, e^{-JG} \tag{1.3}$$

Here $\mathcal{N}$ continues to normalize $Z[0] = 1$.

Since one intends using $\Delta(A)$ "on-gauge-shell", that is next to $\delta[f(A)]$, one often computes instead the "gauge-shell" determinant Δ_o,

$$1 = \Delta_o(A)\int \mathcal{D}\Lambda\, \delta\left[f(A^\Lambda) - f(A)\right]. \tag{1.4}$$

If both exist, Δ and Δ_o will agree when $f(A) = 0$.

Recently Gribov[2] has emphasized that there may, in general, be more solutions than $\Lambda = 0$ to $f(A^\Lambda) - f(A) = 0$. That is, there may be non-trivial gauge transformations Λ connecting configurations in the gauge $f(A) = 0$. If a set of configurations A_i, all satisfying $f(A_i) = 0$, can be gauged into one another, we shall refer to the set $\{A_i\}$ as "Gribov copies."

It is our viewpoint in this paper that <u>the existence of the copying phenomenon</u> (for certain gauges) <u>does not necessarily invalidate the gauge</u>. At least in the action formalism,(*) and at least in our simple examples, one need only compute Δ, Δ_o exactly and procede; the determinant merely provides the correct measure for the copies. It can, in realistic models, be extremely difficult to compute Δ, or Δ_o exactly, and Gribov has also proposed a <u>truncation procedure</u>, keeping only the first $(\Lambda = 0)$ copy's contribution to Δ_o. In some examples we will see that this is also a <u>viable procedure</u>. It is important to notice however that <u>the copying phenomenon is highly model-and gauge-dependent</u>: we do not know if

(*) When $\Delta = 0$, one no longer has an obviously well-defined canonical formalism.[3]

any of our examples resemble the situation in, say, the Landau gauge of quantum chromodynamics (QCD).

The properties of Δ and Δ_o tell us much about the gauge choice. We summarize what can be seen in our examples below.

<u>Properties of Δ and Δ_o</u>

1. The computation of $\Delta(A)$ is equivalent to finding the gauge transformation that brings the configuration A into one (or more) configurations in the chosen gauge.
2. If $\Delta = \infty$, even on a sub-manifold, the gauge cannot be reached for configurations in that sub-manifold.
3. The computation of Δ_o is a search for gauge transformations which return a configuration to the chosen gauge, <u>starting from that gauge</u>. Δ_o will exist even when $\Delta = \infty$ on a sub-manifold.
4. If there is a sub-manifold of "continuous" copies (copies connected to each other by infinitesimal gauge transformations), then $\Delta_o = 0$ on that sub-manifold. "Discrete" Gribov copies (separated by finite gauge-transformations) will <u>not</u> in general show up as $\Delta_o = 0$: <u>$\Delta_o = 0$ is not necessary for (discrete) copies</u>.
5. We also have examples in which (a) $\Delta = \Delta_o = 0$ identically, but there are no copies; instead the gauge cannot be reached from arbitrary configurations. (b) $\Delta_o = 0$ on a sub-manifold, yet there are no copies: <u>$\Delta_o = 0$ is neither necessary nor sufficient for copies</u>.

II. AN ABELIAN MODEL

We define a simple model with an Abelian gauge invariance as

$$Z = \int \prod_{\ell} dx_{\ell}\, e^{-\frac{1}{2}F_{ij}F_{ij}} \qquad (2.1)$$

$$F_{ij} \equiv d_i\, x_j - d_j x_i$$

Here x_i are ordinary variables (for simplicity, we will discuss the model only for $i = 1, 2, 3$), and d_i are a set of fixed numbers ("derivatives" on the finite lattice). The field-strengths F_{ij} are invariant under the position-dependent translation

$$\underset{\sim}{x} \to \underset{\sim}{x} + \underset{\sim}{d}\, \lambda(\underset{\sim}{x}) \qquad (2.2)$$

where $\lambda(\underset{\sim}{x})$ is the gauge function. Z is then (multiplicatively) infinite, and we may gauge-fix. We list a number of simple gauge choices and remarks.

(1) <u>Axial gauges</u>: $\hat{n} \cdot \underset{\sim}{x} = 0$.

Here $\hat{n}$ is a fixed direction. The gauge condition specifies a surface, here a plane. These gauges can have no copies: if you leave the plane $\hat{n} \cdot \underset{\sim}{x} = 0$ in the $\pm\underset{\sim}{d}$ direction, you will never reach another gauge surface. We compute

$$\Delta^{-1} = \int_{-\infty}^{+\infty} d\lambda\ \delta(\hat{n} \cdot \underset{\sim}{x} + \hat{n} \cdot \underset{\sim}{d}\lambda) \to \Delta = |\hat{n} \cdot \underset{\sim}{d}|$$

$$\Delta_o^{-1} = \int_{-\infty}^{+\infty} d\lambda\ \delta(\hat{n} \cdot \underset{\sim}{d}\lambda) \to \Delta_o = |\hat{n} \cdot \underset{\sim}{d}| \ . \qquad (2.3)$$

The unique root $\lambda = -\hat{n} \cdot \underset{\sim}{x}(\hat{n} \cdot \underset{\sim}{d})^{-1}$ of the first δ-function states that the gauge is uniquely reached (by this λ) from arbitrary configuration λ. The unique root $\lambda = 0$ of the second δ-function shows that there are no Gribov copies.

Note that, varying $\hat{n}$ until $\hat{n} \cdot \underset{\sim}{d} = 0$, we reach $\Delta = 0$; when $\hat{n} \cdot \underset{\sim}{d} = 0$, it is clear geometrically that points off the plane cannot be gauged onto it. This is an example then that <u>$\Delta = 0$ does not always imply copies</u> (instead this gauge is not accessible for arbitrary configurations).

(2) <u>Landau gauge</u>: $\underset{\sim}{d} \cdot \underset{\sim}{x} = 0$.

This gauge surface is also a plane, this time perpendicular to $\underset{\sim}{d}$. It is clear geometrically that this gauge is accessible from arbitrary configurations and without copies. One easily computes $\Delta = \Delta_o = |\underset{\sim}{d}^2|$.

(3) <u>A gauge with copies</u>: $\delta\left[\sin(ez)\right]$.

Here $z = x_3$ and e may be the gauge coupling. The gauge surface is an infinite series of planes at

$$z_n = \frac{n\pi}{e} \qquad (\text{all } n) \ . \qquad (2.4)$$

Copies are obvious in this gauge; pick a point say in the $z = 0$ plane and translate it along $\pm\underset{\sim}{d}$ until it hits $z_n = \frac{n\pi}{e}$. These points are the (discrete) copies of the original point.

We compute

$$\Delta^{-1} = \int_{-\infty}^{+\infty} d\lambda\, \delta\left[\sin\left(e\{z + d_3\lambda\}\right)\right] \tag{2.5}$$

The roots

$$\lambda_n = \frac{1}{d_3}\left(\frac{n\pi}{e} - z\right) \tag{2.6}$$

tell us that there are an infinite number of ways to get into the gauge from an arbitrary configuration $\underset{\sim}{x}$ (by these λ's). This is the copying phenomenon. Each copy contributes one unit of $|ed_3|^{-1}$ to Δ^{-1}, so

$$\Delta = \frac{|ed_3|}{N}, \qquad N = \sum_{-\infty}^{+\infty} . \tag{2.8}$$

The same result is obtained for Δ_o, the roots of the appropriate δ-function being the gauge maps from copy to copy.

In this gauge, the copies have forced $\Delta = 0$ identically, and it is somewhat academic whether tha gauge is legitimate. The structure $(S = \frac{1}{4} F_{ij}F_{ij})$

$$\bar{Z} = \int dx\, dy\, dz\, \delta(\sin(ez))\, \frac{|ed_3|}{N}\, e^{-S} \tag{2.9}$$

can easily be defined as follows. We truncate the z integration somewhere between the nth and $n + 1^{st}$ copies, say by inserting

$\theta((n + \frac{1}{2})^2 \frac{\pi^2}{e^2} - z^2)$, and keep only the first $2n + 1$ roots in the computation of Δ_o (thus $\Delta_o \approx (2n+1)^{-1}|ed_3|$). Then for every n, it is clear on inspection that $\bar{Z}$ agrees with the axial gauge $(\delta(z))$. As $n \to \infty$, this defines the original gauge. If we stay at $n = 0$ (keeping only the first copy), this is Gribov's truncation.[3] Either prescription is valid.

This gauge is also the limit of the following "smoothed" gauge.

(4) <u>A gauge with copies and $\Delta \neq 0$</u>

Consider the gauge choice $\delta\left[e^{\beta|z|} \sin(ez)\right]$ with $\beta e > 0$. The copy geometry is the same as the previous gauge. We compute now

$$\Delta = \Delta_o = \frac{|ed_3|}{\sum_{n=-\infty}^{+\infty} e^{-|n|\pi\beta/e}} = |ed_3| \tanh\left(\frac{\pi\beta}{2e}\right) \tag{2.10}$$

Δ is finite. <u>Discrete Gribov copies do not necessarily show up as $\Delta = 0$</u>. Again the truncation procedure works: keep only the first copy's contribution to Δ_o $(\Delta_o \approx |ed_3|)$, and truncate with $\theta\left(\frac{\pi^2}{4e^2} - z^2\right)$. Then only the first zero of the sin contributes and we are in agreement with the axial gauge.

(5) <u>A gauge with more copies at higher coupling.</u>

Consider the gauge choice $\delta\left[\sin\left(\frac{2\pi ez}{1 + z^2}\right)\right]$ where $z = x_3$, $d_3 \neq 0$, and e is to be identified with a coupling constant.

Note that $f(z) = 2z(1 + z^2)^{-1}$ is an odd function with maximum (minimum) value $+1(-1)$. Consider the equation $f(z) = \frac{n}{e}$, which locates the gauge surfaces (or $f(z + d_3\lambda) = \frac{n}{e}$ which gives the gauge transformations into the gauge). For $|e| < 1$, the only solution is with $n = 0$. $\sin(\pi e f(z))$ vanishes only at $z = 0$; the gauge has no copies and is equivalent to the axial gauge ($\delta(z)$).

For $|e| = 1$, there are solutions for $n = 0, \pm 1$; the gauge surfaces are three planes, and there are three copies (for every point). For $1 < |e| < 2$, there are five copies|point (still from $n = 0, \pm 1$). In general for $k < |e| < k + 1$, there are $4k + 1$ copies. Gribov[2] argues that similar behaviour appears in the Coulomb gauge choice for QCD.

(6) <u>A gauge with copies, and $\Delta = 0$ on a sub-manifold</u>

Consider the gauge

$$\delta(xy) = \frac{\delta(x)}{|y|} + \frac{\delta(y)}{|x|} . \tag{2.11}$$

This choice forces pairs of copies: Start at, say, $x = 0$ and translate along $\pm \underset{\sim}{d}$ until you hit $y = 0$. These points are each others Gribov copies. We compute explicitly (there are two roots for λ),

$$\Delta = \frac{1}{2} |F_{xy}| = \frac{1}{2} |d_x y - d_y x| . \tag{2.12}$$

The overlap of $\Delta = 0$ and $xy = 0$ is the z-axis. The z-axis is where Gribov copies approach infinitesimally close to one another. <u>Infinitesimally close copies show up as $\Delta = 0$</u>. (This is easy to prove in general). All the other (discrete) copies do <u>not</u> show as $\Delta = 0$.

(7) <u>Nambu gauge</u>[4] : $\underset{\sim}{x}^2 = \eta^2$

We compute

$$\Delta_o^{-1} = \int_{-\infty}^{+\infty} d\lambda\, \delta\left[(\underset{\sim}{x} + \underset{\sim}{d}\lambda)^2 - \eta^2 - (\underset{\sim}{x}^2 - \eta^2)\right] \tag{2.13}$$

There are two roots and we obtain $\Delta_o = |\underset{\sim}{x} \cdot \underset{\sim}{d}|$. Clearly the gauge has pairs of copies; start at a point on the spherical gauge surface and translate along $\pm\underset{\sim}{d}$ until you reach the surface again. In general, as above, the copies do not show up as $\Delta_o = 0$. However, as above, the overlap of $\Delta_o = 0$ and the sphere is the circle along which the copies have approached each other infinitesimally.

Unfortunately, this gauge cannot be reached from all configurations. This is obvious geometrically: For simplicity imagine that $\underset{\sim}{d}$ is in the z-direction. Then any point outside the cylinder $x^2 + y^2 = \eta^2$ cannot be gauge-transformed to the surface of the sphere. Formally

$$\Delta^{-1}(\underset{\sim}{x}) = \int_{-\infty}^{+\infty} d\lambda\, \delta\left[(\underset{\sim}{x} + \underset{\sim}{d}\lambda)^2 - \eta^2\right] . \tag{2.14}$$

For points outside the cylinder the argument of the δ-function never vanishes and $\Delta = \infty$. $\Delta = \infty$ on a manifold heralds inaccessibility of the gauge for that manifold. In general this will not show in Δ_o. This gauge surface can be included in legitimate (and copy-filled) gauges such as $\delta\left[\sin(x^2 - \eta^2)\right]$.

(8) 't Hooft-Veltman gauge: $\underset{\sim}{d} \cdot \underset{\sim}{x} + e\underset{\sim}{x}^2 + \gamma = 0$.

The gauge surface is a distorted sphere, and has the same problem as the Nambu gauge above. We compute $\Delta_o = |\underset{\sim}{d}^2 + e\underset{\sim}{x} \cdot \underset{\sim}{d}|$ (whose zeroes locate the infinitesimally close copies), but $\Delta = \infty$ for large $|\underset{\sim}{x}|$. The gauge is inaccessible from arbitrary configurations.

What is curious about this gauge is that it can be reached in a perturbation expansion in e -- i. e., a (non-convergent) expansion about the Landau gauge.

We have not explored this further, but the phenomenon brings to mind the notion of gauge-space breaking up into inequivalent "gauge-balls". Consider a gauge, such as this one, that is not reachable from, say, the axial gauge. By gauge-change of variables we can easily surround this gauge by its own equivalent "ball of gauges", the entire ball inequivalent to the "axial-ball". What would then be the criteria for such an inequivalent gauge-ball to be a theory in its own right? (a canonical formulation? a perturbation theory which agrees with the axial gauge?)

Remarks

In general in this model, we have seen that gauges with copies are valid. $\Delta_o = 0$ locates the continuous copies, and,

more generally, Δ_o provides the correct measure for the copies: In all our examples, all copies of a given configuration map onto a single configuration in (say) the axial gauge.

III. NON-ABELIAN MODEL WITH CONTINUOUS COPIES

Consider the simple model

$$Z[J,K,L] = \mathcal{N}\int \mathcal{D}\underset{\sim}{A}(t)\mathcal{D}\underset{\sim}{B}(t)\, e^{-S-\Delta}$$

$$S = \int dt\, \{(\underset{\sim}{A}^2 + \underset{\sim}{B}^2 + \varepsilon\, \underset{\sim}{A}\cdot\underset{\sim}{B})\} \tag{3.1}$$

$$\Delta = \int dt\{J\, \underset{\sim}{A}^2 + K\, \underset{\sim}{B}^2 + L\, \underset{\sim}{A}\cdot\underset{\sim}{B}\}$$

where $\mathcal{N}$ is fixed so that $Z[0,0,0] = 1$. The action S has a non-Abelian gauge-invariance

$$A_a(t) \to O_{ab}(t)\, A_b(t) \qquad (a,b = 1,2,3) \tag{3.2}$$

(and similarly for $\underset{\sim}{B}$). Here O is an orthogonal matrix. Thus the numerator (and denominator) of Z has a multiplicative infinity of the form $(\int d\,\Omega)^N = (4\pi)^N$ where $d\Omega$ is the integration over the group volume and N is the number of points on the time grid. In this simple case, we could do perturbation theory (say in ε) without gauge-fixing, but we choose to gauge-fix anyway.

We will choose our gauge geometrically. First, at every point in time, rotate $\underset{\sim}{A}(t)$ onto the positive x-axis in color space. Then make rotations around the x-axis until $\underset{\sim}{B}(t)$ has no z-component, and positive y-component. The gauge-choice is then

$$\delta[A_2]\delta[A_3]\delta[B_3]\;\theta[A_1]\theta[B_2]\;. \tag{3.3}$$

The gauge is obviously accessible from arbitrary configurations, so we will compute only Δ_o. The full Δ is

$$\Delta^{-1} = \int \mathcal{D}\Omega\;\; \delta[O_{2b}A_b]\,\delta[O_{3c}A_c]\,\delta[O_{3d}\,B_d] \otimes \theta[O_{1e}A_e]\;\theta[O_{2f}B_f] \tag{3.4}$$

but, using the gauge-condition, we have

$$\Delta_o^{-1} = \int \mathcal{D}\Omega\;\; \delta[O_{21}A_1]\;\delta[O_{31}A_1]\;\delta[O_{31}B_1 + O_{32}B_2] \otimes \theta[O_{11}A_1]\;\theta[O_{21}B_1 + O_{22}B_2]\;. \tag{3.5}$$

If A_1 and B_2 are not zero, the roots of the δ-functions specify $O_{ab} = \delta_{ab}$ and, thus,

$$\Delta_o = \prod_t\; A_1^2(t)\;|B_2(t)| \tag{3.6}$$

At A_1 or B_2 equals zero, we will define Δ_o as the limit of (3.6). We must compute then

$$Z_F(J,K,L) = \mathcal{N}_F\int \mathcal{D}A_1\mathcal{D}B_1\mathcal{D}B_2\;\theta(A_1)\,\theta(B_2)A_1^{\,2}\,B_2 \otimes \exp\{-\int dt(A_1^2 + B_1^2 + B_2^2 + \varepsilon A_1B_1)\} \otimes \exp\{-\int dt(JA_1^2 + K(B_1^2 + B_2^2) + LA_1B_1)\} \tag{3.7}$$

where $\mathcal{N}_F$ normalizes $Z_F(0,0,0) = 1$. We shall do so after a brief discussion of the copies.

In this model, all Gribov copies are continuous and can be located from $\Delta_o = 0$. What are the copies when $A_1 = 0$? Consider

$$OV \equiv \begin{bmatrix} O_{11} & O_{12} & O_{13} \\ O_{21} & O_{22} & O_{23} \\ O_{31} & O_{32} & O_{33} \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ 0 & B_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} A_1' & B_1' \\ 0 & B_2' \\ 0 & 0 \end{bmatrix} . \qquad (3.8)$$

It is easy to see that when $A_1 = 0$, OV is of the same form ($A_1' = 0$) as long as $O_{31}\, O_{32}^{-1} = -B_2 B_1^{-1}$: $A_1 = 0$ corresponds to a two-parameter family of rotated Gribov copies. The family of rotations leaves $|\underset{\sim}{B}|$ invariant, and so, of course, is a symmetry of the action.

This family of copies contributes an apparently indeterminate term to the functional integral: a (double) zero (from the measure, A_1^2), times an infinity (number of copies - i. e., volume of the subgroup of rotations to the Nth power), times the exponential of the action for a given member of the copy set. This is in fact the general form expected for continuous copies.

Fortunately the functional integrals (3.1) and (3.7) are easy to do explicitly on the grid. There we face N decoupled three-dimensional integrals. The result is finite on the grid for sufficiently small ε, J, K, L:

$$Z\left[J,K,L\right] = Z_F\left[J,K,L\right] = \prod_t \left\{ \frac{\left[1 - \varepsilon^2/4\right]}{\left[1 + J(t)\right]\left[1 + K(t)\right] - \frac{1}{4}\left[\varepsilon + L(t)\right]^2} \right\}^{\frac{3}{2}} \qquad (3.9)$$

It is particularly clear on the grid that the "copy-space" (say $A_1 = 0$) is contributing nothing. Even though all B_1, B_2 is contributing, $A_1 = 0$ is a set of measure zero. Indeed, we could have neglected the copy space entirely and still gotten the same answer.

We are not certain that Gribov copies are always so harmless. On the basis of this model however, and those of Section II, we can repeat that copies do not necessarily invalidate a gauge. Indeed, the most reasonable hypothesis continues to be: if Z is well defined in any gauge, it will be well defined in a copy-gauge, Δ providing the correct measure for the copies.

A final remark illustrated by this model is that <u>$\Delta_o = 0$ is not sufficient for Gribov copies</u>. To wit, $\Delta_o = 0$ when $B_2 = 0$. The form

$$V = \begin{pmatrix} A_1 & B_1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

is preserved by O a rotation about the 1 direction; yet under this rotation $A_1 \to A_1$, $B_1 \to B_1$ and there are no copies.

IV. EXACT Δ'S IN QCD

To make our points above cleanly, we needed to be able to compute the Faddeev-Popov determinants in closed form. In QCD, beyond the axial gauges, we know of no such cases in the literature.

There is however a large class of (ghost) gauges in QCD for which the determinants are easily computed. These are the "field-strength" gauges, wherein gauge conditions are on the field-strengths instead of the potentials. As an example, we will discuss the so-called "upper-triangular" gauge.[5]

In $O(3)$ QCD, consider the color electric field E_i^a $\left[a = 1,2,3 \text{ is color}, \; i = 1,2,3 \text{ is space}\right]$. In matrix notation $(E)_{ai} \equiv E_i^a$, the gauge rotation is $E \to OE$, with O an orthogonal matrix. Following the gauge-choice (and discussion) of Section III, we choose the upper-triangular gauge[5]

$$E = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ 0 & E_{22} & E_{23} \\ 0 & 0 & E_{33} \end{bmatrix} \tag{4.1}$$

and $E_{11} \geq 0$, $E_{22} \geq 0$. Following Section III, we compute

$$\begin{aligned} &\delta(E_{21})\,\delta(E_{31})\,\delta(E_{32})\,\theta(E_{11})\,\theta(E_{22}) \\ &\to \quad \Delta_o = E_{11}^2 |E_{22}| \quad . \end{aligned} \tag{4.2}$$

It is easy to see also (as in Section III) that there is a two-parameter set of (rotated) Gribov copies for every configuration with $E_{11} = 0$; there is also a one-parameter (now non-trivial) set of copies for each configuration with $E_{22} = 0$.

We have not been able to prove that the copies are again of measure zero (as in Section III) or harmless (as in Section II). The field-strength formulation of QCD[5,6] may be helpful in this investigation.

Also of note about the field-strength gauges is the fact that they cannot be reached in perturbation theory. By the same token, they have no obvious perturbation expansion. The reason is simple: in perturbation theory one is expanding about an Abelian invariance, and such transformations will not rotate the field-strengths. Such gauges deserve further investigation.

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