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WEAK INTERACTIONS OF ULTRA HEAVY FERMIONS*

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ABSTRACT

We study the weak interactions of ultra heavy fermions, their scattering at high energies and the renormalization corrections they induce at low energies.

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Motivated by the experimental proliferation of new quarks and leptons and by the theoretical work of Lee, Quigg and Thacker [1] and of Veltman [2] on the consequences of ultra heavy Higgs bosons, we have studied the effect on the weak interactions of ultra heavy fermions with masses of the order of hundreds of GeV. We have explored the nature of the weak interactions above the threshold for production of ultra heavy fermions and have also searched for large radiative corrections which could be measured at presently accessible energies. We assume the standard $SU(2) \times U(1)$ gauge model [3].

Above threshold we compute partial wave amplitudes in tree approximation to leading order in the large fermion mass m_F. The amplitudes for $F\bar{F} \to F\bar{F}$, W^+W^-, ZZ, ZH, HH all grow like $G_F m_F^2$. When m_F is so large that partial wave unitarity is saturated in tree approximation, these amplitudes become strong in that the higher order terms in the perturbation expansion must be greater than or equal to the lowest order term. F, $W^\pm$, Z and H become "sthenons" in the sense of Appelquist and Bjorken [4]: they couple strongly to one another* but weakly to non-sthenons (i.e., the light particles in the theory). The strong coupling parameter $\sqrt{G_F m_F^2}$ is just the scalar field-$F\bar{F}$ coupling of the unbroken theory, "remembered" in the broken theory by the couplings of the longitudinal vector bosons and the physical Higgs boson to heavy fermions. Amusingly, the weak interaction gauge theory

* Strictly speaking, H must also be ultra heavy so that W, Z and H scattering off one another be strong in tree approximation [1]. As noted below, the existence of ultra heavy fermions means that the Higgs boson may also be ultra heavy.

requires that the weak interaction of ultra heavy quarks be strong, while the asymptotic freedom of the strong interaction gauge theory suggests that their strong interactions will be weak. We show below that the critical mass for quarks is about 550 GeV/$\sqrt{N}$, where N is the number of nearly degenerate quark doublets. For leptons the scale is about 1.2 TeV/$\sqrt{N}$. This is like the 1 TeV mass scale obtained in ref. [1] for a strongly coupled Higgs boson.

Below their threshold, ultra heavy fermions would cause perturbation theory to fail because of the appearance of the parameter $G_F m_F^2$ in high orders. If such a correction occurred in one-loop approximation, it could be observed experimentally at presently accessible energies. Veltman [2] showed that there are no such one-loop effects due to an ultra heavy Higgs boson. We have found four examples of one-loop corrections due to ultra heavy fermions: in the couplings of the Higgs boson to light fermions and to the $W^{\pm}$ and Z bosons, and in the ratio $M_W/M_Z \cos\theta_W$. The corrections to the Higgs couplings will modify predictions for the production and decay of the Higgs boson.* The correction to $M_W/M_Z \cos\theta_W$ is only large if the mass <u>splitting</u> within the doublet is ultra large, i.e., of order 1 TeV. For a doublet of equal masses there is no effect proportional to m_F^2. After the work reported here was completed we learned that Veltman [6] has also gone on to study the radiative corrections due to ultra heavy fermions, obtaining the same result as ours for the above mentioned ratio. Using the most recent neutrino and antineutrino scattering

* Such predictions have been discussed by Ellis, Gaillard and Nanopoulos [5].

data, and taking scaling violations into account we find an upper bound of about 550 GeV (at the two standard deviation level) for a heavy lepton in a doublet with a light partner. This is to be compared with Veltman's earlier estimate of about 850 GeV.

We now briefly present the highlights of our results on the partial wave amplitudes at high energies and the radiative corrections at low energies. A more detailed account will be given elsewhere [7].

In tree approximation the $J = 0$ partial wave amplitudes for $F\bar{F} \to F\bar{F}$ and $F\bar{F} \to F'\bar{F}'$ grow like $G_F m_F^2$ and $G_F m_F m_{F'}$. The amplitudes $F\bar{F} \to W^+W^-$, ZZ, HZ, HH also grow like $G_F m_F^2$ but only for partial waves $J > 0$. In this letter we consider only $J = 0$ partial waves since they yield the most restrictive unitarity constraints.

As a simple example, consider the elastic amplitude $F_+\bar{F}_+ \to F_+\bar{F}_+$ where the subscript denotes helicity $+\frac{1}{2}$. Only the Z and H s-channel exchanges contribute to the $J = 0$ partial wave in proportion to m_F^2 ; we find for $s \gg m_F^2 \gg M_W^2$

$$a_0(F_+\bar{F}_+ \to F_+\bar{F}_+) = i\,\frac{G_F m_F^2}{4\sqrt{2}\,\pi} = i\,\frac{g^2}{32\pi}\,\frac{m_F^2}{M_W^2}\,. \qquad (1)$$

Partial wave unitarity implies that $|a_0| < 1$, so the validity of the perturbation expansion in g requires

$$m_F^2 < \frac{4\sqrt{2}\,\pi}{G_F} = (1.2\ \text{TeV})^2\,. \qquad (2)$$

For $m_F^2 \geqslant 4\sqrt{2}\,\pi/G_F$, the $O(g^4)$ terms must make a larger contribution to a_0 than the Born term given in eq. (2). More generally, if $a_0 = c_1 g^2 + c_2 g^4 + \cdots$ and we require $|c_2 g^4/c_1 g^2| \leqslant r \ll 1$ then,

in place of inequality (2) we would have $m_F^2 \leqslant 4\sqrt{2}\,\pi\, r/G_F$. In this sense inequality (2) is conservative: the perturbation expansion begins to fail for appreciably smaller values of m_F .

We may improve the bound (2) by considering the scattering of different helicity and flavor combinations. As in ref.[1], the most restrictive bound is obtained from the largest eigenvalue of the coupled channel matrix.

Consider first a lepton doublet $\begin{pmatrix} L_1 \\ L_2 \end{pmatrix}$. The $J = 0$ coupled channel matrix is 4×4; the four relevant states are $L_{i+}\bar{L}_{i+}$ and $L_{i-}\bar{L}_{i-}$ where $i = 1, 2$ and $\pm$ denotes helicities $\pm\frac{1}{2}$. The diagonal matrix elements are due to s-channel Z and H exchanges, as already noted, and are just given by eq. (1). The matrix elements which are off-diagonal in both helicity and flavor, $L_{i+}\bar{L}_{i+} \longleftrightarrow L_{j-}\bar{L}_{j-}$ with $i \neq j$, receive contributions both from s-channel Z and H exchanges and from t-channel W exchange. If there is no Cabibbo-like angle, these contributions cancel. All other matrix elements also vanish, so that the coupled channel matrix is already diagonal and therefore there is no improvement on inequality (2). But if there is a Cabibbo-like angle, $\begin{pmatrix} L_1 \\ L_2 \cos\theta + \cdots \end{pmatrix}$, then the cancellation of the off-diagonal elements does not occur. Diagonalizing, we find that the inequality is improved by a factor $1 + \sin^2\theta$, i.e.,

$$m_{L_i}^2 < \frac{4\sqrt{2}\,\pi}{G_F} \quad \frac{1}{1 + \sin^2\theta} \quad . \tag{3}$$

If there are N almost degenerate doublets then the resulting $4N \times 4N$ matrix is approximately a direct product of the previous 4×4 matrix times the $N \times N$ matrix with all entries equal to one; diagonalizing for $\theta = 0$ we have, therefore,

$$m_L^2 < \frac{4\sqrt{2}\,\pi}{G_F} \quad \frac{1}{N} \quad . \tag{4}$$

The improvement of the bound (3) for $\theta \neq 0$ can be understood by noting that $\theta = \pi/2$ corresponds to $N \geqslant 2$.

Next we consider an ultra heavy quark doublet $\begin{pmatrix} Q_1 \\ Q_2 \cos\theta + \cdots \end{pmatrix}$, with three colors denoted by R, B, Y. Although the quark scattering amplitudes are not directly observable if quarks are confined, it is nevertheless correct for our purposes to insist that they satisfy partial wave unitarity. We are interested in whether the perturbation expansion for the weak interactions of the quarks is valid or not. Order by order in g, the perturbation expansion of this field theory is consistent with unitarity, and when the partial wave bounds are saturated by the Born terms, the expansion fails and the weak theory has become strongly interacting. This is all we are using the unitarity constraints to demonstrate.

For a quark doublet the coupled channel matrix is 36×36 because of the nine color channels, $R\bar{R}$, $R\bar{B}$, $\cdots$. But only the color-neutral channels, $R\bar{R}$, $B\bar{B}$, $Y\bar{Y}$ are important, so the matrix of interest is 12×12. The key difference from the lepton matrix is that there is no t-channel W exchange in color off-diagonal matrix elements such as $R\bar{R} \to B\bar{B}$, so that the cancellation of flavor-helicity off-diagonal elements does not occur. The largest eigenvalue is

$$|a_0|_{max} = \frac{G_F}{8\sqrt{2}\,\pi}\left\{3(m_{Q_1}^2 + m_{Q_2}^2) + \sqrt{9(m_{Q_1}^2 - m_{Q_2}^2)^2 + 4\, m_{Q_1}^2\, m_{Q_2}^2 (2+\sin^2\theta)^2}\right\}. \tag{5}$$

For $m_{Q_1} \cong m_{Q_2} \equiv m_Q$, the inequality (3) is improved by the factor $5 + \sin^2\theta$

$$m_Q^2 < \frac{4\sqrt{2}\,\pi}{G_F}\,\frac{1}{5+\sin^2\theta}\,. \tag{6}$$

For N nearly degenerate doublets the right-hand side is decreased by $\sim 1/N$. For $\theta = 0$ the bound in eq. (6) implies that $m_Q < 550$ GeV.

Next we consider the radiative corrections induced by ultra heavy fermions in low energy processes. One-loop corrections proportional to $G_F\, m_F^2$ would be experimentally observable. If corrections proportional to $G_F\, m_F^2$ only appeared in two-loop or higher order, they would be too small to detect experimentally. We find contributions proportional to $G_F\, m_F^2$ from one-loop corrections to the W, Z, and H propagators and to the WWH and ZZH proper vertices.

Consider the renormalization of the Higgs boson mass from an ultra heavy doublet $\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$:

$$m_H^2 = m_{H_0}^2 - \xi\,\frac{G_F}{2\sqrt{2}\,\pi^2}\sum_{i=1,2} m_i^4\left(3\,\ln\frac{m_i^2}{\mu^2} - 1\right). \tag{7}$$

Here $\xi = 1$ for leptons and $\xi = 3$ for quarks and μ is the (arbitrary) renormalization point. Notice that the effect is proportional to m_F^4 and that for fermion masses which saturate inequalities

(2) or (6), the renormalization effect is of order m_i^2 (and negative if $\mu \ll m_i$). Successive terms in the perturbation expansion will be of the same order of magnitude and some will have signs opposite to that of the leading term. This suggests that if ultra heavy fermions exist, then the Higgs boson will also be ultra heavy (a remark made by Weinberg in a somewhat different way [8].). The suggestion is plausible but not necessarily correct, since in these circumstances the perturbation expansion may not even allow a qualitative estimate for the mass of the Higgs boson. In particular, Veltman [2] has emphasized the possibility that strong coupling effects in the Higgs sector could sum to yield a Higgs boson mass well below the ultra heavy mass scale. We regard as an open question whether or not $m_H \ll m_F$ is a consistent possibility.

Now we turn to the $O(G_F\, m_F^2)$ consequences of the W and Z mass renormalizations, of the Higgs wave function renormalization and of the proper HZZ and HWW vertex functions. For the coupling of the Higgs boson to a light fermion f we find

$$\lambda_{Hff} = \frac{g m_f}{2M_W}\left[1 + \xi\,\frac{G_F}{8\sqrt{2}\,\pi^2}\left(\frac{m_1^2\, m_2^2}{m_1^2 - m_2^2}\,\ln\frac{m_2^2}{m_1^2} + \frac{7}{6}(m_1^2 + m_2^2)\right)\right]. \tag{8}$$

Similarly the HWW and HZZ couplings become

$$\lambda_{HWW} = gM_W\left[1 - \xi\,\frac{G_F}{8\sqrt{2}\,\pi^2}\left(\frac{m_1^2\, m_2^2}{m_1^2 - m_2^2}\,\ln\frac{m_1^2}{m_2^2} + \frac{5}{6}(m_1^2 + m_2^2)\right)\right] \tag{9}$$

$$\lambda_{HZZ} = \frac{gM_Z}{\cos\theta}\left[1 - \xi\,\frac{G_F}{6\sqrt{2}\,\pi^2}\,(m_1^2 + m_2^2)\right]. \tag{10}$$

Finally we have obtained the result (also found by Veltman [6]) for the ratio of the masses of $W^{\pm}$ and Z

$$\frac{M_W^2}{M_Z^2} = \cos^2\theta\left[1 + \xi\frac{G_F}{8\sqrt{2}\,\pi^2}\left(\frac{2\,m_1^2\,m_2^2}{m_1^2 - m_2^2}\,\ell n\,\frac{m_2^2}{m_1^2} + (m_1^2 + m_2^2)\right)\right]. \tag{11}$$

Equations (8), (9), and (10) (but not eq. (11)) have a very interesting property: they are physical predictions which depend on how γ_5 is continued to n dimensions when dimensional regularization [9] is used. This is true even though the theory is constructed so that the chiral anomaly cancels. We have resolved this ambiguity by requiring that scattering amplitudes have acceptable high energy behavior to order $g^4 M_F^2$ or, equivalently, by insisting on the validity of the relevant Ward identities [7].* Our conclusion is that in deriving eqs. (8) - (10) a correct prescription is to stipulate that γ_5 anti-commute with all γ^μ in n dimensions. It should be emphasized that this is merely a prescription or mnemonic which summarizes the content of the relevant Ward identities: it is irrelevant whether an anti-commuting γ_5 can actually be constructed in n dimensions. Other prescriptions [10], which were motivated by the known chiral anomaly [11], introduce spurious anomalies into the Ward identities relevant to eqs. (8) - (10) and would imply incorrect physical predictions. These spurious anomalies would destroy the renormalizability of the theory: they are positive definite and could

* We are grateful to Bill Bardeen for a discussion of this point.

not be arranged to cancel. Our analysis, to be presented fully elsewhere [7], suggests that the anti-commuting γ_5 prescription is always satisfactory except for the usual polynomial ambiguities associated with the known chiral anomaly, which can be resolved in the usual way [11]. The point is that the renormalizability of the spontaneously broken gauge theories depends on the validity of the naive canonical Ward identities (which is why the unavoidable chiral anomaly must be arranged to cancel) which are in turn guaranteed by prescribing a γ_5 which is fully anti-commuting in n dimensions.

In the equal mass limit, $m_1 = m_2 \equiv m$ the correction to λ_{Hff} is $(1 + \xi\, G_F\, m^2/6\sqrt{2}\,\pi^2)$ while the corrections to λ_{HWW} and λ_{HZZ} are $(1 - \xi\, G_F\, m^2/3\sqrt{2}\,\pi^2)$. For quarks of equal mass saturating the bound (6), the effect is of order 50% in λ^2_{HWW}. For a doublet with a massless neutrino and a heavy lepton whose mass saturates (2) the effect is of order 25% in λ^2_{HWW}. Therefore if a Higgs boson were discovered with a mass less than a few hundred GeV, its couplings would be very sensitive to the existence of much heavier fermions.

Unlike the corrections to the Higgs couplings in eq. (8)-(10), the correction to M_W/M_Z in eq. (11) vanishes if $m_1 = m_2$ and is only substantial if $(m_1/m_2)^2 \gg 1$ or $(m_1/m_2)^2 \ll 1$. For a doublet consisting of a massless neutrino, $m_1 = 0$, and an ultra heavy lepton, Veltman [6] used eq. (11) and the charged and neutral current ν and $\bar{\nu}$ total cross sections to estimate that $m_2 \lesssim 850$ GeV. Using the more recent and copious BCDHS [12] data and taking into account scaling violations [13] appropriate for the experimental cuts in the BCDHS data, we find that $m_2 \lesssim 550$ GeV at the two standard deviation level.

To conclude, the standard $SU(2) \times U(1)$ model must acquire a strongly interacting ("sthenon") sector if ultra heavy fermions exist. The critical mass scale is about 550 GeV/$\sqrt{N}$ for N nearly degenerate quark doublets and about 1.2 TeV/$\sqrt{N}$ for leptons. Ultra heavy fermions would induce large measurable low energy radiative corrections to the coupling of the Higgs boson to the W and Z bosons and to the light fermions. If the mass splitting in the doublet is ultra large, then there is also a substantial correction to M_W/M_Z. The present experimental limit is about $\frac{1}{2}$ TeV for the mass of a heavy lepton with a massless neutrino.

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