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GALILEAN PRESYMMETRY AND THE TWO FLUID MODEL OF SUPERFLUIDITY*

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ABSTRACT

A theorem of Jauch is used to establish the local gauge properties of the statistical mechanics of persistent flow. Linear response theory yields the two-fluid model.

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† Participating Guest

1. INTRODUCTION

In the Landau theory of superfluidity the Galilean principle has played a fundamental role in the construction of the phenomenological two-fluid model. The concept of a two fluid model has been central to our understanding of superfluidity in Bose and Fermi systems, and has been strikingly effective in predicting the phenomenology well in advance of appropriate microscopic theories. It therefore becomes essential to understand the two-fluid model from as fundamental a viewpoint as possible.

The Landau theory requires two densities ρ_s, ρ_n and two fluid velocities $\underset{\sim}{v}_s$, $\underset{\sim}{v}_n$. The conservation laws impose the following constraints on the density and current density: $\rho = \rho_s + \rho_n$ and $\rho_s \underset{\sim}{v}_s + \rho_n \underset{\sim}{v}_n = \underset{\sim}{j}$, where $\underset{\sim}{j}$ is the total current density. An additional condition required to characterize superfluid motion is that $\underset{\sim}{\nabla} \times \underset{\sim}{v}_s = 0$. The Landau hydrodynamics for non-dissipative flow may be obtained from the above characterization by imposing the conservation laws and the principle of Galilean relativity.(1)

It is the purpose of this paper to obtain the two-fluid model as a consequence of the representation theory of the Galilei Group and the statistical mechanics of persistent flow. In this way we place the Landau theory on a firm theoretical foundation.

2. GALILEAN PRESYMMETRY AND THE EQUILIBRIUM STATE

Let us consider a macroscopic system of N identical particles (the generalization to systems of indefinite particle number will be evident). For our immediate purpose we shall take the

particles to be spinless neutral Bosons. We wish to establish the most general form of the partition function under two conditions: equilibrium, and the unitary implementation of the Galilei Group on the states of the system.

In our previous work, we have explored the consequences of these two conditions for the statistical mechanics of equilibrium flow. Here we shall summarize the previous results.(2)

Consider the following transformation on the coordinate and velocity of a particle: $\underset{\sim}{r}_\alpha \to \underset{\sim}{r}_\alpha$, $\dot{\underset{\sim}{r}}_\alpha \to \dot{\underset{\sim}{r}}_\alpha + \underset{\sim}{\eta}$. The condition that this transformation be unitarily implemented on the Hilbert space of states $\mathcal{H}_N$ shall be called Galilean Presymmetry. A theorem of Jauch establishes that Galilean Presymmetry fixes the form of the hamiltonian such that it exhibits the property of local gauge symmetry.(3)

We have demonstrated that the conditions of equilibrium and Galilean Presymmetry yield the following form for the partition function of a system of identical particles of mass m:

$$Z = \mathrm{tr}_{\mathcal{H}_N} \exp(- \beta H_N[\underset{\sim}{\mathcal{G}}])$$

where

$$H_N[\underset{\sim}{\mathcal{G}}] = \sum_{\alpha=1}^{N} \frac{m}{2} \left[\frac{\underset{\sim}{p}_\alpha}{m} - \underset{\sim}{\mathcal{G}}(\underset{\sim}{r}_\alpha) \right]^2 + U\left((\underset{\sim}{r}_\alpha)_{\alpha=1}^{N} \right).$$

Here we have taken $\underset{\sim}{\mathcal{G}}$ to be time independent. The condition $\underset{\sim}{\nabla} \times \underset{\sim}{\mathcal{G}} = 0$ is required in order that there exist states of fixed particle flux, when particle interactions are translationally invariant.

We see that a consequence of Galilean Presymmetry is the freedom of introducing a vector field which plays the role of a gauge field.(4) It is just this freedom which is the basis of the two-fluid model. The physical identity of the gauge field may be determined under the conditions in which linear response theory is valid by computing the density that is transported by the gauge field. We shall see that unless the local gauge symmetry of the hamiltonian is broken in the statistical state of the macroscopic system, the gauge field is empty in the sense that the associated density vanishes. Moreover, for the case of neutral Bosons, when the local gauge symmetry is broken, the density transported by the gauge field is just the superfluid density ρ_s (as determined within linear response theory). This method is similar to that developed by Baym except that by introducing the gauge field we see that the existence of the superfluid density is a consequence of breaking local, rather than global gauge symmetry. The latter condition has the further consequence that it leads to Bose condensation. Thus the breaking of local gauge symmetry is a weaker condition, and serves to separate the questions of superfluidity and Bose condensation.

3. GALILEAN PRESYMMETRY AND THE STATISTICAL MECHANICS OF PERSISTENT FLOW

The characterization of a state of a fluid in which the equilibrium current density may be specified at each point in space requires a more general ensemble than that considered in the previous section. The construction is standard, however. One extends the ensemble of the absolute equilibrium state in such a

way as to fix $\langle \underset{\sim}{j}(\underset{\sim}{x})\rangle$ as a constraint. We then acquire an additional field as a lagrangian parameter associated with the constraint. This yields the following ensemble:

$$D = \frac{\exp[-\beta H_{eff}[\underset{\sim}{\mathcal{G}}, \underset{\sim}{u}]]}{\mathrm{tr}\,\exp[-\beta H_{eff}[\underset{\sim}{\mathcal{G}}, \underset{\sim}{u}]]}$$

where D is the statistical operator,

$$H_{eff}[\underset{\sim}{\mathcal{G}}, \underset{\sim}{u}] = H[\underset{\sim}{\mathcal{G}}] - \int d^3r\, \underset{\sim}{j}(\underset{\sim}{r}) \cdot \underset{\sim}{u}(\underset{\sim}{r}) - \mu N.$$

and $\underset{\sim}{j}(\underset{\sim}{r})$ is the local current density. $\underset{\sim}{\mathcal{G}}$ is the gauge field and $\underset{\sim}{u}$ is the vector lagrangian parameter-field. We see that even when the gauge field is subject to the condition $\underset{\sim}{\nabla} \times \underset{\sim}{\mathcal{G}} = 0$, local gauge tranformations fail to remove $\underset{\sim}{\mathcal{G}}$. Thus this ensemble describes a statistical state of quasi-equilibrium in which the local gauge symmetry is broken (though global gauge symmetry is maintained).

Let us define $\underset{\sim}{w} \equiv \underset{\sim}{u} - \underset{\sim}{\mathcal{G}}$ and determine the linear response of the current to $\underset{\sim}{w}$ in the quasi-equilibrium state. We are led to examine the current-current correlation function[5]*

$$\chi''_{ij}(\underset{\sim}{r} - \underset{\sim}{r}'; t - t') \equiv \frac{1}{i} \left\langle [\underset{\sim}{j}_i(\underset{\sim}{r}, t), j_j(\underset{\sim}{r}', t')] \right\rangle$$

and

$$\delta\left\langle j_i(\underset{\sim}{r}, t)\right\rangle = \int_{-\infty}^{t} dt' \int d^3\underset{\sim}{r}'\, \chi''_{ij}(\underset{\sim}{r} - \underset{\sim}{r}'; t - t') w_j(r', t')$$

* We have taken $\hbar \equiv 1$.

In terms of the causal response function

$$\chi_{ij}(\underset{\sim}{k}, \omega) = \int \frac{d\omega'}{\pi} \frac{\chi''_{ij}(\underset{\sim}{k}, \omega')}{\omega' - \omega + i\epsilon}$$

the fourier transforms are related by

$$\delta\langle j_i(\underset{\sim}{k}, \omega)\rangle = \chi_{ij}(\underset{\sim}{k}, \omega)\, w_j(\underset{\sim}{k}, \omega)$$

The ensemble average of $\underset{\sim}{j}_i$ with $\underset{\sim}{w} = 0$ is

$$\langle j_i(\underset{\sim}{k}, \omega)\rangle_{\underset{\sim}{w}=0} = \langle\rho\rangle\, \underset{\sim}{\mathcal{G}}_i(\underset{\sim}{k}, \omega)$$

Hence

$$\langle j_i(\underset{\sim}{k}, \omega)\rangle = \langle\rho\rangle\, \underset{\sim}{\mathcal{G}}_i(\underset{\sim}{k}, \omega) + \chi_{ij}(\underset{\sim}{k}, \omega)\, w_j(\underset{\sim}{k}, \omega)$$

Since the fluid is isotropic, $\chi_{ij}(\underset{\sim}{k}, \omega)$ takes the form

$$\chi_{ij}(\underset{\sim}{k}, \omega) = \chi^L(|\underset{\sim}{k}|, \omega)\, \hat{k}_i \hat{k}_j + \chi^T(|\underset{\sim}{k}|, \omega)(\delta_{ij} - \hat{k}_i \hat{k}_j); \quad \hat{\underset{\sim}{k}} = \frac{\underset{\sim}{k}}{|\underset{\sim}{k}|}$$

We shall consider the limit $\omega \to 0$ in what follows. The static susceptibility is given by

$$\chi_{ij}(\underset{\sim}{k}) = \int \frac{d\omega}{\pi} \frac{\chi''_{ij}(\underset{\sim}{k}, \omega)}{\omega}$$

We shall also make use of the longitudinal sum rule for $\chi^L(k)$:

$$\chi^L(|\underset{\sim}{k}|) = \langle\rho\rangle$$

Therefore we may write

$$\chi_{ij}(\underset{\sim}{k}) = \left[\langle\rho\rangle - \chi^T(|\underset{\sim}{k}|)\right] \hat{k}_i\hat{k}_j + \chi^T(|\underset{\sim}{k}|)\delta_{ij}$$

We have the additional sum rule

$$\lim_{|\underset{\sim}{k}|\to 0} \chi^T(|\underset{\sim}{k}|) \leq \langle\rho\rangle$$

Defining $\rho_n \equiv \lim_{k\to 0} \chi^T(k)$ and $\rho_s \equiv \langle\rho\rangle - \rho_n$ we have

$$\lim_{\underset{\sim}{k}\to 0} \chi_{ij}(\underset{\sim}{k}) = \rho_s\hat{k}_i\hat{k}_j + \rho_n\delta_{ij}$$

The limit $\underset{\sim}{k} \to 0$ must be taken in the manner appropriate to an open system,*

$$\lim_{k_j,k_k\to 0} \lim_{k_i\to 0} \frac{k_i k_j}{k^2} = 0$$

Thus we find that

$$\lim_{\underset{\sim}{k}\to 0} \chi_{ij}(\underset{\sim}{k}) = \rho_n\delta_{ij}$$

and therefore

$$\lim_{\underset{\sim}{k}\to 0} \langle \underset{\sim}{j}(\underset{\sim}{k})\rangle = \rho_s \underset{\sim}{\mathcal{V}} + \rho_n \underset{\sim}{u}$$

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* See reference (5)

We may identify the gauge field as the velocity of the superfluid component and the lagrange parameter-field as the normal fluid velocity. The above results are summarized as

$$\underset{\sim}{\mathcal{V}} = \underset{\sim}{v}_s, \quad \underset{\sim}{u} = \underset{\sim}{v}_n$$

$$\langle \underset{\sim}{j}\rangle = \rho_s \underset{\sim}{v}_s + \rho_n \underset{\sim}{v}_n \quad \text{and} \quad \underset{\sim}{\nabla} \times \underset{\sim}{v}_s = 0$$

These are just the original equations of the two-fluid model.

4. CONCLUSION

In this paper we have shown how the basic equations of the two fluid model can be derived in a straight forward way through the complete exploitation of the properties of the Galilei Group within the context of linear response theory. We find that a local gauge field emerges naturally, and that the necessary condition for a system to be superfluid is that the local gauge symmetry is broken in the statistical state. The identification of the superfluid velocity with this gauge field should be contrasted with other theories in which it appears as the gradient of the phase of a macroscopic wavefunction $\langle\psi(\underset{\sim}{x})$, or the gradient of a scalar operator*, both of which imply the existence of a Bose condensate. Whether such a condensate does exist is a precludes the existence of a condensate. The sole criterion for superfluid behavior is that $\rho_s \neq 0$.

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* See Ref. (5), Forster P. 232. Here $\psi(\underset{\sim}{x})$ is the complex scalar Bose field operator.

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   See also the discussion of D. Forster, Hydrodynamic Fluctuations, Broken Symmetry and Correlation Functions, (Benjamin, Reading, 1975).

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