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GENERAL TOPOLOGICAL EXPANSION OF HADRONIC AMPLITUDES

Jean-Pierre Sursock

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GENERAL TOPOLOGICAL EXPANSION OF HADRONIC AMPLITUDES*

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April 26, 1978

ABSTRACT

A classification of products of general amplitude graphs in terms of three indices is presented. The classification is shown to be consistent, i. e., indices corresponding to the product of two amplitudes are completely determined by the indices of the members of the product and the specified intermediate state. In conjunction with the underlying ordered S-matrix, the proposed topological expansion provides the basis for a complete (unitary) theory of strong interactions. In the meson sector, the new topological classification reduces to the "standard" one based on handles and boundaries.

TABLE OF CONTENTS

		Page
1.	Introduction	3
2.	General ordered amplitudes	4
3.	Topological amplitudes	6
4.	Mesonic sector revisited	8
5.	Hadronic amplitudes	11
	5.1 Algebraic notation	11
	5.2 Products of amplitudes	13
	5.3 Boundary structure	14
	5.4 The r-index	16
	5.5 The g-index	19
	5.6 Algebraic notation	22
	5.7 1/N expansion	25
7.	Conclusion	26
8.	References	27
Appendix A:	Edmonds orbits	28
Appendix B:	Edmonds orbits and Algebraic notations	29
Figure captions		33

1. Introduction*

The objective of the Dual Topological Unitarization (DTU) program is to systematically approach complete satisfaction of unitarity for physical transition amplitude through an infinite sum of simpler structures called topological amplitudes. Formally, one may write the transition amplitude between states $|i\rangle$ and $|f\rangle$ as

$$T_{fi} = \sum_{\alpha} (T_{\alpha})_{fi} \qquad (1)$$

where α stands for a number of "indices" completely characterizing the topological amplitudes. The word "indices" is used rather loosely here (for historical reasons). It refers not only to the integers g and r to be discussed in Sec. 5 but also to the boundary structure of the amplitude (cf. Sec. 5) which is a graph, not an integer. (In the early days of the topological expansion only the number of boundaries was considered important for the $1/N$ expansion). Thus, the symbol α represents a couple of integers (g and r) as well as the specification of interconnections between vertices representing particles (the boundary structure). Henceforth, this set will be referred to as the "indices" of a topological amplitude.

* This paper is a continuation of Ref. 1.

The adjective "topological" describes the fact that the analytical structure of the T_{α}'s is related to the topological properties of associated dual-like diagrams. This adjective will be used for all amplitudes T_{α} except those corresponding to the lower order of the expansion (the ordered amplitudes).

2. General ordered amplitude

The leading terms in the expansion of Eq. (1) are called ordered amplitudes.[2,3] The ordered amplitudes are defined in an ordered Hilbert space and obey unitarity.[1,3] In this section we only briefly recall the basic properties of ordered hadronic amplitudes. For more details, the reader is referred to Refs. 1, 3, and 9.

1. The order between particles of a given state is represented by a colored and oriented graph relating each particle to its neighbors. In the ordered Hilbert space, two states with the same particles (same spin and momenta) but differing by the order i. e., by the graph representing the connections between particles are considered different.

2. Not all graphs are allowed to represent ordered states. It can be shown[1] that the most general graphs are those constructed starting with cubic trees - i. e., such that all internal

vertices are of degree three - and expanding according to the following operation (cf. Fig. 1):

(a) Select an edge and cut it in two parts (color i).

(b) Connect the two vertices newly created with two edges colored j and k ($j \neq k \neq i$) and with opposite orientation.

(c) Iterate as many times as necessary with any edge.

It can be shown that in such graphs all edges are distinguishable.

3. Particles (poles) are represented by subgraphs of these graphs. In general particles are represented by subtrees. Mesons could be represented either by 2-vertices or by the graph of Fig. 1c. In this paper we use the latter for convenience of presentation (all vertices are 3-vertices) but the 2-vertex representation can be shown to be equivalent.

4. A colored oriented tree graph and all graphs derived from it by the operation above form a sector of the ordered Hilbert space. The tree graph is called the skeleton of the sector.

5. Ordered amplitudes are non-vanishing only between states belonging to the same sector. The ordered amplitude graph (OAG) is obtained by matching the "corresponding outer edges" of each state. In this context, two "corresponding" outer edges refer to the same outer edge of the common skeleton graph.

6. Among many topological properties of the OAG one of the most important is the existence of mates. Given a vertex A, there exists a unique vertex A' such that A and A' are connected by three disjoint paths (equivalently, A and A' are on the same three faces). A and A' are called mates. An example is shown on Fig. 2. A_1 and A_2 are mates as well as B_1 and B_2; and C_1 and C_2.

7. OAG's are planar, 3-colorable and bipartite. As a result, without loss of generality, the graphs can be imbedded on a sphere such that the orientation of the vertices alternates on any cycle. (By convention, a vertex is positively oriented if the cyclic order of the colored edges is clockwise and negatively oriented if the cyclic order is anticlockwise). For instance, in the graph of Fig. 2, A_1, C_1 and B_2 are positively oriented whereas A_2, B_1, and C_2 are negatively oriented. Once imbedded on an orientable surface (e. g., sphere), the orientation of the edges (i. e., arrows) becomes redundant and henceforth will be dropped.

3. Topological amplitudes

In order to get rid of the order degree of freedom, one is led to define a new amplitude as the sum of all possible ordered amplitudes containing the relevant particles.[3] For historical

reasons, this new amplitude is called "planar amplitude". An example is shown on Fig. 3.

One quickly realizes that planar amplitudes do not obey unitarity since the product of two planar amplitudes includes unordered products.* The unordered products can be classified in terms of their topological structure. More specifically, it will be shown that they can be classified in terms of three "indices". The topological structure of a product being related to the analytic structure of the associated complex function. All products belonging to a given classification (i. e. with a specific set of "indices") are then attributable to the discontinuity formula of a topological amplitude characterized by the same "indices."

Since the planar amplitude is not unitary it must be corrected. The corrections include these topological amplitudes generated through unordered products by the mechanism described above. One expects a better approximation (i. e., closer to unitarity) to be achieved by adding to the planar amplitude

* The expression "ordered product" refers to products of amplitudes that could appear in discontinuity formulae of ordered amplitudes. The "unordered products" never appear in such formulae.

all the topological amplitudes found in products of planar amplitudes. This new amplitude is still not unitary because a product of two of these new amplitudes will generate still higher order topological amplitudes. The process needs to be repeated indefinitely. Of course, to be meaningful this topological expansion must comverge at a reasonable rate. There are numerous indications - although no rigorous proof - that such is the case.

4. Mesonic sector revisited

Mesonic amplitudes, with simple sequential order have been extensively studied. The original concept, due to Veneziano[4], made use of dual diagrams in the Feynman spirit and interpreted the expansion in terms of the parameter $1/N$ where N is the number of flavors. Two topological properties of the graph were identified as affecting the power of $1/N$.

(a) The genus of the graph (h_v)

(b) The number of boundaries (B_v). This latter number is essentially the number of orbits (see Appendix A) to which external particles are attached. Later, Chew and Rosenzweig[2] reinterpreted the diagrams in the S -matrix spirit and have shown that

(1) the genus of the graph is related to the strength of the discontinuity

(2) the number of boundaries is related to the pole structure.

Thus, h_v and B_v characterize the analytic structure of the function associated to the graph.

Three types of graphs have been used to represent the topological amplitudes of mesons. In the mesonic sector they are all equivalent. Examples are shown in Fig. 4. Figure 4a is a representative of the quark (or dual) diagrams(lines represent quarks); whereas Fig. 4c is the equivalent particle diagram (lines represent particles - they are not oriented). Figure 4b is the equivalent hybrid diagram (oriented lines represent quarks, non-oriented lines represent particles). It turns out that the most useful type of diagram for generalization purposes is a modified version of the hybrid diagrams. Figure 5 is equivalent to the diagrams of Fig. 4 ($h_v = 1$; $B_v = 1$) except that each particle line is replaced by two particle lines.* At each vertex, the cyclic order is alternatively quark line and particle line. A particle line either starts and ends "outside"

* It is tempting to associate the pair of particle lines with quarks as for dual diagrams. However, such association will not be made in this paper; our particle lines are not oriented.

the quark rings or starts and ends "inside" the quark rings. ("Outside the ring" means that at any vertex, a clockwise rotation goes from the incoming quark edge to the outgoing quark edge. The situation is opposite if we are "inside the ring" as in Fig. 5 for instance).

The genus, h, of the graph in Fig. 5 and the number of orbits, f, are higher than in the corresponding graphs of Fig. 4. However, it is easy to see that we have a general relation relating the new and old quantities:

$$\begin{aligned} h &= 2h_v + 1 \\ f &= 2f_v . \end{aligned} \tag{2}$$

Now, the orbits of the new graphs can be coupled in pairs such that both members of a given pair have the same particle structure (number <u>and</u> order). The boundary is now redefined as being a couple of orbits with the same particle structure. Further, if the number of quark rings (i. e., the number of connected graphs when all particle lines are removed) is d instead of 2 as in the previous example, then we have the more general formula:

$$\begin{aligned} h &= 2h_v + d - 1 \\ B &= B_v \end{aligned} \tag{3}$$

As will be discussed later, the amplitude is still characterized by h_v (and B_v) and not h. (More specifically, the index g to be discussed in Sect. 5 is equal to $2h_v$ for mesonic amplitudes).

5. Hadronic Amplitudes

5.1. Construction of Topological Amplitudes

The construction of higher order topological amplitudes is carried through products of amplitudes of lower order. In as much as these products define the higher amplitudes, a topological amplitude is defined by its discontinuities.

A necessary and sufficient set of conditions to define a consistent topological expansion can be stated as follows:

(a) The first condition is to define a multiplication rule for the OAG consistent with the unitarity requirements at the ordered level. In other words, the multiplication rule should be such that the ordered product of two OAG can be interpreted as representing the discontinuity of an ordered amplitude in an ordered discontinuity formula.

(b) The next condition is to characterize the graphs representing the product of two OAG by a set of "indices" such that substitution of an ordered subgraph of the intermediate

state* of the product by its skeleton (single particle) - or conversely - does not alter the indices and substitution of unordered subgraphs by their skeletons alters at least one index. In other words, these "indices" completely determine the product relative to an ordered product.

(c) The particular product - or discontinuity - can then be attributed to a topological amplitude carrying the same "indices." Therefore, by definition, the topological amplitude can be represented by any (product) graph carrying the relevant "indices". In practice however one chooses the simplest graph carrying these "indices". The algebraic notation, to be introduced later, automatically incorporates this feature.

(d) The product of two general topological amplitudes must be defined by the same multiplication rule as in condition (a) and carry the same set of "indices" as found in (b). Furthermore, knowledge of the "indices" of both members of the product and

* A subgraph of the intermediate state is a connected subgraph such that all vertices belong to the intermediate state. This subgraph is "ordered" if the corresponding vertices on either side of the product are connected in exactly the same way (ordered product) and is "unordered" otherwise.

specification of the particular intermediate state should be sufficient to determine the set of "indices" characterizing the product.

In summary, the topological amplitudes are defined through products of lower order amplitudes. They are characterized by a set of "indices" related to the topological structure of the associated graphs. These "indices" are selected by the multiplication rule of the lower order graphs and by the requirements of consistency and completeness (i. e., products of topological amplitudes are defined by the same set of "indices" as either member of the product and this set is completely determined by the indices of the members of the product and the intermediate state under consideration).

5.2 Product of two amplitudes

Generalizing the rule for products of mesonic amplitude graphs discussed in Sec. 4 to general ordered amplitude graphs (regular cubic graphs), we connect corresponding vertices belonging to the intermediate states (henceforth called intermediate vertices) with <u>three</u> "particle lines"; each starting

between a different couple of "quark lines" (i. e., colored edges) (i,j), (j,k) or (k,i) and ending between the corresponding couple of quark lines on the other graph. Simple examples are shown in Figs. 6 and 7. When the detailed structure of the intermediate lines (particle lines) is not needed, a wavy line will be drawn instead of three particle lines (e. g., Fig. 8).

We now discuss the topological indices in details. These indices are first obtained in products of amplitude graphs as discussed in Sec. 5.1. They are then - and only then - associated with amplitude graphs.

5.3. The boundary structure

As in the mesonic case, the boundary structure is defined in terms of Edmonds' orbits (Appendix A). We shall say that two orbits are <u>connected</u> if they have at least one vertex in common.

Then, given a graph representing either an amplitude or a discontinuity we group the orbits in sets of connected orbits. Each set will be called a <u>boundary</u>. The number of boundaries will be designated by B and the external vertices belonging to a given boundary are unambiguously interconnected. Vertices (and particles) belonging to different boundaries are completely disconnected. As an example, the graph of Fig. 6 has a single

boundary whereas the graph of Fig. 7 has two boundaries. In general, every boundary can be represented by a regular cubic graph. Note that the boundary structure need not be an ordered graph or even a planar graph but it must be bipartite. It is, however, very important to recognize that once the connections between vertices in a boundary are specified, the number of orbits of the boundary is completely determined.*

As was true for the mesonic amplitudes, the boundary structure is related to the pole structure of the amplitude.[2] The amplitude has a pole in a particular channel - defined by a specific partition of the external vertices into initial and final states - if and only if there exist a graph representing the amplitude (i. e., with the relevant "indices") that could be cut into two connected subgraphs such that one connected subgraph contains the vertices of the initial (or final) state only and the other subgraph contains all other vertices (i. e., all intermediate vertices and all vertices of the final (initial) state). For instance, the amplitude of Fig. 8 has poles in the channel $AD \to BC$ but not in the channel $AB \to CD$.

* In contrast with a discontinuity graph, a graph representing an amplitude contains by convention the minimum number of orbits compatible with the boundary structure.

5.4. The mate-remoteness index (r-index)

We first define the concept of mate walk. All graphs we are concerned with are composed of a certain number of ordered amplitude graphs (OAG) connected to one another by particle lines (wavy lines in Fig. 9). In each OAG, the mating is well-defined. However, some vertices may be mated to intermediate vertices; (i. e., to which wavy lines are attached). For instance, in Fig. 9a, A is mated to B. A thereby "loses" its mate but eventually retrieves one in F. The path ABCDEF is called a mate walk. Similarly, the path MNPQRSTU is a mate walk. A and F, M and U are called remote mates.

Formally, the operation of finding the remote mate of A is to go to its mate in the OAG (B); then jump to the counterpart of that mate in the other OAG (C); go to its mate (D), and so on, until one reaches an external vertex (no particle - or wavy line attached).

Since no such operation is required for X and Y in Fig. 9a, they are called close mates.

In Fig. 9b similar situations are displayed except that no external vertex appears in the walk. Such a closed walk will be called circuit. Thus ABCD is a circuit and EFGHIJKL is another. X and Y as well as Z and T are close mates.

Finally a circuit with only four vertices, as ABCD in Fig. 9b, is called a trivial circuit or <u>ordered circuit</u> because it involves an ordered product.

We can now define the remoteness index of the product of two OAG as

$$r \equiv \sum_{w} \left[\frac{b}{2}\right]_{w} - C_{o} \qquad (4)$$

where the sum is over all walks (open and closed), b is the number of wavy lines belonging to a given walk and C_o is the number of ordered circuits. The notation $[x]$ is defined by

$$[x] \equiv \sup \{n \in \mathbb{N} / n \leq x\} \qquad (5)$$

for any real number x. That is, $[x]$ is the largest integer smaller than or equal to x. (e. g., $[3/2] = 1$)

Note that r is completely determined by the boundary structure of the members of the product and by the specification of the intermediate state.

A discontinuity with $r \neq 0$ belongs to an unordered topological amplitude with corresponding r-index. We can now define the remoteness index for a general product. Consider

a product of two amplitudes M_1 and M_2 with respective indices r_1 and r_2. Then the resulting remoteness index is

$$r = r_1 + r_2 + r_i \qquad (6a)$$

where

$$r_i \equiv \sum_{w_i} \left[\frac{b}{2}\right]_{w_i} - C_{oi} \qquad (6b)$$

the sum on the right hand side of Eq. (6b) and the term C_{oi} refer, respectively, to the walks and ordered circuits created by the <u>product</u> of M_1 and M_2. Note again that r_i is completely determined by the boundary structure of M_1 and M_2 and by the specification of the intermediate state. We also define for future needs the index r':

$$r' = r_1 + r_2 + \sum_{w_i} \left[\frac{b}{2}\right]_{w_i} - C_i \qquad (7)$$

where C_i is the total number of intermediate circuits created by the product.

In general, $2r$ represents the minimum number of additional 3-vertices (uncorrelated baryons*) in the simplest discontinuity

of the amplitude relative to the corresponding ordered amplitude. Thus, a topological amplitude with index r will not have a discontinuity with less than $2r$ 3-vertices in the channel graph representing the intermediate state.

5.5 The g-index

The g-index can be thought of as a renormalized handle or genus index of the mesonic amplitudes. The g-index of a product of two graphs representing amplitudes is defined by

$$g = h - b + c - d + 1 \qquad (8)$$

where h is the topological number of handles (genus) computed with Euler's formula and Edmonds' orbits; b is the number of wavy lines (intermediate 3-vertices on either side of the product); c is the number of circuits and d is the number of OAG included in the product (or number of connected parts if all particle lines are eliminated).

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* Uncorrelated baryons are pairs of baryons antibaryon that cannot be reduced to a single meson. Prof. G. F. Chew has suggested a potential suppression mechanism related to this fact.

For ordered products, g is always equal to zero.

As was true for the h-index, to each topological amplitude we associate an index g such that all its discontinuities possess the same value of g.

When performing the product of two amplitudes M_1 and M_2 with respective indices g_1 and g_2 we generally get a graph with a new index g. We now shown how to compute g in terms of g_1, g_2 and the boundary structure of M_1 and M_2. In this section, the subscripts 1, 2 and i refer to the members of the product M_1 and M_2 and to the intermediate state respectively. The symbols without subscripts refer to the graph product. The letters h, v, e, f symbolize the number of handles, vertices, edges and orbits (faces) while g is defined by Eq. 8. We shall extensively use Euler's formula:

$$2h = 2 - v + e - f . \qquad (9)$$

We first determine h as a function of h_1 and h_2. Starting from Eq. 9 applied to M_1 and M_2 respectively and adding, we get,

$$2(h_1 + h_2) = 4 - (v_1 + v_2) + (e_1 + e_2) - (f_1 + f_2).$$

Now, subtracting from Eq. (9) and using $v = v_1 + v_2$; $e = e_1 + e_2 + e_i$; we get:

$$h = h_1 + h_2 + \frac{1}{2}(e_i - f_i - 2) \tag{10}$$

with the formal definition:

$$f_i \equiv f - (f_1 + f_2). \tag{11}$$

We can also obviously write:

$$b = b_1 + b_2 + b_i \tag{12}$$

$$c = c_1 + c_2 + c_i \tag{13}$$

$$d = d_1 + d_2 \quad , \tag{14}$$

and therefore, after substitution of Eq. (10) through (14) in Eq. (8) and some straightforward algebra we get:*

* Note: using the formal definition $v_i \equiv 4$; $d_i \equiv 2$; $2h_i \equiv 2 - v_i + e_i - f_i$ and $g_i \equiv h_i - b_i + c_i - d_i + 1$ we get the formulae $h = h_1 + h_2 + h_i$ and $g = g_1 + g_2 + g_i$ (comparable to Eq. (6a)).

$$g = g_1 + g_2 + \frac{1}{2}(e_i - f_i - 4) - b_i + c_i. \tag{15}$$

Now, e_i, b_i and c_i are completely determined by the specification of the intermediate state (note the identity $e_i = 3b_i$), f_1 and f_2 are determined by the boundary structure of M_1 and M_2 and f is determined by the boundary structure and the specification of the intermediate state since, to determine the Edmond orbits we only need to know the connections between the vertices. Thus, f_i is completely determined by the boundary structure of M_1 and M_2 and by the specification of the intermediate state. Therefore, given g_1 and g_2, the boundary structures of M_1 and M_2 and the intermediate state of the product, g is completely specified by Eq. (15).

5.6. Algebraic notation

We now discuss how to write algebraically (as opposed to graphically) the boundary structure of an amplitude and its r and g indices. The major advantage of this notation is to exhibit the fact that topological amplitudes are independent of their graphical representation. Only the essential features common to all graphs (i. e., the connections between external vertices) are preserved in this notation.

(a) Representation of boundary structure: We recognize that, within a boundary, only the connections between vertices are significant. Therefore, we associate to each vertex V a set of three indices: $\alpha_i, \alpha_j, \alpha_k$ (representing the three edges attached to V) and we write $V(\overrightarrow{\alpha_i\alpha_j\alpha_k})$. The order or the α-indices is important as it corresponds to the color index of the associated edge. The left-to-right arrow corresponds to a positively oriented vertex and the right-to-left arrow $[V(\overleftarrow{\alpha_i\alpha_j\alpha_k})]$ corresponds to a negatively oriented vertex.

The boundary structure is defined by listing all vertices of the amplitude with the relevant α-indices. Two vertices connected by k edges in the graphical representation will have k α-indices in common in the algebraic notation (k = 1, 2 or 3). Since different boundaries have no α-index (edge) in common they are separated by a semi-column in the list.

Finally, it is useful to specify pairs of mates by an arbitrary integer. The two members of the pair would then carry the same number and would be readily identified. For example, $\{V(n,\overrightarrow{\alpha_1\alpha_2\alpha_3})\ W(n,\overleftarrow{\alpha_1\alpha_2\alpha_5})\}$.

(b) r and g indices. The remoteness index r and the g index of a topological amplitude M will be specified by a superscript and a subscript respectively (e. g., M_g^r {boundary structure}). Furthermore, the remote mates must be specified since two adjacent remote mates correspond to a different pole structure than two close mates. The specification is done by connecting the remote mates by a bar over the amplitude.

As an example, the algebraic notation corresponding to the graph of Fig. 10 is:

$$M_0^1\ \{\overline{A(1,\overrightarrow{\alpha_1\alpha_2\alpha_3})\ B(1,\overleftarrow{\alpha_1\alpha_2\alpha_3})}\ ;\ C(2,\overrightarrow{\alpha_4\alpha_5\alpha_6})D(2,\overleftarrow{\alpha_4\alpha_5\alpha_6})\}$$

Products of two amplitudes can be computed with this notation. The procedure is detailed in Appendix B. This fact clearly shows that all the necessary ingredients to determine the product of two topological amplitudes are included in r, g and the boundary structure.

Another advantage of the algebraic notation is that it enables a sharp distinction between amplitudes and products of amplitudes or discontinuities. Although they are both represented by the same type of graphs it is crucial - in order to avoid confusion - to have a clear idea whether a given graph is supposed

to represent an amplitude or a discontinuity. In the latter case, the intermediate state is well defined and cannot be altered while in the former case, there is no intermediate state - or particles and some particle lines may be removed - so long as the indices are not altered. Conversely, analytic continuation from one reaction to a crossed reaction is a property of amplitudes, not discontinuities. Failure to recognize these simple facts could lead to a paradoxical situation. The consistent use of the algebraic notation guarantees a clear distinction between amplitudes and discontinuities.

5.7 The 1/N expansion

We conjecture* that the suppression of non-ordered products relative to the corresponding ordered products (i. e., the ordered product with the same particles in the initial, final and intermediate states) can be understood as a 1/N suppression as defined by Veneziano.[4] If the product is characterized by g and r' defined by Eq. (8) and (7) respectively and by B boundaries then in all the examples we have studied there are $g + (B - 1) + r'$ fewer free quark lines than in the corresponding ordered amplitude. Thus the suppression factor is $(1/N)^{g+(B-1)+r'}$.

* This conjecture was originally suggested by Prof. G. F. Chew.

6. Conclusion

A classification of the product of general amplitude graphs in terms of three topological indices was presented. The classification was shown to be self-consistent; i. e., the indices corresponding to the product of two amplitudes are completely determined by the indices of the members of the product and by the specification of the intermediate state. The indices completely characterize the graph product and are related to the singularity structure of the associated complex function.

The new classification reduces to the "standard" one in the mesonic sector.

The next phase of the theory should be to analyze the spectrum of particles generated by this approach, to determine more accurately the connection with the analytical structure of the indices, to investigate the mechanisms of convergence and to apply it to practical considerations.

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Appendix A: Edmonds' orbits

The following technique developed by Edmonds[7] and by Young[8] enables one to determine the faces (orbits) of a graph imbedded on any orientable two-dimensional surface.

The principle is to start from some point and to move alongside an edge such that the edge is always on the right (say), until a vertex is reached. At the vertex, we make a clockwise rotation until we hit the next edge* and we "travel" again with the edge to the right until the next vertex where we make a clockwise rotation, and so on. In the end, we come back to the starting point. This round trip is called an orbit. When all the orbits of the graph are determined we should have traversed each edge twice; once in each direction.

The number of orbits, being the number of faces of the graph enables us to compute the genus of the graph through Euler's formula (Eq. (9)) and also to specify the connections between vertices by recording the sequence in which they appear on each orbit.

* This is why the position of particle edges with respect to quark edges is crucial. In fact, when we are on a quark edge of a given color in one OAG, the prescription will keep us on the same color after "landing" on the other OAG.

Appendix B: Edmonds' orbits and algebraic notation

It is possible, although tedious, to determine Edmonds' orbits from the algebraic notation alone. This method is analogous to the one introduced in Ref. 5. We shall describe the general rules and illustrate them with the simple product of Fig. B1. In the algebraic notation, this product is

$$P = M_0^0 \; \{A(1,\overleftarrow{\alpha_1\alpha_2\alpha_3})B(1,\overrightarrow{\alpha_4\alpha_2\alpha_3})F(2,\overleftarrow{\alpha_4\alpha_5\alpha_6})$$
$$E(2,\overrightarrow{\alpha_1\alpha_5\alpha_6})\} \otimes M_0^0 \; \{E(1,\overleftarrow{\beta_1\beta_2\beta_3})F(1,\overrightarrow{\beta_1\beta_2\beta_4})$$
$$C(2,\overleftarrow{\beta_5\beta_6\beta_4})D(2,\overrightarrow{\beta_5\beta_6\beta_3})\} \; . \qquad \text{(B1)}$$

We first introduce some definitions. At each step described below two α-indices are crossed. The first one is called <u>in-index</u> and the second <u>out-index</u>. There are also two types of vertices: those without particle lines (e. g., A, B, C, D) called external vertices and those with particle lines (e. g., E, F) called <u>intermediate vertices</u>. The <u>correspondent</u> of an upper (lower) α-index is the same α-index but appearing as a subscript (superscript) in the <u>same</u> amplitude. The <u>counterpart</u> of an α-index is the α-index appearing in the same position (i. e., corresponding intermediate vertex and same order in the <u>other</u>

amplitude, e. g., the counterpart of α_5 is β_2) and the <u>follower</u> of an index is the next index (cyclic order specified below) on the <u>same</u> vertex. We now specify the general rules to determine the orbits:

(1). The clockwise order is followed on vertices with negative orientation as the counterclockwise order is followed on vertices with positive orientation (hence, the directions of the arrows in Eq. (B1)).

(2) To start, select any index in one amplitude as the first in-index of the sequence.

(3) Now cross the indices two by two in the following sequence:

(a) If you are on an external vertex
- after the in-index go to the follower.
- after the out-index go to the correspondent.

(b) If you are on an intermediate vertex
- after the in-index go to the counterpart.
- after the out-index go to the correspondent.

Thus the "correspondent" is always an in-index; the "follower" or the "counterpart" are always out-indices.

(4) An orbit is completed when the starting index is reached again as an in-index. The sequence should contain an even number of indices.

(5) At the end of the operation, every index should be crossed twice; once as an in-index and once as an out-index.

As an example, the sequences for the product introduced above are given below. The notation $\alpha(V)$ means the index appearing on vertex V. We thus have:

1st orbit = $\alpha_2(A)\alpha_1(A)\alpha_1(E)\beta_1(E)\beta_1(F)\alpha_4(F)\alpha_4(B)\alpha_2(B)$

2nd orbit = $\alpha_3(A)\alpha_2(A)\alpha_2(B)\alpha_3(B)$

3rd orbit = $\alpha_1(A)\alpha_3(A)\alpha_3(B)\alpha_4(B)\alpha_4(F)\beta_1(F)\beta_1(E)\alpha_1(E)$

4th orbit = $\beta_5(C)\beta_4(C)\beta_4(F)\alpha_6(F)\alpha_6(E)\beta_3(E)\beta_3(D)\beta_5(D)$

5th orbit = $\beta_6(C)\beta_5(C)\beta_5(D)\beta_6(D)$

6th orbit = $\beta_4(C)\beta_6(C)\beta_6(D)\beta_3(D)\beta_3(E)\alpha_6(E)\alpha_6(F)\beta_4(F)$

7th orbit = $\alpha_5(E)\beta_2(E)\beta_2(F)\alpha_5(F)$

8th orbit = $\alpha_5(F)\beta_2(F)\beta_2(E)\alpha_5(E)$.

Since there are eight orbits*, Euler's formula yields $h = 2$ and Eq. (8) yields $g = 0$. But we see that $B = 2$ because A and B do not have any orbit in common with C and D. As a result, the amplitude to which the discontinuity belongs can be written as:

$$M_0^0 \ \{A(1,\overleftarrow{\alpha_1\alpha_2\alpha_3})B(1,\overrightarrow{\alpha_1\alpha_2\alpha_3});C(2,\overleftarrow{\alpha_4\alpha_5\alpha_6})D(2,\overrightarrow{\alpha_4\alpha_5\alpha_6})\}.$$

The remote mates can be obtained simply by following a walk of mates as discussed earlier. In our example, all mates are close and the only circuit is ordered.

* Note that they are composed of an <u>even</u> number of α-indices and an <u>even</u> number of β-indices.

FIGURE CAPTIONS

Fig. 1: Elementary operation generating all ordered state graphs

Fig. 2: An ordered amplitude. A_1, B_2, C_1 are positively oriented and A_2, B_1, C_2 are negatively oriented (color cycles). (A_1, A_2); (B_1, B_2); (C_1, C_2) are mates.

Fig. 3: Planar amplitude of four baryons.

Fig. 4: Three equivalent representations of mesonic amplitudes: (a) dual diagram; (b) hybrid diagram; (c) particle diagram.

Fig. 5: New version of hybrid diagram with two particle lines at each intermediate vertex

Fig. 6: Product of ordered amplitudes. The particle lines start and end between the same color couples (i,j).

Fig. 7: Same graph as Fig. 6, but with two boundaries.

Fig. 8: In this product, the two mates A and B (and C and D) are not adjacent. There is no pole in the channel $AB \to CD$.

Fig. 9: Walks of mates: (a) open walks. (b) closed walks (circuit).

Fig. 10: In this product A and B are remote mates. C and D are close mates.

Fig. B1: Product of two ordered amplitudes yielding B = 2.

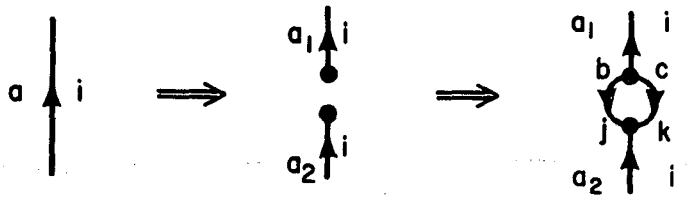

Fig. 1

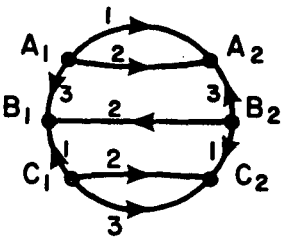

Fig. 2

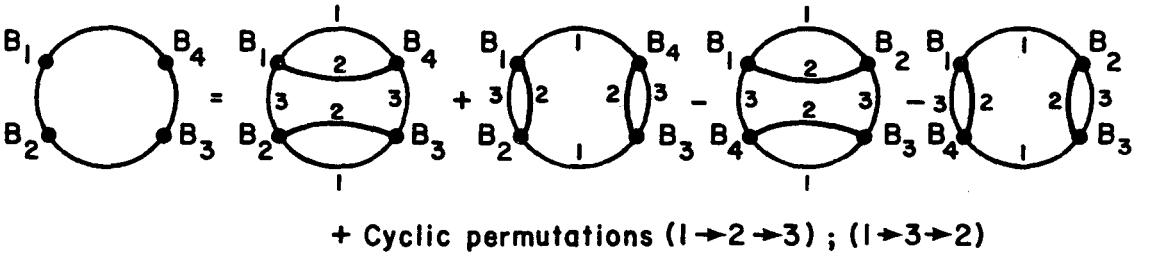

Fig. 3

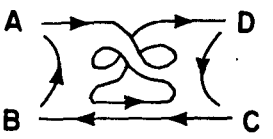

Fig. 4a

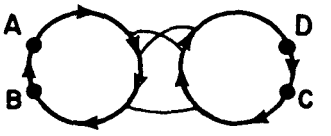

Fig. 4b

Fig. 4c

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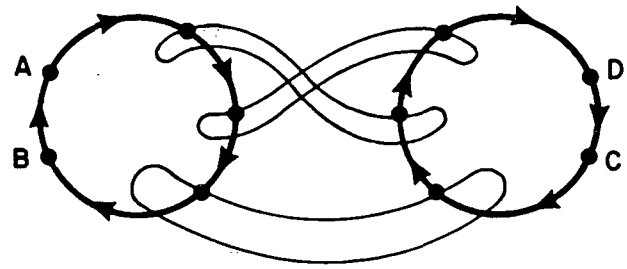

Fig. 5

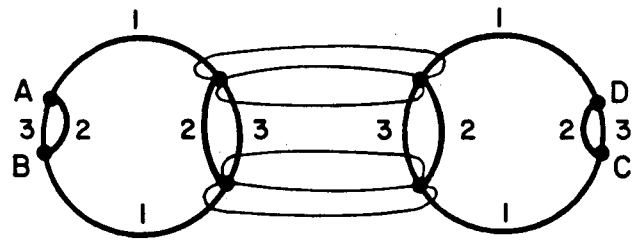

Fig. 6

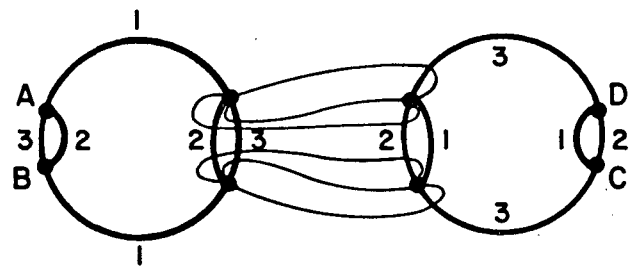

Fig. 7

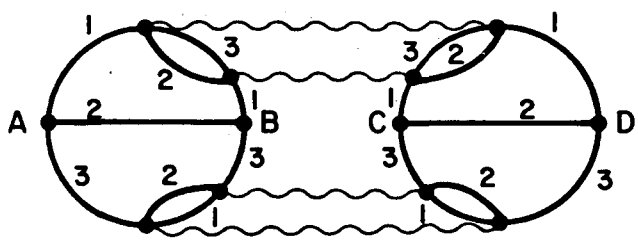

Fig. 8

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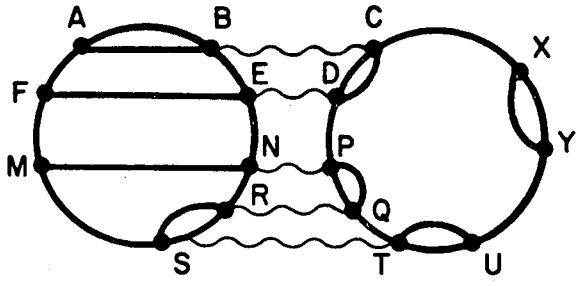

Fig. 9a

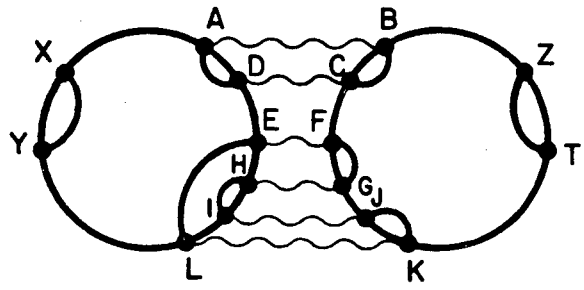

Fig. 9b

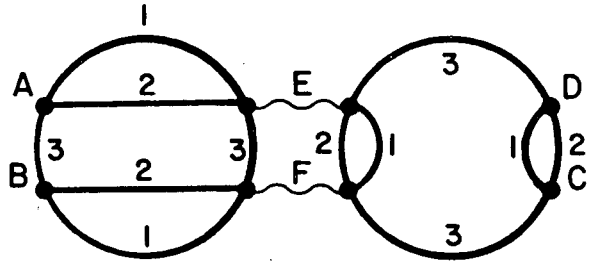

Fig. 10

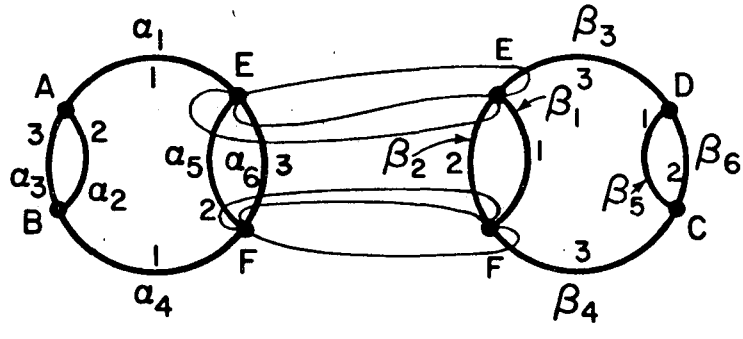

Fig. B1

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