Submitted to Physics Letters

LBL-6406
Preprint c.1

BARRELET ZEROS AND THE FORWARD DIFFRACTION PEAK

D. M. Chew

March 1977

Prepared for the U. S. Energy Research and
Development Administration under Contract W-7405-ENG-48

For Reference

Not to be taken from this room

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY

LBL-6406
c.1

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

0 0 0 0 4 8 0 1 4 3 4

(Submitted for publication) LBL-6406

BARRELET ZEROS AND THE FORWARD DIFFRACTION PEAK*

D. M. Chew+

Lawrence Berkeley Laboratory
University of California
Berkeley,Ca. 94720

March, 1977

ABSTRACT

It is generally supposed that the sharp forward diffractive peak in high energy elastic scattering cannot be represented by meaningful Barrelet zeros and correspondingly that Barrelet-moment analysis of scattering data cannot deal effectively with the full angular interval $-1 < \cos\theta < +1$.We have made a Barrelet analysis of π^+p elastic scattering and polarization data at P_{lab}= 1.770 GeV/c, including the entire forward peak but with a weight function for the determination of moments that assigns comparable importance to all angular intervals.We thereby achieve a 10-zero polynomial representation that adequately fits all the data.Eight of the 10 zeros lie well inside the region of convergence of a polynomial expansion and the last two ,while closer to the estimated boundary,are still inside.We are led to suggest that the diffraction peak is associated with true amplitude zeros.

+ On leave from the University of paris VI,Paris,France.

* This work was done with support from the United States Energy Research and Development Administration.

To the extent that the forward high-energy diffraction peak corresponds to exponential dependence on cos θ_{cm} ,one does not expect this peak to be associated with zeros of the scattering amplitude.Angular distributions of finite accuracy can of course always be represented by a finite-order polynomials in cosθ and a polynomial of order N is equivalent to N zeros, so any experimentally-measured forward peak may be represented through a sufficiently large number of zeros.But Barrelet has emphasized the distinction between true zeros of the amplitude and zeros that are artifacts of data fitting[1].Roughly speaking,the latter lie outside the domain of convergence of a polynomial expansion.Thus one might expect that a polynomial representation of a sharp forward elastic peak would involve many "distant" zeros that do not correspond to true amplitude zeros.The Barrelet program[2]--based on moment analysis of partial intervals in cosθ -- in fact has recommended that the extreme forward interval be ignored.The strong forward peak,it was argued,has no relation to true("nearby") amplitude zeros and can only obscure the information residing in other angular regions[3].

In practice,however, "nearby" zeros are found to occur in the neighborhood of cosθ =+1 and their determination requires attention to the region where the diffraction peak begins to be felt.It is unclear how much of the forward peak should be omitted in order to optimize information about true zeros.It has furthermore been found that,even when the interval containing most of the forward peak is omitted,the nearby-zeros polynomial representation of the remaining data extrapolates reasonably well(e.g.within 10%) to the forward data*.It is therefore tempting to apply Barrelet's moment analysis to the entire interval, including the smallest angles.The large forward peak may be prevented from overwhelming the other angles in thecomputation of moments by employing a suitable weight function.We are here rep orting the results of such an analysis of π^+p elastic scattering at 1.77 GeV/c ,where many different measurements are available.Even though the diffraction peak has a width of only ~ 0.1 in cosθ ,and looks e ponential in shape,Barrelet's moment analysis lead to a 10-zero polynomial that fits all the data including the forward peak.The surprising aspect of our result is that none of the ten zeros is really "distant".Eight of the ten are well inside the domain of convergence of a polynomial expansion,and the remaining two although further from the physical region are still inside the usually estimated boundary.

*At least up until the Δ(1900) mass region in the elastic π^+p data analysis described in Reference 4 (See Ref.5).

Our selection of data at 1.77 GeV/c is described in detail in Ref. (6). A key ingredient in this selection was the extreme forward direction differential cross section that has been inferred from total cross section measurements in combination with dispersion relations [7]. The accurate normalization thereby given allowed data with systematic errors to be identified and eliminated. Our final selection of data is shown in Fig. 1, where one observes the exponential-shaped forward peak in the differential cross section with a width of about 0.1 in cos θ.

The standard Barrelet method of moment analysis[1,2] was applied[6]. To represent an angular interval $a < \cos\theta < b$ with a weighting norm $n_p(\cos\theta)$, Barrelet employs a set of orthonormal pseudopolynomials $p_\ell(\cos\theta)$ such that:

$$\int_a^b d(\cos\theta)\, p_\ell(\cos\theta)\, p_{\ell'}(\cos\theta)\, n_p(\cos\theta) = \delta_{\ell\ell'}$$

The differential cross section in this interval is then represented as

$$\frac{d\sigma}{d\Omega} = \sum_{\ell=0}^{N_1} A_\ell p_\ell(\cos\theta)$$

the moments A_ℓ being calculated from the formula:

$$A_\ell = \int_a^b \frac{d\sigma}{d\Omega}(\cos\theta)\, p_\ell(\cos\theta)\, n_p(\cos\theta)\, d(\cos\theta)$$

When the latter quadrature is carried out with appropriate care, using data sufficiently dense in cosθ, the statistical error on individual data points translates into statistical errors for the moments A_ℓ. A corresponding analysis may be made of data on $P\frac{d\sigma}{d\Omega}$, P being the polarization, leading to a second set of moments B_ℓ, also with statistical errors. Barrelet then chooses the truncation points N_1 and N_2 for his expansions on the basis that A_ℓ and B_ℓ are expected to tend smoothly toward zero for ℓ sufficiently large. When the error in the moments begins to overlap zero, Barrelet terminates the expansion. To go beyond this point is statistically meaningless.

The result of such analysis may be expressed through the zeros of a single function of the variable $w \equiv e^{i\theta}$, a function whose even and odd parts are, respectively, $\frac{d\sigma}{d\Omega}$ and $P\,\frac{d\sigma}{d\Omega}$. The total number of zeros corresponds to the maximum ℓ values included in the moment expansion. Barrelet originally developed his method for use with individual experiments that each covers a finite portion of the physical interval in $\cos\theta$. The zeros in complex $\cos\theta$ (or w) which may be reliably determined are those near the part of the physical region that has been experimentally studied. Barrelet stressed that individual "nearby" zeros may be unambiguously established* on the basis of data inadequate to a full amplitude analysis.

For locating nearby zeros Barrelet remarked that the forward diffraction peak may be an impediment if the weighting norm $n_p(\cos\theta)$ is chosen to be a constant. The strong forward peak then tends to dominate the moments -- with the consequence that data outside the forward peak is not well represented. Barrelet, however, also stressed the possibility of a norm that assigns comparable importance to all angular intervals. For the data in question we have chosen

$$n_p(\cos\theta) = \frac{.02}{e^{-10\,(1-\cos\theta)} + .02}$$

corresponding to the fact (see Fig. 1b) that the differential cross section outside the forward peak oscillates about an average magnitude $\sim 1/50$ that in the forward direction. We have verified that the precise choice of norm is unimportant provided all angular intervals give comparable contributions

*Apart from the so-called "discrete ambiguity", which cannot distinguish a zero at w=a from a zero at $w=(\overline{a^{-1}})$. We are not concerned in this paper with resolution of the discrete ambiguity, although such resolution is of course essential to determination of the amplitude. We deal elsewhere (8) with the discrete ambiguity.

to the moment integrals.

Our analysis of the entire set of data in Fig. 1 leads to the ten zeros in Fig. 2,* where we indicate the ring of convergence of a polynomial expansion if the nearest singularity in $\cos\theta$ corresponds to rho-meson exchange. One sees that all ten lie inside this ring, although one is close to the boundary and has a position that is only roughly determined. The corresponding polynomial representation of the data, shown in Fig. 1, has a chi-squared per data point of 1.2 for $\frac{d\sigma}{d\Omega}$ and 1.4 for P There is no difficulty in fitting the sharp forward peak. Extrapolation to $\theta=0$ gives $\frac{d\sigma}{d\Omega} = 12.63$, compared to 12.5 from the combination of total cross sections and dispersion relations[7].

We are thus led to suggest that the "exponential" diffraction peak may be associated with true zeros of the scattering amplitude. By extending our analysis to higher energies we should be able to verify whether the two relatively-distant zeros in Fig. 2 are stable or whether they are phenomenological artifacts. In any case, we have established that Barrelet-moment data analysis may be applied to the entire physical region without degrading the information contained in partial angular intervals.

Aknowledgement:

We are very grateful to E.Barrelet and G.F.Chew for helpful and enlightening discussions,and to H.P.Jakob whose criticism stimulated this work.We thank R.Ely and G.Gidal for their kind encouragement.

---

*Our resolution of the discrete ambiguity may be regarded as arbitrary, although the choice shown is in fact suggested by unitarity - causality considerations that will be described elsewhere(8).

REFERENCES.

1. E.Barrelet,Thesis(Paris), 1970(unpublished) ; Nuovo Cimento 8A, 331 (1972).

2. E.Barrelet and M.G.Fouque,Program Moulin,Ecole Polytechnique (1969).

3. E.Barrelet (Septembre 1976),submitted for publication.

4. D.M.Chew and M.Urban,LBL 4656 (June 1976);Proceedings of the Topical Conference on Baryon Resonances,Oxford 5-9 July 1976,p.156.

5. H.P.Jakob,LBL 6151 (February 1977).

6. D.M.Chew, LBL 6102 (March 1977).

7. G.Hohler,H.P.Jakob and F.Kaiser,Tables of pion-Nucleon Forward amplitudes, TKP 7617,University of Karlsruhe, May 1976.

8. D.M.Chew, E.Barrelet and M.Urban,LBL 4644(revised March 1977);see Ref.(4) for preliminary results.

9. J.F.Martin et al.Nucl.Phys. B89 ,253 (1975).

10. (a)K.Abe et al. Phys.Rev. D10 ,3556 (1974).
    (b)A.S.Carroll et al.Phys.Rev.177, 2047(1968).
    (c)R.J.Ott et al.Phys.Letters 42B,133(1972).

Figures

1.(a) Polarization data from Ref.(9) .

(b) Differential cross section with data selected from Ref(10),according to Ref.(4). The arrow indicates the value of $(d\sigma/d\Omega)_{\theta=0}$ from Ref. (7).

2. Location of the 10 Barrelet zeros in the w complex plane.The dashed boundaries correspond to the rho pole as the nearest singularity.The choice of the resolution of the discrete ambiguity corresponds to the results of Ref.(8), (-△-) for (a), (-□-) for (b), and (-◇-) for (c).

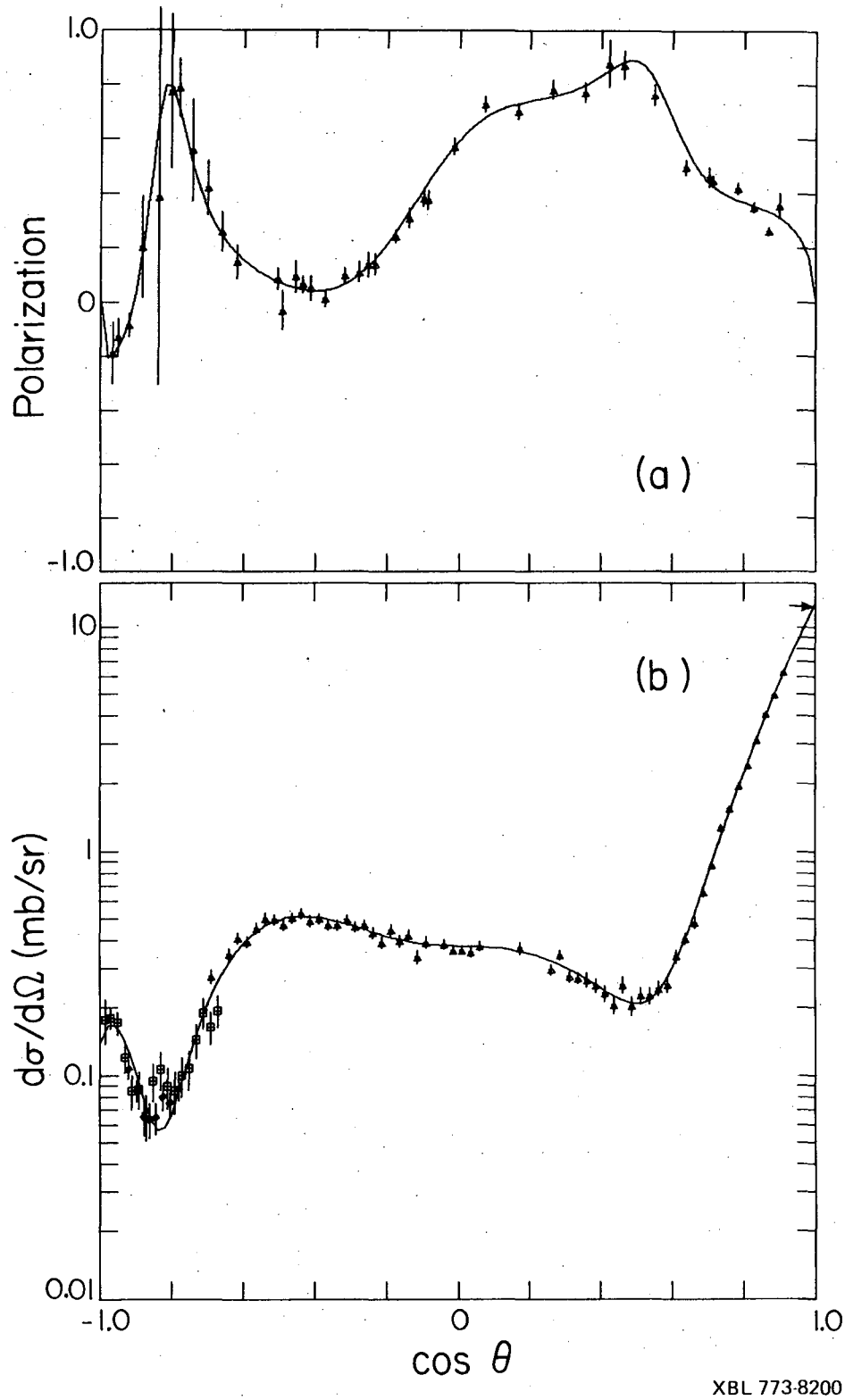
1.0
0
-1.0
Polarization
(a)
10
1
0.1
0.01
dσ/dΩ (mb/sr)
(b)
-1.0
0
1.0
cos θ
XBL 773-8200

Figure 1.

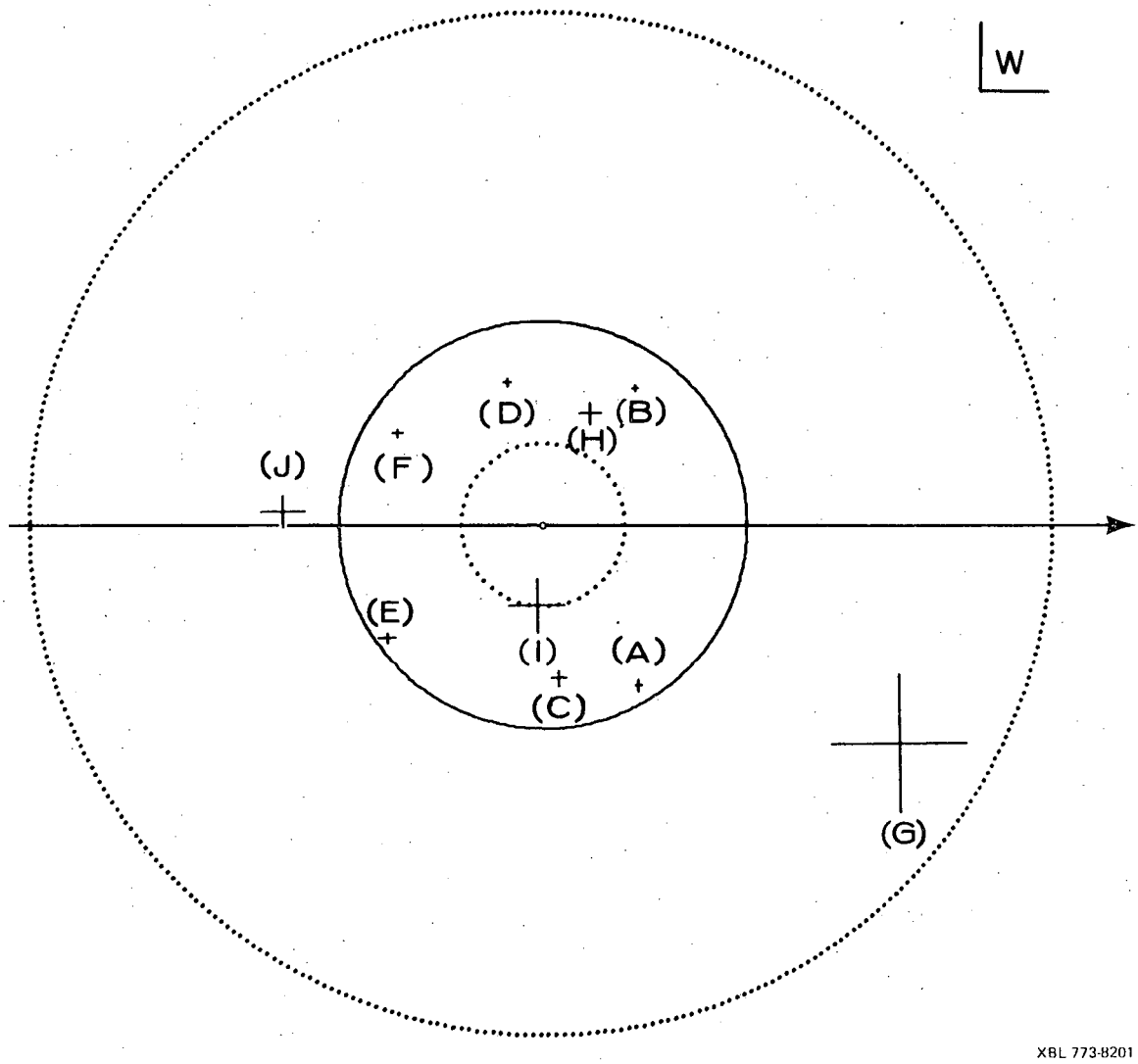

Figure 2.

This report was done with support from the United States Energy Research and Development Administration. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the United States Energy Research and Development Administration.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720