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Mohammad Aslam, William G. Godden, and D. Theodore Scalise

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EARTHQUAKE SLOSHING IN ANNULAR AND CYLINDRICAL TANKS

by

Mohammad Aslam,[1] A.M. ASCE; William G. Godden,[2] M. ASCE
and
D. Theodore Scalise[3]

SUMMARY

This paper presents an analytical and experimental study of the earthquake induced sloshing response of water in annular tanks which are being use in Boiling Water Reactors. The theory developed here is equally applicable to cylindrical tanks.

[1]Associate Engineer, Dept. of Civ. Engrg., Univ. of California, Berkeley, CA.

[2]Professor, Dept. of Civ. Engrg., University of California, Berkeley, CA.

[3]Dept. Head, Engineering Sciences Dept., Lawrence Berkeley Laboratory, University of California, Berkeley, CA.

KEYWORDS

Annular tanks; Boiling Water Reactors (BWR); Cylindrical tanks; Dynamic pressures; Earthquake response; Irrotational flow; Nuclear reactors; Pressure-Suppression pool; Sloshing response; Velocity potential.

ABSTRACT

The sloshing response of water in annular as well as in cylindrical tanks under horizontal earthquake ground motions is studied. A linear analysis, developed for the general annular tank problem is based on potential flow theory; and the predicted values of natural frequencies, surface displacements, and dynamic pressures are compared with measured data from shaking table tests. Experimental studies were conducted on 1/80th and 1/15th scale models of a 120 ft. (36.6 m) annular tank, and correlation data is presented for both sinusoidal and simulated earthquake motions. Test results from the sloshing of water in a 12 ft. (3.66 m) diameter cylindrical tank show that the annular tank solution is also applicable in this case. The range of surface displacement within which linear theory gives satisfactory results was established experimentally.

INTRODUCTION

The problem of the sloshing of a liquid in an annular cylindrical tank (Fig. 1) under horizontal ground motions is encountered in the possible effect of an earthquake on water surface level in the pressure-supression pool of a Boiling Water Reactor (BWR). A cross-section through the annulus of a typical suppression pool design is shown in Figure 2; the inner and outer diameters of the pool are 80 ft. (24.4 m) and 120 ft. (36.6 m) respectively, the water depth is approximately 20 ft. (6.1 m), and the highest inlet opening is 5 ft. 10 in. (1.8 m) below the water surface. The sloshing probelm arises due to the danger of superheated steam escaping if under dynamic conditions the water level at the inner wall were to drop low enough to expose the inlet pipe (level c-c in Figure 2). Hence it is important to be able to predict the maximum water surface displacements for any prescribed earthquake, particularly how far it may drop.

The sloshing of liquid in simple cylindrical tanks has received considerable attention, and both experimental and theoretical studies have been reported in the literature (1-4, 6,8-10, 12-15). Most of this deals with steady-state response and little with response under arbitrary ground motions. The annular tank problem has not been studied to this extent. Bauer's analysis (5) deals with steady-state harmonic ground motion, but neither analytical nor exerimental data has been found for the arbitrary ground motion problem of the kind that exists under earthquake condtions.

This investigation presents the results of both analytical and experimental studies on the sloshing of liquid in annular tanks subjected to both harmonic and arbitrary ground motions. The primary object of the study was to determine the time-history of water surface displacements for any applied ground motion; as in the BWR suppression-pool this represents the greatest potential danger in an earthquake. Experimentally the response characteristics were first studied on a 1/80th scale plexiglass tank mounted on a small table capable of applying horizontal sinusoidal motions (Figure 5a). Subsequently a 1/15th scale steel model with glass viewing windows was used on a large shaking table to study the response of the water to seismic-type horizontal ground motions (Figure 5b). The analytical study is based on linear theory, and the limits for small-displacement response were studied experimentally.

ANALYSIS

Assumptions

The following analysis is based on three assumptions:

1. Displacements are small and hence linear theory is applicable.
2. The tank is assumed to be rigid, the heavy structures used for suppression pools makes this a realistic assumption and even in more flexible tanks the assumption may still be valid as the primary sloshing response is a low frequency phenomenon.
3. Water is assumed to be an incompressible and non-viscous fluid, and thus the flow remains irrotational.

Basic Equations and Boundary Conditions

As the flow is assumed to be irrotational, there exists a velocity potential ϕ which will satisfy the Laplace equation which in a 3-D cylindrical coordinate system (Figure 1) is

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Let a and b be the outer and inner radii of the annular tank and h be the depth of water, then the following boundary conditions must be satisfied

$$\left.\frac{\partial\phi}{\partial r}\right|_{r=a} = \dot{x}\cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\left.\frac{\partial\phi}{\partial r}\right|_{r=b} = \dot{x}\cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=-h} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where $\dot{x} = \frac{dx}{dt}$ = tank wall velocity, and t = time. Also, the linearized free-surface boundary condition is given by the following equation [16]

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \qquad \text{at} \qquad z = 0 \quad \ldots\ldots\ldots\ldots\ldots (5)$$

where g is the acceleration of gravity.

Solution of the Eigenvalue Problem

To determine the natural sloshing frequencies and mode shapes, it is necessary to solve the free vibration problem with homogeneous boundary conditions but non-zero initial conditions.

Assuming

$$\phi = R(r)\ \Theta(\theta)\ Z(z)\ T(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

in which T(t) is harmonic. The solution to Eq. (1) (substituting for ϕ into Eq. (1) and using separation of variables) subject to the following boundary condtions

$$\left.\frac{\partial\phi}{\partial r}\right|_{r=a} = \left.\frac{\partial\phi}{\partial r}\right|_{r=b} = \left.\frac{\partial\phi}{\partial z}\right|_{z=-h} = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

is given by the following equation.

$$\phi = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} e^{i\omega_{mn}t}\left[C_{mn}\sin(m\theta) + D_{mn}\cos(m\theta)\right]\frac{\cosh\xi\left(\frac{z}{a}+\frac{h}{a}\right) C_m\left(\xi_{mn}\frac{r}{a}\right)}{\cosh\left(\xi_{mn}\frac{h}{a}\right) Y_m'(\xi_{mn})} \quad \ldots\ldots \quad (8)$$

in which

$$C_m\left(\xi_{mn}\frac{r}{a}\right) = J_m\left(\xi_{mn}\frac{r}{a}\right) Y_m'(\xi_{mn}) - J_m'(\xi_{mn})\ Y_m\left(\xi_{mn}\frac{r}{a}\right) \quad \ldots\ldots \quad (9)$$

and $T(t) = e^{i\omega_{mn}t}$. The values of ξ_{mn} are given by

$$J_m'(\xi_{mn})\ Y_m'(K\xi_{mn}) - Y_m'(\xi_{mn})\ J_m'(K\xi_{mn}) = 0 \quad \ldots\ldots\ldots\ldots \quad (9a)$$

in which K = b/a

To determine the natural frequencies, we apply the free-surface boundary condition

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial Z} = 0 \qquad \text{at} \qquad Z = 0$$

Substituting for $\partial^2\phi/\partial t^2$ and $\partial\phi/\partial z$, gives

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} - \omega_{mn}^2 e^{i\omega_{mn}t}[C_{mn}\sin m\theta + D_{mn}\cos m\theta]\frac{C_m\left(\xi_{mn}\frac{r}{a}\right)}{Y_m'(\xi_{mn})}$$

$$+ \frac{g}{a}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} e^{i\omega_{mn}t}\,\xi_{mn}[C_{mn}\sin m\theta + D_{mn}\cos m\theta]\frac{C_m\left(\xi_{mn}\frac{r}{a}\right)\tanh\left(\xi_{mn}\frac{h}{a}\right)}{Y_m'(\xi_{mn})} = 0$$

and comparing the coefficients, this gives

$$\omega_{mn}^2 = \left(\frac{g}{a}\right)\xi_{mn}\tanh\left(\xi_{mn}\frac{h}{a}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Equations (10) and (9) give sloshing frequencies and mode shapes of water in a circular annular tank. Equation (8) gives the velocity potential for free vibration and the values of the constants C_{mn} and D_{mn} will be determined by initial conditions. Equations (8), (9) and (10) are similar to the ones derived earlier in reference (5).

Velocity Potential Under Arbitrary Ground Motions

The general solution for sloshing response due to arbitrary horizontal ground motion is derived by solving equation (1) for the velocity potential ϕ for the boundary conditions given in equations (2), (3) and (4). ϕ can be used to derive displacements, velocities and pressures anywhere in the fluid.

As the boundary conditions are non-homogeneous, assume a solution of the form

$$\phi(r,\Theta,z,t) = \phi_1(r,\Theta,z,t) + \phi_c(r,\Theta,t) \quad \ldots\ldots\ldots\ldots\ldots (11)$$

ϕ_c is taken independent of z because of the rigid tank walls.

$$\frac{\partial\phi}{\partial r} = \frac{\partial\phi_1}{\partial r} + \frac{\partial\phi_c}{\partial r} = \dot{x}\cos\theta \quad \text{at} \quad r = a \quad \text{and} \quad r = b$$

Let

$$\frac{\partial\phi_c}{\partial r} = \dot{x}\cos\theta$$

then

$$\frac{\partial\phi_1}{\partial r} = 0 \quad \text{at} \quad r = a \quad \text{and} \quad r = b$$

Substituting Eq. (11) into Eq. (1), we have

$$\left[\frac{\partial^2\phi_1}{\partial r^2} + \frac{\partial^2\phi_c}{\partial r^2}\right] + \frac{1}{r}\left[\frac{\partial\phi_1}{\partial r} + \frac{\partial\phi_c}{\partial r}\right] + \frac{1}{r^2}\left[\frac{\partial^2\phi_1}{\partial\theta^2} + \frac{\partial^2\phi_c}{\partial\theta^2}\right] + \frac{\partial^2\phi_1}{\partial z^2} = 0$$

or

$$\frac{\partial^2\phi_1}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi_1}{\partial\theta^2} + \frac{\partial^2\phi_1}{\partial z^2} = 0 \quad \ldots\ldots\ldots\ldots (12)$$

with the boundary conditions

$$\left.\frac{\partial\phi_1}{\partial r}\right|_{r=a} = \left.\frac{\partial\phi_1}{\partial r}\right|_{r=b} = \left.\frac{\partial\phi_1}{\partial z}\right|_{z=-h} = 0. \quad \ldots\ldots\ldots\ldots\ldots (13)$$

and

$$\frac{\partial^2\phi_c}{\partial r^2} + \frac{1}{r}\frac{\partial^2\phi_c}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi_c}{\partial\theta^2} = 0. \quad \ldots\ldots\ldots\ldots\ldots (14)$$

with the boundary conditions

$$\left.\frac{\partial\phi_c}{\partial r}\right|_{r=a} = \left.\frac{\partial\phi_c}{\partial r}\right|_{r=b} = \dot{x}\cos\theta \quad \ldots\ldots\ldots\ldots\ldots (15)$$

The solution satisfying equations (14) and (15) is given by

$$\phi_c = \dot{x}\, r\cos\theta \quad \ldots\ldots\ldots\ldots\ldots (16)$$

Now consider the free-surface boundary condtion

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \qquad \text{at} \qquad z = 0$$

$$\frac{\partial^2\phi_1}{\partial t^2} + \frac{\partial^2\phi_c}{\partial t^2} + g\frac{\partial\phi_1}{\partial z} = 0$$

$$\left.\frac{\partial^2\phi_1}{\partial t^2} + g\frac{\partial\phi_1}{\partial z}\right|_{z=0} = -\frac{\partial^2\phi_c}{\partial t^2} = -\dddot{x}\, r\cos\theta \quad \ldots\ldots\ldots\ldots (17)$$

Equation (12) must be solved with the additional boundary condition at the free surface given by Eq. (17).

Equation (12) is similar to Eq. (1) and the boundary conditions at the walls and bottom of the tank are homogeneous in both cases. Therefore the general form of the solution in these two cases will be similar except that the time function T(t) will be different for the solution of Eq. (12) which was simply harmonic for the free vibration case. Thus the solution of Eq. (12) will be of the form

$$\phi_1 = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} [C_{mn} \sin m\theta + D_{mn} \cos m\theta] \frac{\cosh \xi_{mn}\left(\frac{z}{a} + \frac{h}{a}\right) C_m\left(\xi_{mn} \frac{r}{a}\right) T_{mn}(t)}{Y_m'(\xi_{mn}) \cosh\left(\xi_{mn} \frac{h}{a}\right)} \quad \ldots\ldots (18)$$

This is the general solution and covers all types of modes. It should be noted, however, that the horizontal ground motion under consideration does not excite all types of modes. It will excite only antisymmetric modes which correspond to an m value equal to 1. Therefore the sum of m can be dropped and it is understood that all subsequent equations refer only to the class of modes which correspond to m = 1 and which are antisymmetric about an axis perpendicular to the axis of excitation. Thus ϕ_1 becomes

$$\phi_1 = \sum_{n=0}^{\infty} \frac{[C_n \sin\theta + D_n \cos\theta] \cosh \xi_n\left(\frac{z}{a} + \frac{h}{a}\right) C_1\left(\xi_n \frac{r}{a}\right) T_n(t)}{Y_1'(\xi_n) \cosh\left(\xi_n \frac{h}{a}\right)} \quad \ldots (19)$$

in which

$$C_1\left(\xi_n \frac{r}{a}\right) = J_1\left(\xi_n \frac{r}{a}\right) Y_1'(\xi_n) - J_1'(\xi_n)\, Y_1\left(\xi_n \frac{r}{a}\right) \quad \ldots\ldots\ldots\ldots (20)$$

and ξ_n are the roots of the equation.

$$J_1'(\xi)\; Y_1'(K\xi) - J_1'(K\xi)\; Y_1'(\xi) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

$T_n(t)$ are the time functions yet to be determined. C_n and D_n are the constants to be determined from the initial conditions. Applying the free-surface ($z = 0$) boundary conditions given by Eq. (17), gathering like terms and dividing by a gives

$$\frac{1}{a}\sum_{n=0}^{\infty} C_n \sin\theta\; C_1\left(\xi_n \frac{r}{a}\right)\frac{\left[\ddot{T}_n + \frac{g}{a}\xi_n \tanh\left(\xi_n \frac{h}{a}\right) T_n\right]}{Y_1'(\xi_n)}$$

$$+ \frac{1}{a}\sum_{n=0}^{\infty} D_n \cos\theta\; C_1\left(\xi_n \frac{r}{a}\right)\frac{\left[\ddot{T}_n + \frac{g}{a}\xi_n \tanh\left(\xi_n \frac{h}{a}\right) T_n\right]}{Y_1'(\xi_n)} = -\frac{r}{a}\cos\theta\; \dddot{x} \quad \ldots\ldots\ldots (22)$$

Using the orthogonality property of Bessel functions

$$\frac{r}{a} = \sum_{n=0}^{\infty} A_n\, C_1\left(\xi_n \frac{r}{a}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which

$$A_n = \frac{2\left[\frac{2}{\pi\xi_n} - KC_1(K\xi_n)\right]}{\frac{4}{\pi^2}\xi_n^2(\xi_n^2 - 1) + C_1^2(K\xi_n)[1 - K^2\xi_n^2]} \quad \ldots\ldots\ldots\ldots (24)$$

Substituting for r/a into Eq. (22), and comparing the coefficients,

$$\frac{D_n}{a}\,\frac{\left[\ddot{T}_n + \frac{g}{a}\xi_n \tanh\left(\xi_n \frac{h}{a}\right) T_n\right]}{Y_1'(\xi_n)} = -A_n \dddot{x} \quad \ldots\ldots\ldots\ldots (25)$$

$$\ddot{T}_n + \omega_n^2 T_n = -\frac{a}{D_n} Y_1'(\xi_n) A_n \dddot{x} = E_n \dddot{x} \quad \ldots\ldots\ldots\ldots (26)$$

in which

$$\omega_n^2 = \frac{g}{a}\xi_n \tanh\left(\xi_n \frac{h}{a}\right) \quad \ldots\ldots\ldots\ldots (27)$$

and

$$E_n = -\frac{a}{D_n} Y_1'(\xi_n) A_n \quad \ldots\ldots\ldots\ldots (28)$$

The general solution for $T_n(t)$ is given by

$$T_n(t) = \left[F_n \sin \omega_n t + G_n \cos \omega_n t + \frac{E_n}{\omega_n} \sin \omega_n t \int_0^t \dddot{x}(\tau) \cos \omega_n \tau \, d\tau - \frac{E_n}{\omega_n} \cos \omega_n t \int_0^t \dddot{x}(\tau) \sin \omega_n \tau \, d\tau\right] \quad \ldots\ldots\ldots\ldots (29)$$

Thus the solution for ϕ_1 becomes

$$\phi_1 = \sum_{n=0}^{\infty} \frac{\cos\theta \cosh \xi_n \left(\frac{z}{a} + \frac{h}{a}\right) C_1\left(\xi_n \frac{r}{a}\right) T_n(t)}{Y_1'(\xi_n) \cosh\left(\xi_n \frac{h}{a}\right)} \quad \ldots\ldots\ldots\ldots (30)$$

Using Eqs. (11), (16) and (29)

$$\phi = r \cos\theta\, \dot{x} + \sum_{n=0}^{\infty} \frac{\cos\theta \cosh \xi_n \left(\frac{z}{a} + \frac{h}{a}\right) C_1\left(\xi_n \frac{r}{a}\right)}{Y_1'(\xi_n) \cosh\left(\xi_n \frac{h}{a}\right)} \cdot T_n(t) \ldots\ldots (31)$$

Applying the initial ("at rest") conditions, namely $\phi(r,\theta,z,0) = 0$ and $\frac{\partial\phi}{\partial t}(r,\theta,z,0) = 0$, $F_n = G_n = 0$. Thus

$$T_n(t) = \left[\frac{E_n}{\omega_n} \sin \omega_n t \int_0^t \dddot{x} \cos \omega_n \tau \, d\tau - \frac{E_n}{\omega_n} \cos \omega_n t \int_0^t \dddot{x} \sin \omega_n \tau \, d\tau\right]. (32)$$

Substituting for E_n and rearranging Eq. (31)

$$\phi = \cos\theta \left\{ r\dot{x} - a \sum_{n=0}^{\infty} A_n \frac{\cosh \xi_n \left(\frac{z}{a} + \frac{h}{a}\right) C_1\left(\xi_n \frac{r}{a}\right)}{\omega_n \cosh\left(\xi_n \frac{h}{a}\right)} \cdot T_n(t) \right\} \ldots\ldots (33)$$

in which

$$T_n(t) = \left[\sin \omega_n t \int_0^t \dddot{x} \cos \omega_n \tau \, d\tau - \cos \omega_n t \int_0^t \dddot{x} \sin \omega_n \tau \, d\tau\right] \ldots (34)$$

Equation (33) is the general expression for the velocity potential in an annular-circular tank.

Displacements, Velocities and Dynamic Pressures

Once the expression for the velocity potential is known, the fluid displacements, velocities, and dynamic pressures anywhere in the fluid can easily be derived. Let

δ = fluid displacement

U_r = fluid velocity in the direction of r

U_θ = fluid velocity in the direction of θ

U_z = fluid velocity in the direction of z

p = dynamic pressure

then

$$\delta(r,\theta,z,t) = -\frac{1}{g}\frac{\partial\phi}{\partial t}(r,\theta,z,t)$$

$$U_r(r,\theta,z,t) = \frac{\partial\phi}{\partial r}(r,\theta,z,t)$$

$$p(r,\theta,z,t) = -\rho\frac{\partial\phi(r,\theta,z,t)}{\partial t}$$

in which ρ = density of the fluid. From Eq. (33) we have the following general expressions for δ and p

$$\delta(r,\theta,z,t) = -\frac{\cos\theta}{g}\left\{r\ddot{x} - a\sum_{n=0}^{\infty}A_n\frac{\cosh\xi_n\left(\frac{z}{a}+\frac{h}{a}\right)c_1\left(\xi_n\frac{r}{a}\right)}{\cosh\left(\xi_n\frac{h}{a}\right)}\right.$$

$$\left.\cdot\left[\cos\omega_n t\int_0^t\ddot{x}\cos\omega_n\tau\,d\tau + \sin\omega_n t\int_0^t\ddot{x}\sin\omega_n\tau\,d\tau\right]\right\} \quad \ldots\ldots\ldots (35)$$

$$p(r,\theta,z,t) = -\rho\cos\theta\left\{r\ddot{x} - a\sum_{n=0}^{\infty} A_n \frac{\cosh \xi_n\left(\frac{z}{a}+\frac{h}{a}\right) c_1\, \xi_n \frac{r}{a}}{\cosh\left(\xi_n \frac{h}{a}\right)}\right.$$

$$\left.\cdot\left[\cos\omega_n t\int_0^t \dddot{x}\cos\omega_n\tau\, d\tau + \sin\omega_n t\int_0^t \dddot{x}\sin\omega_n\tau\, d\tau\right]\right\} \quad \ldots\ldots\ldots (36)$$

Similar expressions can be derived for velocity components U_r, U_θ and U_z.

Steady-State Response

Let the ground acceleration $\ddot{x} = x_o \sin\omega_o t$, where x_o = amplitude of the ground acceleration, and ω_o = circular frequency of the ground motion. Substituting for $\ddot{x}$ in Eq. (35) and simplifying, gives

$$\delta(r,\theta,z,t) = -\frac{\cos\theta}{g}\left\{r\ddot{x} - a\sum_{n=0}^{\infty} A_n \frac{\cosh \xi_n\left(\frac{z}{a}+\frac{h}{a}\right) c_1\left(\xi_n \frac{r}{a}\right)}{\cosh\left(\xi \frac{h}{a}\right)}\right.$$

$$\left.\cdot\left[-\frac{x_o \sin\omega_o t}{\left(\frac{\omega_n}{\omega_o}\right)^2 - 1} + \frac{x_o \sin\omega_n t}{\frac{\omega_n}{\omega_o} - \frac{\omega_o}{\omega_n}}\right]\right\} \quad \ldots\ldots\ldots\ldots (37)$$

If the transient term in the above equation is neglected, the expression for the steady state displacement response δ becomes

$$\delta(r,\theta,z,t) = -\frac{\cos\theta}{g}\left\{r\, x_o \sin\omega_o - a\sum_{n=0}^{\infty} A_n \frac{\cosh \xi_n\left(\frac{z}{a}+\frac{h}{a}\right) c_1\left(\xi_n \frac{r}{a}\right)}{\cosh\left(\xi_n \frac{h}{a}\right)}\right.$$

$$\left.\cdot\left[-\frac{x_o \sin\omega_o t}{\left(\frac{\omega_n}{\omega_o}\right)^2 - 1}\right]\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Similarly, the steady-state pressure response will be given by

$$p(r,\theta,z,t) = -\rho\cos\theta\left\{r\,x_o\sin\omega_o - a\sum_{n=0}^{\infty} A_n \frac{\cosh\xi_n\left(\frac{z}{a}+\frac{h}{a}\right)c_1\left(\xi_n\frac{r}{a}\right)}{\cosh\left(\xi_n\frac{h}{a}\right)} \cdot \left[-\frac{x_o\sin\omega_o t}{\left(\frac{\omega_n}{\omega_o}\right)^2-1}\right]\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Mode Shapes and Sloshing Response Analysis

A time-history analysis of sloshing and pressure response under earthquake ground motions was carried out by computer. A program was written to solve for mode shapes and frequencies as well as to print and plot the time-history response of water displacements and pressures along the axis of ground excitation.

Figure 3a shows the first and second anti-symmetric mode shapes and Figure 3b gives the associated water surface profiles for an annular tank in which h:b:a = 1:2:3 at Section A-A Figure 1. The profile at any other section can be determined by multiplying with cosθ. For a typical prototype structure [a = 60 ft. (18.3 m), b = 40 ft. (12.2 m), h = 20 ft. (6.1 m)], the first and second natural frequencies are 0.079 and 0.36 Hz respectively. Figure 4a shows the time-history of water displacements at the inner and outer walls of the prototype tank under El Centro 1940 earthquake. Similar plots were obtained for the pressure response and Figure 4b shows the impulsive dynamic pressure variation with depth at the inner and outer walls of the prototype structure at the time of peak ground acceleration of 0.34 g.

MODEL STUDY

To verify the analytical results, tests were carried out on both small and large scale models as follows:

1. Small Scale Models

These tests were conducted on a 3 x 4 ft. (.92 x 1.22 m) shaking table capable of applying harmonic motions (Fig. 5a). Two 18 in. (45.71 cm) outside diameter plexiglass models were studied: model 1 had an inside diameter of 12 in. (30.5 cm) and was thus a 1/80th scale model of the prototype; model 2 had an inside diameter of 6 in. (15.24 cm). In both tests the water depth was 3 in. (7.62 cm).

The models were instrumented to measure the surface wave heights near the inner walls. The frequency of the table motion was known from the function generator and the amplitude of table motion was measured with an accelerometer. Natural sloshing frequencies and water displacements under steady state sinusoidal table motion were determined and the measured and equivalent analytical results are given in Tables 1 and 2. These forced vibration tests show a good agreement with the analytical values.

2. Large Scale Tests

These tests were conducted on the 20 ft. x 20 ft. (6.1 x 6.1 m) shaking table at the Earthquake Engineering Research Center at the University of California, Berkeley (Figure 5b). An 8 ft. (2.44 m) diameter steel tank with plexiglass observation windows and an inside diameter of 5 ft. 6 in. (1.67 m) represented a 1/15th scale model of the prototype suppression pool. Tests were conducted using time-scaled accelerograms of the El Centro (1940) and Parkfield earthquakes. Wave

heights and dynamic pressures were recorded at the locations shown in Figures 6 and 7 at a sampling speed of 100 per second.

Figures 8, 9, and 10 show the wave-height comparisons for increasing intensities of the El Centro earthquake, the form and duration of the earthquake remaining the same in all cases and the peak accelerations for the three tests being 0.24g, 0.31g, and 0.44g respectively. Figure 11 shows the computed and measured dynamic pressures at gage 1 at the bottom of the outer wall for the El Centro earthquake with 0.31g peak acceleration.

The wave-height tests indicate that even for a strong earthquake of the form of the El Centro accelerogram with significant long period energy the linear theory provides a reasonably accurate means of computing water surface displacements. In Figure 8 for a peak acceleration of 0.24g the correlation is extremely good indicating that the response is well within the linear range. In Figure 9, at the actual El Centro intensity, the results are still acceptable even though the effect of a slight nonlinearity is beginning to show: the response is not as sinusoidal in that the positive upward displacements are greater than the downward and there is a slight increase in period with amplitude. This effect is even more noticeable in Figure 10 which shows the response due to a ground motion of intensity 40% above the actual El Centro motion. Even in this extreme case however the computed values may be adequate for design purposes remembering that the most important response value is the maximum downward displacement at the inner wall.

The range of linear behavior will, of course, depend on the predominant response mode as well as on the geometry of the tank, and the

values shown for the El Centro earthquake indicate the range of linearity for one particular tank when the response is primarily in the second mode. The maximum water surface gradient in this case taken as $2\delta_{max}/(a-b)$ is approximately 1/5 which would indicate a linearity range for cases where radial modes predominate.

APPLICATION TO SIMPLE CYLINDRICAL TANKS

The theoretical results for the general annular tank problem can also be applied to a cylindrical tank as this is a limiting case of an annular tank as the inner radius approaches zero.

In order to check the validity of this solution, a comparison was made with test results obtained from another study(7) conducted on flexible cylindrical tanks. In this study the flexibility of the tanks, though having a significant influence on the stresses in the tank walls, had little effect on the overall sloshing response of the liquid. The test results shown in Figure 12 are for a 12 ft. (3.66 m) diameter tank, and for a water depth of 5 ft. (1.53 m). Both measured and computed time-histories are shown for the water-surface at the outer wall for the El Centro earthquake increased in intensity to a peak of 0.5g. The correlation between the two is very close indicating that the general annular tank solution is applicable to this case also and that the response is essentially within the linear range. It should be noted that in this analysis the inner radius cannot be made zero, and in the solution given it was made small enough (0.1 in.) to have a negligible effect on the overall response.

CONCLUSIONS

A good correlation between the test and computer results shows that the linearized small displacement theory can accurately predict the sloshing displacements and dynamic pressures in annular cylindrical as well as simple cylindrical tanks under earthquake motions. The range of linearity is quite large even in annular tanks of the type under consideration and the analysis gives satisfactory results as long as displacements remain within approximately 30 in. (76.2 cm) for the prototype being studied. The linearity range for simple cylindrical tanks of the same size is much larger.

The second mode is the dominant sloshing mode of vibration in the prototype tank for all of the earthquake motions studied, namely El Centro, Pacoima Dam and Housner's (11) artificial accelerogram A1, A2, B1, B2. The maximum displacement always occurs at the inner wall.

ACKNOWLEDGEMENTS

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TABLE I. Test and analytical frequencies of Models 1 and 2 for Depth of Water h = 3 in.

Mode Number	Model 1		Model 2	
	Test	Theory	Test	Theory
1	0.70	0.70	0.90	0.88
2	3.20	3.17	2.30	2.31
3	4.50	4.47	3.20	3.22
4	5.50	5.47	3.90	3.92

TABLE 2. Sloshing response of water in Models 1 and 2 under sinusoidal ground accelerations (h = 3 in.).

Frequency (Hz)	Acceleration (g)	Maximum Displacement at Inner Wall (Inches)			
		Model 1		Model 2	
		Test	Theory	Test	Theory
0.6	0.00585	0.167	0.153	0.040	0.042
1.2	0.0312	0.184	0.180	0.308	0.321
2.0	0.0530	0.168	0.174	0.580	0.586
3.7	0.0742	0.271	0.237	0.196	0.201

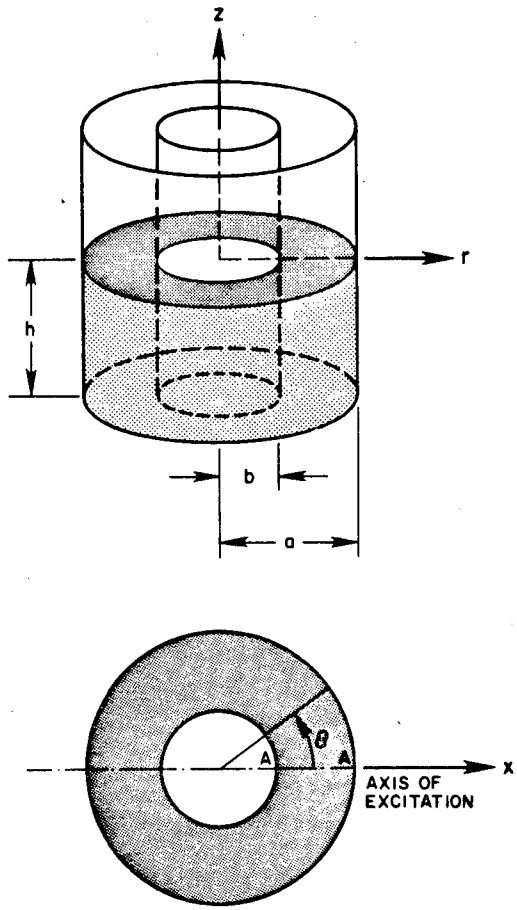

FIG. I ANNULAR TANK SHOWING COORDINATE SYSTEM

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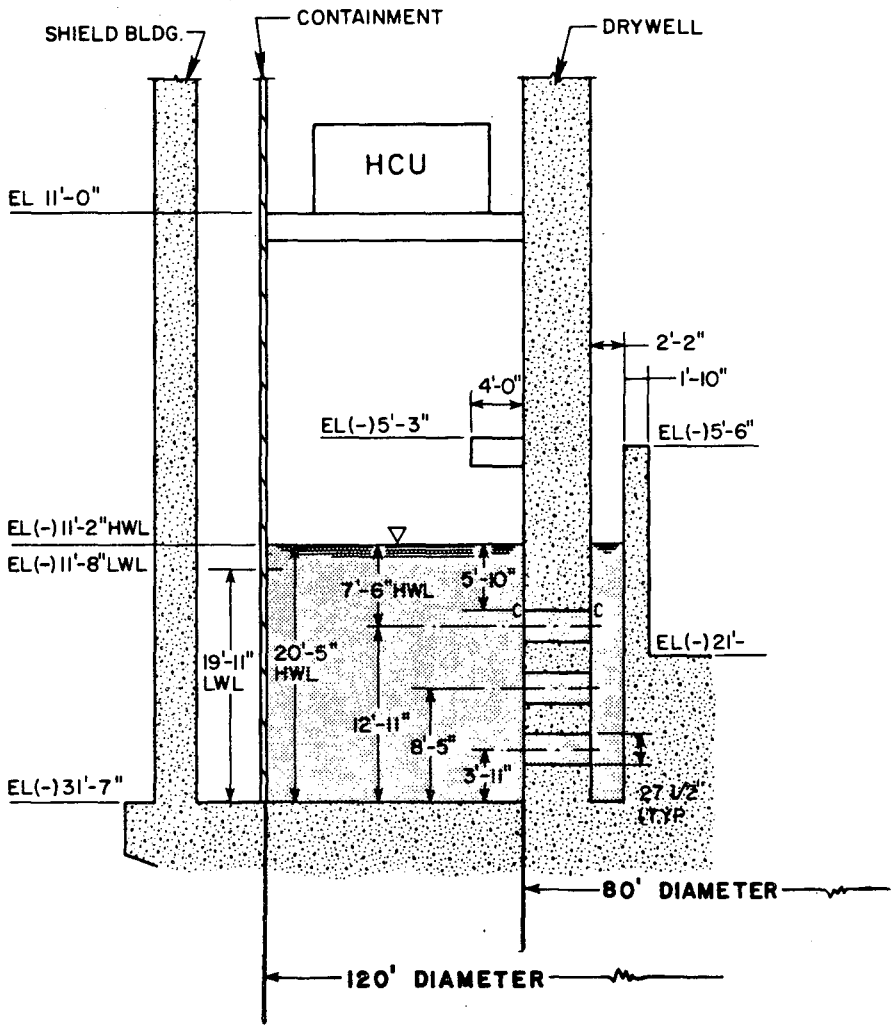

FIG. 2 CROSS-SECTION THROUGH THE ANNULUS OF A TYPICAL PRESSURE SUPPRESSION POOL OF A BOILING WATER REACTOR (MARK III)

(1 FT. = 0.305 M)

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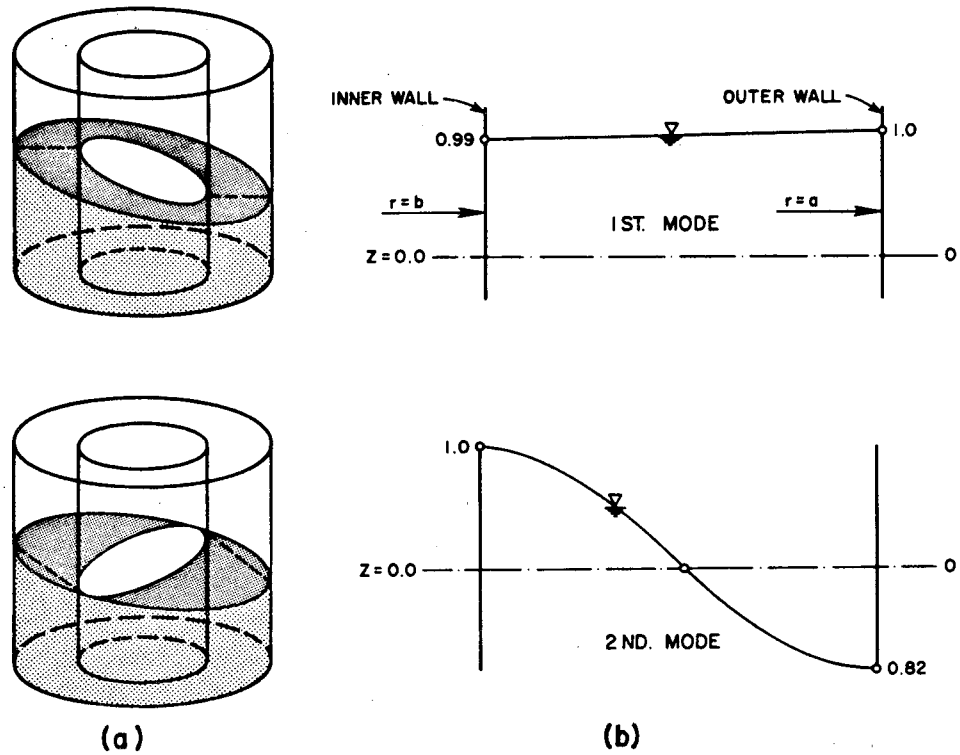

FIG. 3 FIRST AND SECOND ANTI-SYMMETRIC MODE SHAPES

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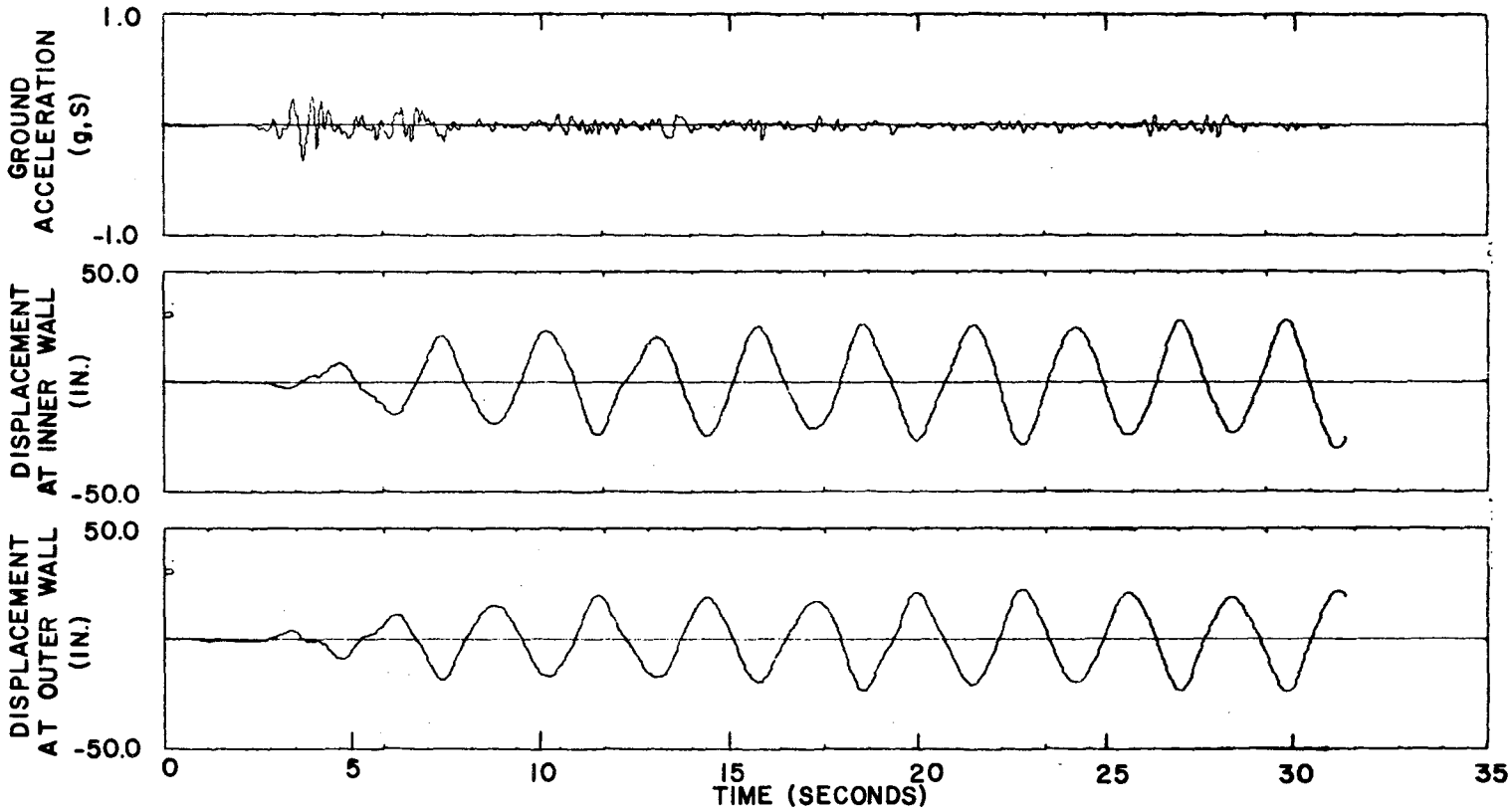


Fig. 4a SLOSHING RESPONSE OF WATER IN ANNULAR TANK (INNER RADIUS = 480 IN., OUTER RADIUS = 720 IN., DEPTH OF WATER = 240 IN.) UNDER EL CENTRO 1940 EARTHQUAKE (1 IN. = 2.54 cm).

XBL783-500

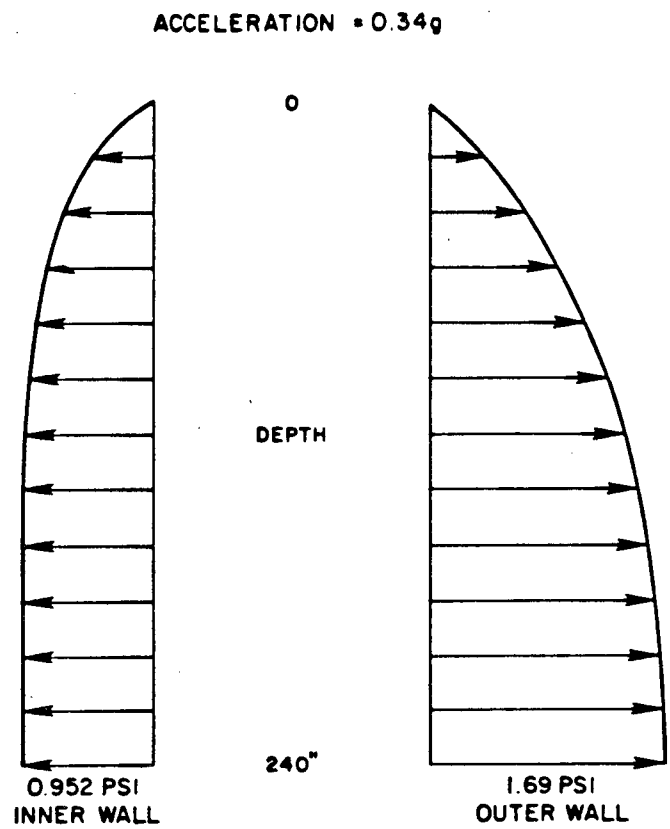
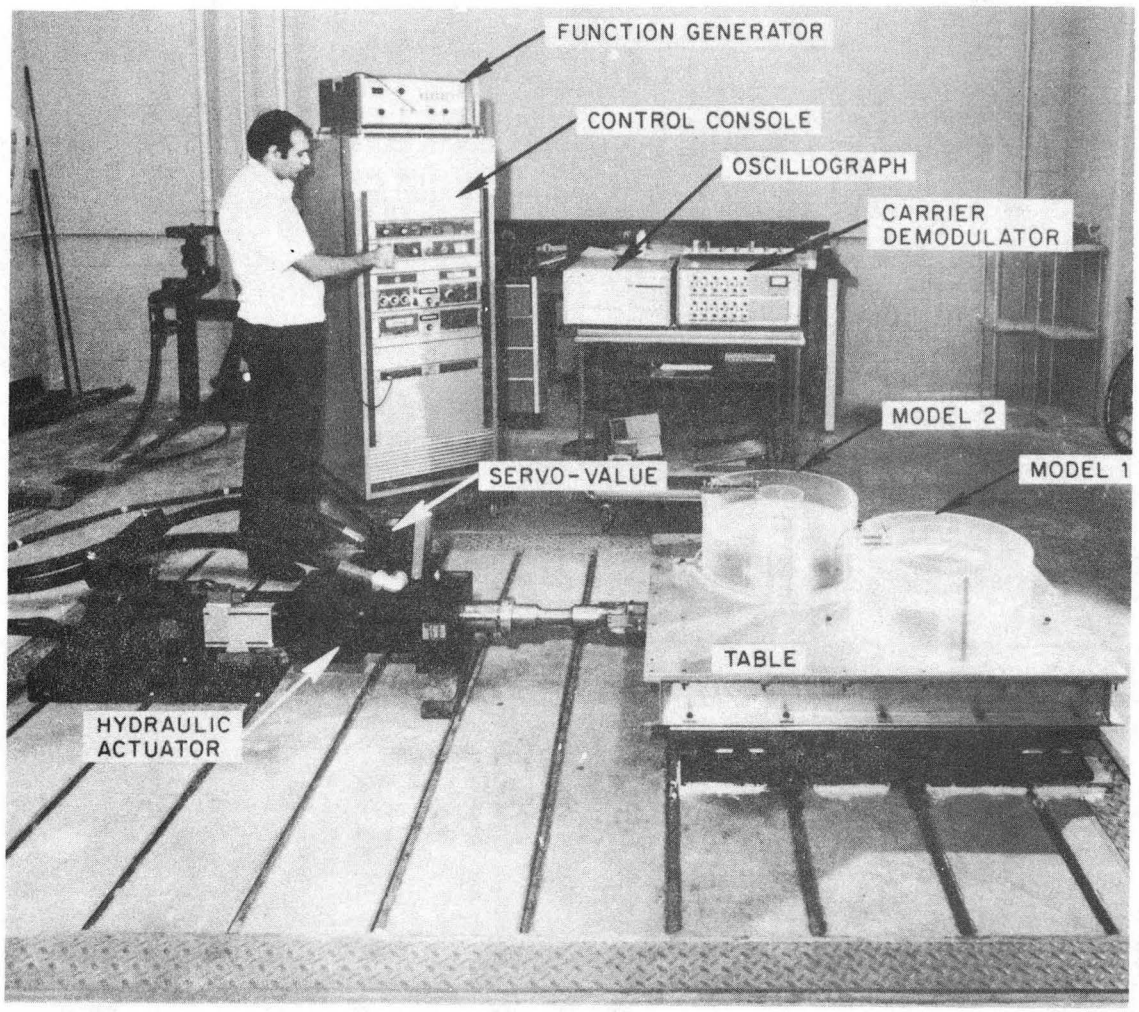


FIG. 4b PRESSURE DISTRIBUTION IN PROTOTYPE
(1 IN. = 2.54 cm, 1 LB. = 4.5 N)

XBL782-7208



CBB 762-1195A

FIG. 5a. TEST SET-UP FOR SMALL SCALE MODEL TESTS

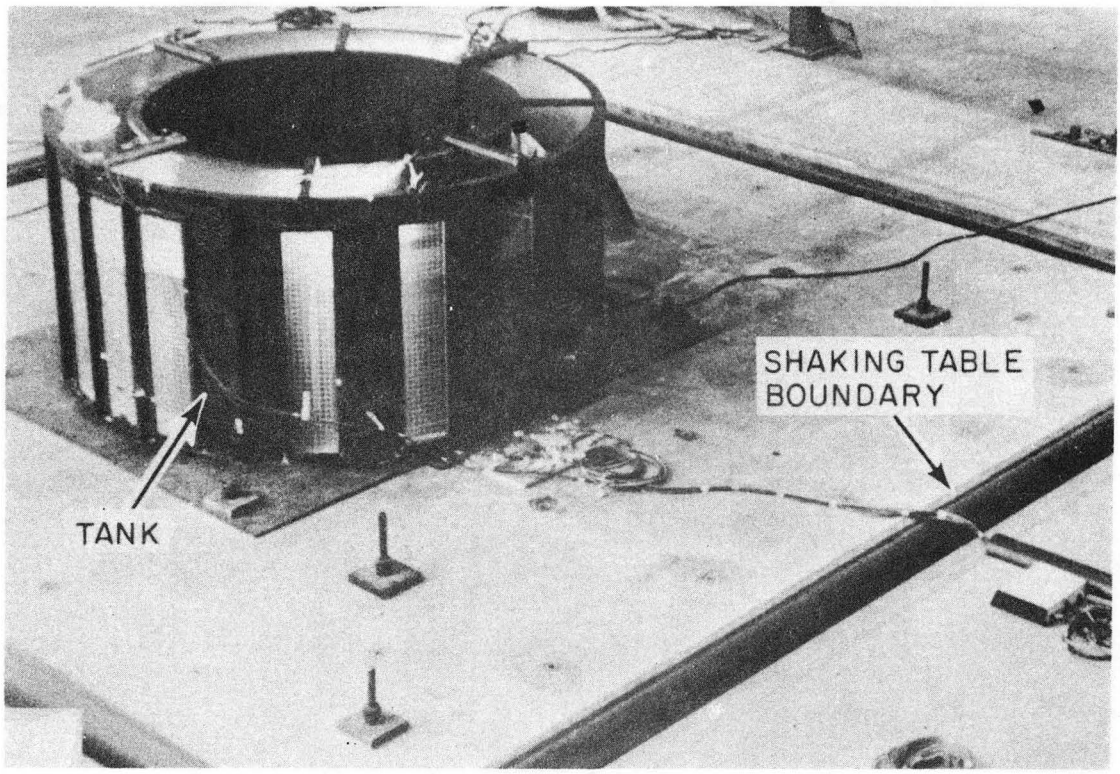


FIG 5b. 8 FT. STEEL TANK ON 20 × 20 FT. SHAKING TABLE (1 FT. = 0.305 M)

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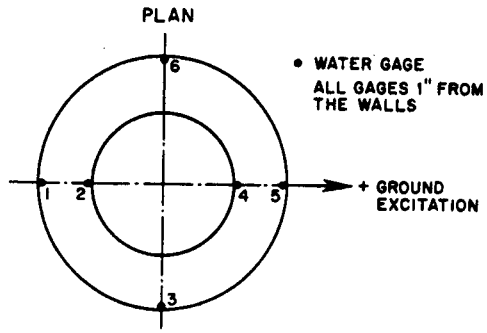

FIG. 6 LOCATION OF WATER DISPLACEMENT GAGES (1 IN. = 2.54 cm)

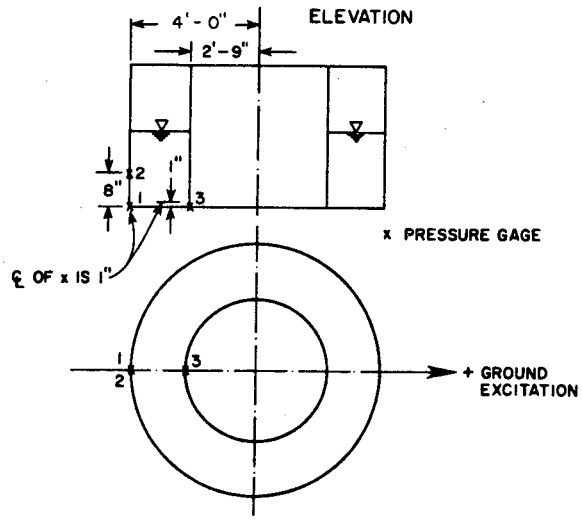

FIG. 7 LOCATION OF PRESSURE GAGES (1 IN. = 2.54 cm)

XBL 782-7215 A

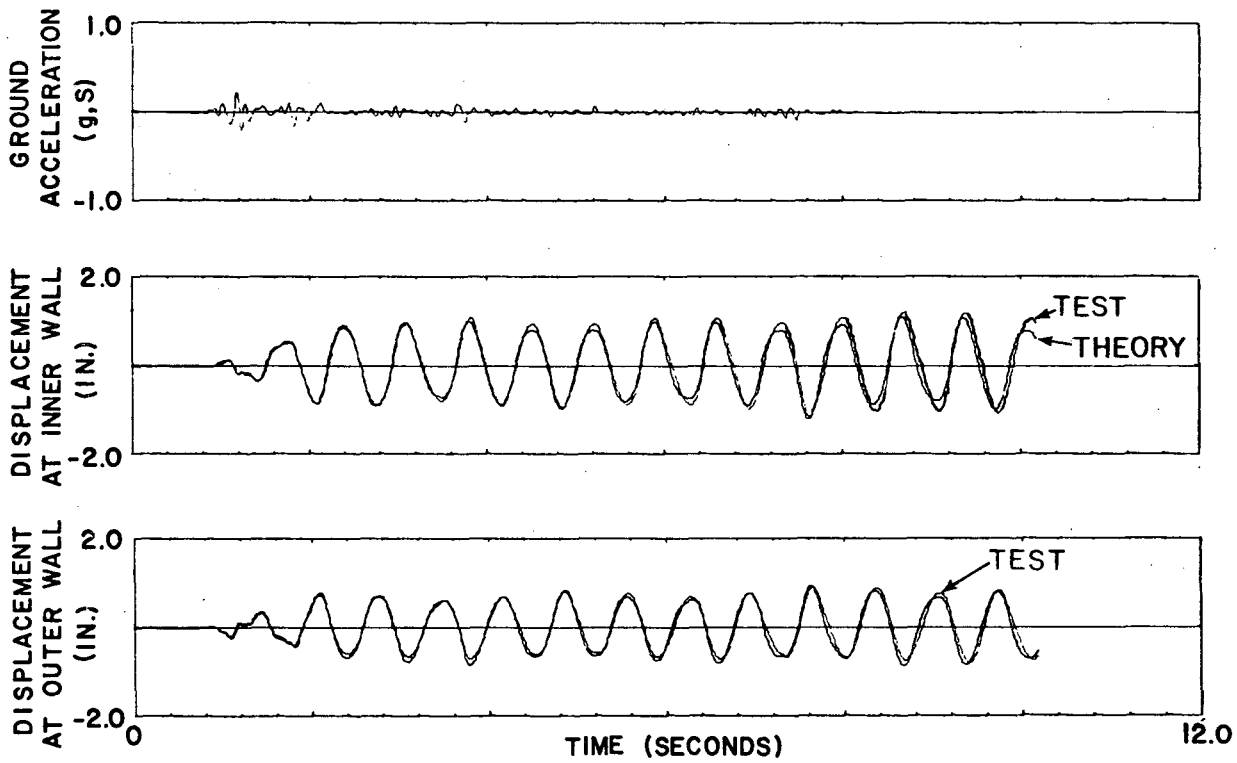

FIG. 8 SLOSHING RESPONSE OF WATER IN ANNULAR TANK (INNER RADIUS = 33.2 IN., OUTER RADIUS = 48.0 IN., DEPTH OF WATER = 16.0 IN.) UNDER EL CENTRO EQ. 1940 (COMPARISON OF THEORY AND TEST 211276.1 (1 IN. = 2.54 cm)

XBL782 7209A

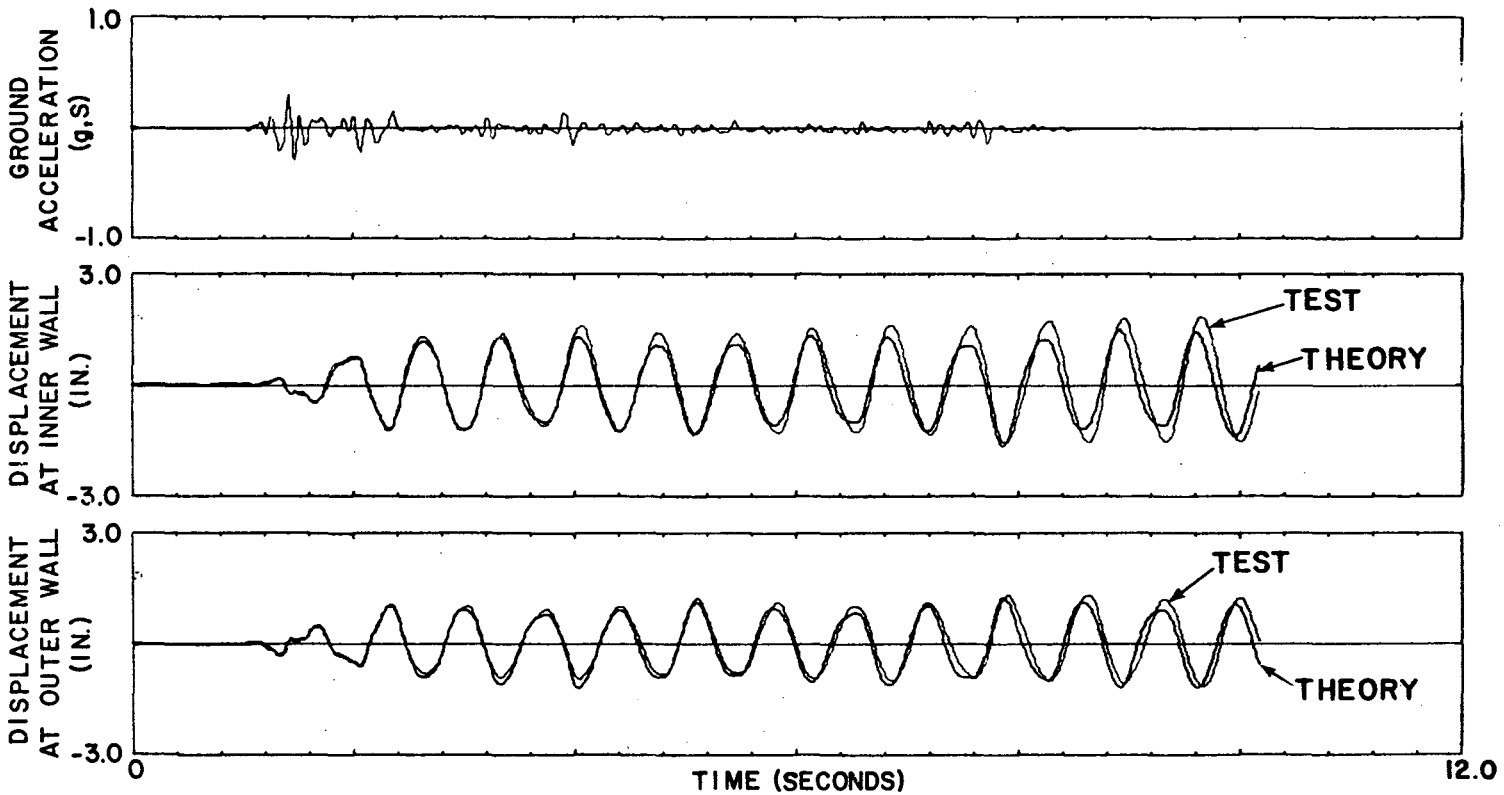

FIG. 9. SLOSHING RESPONSE OF WATER IN ANNULAR TANK (INNER RADIUS = 33.2 IN., OUTER RADIUS = 48.0 IN., DEPTH OF WATER = 16.0 IN.) UNDER EL CENTRO EARTHQUAKE (COMPARISON OF THEORY AND TEST 201276.21) (1 IN. = 2.54 cm)

XBL783-501

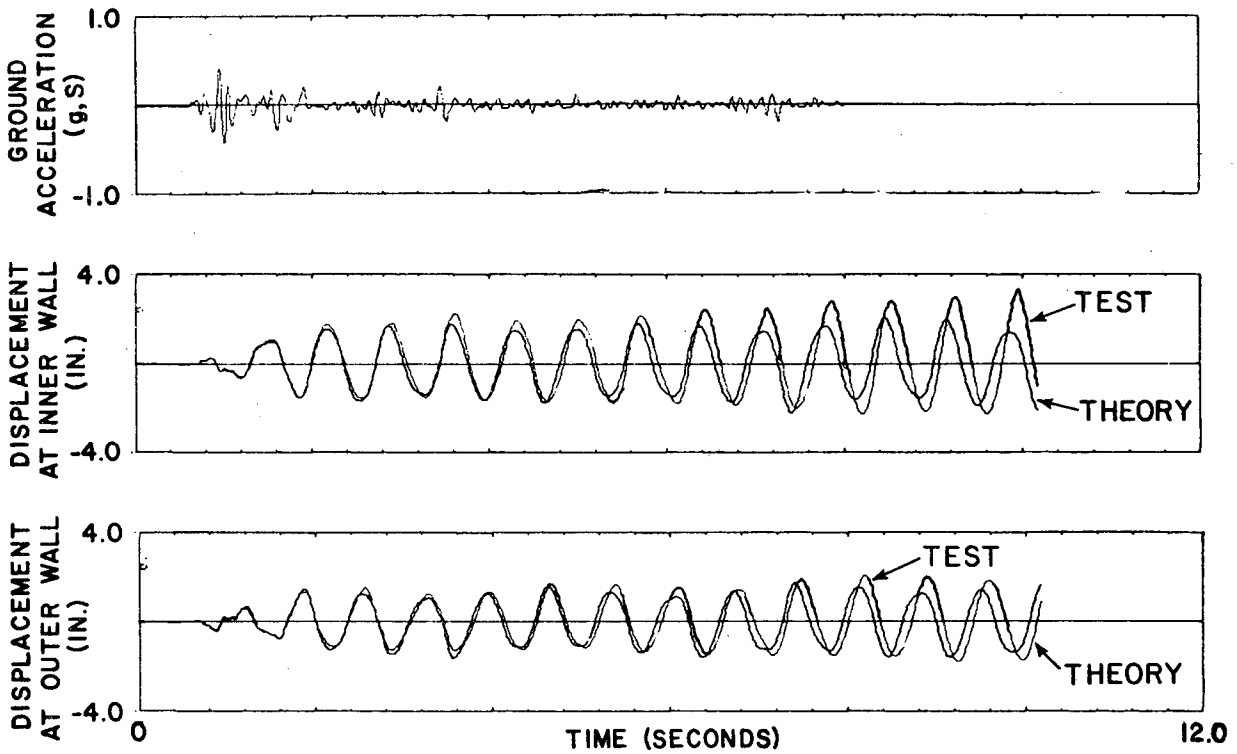

FIG. 10. SLOSHING RESPONSE OF WATER IN ANNULAR TANK (INNER RADIUS = 33.2 IN., OUTER RADIUS = 48.0 IN., DEPTH OF WATER = 16.0 IN.) UNDER EL CENTRO EARTHQUAKE 1940 (COMPARISON OF THEORY AND TEST 211276.2) (1 IN. = 2.54 cm)

XBL782-7216A

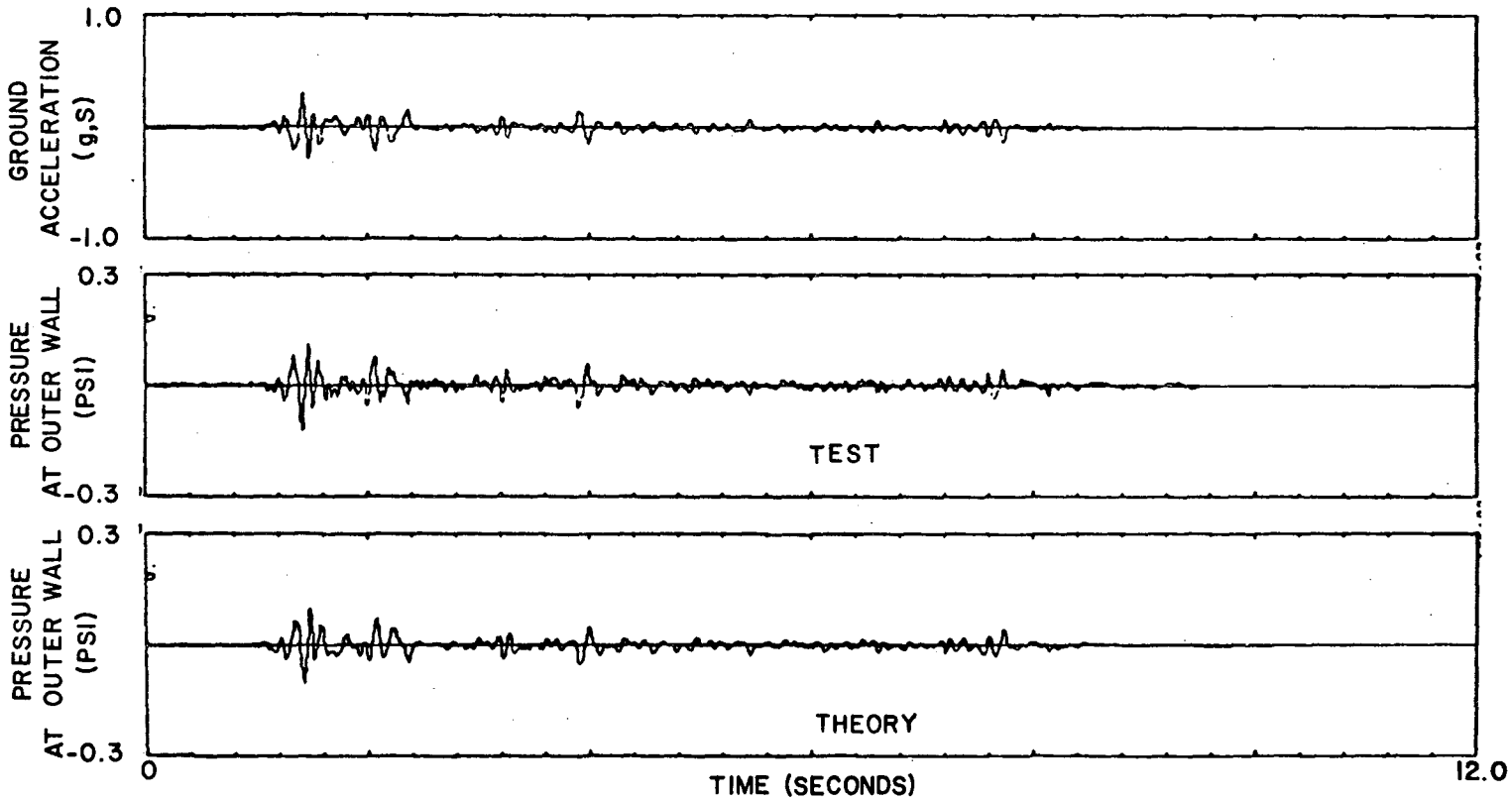

FIG. 11 PRESSURE RESPONSE OF WATER IN ANNULAR TANK (INNER RADIUS = 33.2 IN., OUTER RADIUS = 48.0 IN., DEPTH OF WATER = 16.0 IN.) UNDER EL CENTRO EARTHQUAKE 1940 (COMPARISON OF THEORY AND TEST 201276.21) (1 IN. = 2.54 cm, 1 LB. = 4.45 N).

783-502

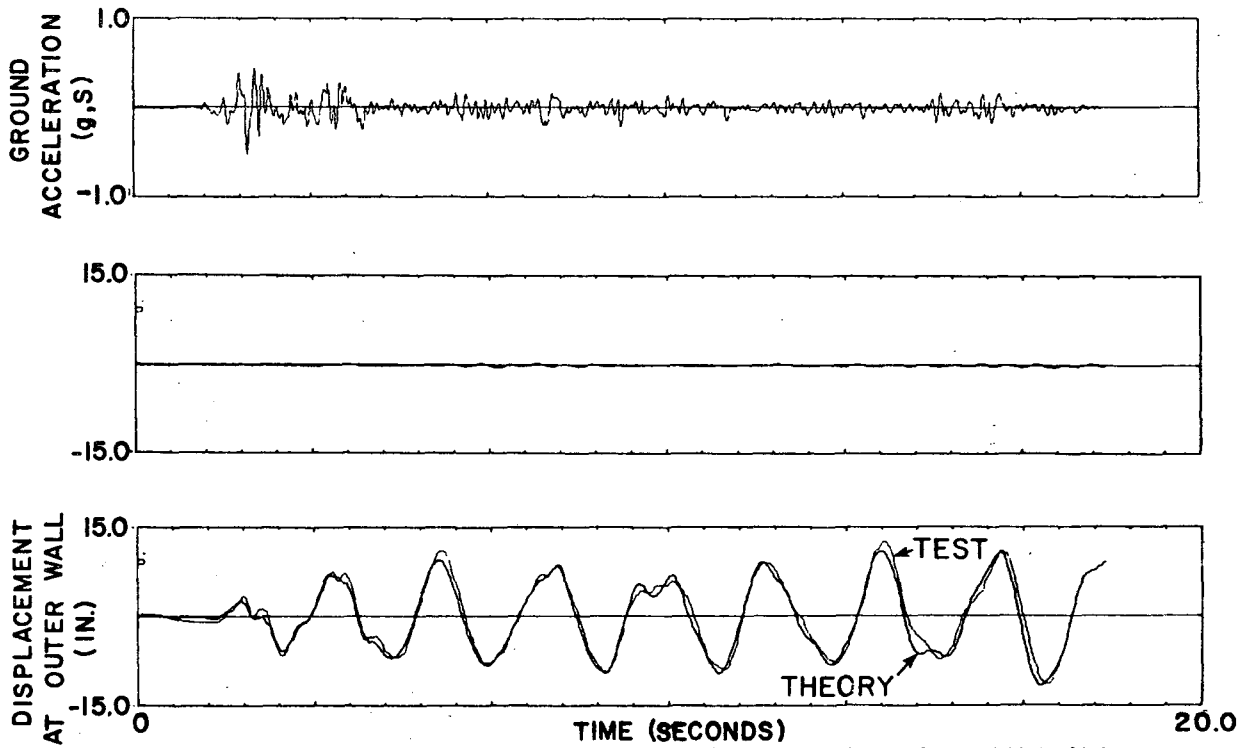

FIG. 12 SLOSHING RESPONSE OF WATER IN ANNULAR TANK (INNER RADIUS = 0.1 IN., OUTER RADIUS = 72.0 IN., DEPTH OF WATER = 60.0 IN.) UNDER EL CENTRO EARTHQUAKE (1940)
TIME SCALE = 1.73 (1 IN. = 2.54 cm)

XBL 782-7213A

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APPENDIX II—NOTATION

a	=	Outer radius of the tank
b	=	Inner radius of the tank
g	=	Acceleration of gravity
h,H	=	Depth of water in the tank
J_m	=	Bessel functions of first kind and order m
J'_m	=	Derivative of J_m
p	=	dynamic pressure
r	=	Radius
t	=	Time
U_r	=	Velocity in r direction
U_θ	=	Velocity in θ direction

U_z	=	Velocity in z direction
x	=	Ground displacement
$\dot{x}$	=	Ground velocity (dx/dt - dot means derivative w.r.t.t)
Y_m	=	Bessel functions of second kind and order m
z	=	z-Coordinate direction
ϕ	=	Velocity potential
θ	=	θ-Coordinate direction
ω_n	=	Sloshing frequency (circular)
ω_o	=	Ground frequency (circular)
δ	=	Water displacement
δ_{max}	=	Maximum water displacement

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