

VERY HIGH ENERGY NUCLEAR COLLISIONS:
 The Asymptotic Hadron Spectrum, Anti-Nuclei,
 Hyper-Nuclei, and Quark Phase *

Norman K. Glendenning

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The Hadrons Who Will Never be Known by Name

This talk could have been given the above more whimsical title. It sounds at first like a sad story. But I think I can convince you that the multitude, perhaps infinity, of hadrons that are presumed to exist at high mass, played a vital role in determining the temperature and composition of the universe at the beginning of time, and that they may play a vital role in particle physics in deciding between different theories of hadronic structure. It is instead a possibly exciting story with a happy ending.

First let us speak of the hadrons that are known by name. There are 56 named hadrons representing about 1000 hadronic states with spin, isospin, baryonic charge and strangeness quantum numbers measured (Table 1)¹⁾. The lightest of these are the three pions at $m_\pi \sim 140$ MeV. They become quite densely spaced as their mass increases to about $10 m_\pi$. Thereafter the spectrum becomes sparse. Presumably however the cutoff is an experimental one. Figure 2 plots the number of hadronic states per pion mass interval. Already at $m = 10 m_\pi$ there are 34 non-strange states per pion mass interval and the average width at this mass is $\Gamma \sim 100$ MeV. Production rates are expected to decrease with m . The experimental problem becomes one of intensity and resolution.

Theories of hadronic structure, in contrast to the known spectrum, imply that it continues indefinitely. The bootstrap hypothesis predicts a spectrum that increases exponentially.²⁾ The hypothesis can be stated simply as follows: From among the known particles or resonances select two (or more) and combine their quantum numbers. The multiplet so obtained are also particles or resonances (at something like the sum of the masses). Add these to the pool of known particles and continue. The spectrum thereby generated by Hamer and Frautschi²⁾ is also shown in Fig. 2. The implication is astonishing.

The number of particles and resonances grows so fast that at only 2.5 GeV the number expected in a pion mass interval, on the basis of the bootstrap hypothesis, is $\sim 10^4$. The number of known particles is $\lesssim 10^2$ at that mass. If new particles were discovered at the rate of one a day it would require about a hundred years to verify the bootstrap prediction by a direct count, and that at only one mass!

The quark bag model, in its simplest form, predicts a slower but none the less rapid increase in the density of states, proportional to a power of the mass. More realistic bag models will yield a more rapidly increasing density as a function of mass.

Is It Interesting?

We have seen that it is out of the question to determine even the general form of the hadronic mass spectrum at even relatively low masses like 2 to 3 GeV, much less in the high mass region, by a direct count of individual particles and resonances. The sheer density of states is not only very large, but the widths are at least a pion mass, so of the order 10^4 or more states fall within the width of any one.

Is it important or even interesting to know the density of hadronic states in the region where they cannot be individually discovered and given a name? I think so. It is both interesting and important. Interesting because it is a fundamental property of matter on the smallest scale, and important for two reasons that I can think of. It is important in particle physics because the density of hadron states at high mass provides an asymptotic constraint on theories of hadronic structure. Let me elaborate. The properties of the low mass particles that can be individually identified provide important clues as to the group structure of the theory. Their quantum numbers (spin, isospin, strangeness) which are determined by the decay modes and so on, and their

masses suggest particular classifications which any theory of hadronic structure must account for. But the symmetries are not perfect. This fact leaves a lot of room for competing theories. Certainly the quark theory is favored by many particle physicists today, but there are a number of quark models. If quarks are the fundamental building blocks, there is still no agreement as to the nature of the glue that holds them together. And because the symmetries are broken, the light particle spectroscopy cannot provide a unique way of discriminating between the theories. Yet any theory of hadronic structure, when sufficiently developed, can be made to yield a prediction of the asymptotic region (i.e., high mass). It is in this sense that the asymptotic behavior of the hadronic spectrum may become decisive in particle theory.

The asymptotic region is important also in cosmology. The thermal history of the universe can be guessed with considerable confidence back to the time of helium synthesis at temperatures of about 1 MeV.* For much earlier times when the energy density was extremely high, the composition of the universe must have been very different in kind, not merely in density and temperature, from what we see today. That there were no nuclei is clear, but that there were no nucleons is likely. What there was was in fact determined by the spectrum of hadrons and leptons that could energetically exist at the energy densities prevalent. At even earlier times, at extreme particle and energy density, the hadrons may have been dissolved into a quark soup which only later condensed into hadrons.

I am sure that I have convinced you by now of two things. The general form of the hadronic spectrum is a most interesting thing to know, and it cannot be discovered by looking for the individual particles of which it is composed.

* 1 MeV \approx 10^{10} °K.

How Then Can it be Discovered?

Perhaps by creating as large a piece of matter as possible at high energy density and studying its properties. This is the only way I can think of. I am sure that you appreciate for example, that the specific heat of material objects depends upon their compositions. The composition of matter at high energy density depends in turn upon the number, type, and masses of hadrons that can energetically exist at that density--both those that are known, and those that are unknown, and never will be known by name!

The only means we have of producing matter at very high energy density is in collisions, by no means an ideal situation for performing calorimetric measurements. Yet it is our only hope.

So we have in mind collisions between large nuclei at high energy. Two questions come immediately to mind. 1) What is the dynamical description of the reaction? and 2) At what energy can different assumptions about the hadronic spectrum be expected to yield observable difference in the outcome of the collision?

For the dynamics I can envision two extremes. Either the collision of two nuclei at high energy 1) develops as a sequence of independent collisions, or 2) it attains thermal equilibrium and then decays.

If the first is true then for the purpose at hand, at least, there is no point in studying nuclear collisions rather than nucleon-nucleon collisions.

But I think it highly unlikely that the first is true. More likely the truth lies between the extremes.

A moment's reflection makes clear that a complete dynamical description of a collision between nuclei at very high energy involves something like the full complexity of a relativistic quantum field theory. Of course if all the ingredients of such a theory were at hand, the question raised by this

paper would be moot. Since however the ultimate theory of particle structure is unlikely to emerge in the near future, it seems reasonable to attempt a model description of the dynamics of a nuclear collision.

Before attempting to explore too elaborate a model it seems prudent to me to assess whether it is worth doing so. For example, it might turn out that the energy at which sensitivity to the hadronic spectrum is achieved is so high as to be out of sight; that by no stretch of the imagination would it ever be possible to produce the required energy in the laboratory.

Therefore, Y. Karant⁴⁾ and I have assumed thermal equilibrium as a model of high energy collisions for the purpose of accessing the prospects of learning from nuclear collisions the form of the hadronic spectrum and the possibility of distinguishing between various theories of hadronic structure. If the results of such a study give an optimistic prognosis, we will feel encouraged to try harder in our treatment of the dynamics.

The attainment of a state of thermal equilibrium in a nuclear collision may seem strange at first. But at the energies in question a very large phase space is opened up by particle production. The high velocity (near c) of the pions and their strong interaction with nucleons provides a fast mechanism for thermalization in addition of course to the hadron-hadron collisions. Indeed computer studies suggest that thermalization can occur already after 3 or 4 collisions.⁵⁾ Chemical equilibrium among the various species π , N , N^* , Δ ... takes longer but may still be fast compared to the disassembly time of the composite. The extended size of the initial nuclear composite for geometrical reasons alone, slows the disassembly of the interior.⁶⁾

There is a very extensive and beautiful literature on the thermodynamic theory of hadronic structure.²⁾ Also for nuclear collisions, at lower energy than we have in mind, a thermodynamic model has been introduced.^{7,8,9)} Inspired by the analogy to hadron thermodynamics, the hot composite system was

Table I. The families of light mass multiplets, their average masses in MeV, and their baryon and strangeness quantum numbers (B, S). Total multiplicity including the unlisted multiplets is indicated in the bottom row for each family.

Family (B, S)	Π (0,0)	K (0,1)	N (1,0)	Λ (1,-1)	Σ (1,-1)	Ξ (1,-2)	Ω (1,-3)
Multiplets	π (138)	K(495)	N(940)	1116	1193	1318	1672
	η (549)	K^* (892)	N^* (1430)	1405	1385	1533	
	ρ (773)	K^* (1421)	N^* (1520)	1519	1670		
	ω (783)		N^* (1515)	1670	1745		
	η' (958)		Δ (1232)	1690	1773		
	:		:	:	:		
Total Multiplicity	103	18	248	38	108	12	4 = 531

referred to as a nuclear fireball. The model has been refined¹⁰⁾ and applied¹¹⁾ to data on pion, proton, and composite particle spectra at various energies between 200 MeV per nucleon to 2 GeV per nucleon laboratory kinetic energy. The overall agreement with such a wide range of data is quite impressive.

Thermodynamics of Hadronic Matter

In this section we discuss the thermodynamics of the nuclear fireball in terms of an ideal relativistic gas. It may seem strange that a strongly interacting hadron system is described in such a way. However, Hagedorn²⁾ has argued convincingly, on the basis of statistical mechanical techniques introduced by Beth and Uhlenbeck¹²⁾ and Belenkiĭ¹³⁾ that the hadronic spectrum is the manifestation of the interactions; that by introducing the complete spectrum one has accounted for their interactions completely.

The partition function and momentum distribution for an ideal relativistic gas of Fermions or Bosons of mass m and statistical weight $g = (2J+1)(2I+1)$ occupying a volume V at temperature T are.²⁾ (Units are $\hbar = c = k_{\text{Boltzman}} = 1$).

$$Z(V, T) = \frac{gV}{2\pi^2} m^2 T \sum_1^{\infty} \frac{(\mp)^{n+1}}{n} k_2\left(\frac{nm}{T}\right), \quad (F_B) \quad (1)$$

$$f(p, T) d^3p = \frac{gV}{2\pi^2} \frac{p^2 dp}{\exp\left(\frac{1}{T} \sqrt{p^2 + m^2}\right) \pm 1}, \quad (F_B) \quad (2)$$

from which the various thermodynamic quantities can be calculated. We want to describe a gas of Baryons and Mesons distributed in mass according to some unknown functions $f_{\alpha}(m)$. (α labels the families of particles, ordinary and

strange baryons and mesons, some of which are shown in Table I). There are two important quantum numbers that have to be conserved, the net baryon number and strangeness. This is achieved as usual in thermodynamics, by introducing chemical potentials. If we specialize to symmetric collisions between $Z = N$ nuclei then the conservation of baryon number and strangeness implies conservation of electric charge, since on the average $\langle Q \rangle = \frac{B+S}{2}$. We therefore make this specialization.

The average number and energy for the family of particles labelled α are

$$N_{\alpha} = \frac{VT}{2\pi^2} \int_{m_{\alpha}}^{\infty} dm \rho_{\alpha}(m) m^2 \sum_1^{\infty} \frac{(\mp)^{n+1}}{n} K_2\left(\frac{nm}{T}\right) \exp\left(\frac{n\mu_{\alpha}}{T}\right) \quad (3)$$

$$E_{\alpha} = \frac{VT}{2\pi^2} \int_{m_{\alpha}}^{\infty} dm \rho_{\alpha}(m) m^3 \sum_1^{\infty} \frac{(\mp)^{n+1}}{n} \left[K_1\left(\frac{nm}{T}\right) + \frac{3T}{nm} K_2\left(\frac{nm}{T}\right) \right] \exp\left(\frac{n\mu_{\alpha}}{T}\right) \quad (4)$$

Here μ_{α} is the chemical potential, m_{α} is the threshold, i.e., lowest mass particle in the family α , and K is a Kelvin function.

The baryonic charge of the system is clearly

$$B = \sum_{\alpha} N_{\alpha} B_{\alpha} \quad (5)$$

the sum being extended over the seven families of particles indicated in Table I ($\alpha = \Pi, K, N, \dots$)* The number of antiparticles of type α and their energy are given by the above two equations (3) and (4) with $\mu \rightarrow -\mu$. We

*As a family label, Π does not designate only the pions, but all the ordinary mesons, π, ρ, η , etc.

indicate by a bar, the antiparticle quantity, e.g., \bar{N} . Then the net baryon charge is

$$A = B - \bar{B}. \quad (6)$$

This quantity is conserved, and equal to the initial number of nucleons in the collision. The net strangeness is also conserved and is zero.

$$0 = S - \bar{S} \quad (7)$$

where

$$S = \sum_{\alpha} N_{\alpha} S_{\alpha} \quad (8)$$

(The sign of the strangeness is opposite for particle and antiparticle.)

The two conditions (6) and (7) expressing baryon and strangeness conservation clearly couple all thermodynamic quantities T, μ_{α} . The reactions possible between the various particles dictate certain relations among the chemical potentials with the result that there are only two independent potentials, that for the nucleon and that for the kaon. The scheme we use to solve for the energy and particle populations as a function of temperature is basically the following. Choose a temperature T and find the values of the two chemical potentials that satisfy equations (6) and (7). When these are found then the populations and energies can be found from (3) and (4) and the total energy is of course

$$E = \sum_{\alpha} E_{\alpha} \quad (9)$$

The initial condition of the fireball is a little more complicated to solve. We consider symmetric collisions in the center of mass frame between

nuclei of atomic number $A/2$. Each nucleus is Lorentz contracted by the factor $1/\gamma = m/E$. If the volume per nucleon in the rest frame of each nucleus is $v_0 = \frac{4}{3}\pi (1.2)^3$, then in the C.M. frame it is v_0/γ . We assume that the collision is perfectly inelastic; that each nucleus is stopped by the other. Then the largest possible volume, in which all nucleons are contained, just after the nuclei have stopped each other is the contracted volume occupied originally by one.¹⁴⁾ So the initial baryon density of the fireball is

$$\rho_{\text{initial}} = \frac{2\gamma}{v_0} = \frac{2E}{mv_0} = \gamma^{-1} \quad (10)$$

and the volume per baryon is the reciprocal. Hence the volume V multiplying all quantities (3), (4), etc. is a function of the as yet to be determined energy. How this problem is solved can be found in Appendix B.

We shall want to look at expansions of the system from its initial contracted state under conditions of constant energy or constant entropy. Therefore to complete this section we mention that the entropy is defined by

$$\mathcal{S} = \frac{1}{T} \left[E + PV - \sum_{\alpha} \mu_{\alpha} (N_{\alpha} - \bar{N}_{\alpha}) \right] \quad (11)$$

where the chemical potentials are given in Appendix A. The pressure can be calculated from

$$P_{\alpha} V = \frac{VT^2}{2\pi^2} \int_{\bar{m}_{\alpha}}^{\infty} dm_{\rho_{\alpha}}(m) m^2 \sum \frac{(\pm)^{n+1}}{n^2} K_2\left(\frac{nm}{T}\right) \exp\left(\frac{n\mu_{\alpha}}{T}\right) \quad (12)$$

$$P = \sum_{\alpha} P_{\alpha}$$

Three Examples of Hadronic Spectra

The object of the rest of the paper is to show how and at what energies the thermodynamic nuclear fireball would differ under the three different assumptions for the hadronic spectrum discussed below. For brevity we shall sometimes refer to the results for different spectra as being different worlds. The ultimate object, toward which this paper is a modest start is to discover which is most like our world.

a) The Known Hadrons: As one extreme case we might suppose that all of the hadrons have already been discovered. They are listed with their properties in the Particle Data Tables¹⁾ and their density is plotted in Fig. 1 with the exception of recent discoveries. There are 56 different multiplets known with a total particle multiplicity of 531. Together with the antiparticles these comprise the 1000 or so known hadronic states mentioned earlier. We include them all by using the average mass and width for each multiplet. For our purpose, they fall into the seven families shown in Table I.

b) Bootstrap Spectrum: There are several mathematical formulations of the bootstrap hypothesis but the thermodynamic theory of Hagedorn²⁾ is most useful to us because it yields an asymptotic form for the bootstrap hadron spectrum. The bootstrap spectrum lies at the opposite extreme for the "known" spectrum since it rises exponentially and without bound. We shall test the consequences of a bootstrap theory by using the Hagedorn form of the spectrum for the non-strange mesons and baryons in the region $m > 12 m_\pi$. Below this mass we use the discrete known particles for these two families and all known strange particles. We normalize the Hagedorn spectrum to agree with the average density of states in five pion mass intervals around $10 m_\pi$. Thus

$$\rho_{\text{Bootstrap}}(m) = \begin{cases} \frac{1.12 e^{m/T_0}}{(m/T_0)^3} / \text{pion mass} & m > 12 m_\pi \\ \text{discrete non-strange particles} & m \leq 12 m_\pi \end{cases} \quad (13)$$

+ all strange particles

$$T_0 = 0.958 m_\pi \qquad m_\pi = 140 \text{ MeV}$$

We assume that there is an equal number of ordinary mesons and baryons in the continuous region. We might, but do not yet include continua for the families of strange particles because there is generally an insufficient number to estimate the normalization of the continua.

c) Rigid Quark Bag: As an intermediate case, and so as to bring out where the sensitivity is achieved under less extreme alternatives than the first two, we consider a naive rigid quark bag. A meson is considered to be composed of 2 quarks, and a baryon of three. The walls are considered rigid and no new quark pairs are created within a hadron. Frautschi¹⁵⁾ finds that the density of such objects rises as m^2 and m^5 respectively. Normalizing at $m = 10 m_\pi$ to the same value as the Hagedorn spectrum at that mass, we have for the continuous spectra for ordinary mesons (π) and baryons (N)

$$\rho_{\text{Bag}} = \begin{cases} \rho_\pi(m) = 0.154 (m/m_\pi)^2 / \text{pion mass} & m > 12 m_\pi \\ \rho_N(m) = 1.36 \times 10^{-4} (m/m_\pi)^5 / \text{pion mass} & m > 12 m_\pi \\ \text{discrete non-strange particles} & m \leq 12 m_\pi \end{cases} \quad (14)$$

+ all strange particles

The Temperature

The first crude indication of differences between hadronic matter constructed from the three assumed spectra is registered in the initial temperature they would be heated to for the same energy content. Since we assume a perfectly inelastic collision, the C.M. collision energy per nucleon including rest energy is the total fireball energy per nucleon. These temperatures are shown in Fig. 2. For matter composed of a hadronic spectrum limited to the known particles, the temperature is by far highest at energies greater than several GeV. Because energy goes into making additional particles in the quark bag spectrum that were not present in the known spectrum, the temperature is lower at any corresponding energy. For the exponentially rising spectrum, as first discovered and emphasized by Hagedorn, the temperature is limited to a maximum value corresponding to the constant T_0 in the spectrum eq. (13).

While T_0 appears to be nearly the pion mass, its value is not determined within the theory of Hagedorn. Instead it is deduced from a comparison with data. While the data often used are p_{\perp} measurements, we chose to fit the Frautschi³⁾ bootstrap iteration on the known particles.

The limiting temperature of matter, if composed of hadrons obeying the bootstrap condition (more precisely the exponential rise) is a truly remarkable property which has no analogies in other physical systems that I know of. (The boiling point of water is sometimes mentioned. This is a false analogue. The temperature of matter is limited even though the energy input is increased indefinitely! The limit to water temperature is reached because the energy is carried off by the steam. It is by comparison a trivial limit and totally different in origin.)

The mathematical nature of the limit can be seen by referring to eq. (4). For large masses, $m \gg T$, the Kelvin functions decay exponentially like

$$K(x) \propto \frac{1}{\sqrt{x}} e^{-x}$$

Inserting the Hagedorn spectrum we find

$$E \propto \int_M \frac{dm}{m^{1/2}} \exp - \left(\frac{T_0 - T}{T_0 T} \right) m \propto \left(\frac{T}{T_0 - T} \right)^{1/2}$$

Thus as long as $T < T_0$ the integral converges; the energy is finite. But for $T \geq T_0$ the integral diverges. It would require infinite energy to raise the temperature to T_0 or beyond!

Composition of the Initial Fireball

Neither the temperature nor composition of the initial fireball are observables because any conceivable experiment must look at the products of the collision after the fireball has disassembled. Nonetheless it is interesting to look at the calculated populations because they are the starting point of the subsequent expansion or decay of the fireball. They also give us a glimpse of what the composition of the universe might have looked like at the beginning of time for very high energy and particle density. Because of the time scales involved we do not have to consider photons and leptons in equilibrium with the hadrons, so this is an important difference from the cosmological problem.

A very immediate impression of how the three worlds differ is given by Fig. 3 which shows the degree to which the ordinary (non-strange) baryon number is depleted. Initially all of the baryonic charge resides in non-strange baryons (the original neutrons and protons). As the energy is increased, the

strange particles begin to be populated. The difference between unity and the plotted curves is this strange baryon population.* Since however there was initially no net strangeness, this is exactly counter balanced by kaon populations (the strange mesons). We see that in all cases, there is a sudden rise in the strange particle populations which however is quenched quickly in the bootstrap world but rises to almost 25% in the case of the "known" world. At about 5 GeV almost 25% of the baryonic charge is converted to strange particles and to corresponding kaons!

Because there are so many discrete particles, not to mention the continua, we make the following arbitrary groupings to display more detailed information. Each family of particles is broken up into light particles comprising the lightest five (when there are that many) and heavy particles comprising all the rest, including continuum particles in the case of the quark bag and bootstrap worlds. We sum the populations in each group and plot only the summed populations. Thus the ordinary (non-strange) mesons are represented by two curves, for light and heavy mesons. There are no heavy kaons, but there are anti-kaons so there are curves for both. The ordinary baryons are represented by four curves, light and heavy baryons and anti-baryons. And so on.

Figures 4-9 show truly remarkable differences of the three fireballs depending on which is the underlying hadronic spectrum. For both the known spectrum and the quark bag, the heavy baryon and anti-baryon populations eventually dominate with heavy mesons the next most populous group. In the case of the quark bag this happens at rather low energy (on a particle creation scale). The heavy mesons follow. The composition at one GeV is of course

*This is a loose statement since the strangeness quantum numbers are not limited to the value unity. The succeeding statement for kaons is exact.

all nucleon, but the light baryon and anti-baryons become less populated than the heavy ones in the bag model at energy above 10 GeV. The Hagedorn or bootstrap world is remarkably different. The light meson population rises to 10% and then falls. The heavy baryon population rises sharply and above 3 GeV the fireball is composed of more heavy baryons than light ones. By 10 GeV about 60% of the baryons are heavy and only 40% are light.

There is another remarkable difference. In the "known" and "bag" worlds, all particle-anti-particle populations approach each other at high energy (with anti-particles slightly less numerous). In the bootstrap world, the anti-particles have microscopic populations. It is a world dominated at high energy and density by heavy baryons. This is an inevitable consequence of the exponential rise in the bootstrap density. At high temperature the system wants to produce heavy particles. Since however baryon conservation is forced, the energy is committed to making heavy baryons to the exclusion of mesons.

Expansion of the Fireball

So far we have considered the fireball in its initial configuration just after being formed. This stage is of course hardly observable. It is the expansion of the fireball that carries many, maybe most of the particles to the counting apparatus. Of course some particles may be radiated from the fireball as it expands. We assume, as in cosmology, that the expansion occurs through a series of equilibrium states. At some point during the expansion, when the density falls below a critical value, thermal contact between the particles is broken. This is called the freezeout¹⁶⁾ (freezein might be a better word). Relative populations do not change thereafter except by decay of isolated particles. Thus we envision the disassembly of the fireball as

taking place in two stages.*

Presumably the freezeout density would not be less than one particle per pion wavelength

$$\rho_\lambda = \left(\frac{4}{3}\pi 1.4^3\right)^{-1} \approx 0.085 \text{ Fm}^{-3}$$

Presumably it would be less than the nuclear density

$$\rho_N = \left(\frac{4}{3}\pi 1.2^3\right)^{-1} \approx 0.17 \text{ Fm}^{-3}$$

i.e.,

$$\rho_N \gtrsim \rho_F \gtrsim \rho_\lambda$$

We shall plot our results for the thermal expansion stage as a function of $1/\rho$ where the density is measured in units of the nuclear density. On such a scale, the freezeout presumably occurs between 1 and 2. (ρ is the total hadron density.)

Of course the temperature falls monotonically during the expansion to the freezeout point. Therefore the temperature characterizing particles that remain in thermal contact until the freezeout is the lowest temperature the fireball possessed. In this connection it is worth remarking that the claim in the literature¹⁷⁾ that the ultimate temperature has been measured in hadron-hadron collisions is possibly unwarranted. Unfortunately no one possesses a thermometer that he can insert into the initially formed fireball, but instead

*An alternative which we will explore at some point was suggested by Swaitecki: that the disassembly takes place by way of radiation from the surface with the interior remaining at its original density until it becomes surface.

he must wait until the fireball expands and its constituents arrive at the counters. It is possible, even probably, that some particles will be radiated from the surface of the fireball during the thermal expansion stage. They do carry pre-freezeout information. However to interpret data one clearly needs a precise dynamical description with which to unfold the pre-freezeout and freezeout particles.

We can envision two extreme equilibrium expansions. One is isoergic which is an expansion with maximum increase in entropy, and the other is an isoentropic expansion. The isoentropic expansion suffers a loss in energy which we interpret as being carried out of the thermal region by particle radiation prior to freezeout.

The fall in temperature during an isoergic expansion at 20 GeV is shown in Fig. 10. For the "known" and rigid bag worlds it drops precipitously while for the bootstrap world it remains nearly constant until a late stage. At somewhat less than normal nuclear density, it begins to fall at the more rapid rate of the other worlds. This merely corresponds to the fact that at low enough temperature all worlds look the same. This figure suggests that temperature measurements near freezeout cannot distinguish between various hadronic spectra, as remarked earlier. The possible measurement of a temperature of $T = 119$ MeV in hadron-hadron collisions at 28 GeV reported in the literature¹⁷⁾ would correspond, on our figure, to a freezeout a little to the right of the frame where all worlds have fallen to virtually the same temperature.

The way in which the ordinary baryon charge is depleted during the expansion at 20 GeV is shown in Fig. 11. The bootstrap world is again remarkably different from the others, but although there was an initial large difference between the bag and "known" world (Fig. 3) it rapidly diminishes. At this point it is worth emphasizing that our quark bag model is a highly

simplistic one and almost certainly vastly underestimates the growth rate of the spectrum. We have included it in our discussion as an intermediate model and will replace it as soon as a more realistic asymptotic quark bag spectrum is derived.

The populations of the various groups during the expansion at 20 GeV are shown in Figs. 12-17. The composition of the bootstrap fireball is remarkably different from the others during the early stage. However if thermal contact were sustained for all time it is clear that all worlds must appear the same at low enough temperature and density. Indeed they would just return to the original neutron proton composition. The breaking of thermal contact interrupts this return however. As stated earlier, we expect freezeout to occur between $1 \lesssim \frac{1}{\rho} \lesssim 2$. It is in precisely this range however that all light particle populations have already become quite similar in the three worlds. The heavy particle populations the N, \bar{N} and the heavy strange baryons are however still quite different.

It has now become clear that if thermal contact between all constituents is sustained during an expansion to a freezeout density equal to the nuclear density or less, the problem of distinguishing the three worlds is severe, and rests on exotic products alone. On the other hand it seems most likely that some of the constituents of the fireball will escape prior to the freezeout.

Isentropic Expansion of the Fireball

As remarked earlier, in the isentropic expansion, energy is lost to that part of the fireball that remains in thermal contact. I interpret the loss to be balanced by radiation of particles from the surface during the course of the expansion. This radiation produces the pressure against which the

particles remaining in the fireball work during the expansion.

Figure 18 shows the energy remaining in the fireball for an isentropic expansion starting from an energy of 20 GeV. What this picture immediately suggests is that the pre-freezeout radiation is much more copious for the "known" and bag worlds than for the bootstrap world. The energy is trapped in massive baryons till a later stage in the expansion. Otherwise the qualitative difference between the worlds discussed in connection with the isoergic expansion appear in this expansion also.*

Pre-Freezeout Radiation

I have already remarked that it is likely that some particles will escape from the fireball during the course of the expansion. They carry information on the conditions of the fireball prior to freezeout, which information is characteristically lost in the equilibrium expansion to freezeout. Therefore, this component of the products of the collision is of vital importance. We notice that the early history in the expansion of the bootstrap fireball is very different from that of the other worlds. The early bootstrap world is composed almost entirely of heavy baryons. The other worlds have high populations of both ordinary and strange mesons (π and K families). Pre-freezeout radiation will be profoundly different. This serves to motivate a dynamical theory of the expansion stage.

*In connection with the bootstrap and bag worlds, keep in mind that we have not yet included a continuous distribution for the strange particles. That would raise the populations of all heavy strange particles.

Anti-Nuclei, Hyper-Nuclei, and Quark Phase

We come now to a most remarkable difference between nuclear collisions and hadron-hadron collisions. To the extent that thermodynamics applies to each, then all I have said until now applies to nuclear fireballs as to hadron fireballs.

A reexamination of the populations reveals that the anti-baryons and also strange baryons have significant populations. As an example, for the bootstrap world during the expansion phase at densities below nuclear density, (1 on the ordinate) the population of the light Σ family is ~ 0.2 per baryon (Fig. 12). That means that for a collision involving a hundred nucleons, 20 Σ 's appear at the freezeout! There are even more light anti-baryons present, about 27. Thus although we have not yet calculated composite particle populations, we can anticipate significant production of light anti-nuclei and hyper-nuclei and possibly even strange nuclei; i.e., nuclei composed entirely of strange baryons. This appears to be a fascinating possibility. I presume little is known of the binding properties of such objects, except of course anti-nuclei, which would have to be the same as ordinary nuclei.

Moreover, we note that pre-freezeout radiation of these as with single particles would be quite different in the worlds examined.

A quark phase can also be discussed. If the whole system remains in thermal contact until the density has fallen to a freezeout density below which interactions cease, then a quark phase would be hidden. (Unless of course quarks can exist as free particles in which case some of them may not find a partner to recombine with before freezeout.) The total energy, since it is still shared by the whole system, insures that when the quarks recondense into the hadron phase, the composition will evolve with density to the freezeout in

exactly the same way as if the quark phase had never existed. However, if some particles do escape from the equilibrated region before freezeout, which does seem plausible, then the numbers and types of escaping particles would depend upon whether, during part of the expansion stage, the matter was in a quark phase. During the quark phase there would be no radiation (assuming no asymptotically free quarks), or if there were a mixture of the two phases, say quark matter in the interior surrounded by a hadron halo, the radiation would likely be different than if only the hadron phase existed throughout. Assuming that quarks cannot exist as asymptotically free particles, I conclude that detection of a quark phase may be possible, but its detection would depend upon an accurate description of the disassembly stage of the fireball.

Summary

My impression is that I have raised many more questions than I have answered. But that has made the writing of this paper all the more interesting to me, as I hope its presentation has been to you. We did answer the one question posed at the outset ⁴⁾. We can anticipate a high degree of sensitivity to the asymptotic form of the hadronic mass spectrum at energies from about 5 GeV in the center of mass, with sensitivity increasing at higher energy. These are high energies but they can be attained in conceivable accelerators and possibly already at CERN. One of the remarkable products of nuclear collisions will be emission of light exotic nuclei which can be used as an important signal as to the nature of the underlying hadron spectrum, provided that the dynamics of the disassembly is well understood. Indeed it ought to be emphasized that the dynamics of high energy nuclear collisions is undoubtedly more complicated than thermodynamics. But I can see no reason to

expect that a complete dynamical description would exhibit much less sensitivity of the outcome of a nuclear collision to the underlying hadronic spectrum, than our simple model suggests. Clearly the problems that have to be overcome are many and serious but the goal is well worth an enormous effort.

Acknowledgments

I am indebted to many people. First to N. Vin Mau, Head of the Division de Physique Théorique, Institut de Physique Nucléaire, Orsay, France for the hospitality extended during a short leave in the Fall of 1976 where the idea for this research was conceived. My student, Yasha Karant, has provided much stimulation and provided spot checks of some of the calculations which he performed entirely independently. We have received helpful comments and encouragement from A. Goldhaber, M. Gyulassy, B. G. Harvey, J. I. Kapusta, A. K. Kerman, J. R. Nix, R. Stock, and W. Swiatecki. The beautiful literature by Hagedorn on hadron thermodynamics and the original letter on the nuclear fireball provided the initial stimulation (Ref. 8).

Appendix A: Chemical Potentials

The chemical potentials, as is usual in thermodynamics, must obey certain relations that are dictated by the possible reactions. For example

$$2p \rightarrow 2p + \pi^0 \text{ implies } \mu_{\pi} = 0$$

and

$$pn \rightarrow p\Delta^0$$

$$pn \rightarrow n\Delta^+$$

$$pp \rightarrow p\Delta^+$$

$$pp \rightarrow p\Delta^{++}$$

$$nn \rightarrow n\Delta^-$$

imply that

$$\mu_{\Delta^0} = \mu_n, \mu_{\Delta^+} = \mu_p, \mu_{\Delta^{++}} = 2\mu_p - \mu_n, \mu_{\Delta^-} = 2\mu_n - \mu_p$$

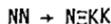
We will ignore the proton, neutron mass difference so that $\mu_n = \mu_p = \mu_B$.

Then

$$\mu_{\Delta} = \mu_B$$

and in general all multiplets belonging to the same family have the same chemical potential.

There are also relationships between the chemical potentials of different families. The reactions



imply

$$\mu_B = \mu_K + \mu_\Lambda = \mu_K + \mu_\Sigma - \mu_\Lambda + \mu_\Sigma = \mu_\Xi + 2\mu_K = \mu_\Omega + 3\mu_K$$

Call $\mu_K = \mu_S$; then a solution to the equations is:

$$\mu_\Lambda = \mu_\Sigma = \mu_B - \mu_S$$

$$\mu_\Xi = \mu_B - 2\mu_S$$

$$\mu_\Omega = \mu_B - 3\mu_S$$

Now all the chemical potentials are expressed in terms of the two, μ_B and μ_S .

Appendix B: Contracted Initial Fireball

As indicated in eq.(10) the initial volume, which multiplies all quantities, depends on the as yet undetermined energy because of the Lorentz contraction. To solve the initial fireball equations, write eqs. (3), (4), (10) as

$$N_\alpha = V n_\alpha(\mu, T) \qquad E_\alpha = V \epsilon_\alpha(\mu, T) \qquad (B1)$$

$$E = V \epsilon(\mu, T) \qquad \epsilon = \sum \epsilon_\alpha$$

where μ stands for μ_S, μ_B , the two independent chemical potentials. Since, eq. (10)

$$V = Av = A \frac{mv_0}{2E} \qquad (B2)$$

we find from (B1) and (B2)

$$E = \left(\frac{v_0 m \epsilon}{2} \right)^{1/2} \qquad (B3)$$

$$V = A \left(\frac{mv_0}{2\epsilon} \right)^{1/2}$$

So the equation for baryon conservation, eq. (6), becomes

$$1 = \left(\frac{m v_0}{2g(\mu, T)} \right)^{1/2} (b(\mu, T) - \bar{b}(\bar{\mu}, T)) \quad (14)$$

The equations that define the initial contracted fireball are (7) and (14) which are to be solved for μ_B and μ_S for chosen T .

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Figure Captions

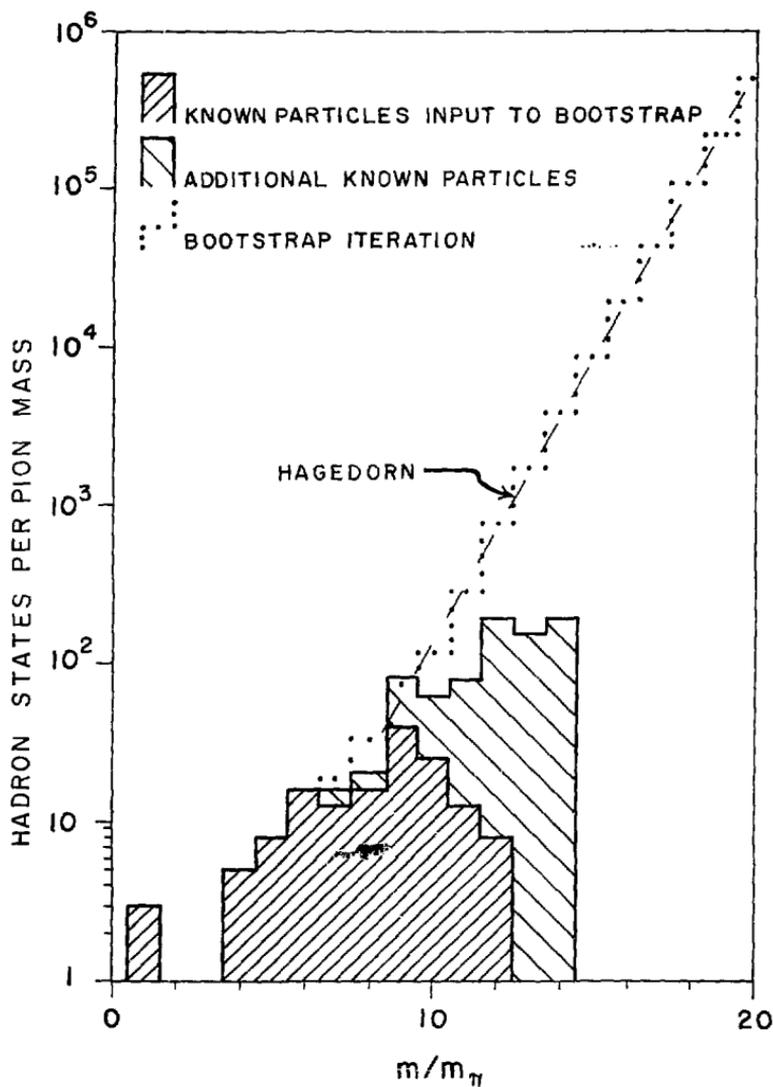
- Fig. 1. The density of different hadrons, ρ , in unit interval is plotted as a function of the mass in units of the pion mass. The experimentally known particles and resonances with their multiplicities are shown in the shade areas. The dotted histogram is a bootstrap iteration³⁾ on the known spectrum and the solid curve is a Hagedorn type spectrum, fitted to the bootstrap.
- Fig. 2. For the three "worlds" considered, the temperature of hot hadronic matter assumed to be produced in a symmetric nuclear collision is plotted as a function of the C.M. total energy per nucleon of the colliding nuclei for a volume corresponding to the initial Lorentz contracted fireball.
- Fig. 3. The number of ordinary baryons is depleted owing to creation of strange baryons. The difference between unity and the curve corresponds to the baryon change resident in strange particles, and is also equal to the strange meson (K) population, because the net strangeness is zero.
- Figs. 4-6. Corresponding to the three "worlds" investigated, the populations of the light and heavy members of the family of ordinary mesons (Π), strange-baryons (K) and ordinary barons (N), are plotted as a function of energy. Light, refers to the first five multiplets (if that many) of each family, and are denoted by <. Heavy, refers to all others, including the continuum where applicable, and are denoted by >. Anti-particles, except for the bootstrap, approach the particle populations at high energy. Refer to Table I for some of the members of the families.
- Figs. 7-9. The populations of the strange baryons as a function of energy for the three "worlds". Notation similar to above.

Fig. 10. The temperature of the fireball as it expands with constant energy equal to 20 GeV. The ordinate is the reciprocal of the total hadron density in units of the density of normal nuclei (0.17 Fm^{-3}). On this scale, 2 corresponds to a density such that each hadron has a share of the volume corresponding to a sphere with radius equal to the pion Compton wavelength 1.4 Fm.

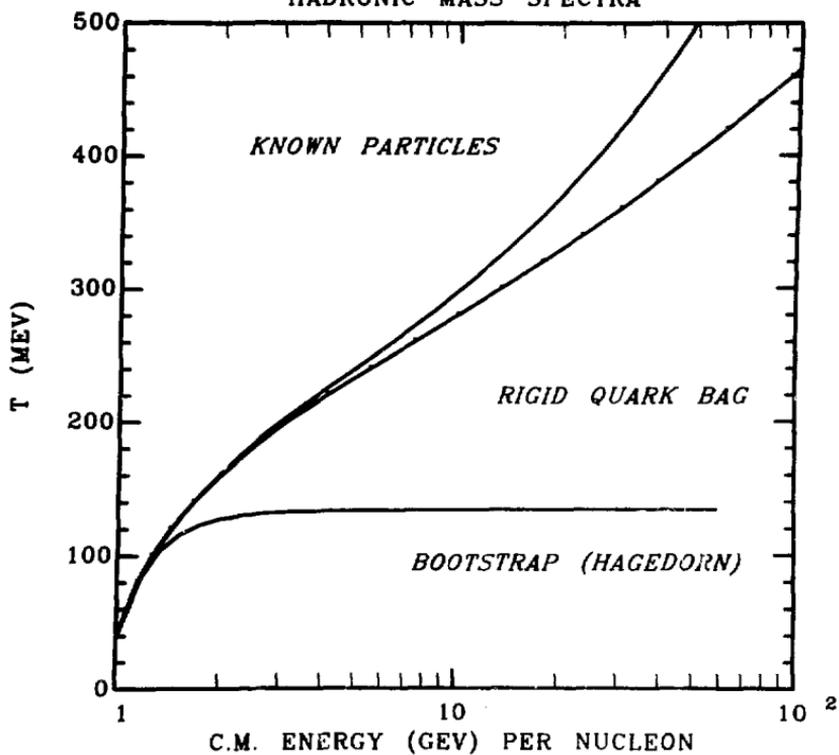
Fig. 11. Similar to Fig. 3 for the expansion at 20 GeV.

Fig. 12-17. For the expansion at 20 GeV the populations for the three "worlds".

Fig. 18. The energy of the fireball decreases during an isentropic expansion. The energy lost to the thermal region is assumed to be carried off by particle radiation. No strange particles were included in this particular calculation.

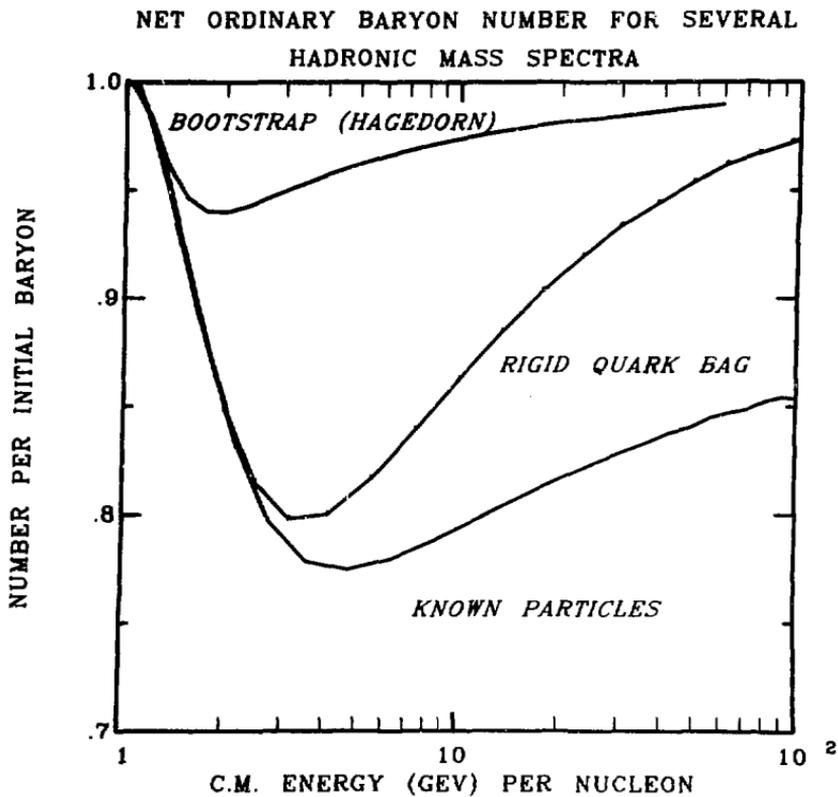


FIREBALL TEMPERATURE FOR SEVERAL
HADRONIC MASS SPECTRA



XBL 781-6713

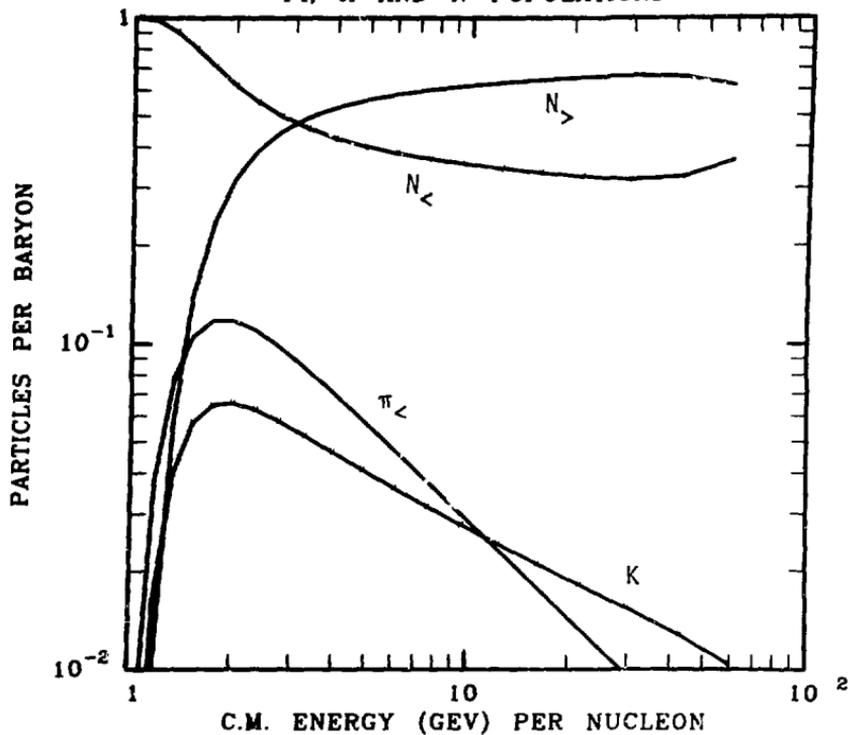
Fig. 2



XBL 781-6708

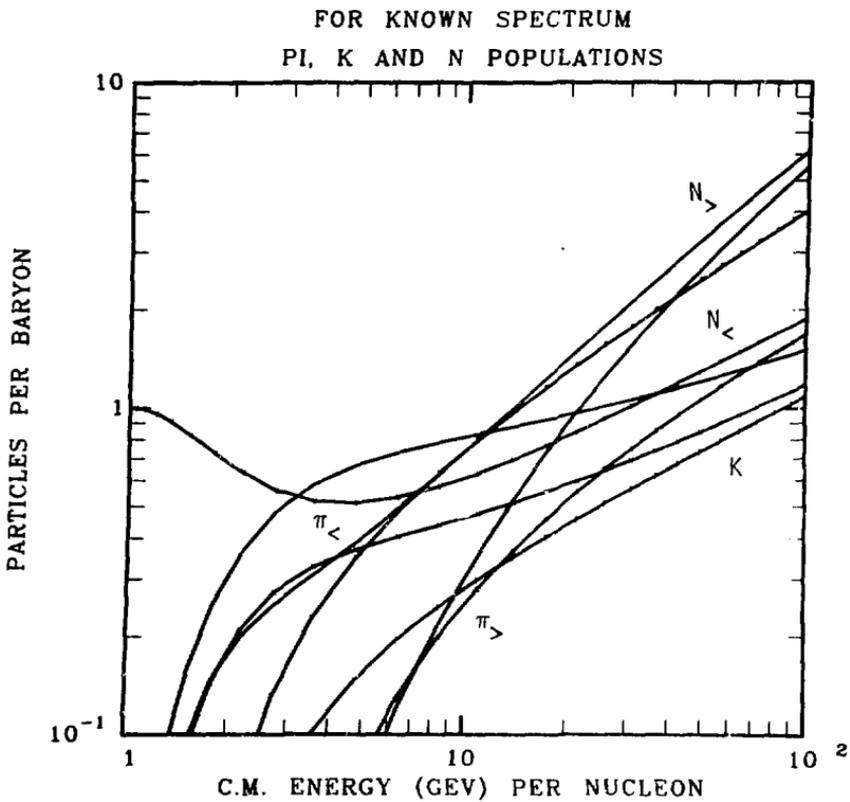
Fig. 3

FOR HAGEDORN SPECTRUM
PI, K AND N POPULATIONS



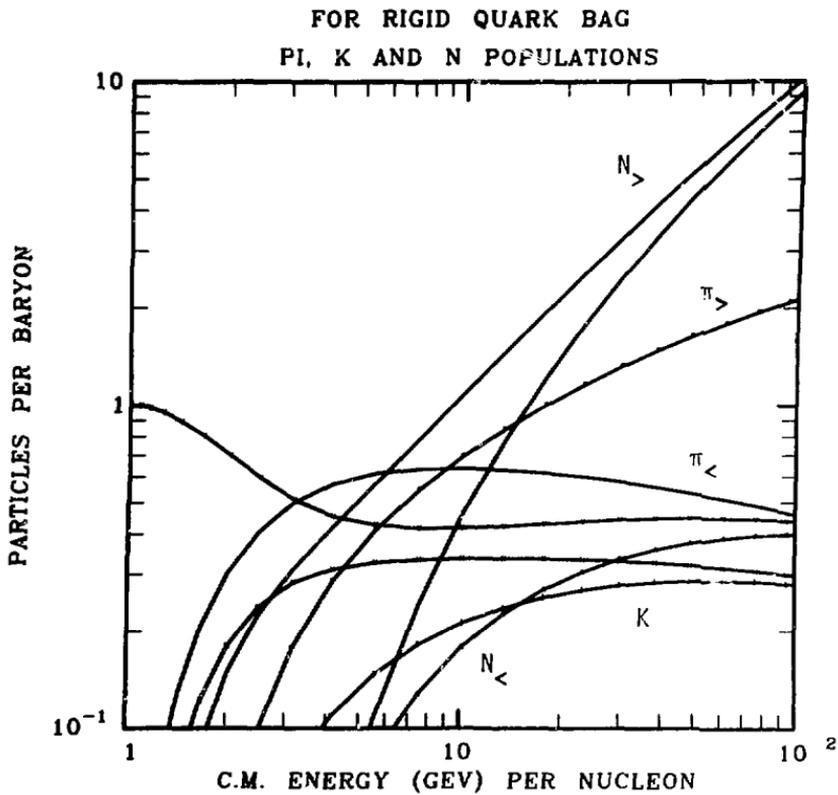
XBL 781-6711

Fig. 4



XBL 781-6712

Fig. 5



XBL 781-6709

Fig. 6

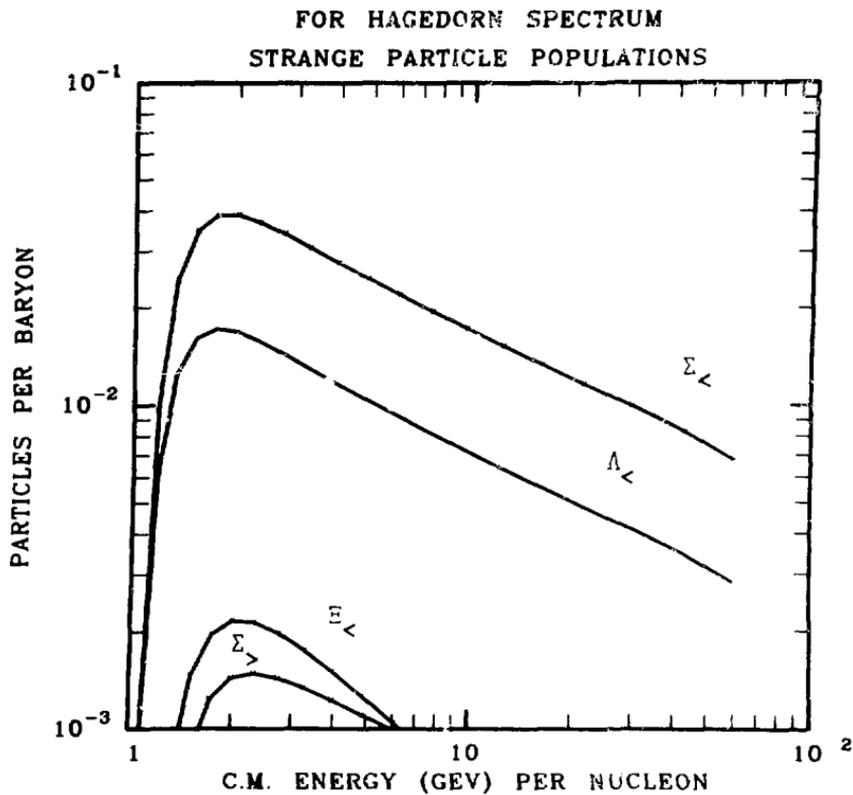
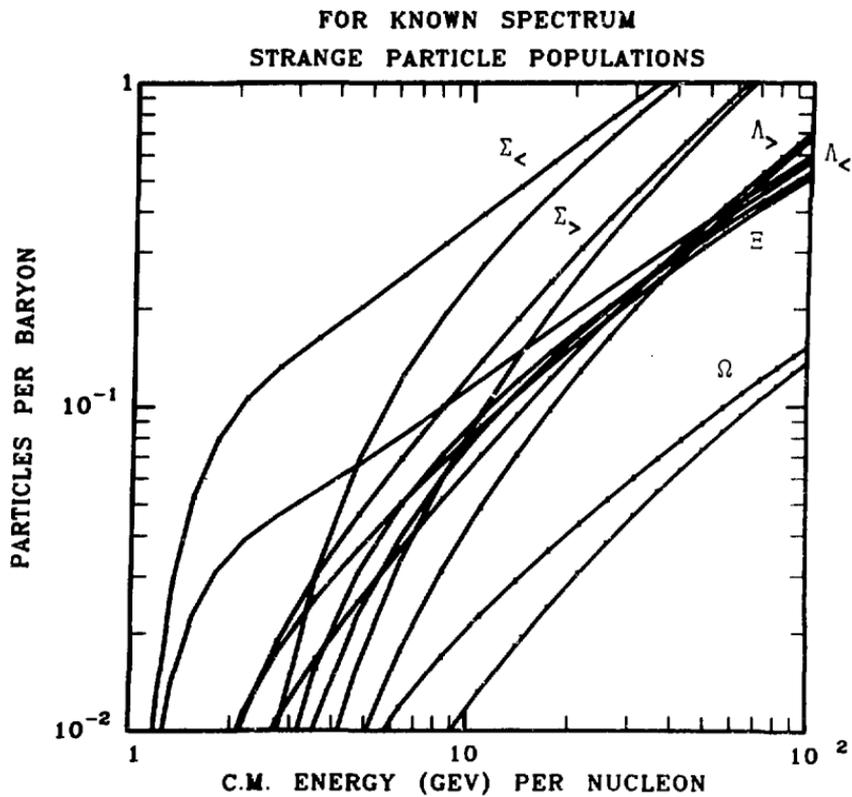


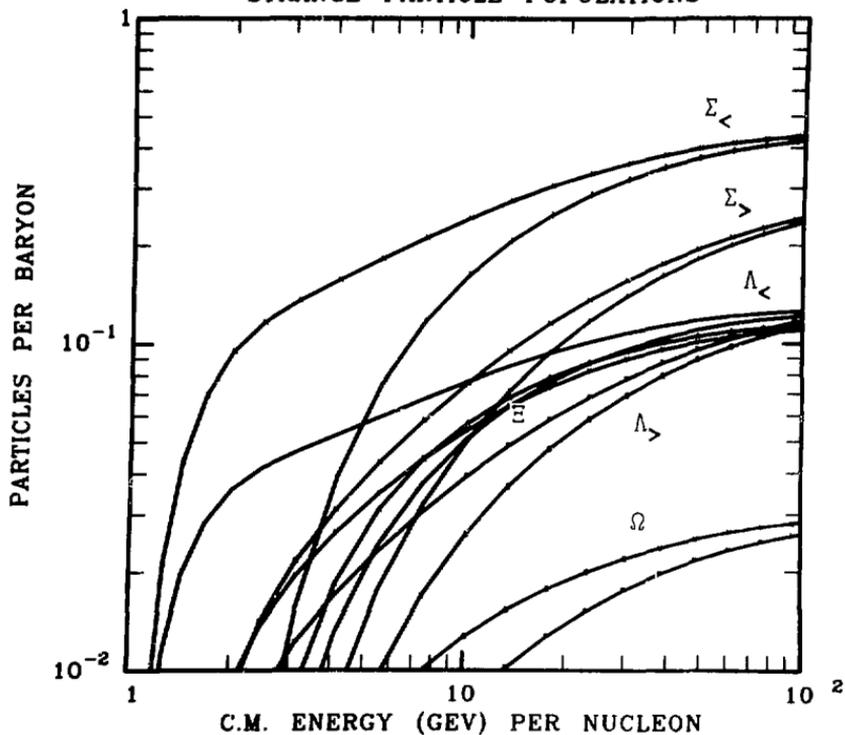
Fig. 7



XBL 781-6707

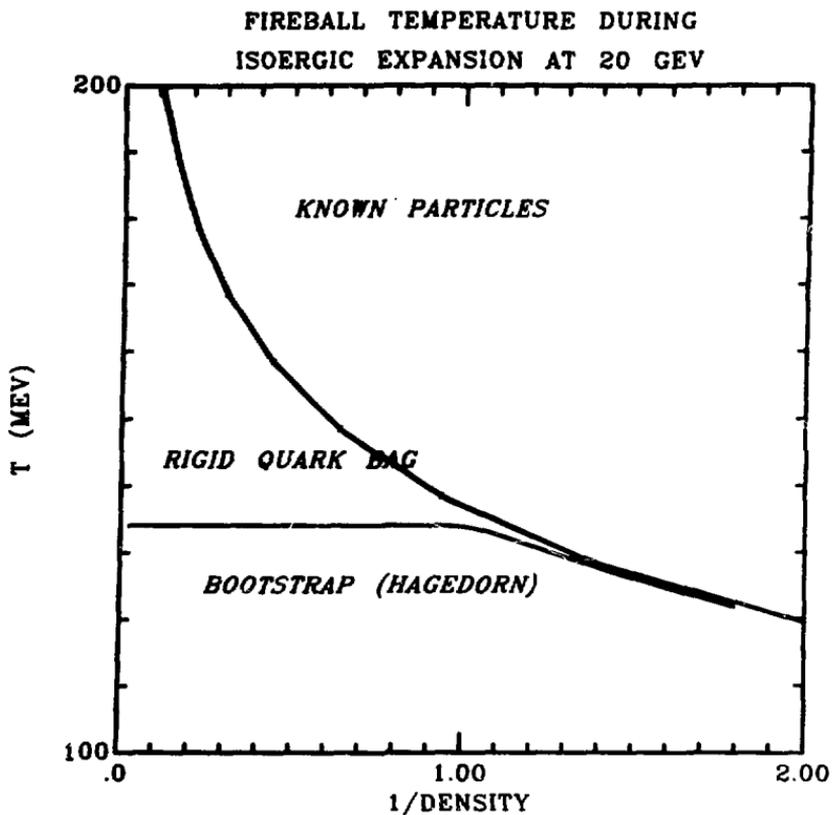
Fig. 8

FOR RIGID QUARK BAG
STRANGE PARTICLE POPULATIONS



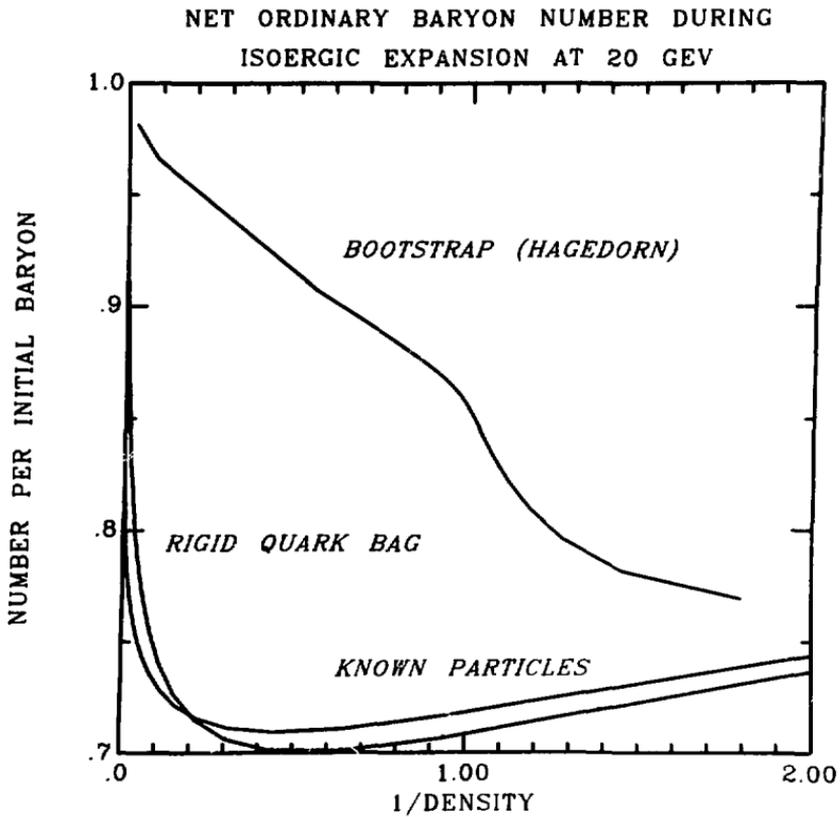
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Fig. 9



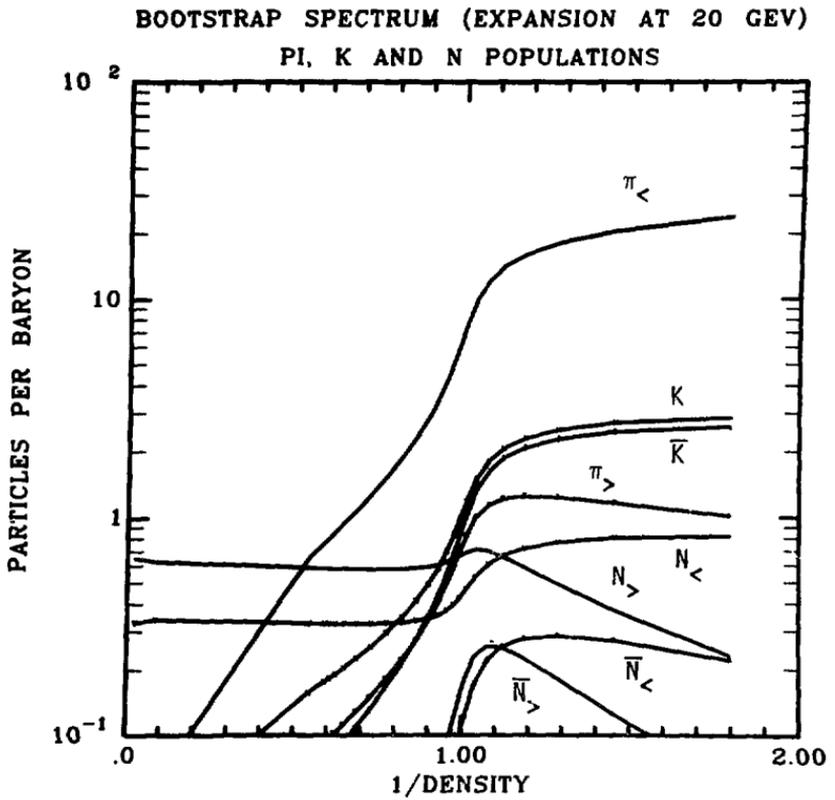
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Fig. 10



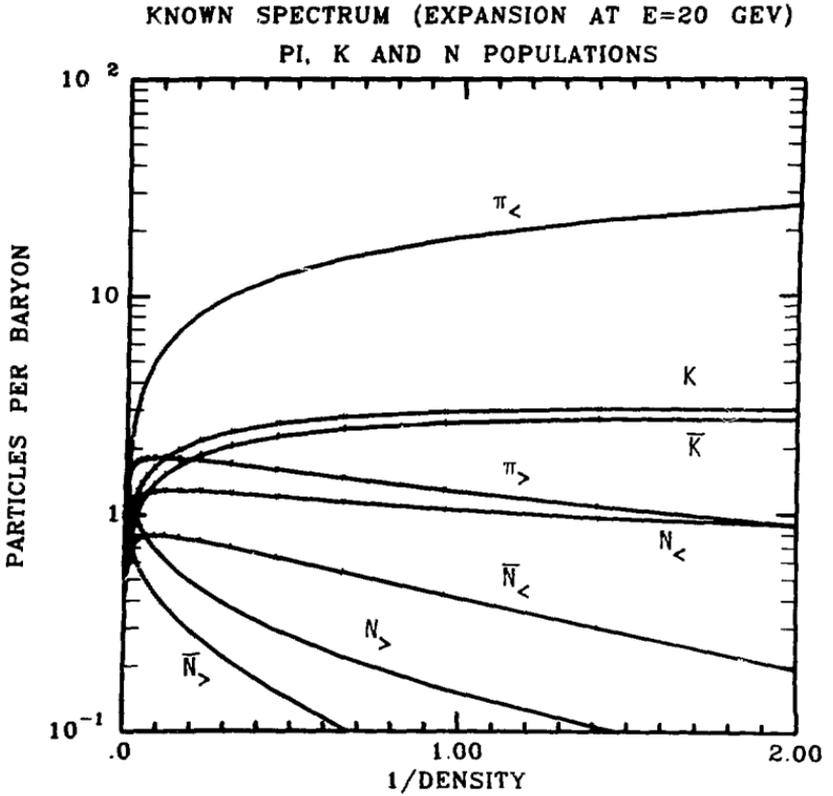
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Fig. 11



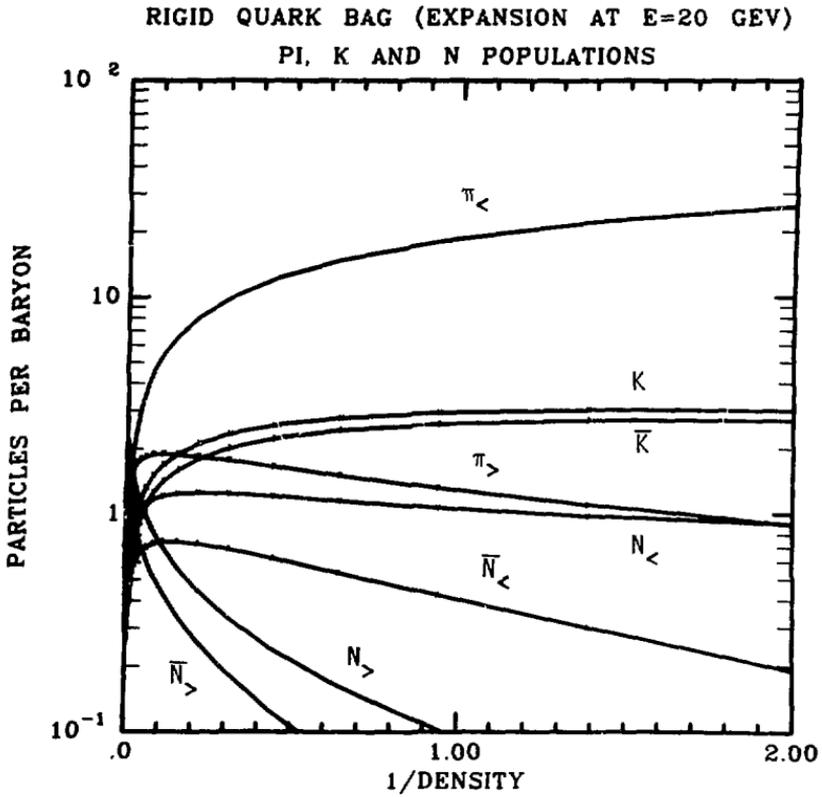
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Fig. 12



XBL 781-6703

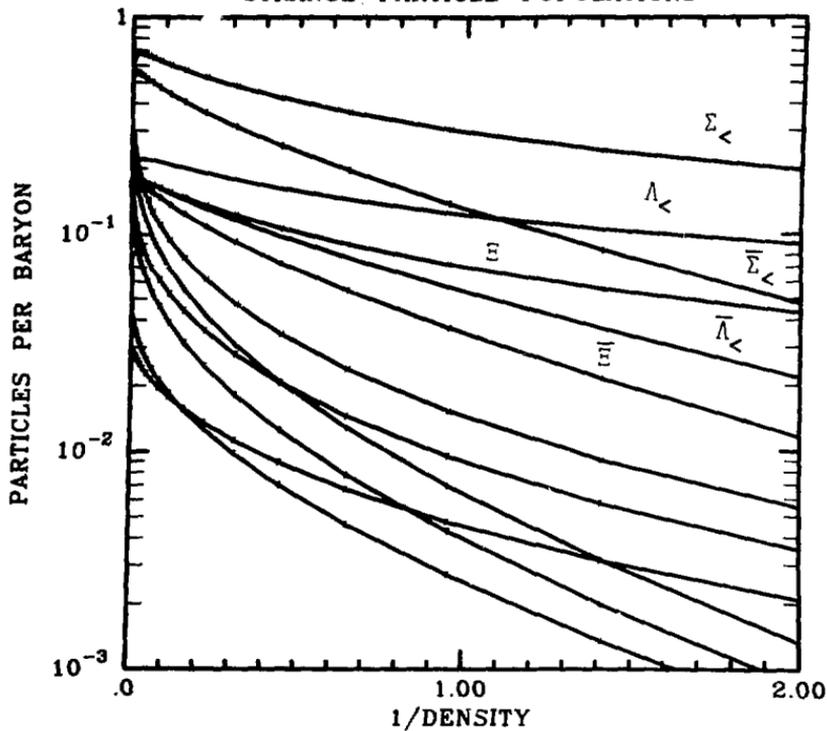
Fig. 13



XBL 781-6702

Fig. 14

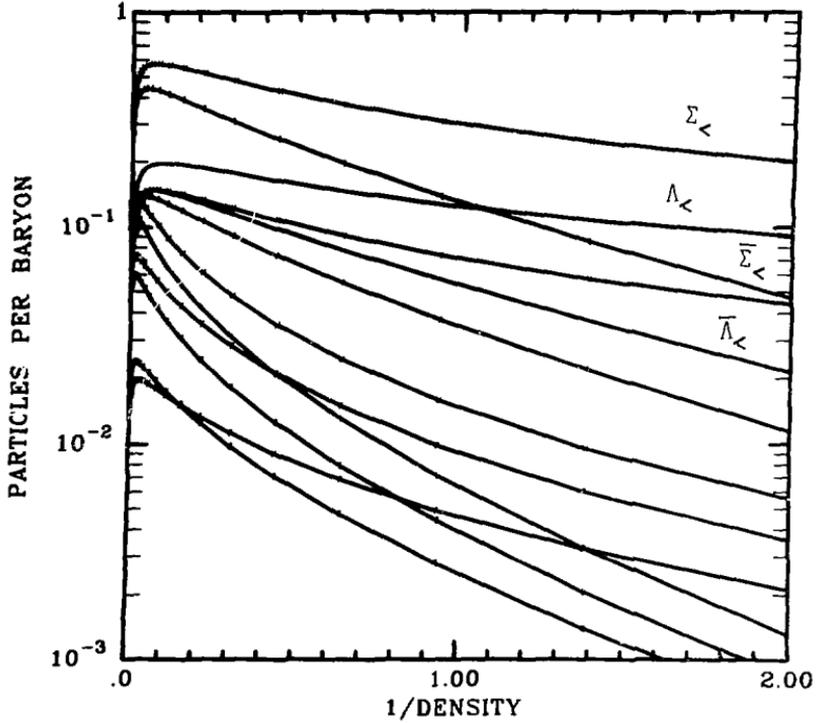
KNOWN SPECTRUM (EXPANSION AT E=20 GEV)
STRANGE PARTICLE POPULATIONS



XBL 781-6701

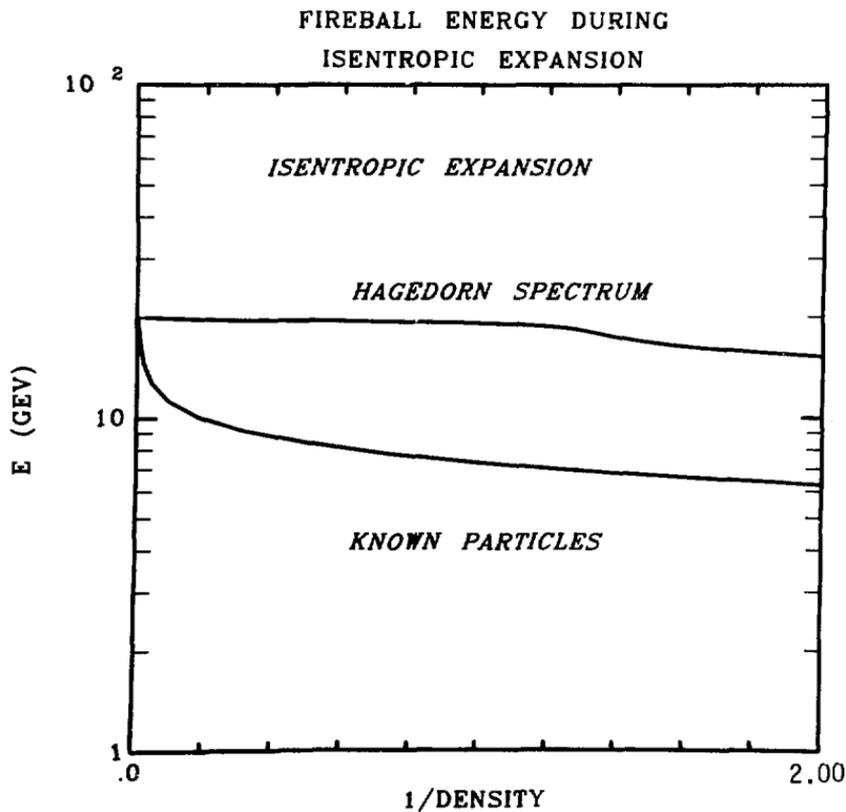
Fig. 16

RIGID QUARK BAG (EXPANSION AT E=20 GEV)
STRANGE PARTICLE POPULATIONS



XBL 781-6700

Fig. 17



XBL 781-6716

Fig. 18