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IMPORTANCE OF MOMENTUM DEPENDENT TERMS IN RADIATIVE PION CAPTURE†

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The contribution of the momentum dependent terms in radiative pion capture from bound atomic orbits is investigated. A shell model calculation for ^{6}Li shows that the momentum dependent terms represent ~ 2% of the 1s capture rate and up to 40% of 2p capture rates.

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Radiative pion capture has been found [1] to provide an interesting new probe for obtaining nuclear structure information. However, before the detailed comparison between theory and experiment can be relied upon, an important point needs to be clarified. This concerns the role of the terms in the effective interaction [2] responsible for radiative pion capture which depend on the pion momentum. Estimates by Delorme and Ericson [3] based on a nuclear Fermi gas model indicated that such terms make only very small contribution to the capture rates from 1s, 2p and 3d pionic Bohr orbits. Thus nearly all subsequent authors have neglected such terms. Skupsky [4], however, has suggested that for 2p capture in ^{12}C the momentum dependent terms make a sizable contribution. Unfortunately from his published results it is not clear exactly how big such constributions are. Since it is now known [6] that pion absorption even in light nuclei occurs predominantly from the 2p atomic orbit, the settlement of this question is of great importance for nuclear structure investigations.

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Furthermore, it is important for understanding the relationship of radiative pion capture to other weak and electromagnetic interactions such as β-decay, γ-decay, electron scattering and muon capture. With the above motivation in mind we have performed a calculation for ^{6}Li. The ^{6}Li (g.s.) to ^{6}He (g.s.) transition was chosen since the recent measurements of the Berkeley group [5] have isolated this transition thereby permitting an unambiguous comparison with experiment. Our results support the conclusion that the momentum dependent terms are important.

The effective Hamiltonian responsible for pion capture was taken to be the nonrelativistic reduction of the pion photoproduction amplitude known as CGLN (Chew-Goldberger-Low-Nambu) interaction [2] which in the impulse approximation can be written as follows [3]:

$$H_{eff} = \sum_{j=1}^{A} e^{-i\vec{k}\cdot\vec{r}_j} H(j)\, \Phi^{n_o}_{\ell,m}(\vec{r}) \tag{2a}$$

$\Phi^{n_o}_{\ell,m}(\vec{r})$ is the wave function for a pion bound in an atomic orbit and

$$H(j) = 2\pi i\, t^+(j)[A\, \sigma_j\cdot\hat{\varepsilon}_\lambda + B(\sigma_j\cdot\hat{\varepsilon}_\lambda)(\vec{q}\cdot\vec{k}) + C(\sigma_j\cdot\vec{k})(\vec{q}\cdot\hat{\varepsilon}_\lambda) + iD\, \vec{q}\cdot(\vec{k}\times\hat{\varepsilon}_\lambda)]\delta(\vec{r}-\vec{r}_j) \tag{2b}$$

where σ_j and $\vec{r}_j$ are the Pauli spin and coordinate of the jth nucleon and $t^+(j)$ is the ordinary isospin raising operator, which transforms a proton into a neutron. The photon polarization and propagation vectors are denoted by $\hat{\varepsilon}_\lambda$ and $\vec{k}$ and the pion momentum and coordinate are denoted by $\vec{q}$ and $\vec{r}$. The effective coupling constants A, B, C, D, which are functions of the four-momentum transfer, are related to the various multipole amplitudes in photoproduction at threshold [2,3]. Values of these constants have been given by Delorme and Ericson [3], Skupsky [2], Kawaguchi, et al. [7], and others, with no two sets being alike. Since

these constants have not been uniquely determined, we performed our calculations employing the coefficients of Delorme and Ericson [3] (I) and of Kawaguchi, et al. [7] (II):

$$\text{I.}\quad A = (-.034 \pm .003)\, M_\pi^{-2};\ B = .019\, M_\pi^{-4};\ C = -.017\, M_\pi^{-4};$$
$$D = -.010\, M_\pi^{-4} \quad (\text{in units of } \hbar = c = 1)$$
$$\text{II.}\quad A = -.032\, M_\pi^{-2};\ B = .0075\, M_\pi^{-4};\ C = -.037\, M_\pi^{-4};\ D = -.014\, M_\pi^{-4}$$

The pion wave function is written $\Phi^{n_o}_{\ell,m}(\vec{r}) = R^{n_o}_{\ell}(r)\, Y^{\ell}_{m}(\hat{r})$, where $Y^{\ell}_{m}(\hat{r})$ is the spherical harmonic of rank ℓ. In the present calculations pure hydrogenic wave functions are used. It is well known that the radial part $R^{n_o}_{\ell}(r)$ is distorted due to strong interactions and the finite nuclear size. However, as discussed later, this distortion can be taken into account by a multiplicative constant applied to the computed capture rates. But even the Krell-Ericson wave function [8] obtained by solving the Klein-Gordon equation in an optical potential can be expanded in the basis of hydrogenic wave functions and incorporated into our formalism, if necessary.

The nuclear wave functions were obtained by diagonalizing the nuclear Hamiltonian within the p-shell. The recent realistic Kuo-Lee [9] energy dependent matrix elements were used, while the single particle energies were taken from experiment ($\varepsilon_{p_{3/2}} = 0$, $\varepsilon_{p_{1/2}} = 6.24$ MeV). The wave functions thus obtained are:

$$^6\text{He(g.s.)}:\quad |J^\pi = 0^+,\ T = 1\rangle = .97841\, |p^2_{3/2}\rangle + .20670\, |p^2_{1/2}\rangle$$
$$^6\text{Li(g.s.)}:\quad |J^\pi = 1^+,\ T = 0\rangle = .79955\, |p^2_{3/2}\rangle - .02580\, |p^2_{1/2}$$
$$- .60041\, |p_{3/2}\, p_{1/2}\rangle$$

It is instructive to write these wave functions in the L-S coupling scheme. One gets:

$$|J^{\pi} = 0^{+}, T = 1\rangle = .91821\ |L = 0, S = 0\rangle - .39612\ |L = 1, S = 1\rangle$$

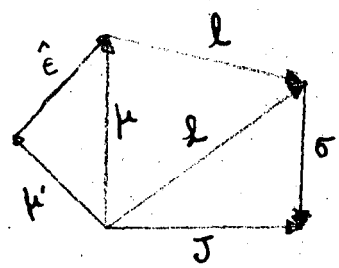
$$|J^{\pi} = 1^{+}, T = 0\rangle = .93347\ |L = 0, S = 1\rangle + .01840\ |L = 2, S = 1\rangle$$

$$+ .30093\ |L = 1, S = 0\rangle$$

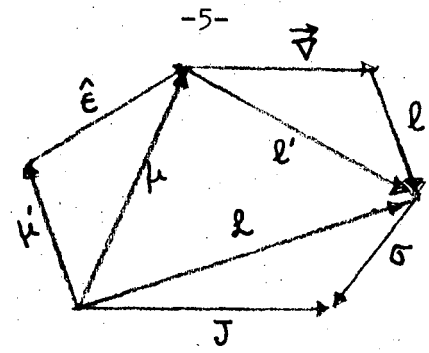
This model space is identical with the one employed by Cohen and Kurath [10]. It is interesting to note that the Kuo-Lee interaction [9] mixes the spatially symmetric and antisymmetric states appreciably.

Perhaps the above model space is rather restricted in the sense that it ignores core excitations. The latter might be important in determining precisely how the capture strength is distributed among the different states. But for the purposes of the present investigation, which is to study the effects of the momentum dependent terms on the pion capture rate, the p-shell model space should be adequate; e.g., for the β-decay (^{6}He → ^{6}Li), the M1 transition $|0^+\ T = 1\rangle \rightarrow |1^+\ T = 0\rangle$, and the g.s. magnetic moment of ^{6}Li one obtains $f_o t = 1030$ sec, $B(M1) = 16.9\ (\frac{e\hbar}{2mc})^2$, $\mu\ (1^+) = .788\ \frac{e\hbar}{2mc}$. These compare very well with the experimental values [11] of 802 sec, $17.2\ (\frac{e\hbar}{2mc})^2$, and $.822\ \frac{e\hbar}{2mc}$, respectively.

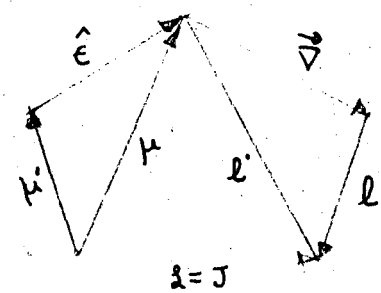
The effective Hamiltonian given by Eqs. (2a) and (2b) is decomposed into irreducible spherical tensors in the usual way. First the retardation factor $e^{-i\vec{k}\cdot\vec{r}}$ is expanded into spherical harmonics (rank denoted by μ'). This spherical harmonic is coupled with the polarization vector to yield a vector spherical harmonic of rank μ. The momentum operator (i.e., $-i\vec{\nabla}$) operates on the pion wave function and yields a spherical harmonic of rank ℓ'. Then the following coupling scheme was used:



Term A



Terms B,C



Term D

Once the operators are written in the above form the matrix element between the nuclear states can be calculated applying the Racah techniques. The transition probability (capture rate) in units of $\hbar = c = 1$ is given by

$$\Lambda_\gamma(n\ell;\ i \to f) = \frac{2k^2}{\pi} \frac{1}{(2J_i + 1)(2\ell + 1)} S_{i\to f} \tag{4a}$$

$$S_{i\to f} = \sum_{m's} \int \frac{d\hat{k}}{4\pi} |\langle J_f M_f | H_{eff} | J_i M_i \rangle|^2 \tag{4b}$$

In the above summation care must be taken so that the polarization and propagation vector for the photon are perpendicular. So the formulas are first derived in a frame in which the Z-axis coincides with the direction of the photon momentum (then $\lambda = \pm 1$). The results are generalized in an arbitrary frame by a mere rotation of the coordinate system.

The photon energy is $E_\gamma = 135$ MeV and its momentum in natural units is $k = .9677$. Since this momentum is fairly high, the multipole expansion of the retardation term converges rather slowly. So all μ-multipoles were retained. Also the long wavelength approximation is no longer applicable and the spherical Bessel functions must be used.

The importance of the momentum dependent terms comes partly from the nuclear structure and the geometry. In addition it depends crucially on the radial integrals involved. The radial integrals are of two types:

(i) $\langle NL | j_\mu(kr)\, R_\ell^{n_o}(r) | N'L' \rangle$ (in term A)

(ii) $\langle NL | j_\mu(kr)\, d_{\ell',\ell}^{n_o}(r) | N'L' \rangle$ (in terms B, C, D)

where $|NL\rangle$, $|N'L'\rangle$ are the harmonic oscillator single particle states, $R_\ell^{n_o}(r)$ is the radial part of the pion wave function and $d_{\ell',\ell}^{n_o}$ is given by

$$d_{\ell',\ell}^{n_o}(r) = \begin{cases} \left(\frac{d}{dr} - \frac{\ell}{r}\right) R_\ell^{n_o}(r) & \ell' = \ell + 1 \\ \left(\frac{d}{dr} + \frac{\ell + 1}{r}\right) R_\ell^{n_o}(r) & \ell' = \ell - 1 \end{cases}$$

$$n_o + \ell = n, \quad \ell = 0,1,2,\ldots,n-1$$

n is the principal quantum number specifying the energy of the Bohr orbit. One easily obtains

$$d_{\ell',\ell}^{n_o}(r) = \frac{2}{n_o + \ell}\left(\frac{Z}{a}\right)\Big[\,[n_o + \ell - 1 + \alpha(\ell')]\, R_\ell^{n_o}(u)/u - 1/2\, R_\ell^{n_o}(u)$$

$$- \left(1 - \frac{1}{n_o + \ell}\right)^2 \sqrt{(n_o + 2\ell)(n_o - 1)} \cdot R_\ell^{n_o - 1}(u)/u\Big]$$

where Z is the atomic number, $a = \frac{\hbar^2}{me^2}$, $u = \frac{2Z}{n_o + \ell}\left(\frac{r}{a}\right)$ and

$$\alpha(\ell') = \begin{cases} -\ell\ , & \ell' = \ell + 1 \\ \ell + 1\ , & \ell' = \ell - 1 \end{cases}$$

The important terms are those which contain the radial integral divided by u. We note that for spherical orbits ($n_o = 1$) the last term vanishes. The first term is also absent when $n_o = 1$ and $\ell' = \ell + 1$ (which is always the case for 1s-pions). This is the reason why for 1s absorption the momentum dependent terms are negligible.

To see why the momentum dependent terms are important for 2p absorption, two points must be considered: First the contribution to every term of the CGLN Hamiltonian of each individual operator characterized by the set of quantum numbers $\{\mu',\mu,\ell,\ell',\mathcal{L},\sigma,J\}$; second, the interference between such contributions. E.g., for 2p absorption the most important contribution to the $|A|^2$ comes from $\mu'=1$, $\mu=1$, $\ell=1$, $\mathcal{L}=0$, $J=1$, while the most important contribution to the $|C|^2$ arises from $\mu'=1$, $\mu=0$, $\ell'=0$, $\ell=1$, $\mathcal{L}=0$, $J=1$. (The $\mathcal{L}=0$ term dominates in both cases because the nuclear wave functions are characterized by $L=0$.) Hence, both terms are characterized by the same "angular" nuclear matrix element. The ratio of the radial integrals involved satisfies $|R_C|R_A|^2 = 5.49\ m_\pi^2$, while the geometrical factors, resulting after the m-sums in Eq. (4b) have been performed, yield $\Omega_C/\Omega_A = 2/9$. So the two terms, apart from the values of the coupling constants, contribute equally. Since the above contributions are the dominant ones in the $|A|^2$ and $|C|^2$ terms, we get $\frac{C^2}{A^2}\frac{\Omega_C}{\Omega_A}\left|\frac{R_C}{R_A}\right|^2 \approx 1$ (in agreement with the exact result of Table 1, II). In some cases, however, interference becomes important. The small value of the term A^*C of Table 1 is the result of destructive interference, while the large value of the A^*B term is the result of constructive interference. We have not been able to understand the physical reasons for these interference results.

Since the pion wave-function is more or less uniform inside the nucleus, the following approximation is usually made [3],

$$\langle NL|j_\mu(kr)R_\lambda^{n_o}(r)|NL\rangle^2 = \langle (R_\ell^{n_o}(r))^2\rangle\langle NL|j_\mu(kr)|NL\rangle^2 \quad . \qquad (5b)$$

With hydrogenic wave functions, this approximation is good to within 10%. The above approximation is the equivalent of the Konopinski-Uhlenbeck approximation used in β-decay and allows one to take care of the distortions of the pion wave-function in an average way [3,4]. This approximation is expected to hold for all n_o, i.e., even for the Krell-Ericson [8] wave functions. It might be questionable, however, when off diagonal nuclear radial integrals are encountered.

Our results are presented in Table 1, columns I and II corresponding to the two sets of coefficients I and II described above. If one defines the fraction f as the ratio of the momentum dependent to the momentum independent terms, i.e.,†

$$f(n\ell) = \frac{|B|^2+|C|^2+|D|^2+A^*B+B^*A+A^*C+C^*A+A^*D+D^*A+B^*C+C^*B+B^*D+D^*B+C^*D+D^*C}{|A|^2}$$

one gets

$$f_I(1s) = -.026 \qquad f_I(2p) = -.269$$
$$f_{II}(1s) = -.015 \qquad f_{II}(2p) = .666$$

Thus we see that the contribution of the momentum dependent terms for 1s absorption is small (in agreement with reference [3]) but important for 2p absorption. This depends crucially on the choice of A, B, C, D coefficients (there is even a sign change in $f(2p)$ between I and II).

In the radiative capture experiments performed to date, the capture rates from isolated pionic orbits are not measured. Rather, the measured quantity R_γ (in % of π^- atoms formed) represents the summed contribution of photons from all capture orbits. Thus R_γ for any final nuclear state or group of states is related to the computed transition rates by

$$R_\gamma = \sum_{n\ell} \frac{\Lambda_\gamma(n\ell)}{\Lambda_a(n\ell)} \omega(n\ell) \tag{7}$$

where $\Lambda_\gamma(n\ell)$ and $\Lambda_a(n\ell)$ are the radiative and total absorption rates, respectively, for the $n\ell$ orbit. The $\omega(n\ell)$ are the probabilities that the pion is absorbed into the nucleus from orbit $n\ell$. The capture schedules for light

†In our notation $|A|^2$ means the contribution to Eqs. (4a) and (4b) resulting from the first term of the CGLN Hamiltonian, etc.

nuclei [6], which are deduced from the pionic X-ray intensity data, show that only $\ell = 0$ and 1 orbits make appreciable contributions to the above sum. However, several values of n for each ℓ contribute. We make the assumption that $\Lambda_\gamma(n\ell)/\Lambda_a(n\ell)$ is independent of n. This reduces Eq. 7 to

$$R_\gamma = \frac{\Lambda_\gamma(1s)}{\Lambda_a(1s)}\,\omega_s + \frac{\Lambda_\gamma(2p)}{\Lambda_a(2p)}\,\omega_p \tag{8}$$

where $\omega_s = \sum_n \omega(ns)$ and $\omega_p = \sum_n \omega(np)$. The values obtained by Sapp et al. [6] for ^{6}Li are $\omega_s = 0.40 \pm 0.09$ and $\omega_p = 0.60 \pm 0.09$. The measured total level width are for the 1s state [12] 0.15 ± 0.05 keV (2.3×10^{17} sec^{-1}) and for the 2p state [6] 0.015 ± 0.004 eV (2.3×10^{13} sec^{-1}). Inserting these values into Eq. 8 together with the calculated values, we obtain

$$R_\gamma(\mathrm{I}) = .515\% \quad \text{and} \quad R_\gamma(\mathrm{II}) = .505\% \quad .$$

The Berkeley experiments [5], which will be reported in detail elsewhere, yield $R_\gamma = 0.39 \pm 0.10\%$. Deutsch et al. [13] reported a value $R_\gamma = 1.0 \pm 0.1\%$.

Although the effective Hamiltonian given by Eqs. (2a) and (2b) is most commonly used,for more precise calculations up to order m_π/m_N [14], eq. (4a) must be multiplied by $1 + m_\pi/m_N$. Furthermore, if realistic pion wave functions are used, this will effectively reduce the contribution of the 1s state and increase the contribution of the 2p state. If we take this into account by multiplicative factors of .65 (1s) and 1.10 (2p) and include the factor of $1 + m_\pi/m_N$, we obtain R_γ (I) = .407% and R_γ (II) = .419%. Both of these compare very well with the results of the Berkeley group [5] but are in disagreement with the results of Deutsch et al. [13].

One additional point must be mentioned. Light nuclei are characterized by a diffuse surface, consequently the energy of the harmonic oscillator, which

provides the single particle wave functions, is not well defined. Energy calculations are consistent with $\hbar\omega = 15$ MeV, while rms radius of 2.54 fm obtained by elastic electron scattering on ^{6}Li [15] is best fitted with a size parameter a = 1.94 fm for the p-shell harmonic oscillator, i.e., $\hbar\omega = 11$ MeV. Our results given above were obtained using the first of the above energies which correspond to a size parameter of 1.66 fm. Hence the whole calculation, including the nuclear eigenvectors, was repeated with $\hbar\omega = 11$ MeV. The results obtained are:

$$f_{1s}(\mathrm{I}) = -.032 \qquad f_{1s}(\mathrm{II}) = -.012$$
$$f_{2p}(\mathrm{I}) = -.237 \qquad f_{2p}(\mathrm{II}) = .281$$
$$R_\gamma(\mathrm{I}) = .307\% \qquad R_\gamma(\mathrm{II}) = .317\%$$

These values of R_γ, which include both corrections mentioned above, are in as good an agreement with the Berkeley experiment as the earlier ones. The quantity R_γ does not seem to be as sensitive to the variations considered here as the individual transition probabilities. But this may be a special feature of ^{6}Li.

In conclusion, we can say that the momentum dependent terms contribute significantly to the radiative pion capture rate from the 2p orbit, the dominant capture orbit for light nuclei. Also a precise determination of the values of the A, B, C, D coefficients is in order if detailed agreement with experiment is to be sought.

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Table 1. Calculated Radiative Capture Rates for the $\pi^- + {}^6Li \rightarrow \gamma + {}^6He$ (g.s.)

TERM [a]	I Delorme and Ericson		II Kawaguchi, et al.	
	$\Lambda_\gamma(1s)$ 10^{15} sec^{-1}	$\Lambda_\gamma(2p)$ 10^{10} sec^{-1}	$\Lambda_\gamma(1s)$ 10^{15} sec^{-1}	$\Lambda_\gamma(2p)$ 10^{10} sec^{-1}
A^2	2.788336	2.012230	2.469908	1.782433
B^2	.000523	.138319	.000081	.021550
C^2	0	.293704	0	1.391276
D^2	-----	-----	-----	-----
A^*B	-.037461	-.526845	-.013917	-.195723
A^*C	.000978	.040071	.002003	.082081
A^*D	-----	-----	-----	-----
B^*C	-.000013	.000071	-.000011	.000061
B^*D	-----	-----	-----	-----
C^*D	-----	-----	-----	-----
TOTAL	2.71587	1.47085	2.434139	2.968097

Columns I and II correspond to the sets of coefficients I and II as defined in the text.

[a] Since $A^*B = AB^*$, etc., terms such as AB^*, etc., are not listed. Not all digits appearing in this table are significant. Many figures were included to give an idea of the order of magnitude of each term.

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