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LBL Report
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Gluon Correction to Semileptonic Decays of Heavy Quarks*

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The lowest order correction is calculated to the simple quark model of semileptonic heavy quark decays in quantum chromodynamics. The semileptonic decay rates are suppressed substantially, as much as 35 % for the charmed quark, and the inclusive energy spectra of final quarks and leptons are softened enough to reproduce well the inclusive lepton spectrum observed for the charmed hadron decays.

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July 1, 1978

1. Introduction

Weak decays of heavy hadrons with new flavor have been discussed in a quark model without including strong interaction corrections [1-4]. They are treated as occurring through the decay of a heavy quark with light quarks acting as spectators. For nonleptonic decays strong interaction corrections at short distances have been shown to contribute significantly to the enhancement of the strange particle decays [4-6], but very little to that of the heavy hadrons with new flavor [4,7]. By contrast, we do not expect such a short distance enhancement or suppression in semileptonic decays because vector and axial vector hadronic weak currents have no anomalous dimension of renormalization group. The semileptonic branching ratio of the charmed mesons has been measured through the dimuon production by neutrinos and anti-neutrino [8-10] and more directly through the inclusive lepton production above the charm threshold [11-13]. The averaged branching ratio for the D mesons has been given as

$$B = \Gamma(c \to \ell \nu X)/\Gamma(c \to \text{all}) = \begin{cases} 12 \sim 15\ \% & [8,9], \\ 8 \sim 11\ \% & [11\text{-}13], \end{cases} \qquad (1.1)$$

while B= 0.20 is predicted in the simplest quark model that ignores the masses of the u and d quarks as well as all of gluon corrections. The e^+e^- annihilation provides us with the inclusive lepton energy spectrum which is reasonably well explained by $D \to K \ell \nu$ and $K^* \ell \nu$ with about equal abundance [14-17]. It is softer than that of the four-fermion decay without gluon corrections.

It can be understood qualitatively why gluon corrections tend to improve the agreement of the quark model with experiment. Gluon loop correction causes a damping effect of weak current form factors to suppress the decay rate while emission of gluon by inner bremsstrahlung increases the decay rate. But the form factor damping dominates over the gluon emission to the order of α_s $(= g^2/4\pi)$ as we know in the quantum electrodynamic correction to the $\mu \to e\,\nu\,\bar{\nu}$ decay [18]. The damping form factor depletes the high energy end of the lepton spectrum and the emitted gluon takes away some share of the lepton energy. We will examine these effects to the lowest order in the gluon coupling.

2. Lowest order gluon corrections

We will work in the quark model of weak decays in which a heavy quark decays with a light quark acting as a spectator. No binding nor confining effect of quarks is taken into account. Though there is no rigorous proof for validity of this assumption, it seems reasonable as weakly decaying hadrons are only loosely bound [1-4].

To make a perturbative expansion in the gluon coupling meaningful, it is desirable to know how important the higher order diagrams are in the expansion. Unfortunately we have no convincing argument for our perturbative expansion on the basis of rigorous renormalization group argument. For the decays such as $c \to s + \ell + \nu$ and $b \to c + \ell + \nu$, the situation is somewhat similar to that of quarkonium states, $c\bar{c}$ and $b\bar{b}$ [19], if we treat b, c, and s as heavy particles. The region of integral for soft gluon momenta is negligible in the entire phase

volume of physical region and the dominant contribution comes from the region of large momenta of the order of the initial quark mass where the effective gluon coupling is small because of asymptotic freedom. Higher order diagrams exist with larger values of the effective gluon coupling. Whether these multigluon diagrams are really unimportant or not is not clear because quarks are on the mass shell, but we usually postulate it to explain the phenomenological Okubo-Zweig-Iizuka rule in quarkonium transitions. We will follow this line of argument to justify our approximation. It is fair to say that our perturbative calculation is based largely on the optimism which has so far been in no serious trouble. It is remarked here that not only the short distance enhancement, but also the long distance enhancement are unlikely in the semileptonic decays of heavy quarks since the processes relevant to the latter are suppresses by the OZI rule.

3. Decay distributions in the lowest order QCD corrections

We consider to the order of α_s

$$q_1(P) \to q_2(p) + \ell(k_\ell) + \nu(k_\nu), \tag{3.1}$$

$$\to q_2(p) + \ell(k_\ell) + \nu(k_\nu) + g(k_g), \tag{3.2}$$

where the letters in the parentheses stand for the momenta satisfying $P^2 = M^2$, $p^2 = m^2$, and $k_\ell^2 = k_\nu^2 = k_g^2 = 0$. We have in mind the processes like $c \to s$, $b \to c$, and possibly $b \to u$. The differential decay rate is a sum of two terms

$$d\Gamma = d\Gamma(q_1 \to q_2 + \ell + \nu) + d\Gamma(q_1 \to q_2 + \ell + \nu + g). \tag{3.3}$$

By integrating over the gluon momentum in the second term, one obtains the form of

$$d\Gamma = \left\{ (2\pi)^4 \delta(P-p-k_\ell-k_\nu) \left|M_1\right|^2 + 2\pi\, \delta((P-p-k_\ell-k_\nu)^2) \left|M_2\right|^2 \right\},$$
$$\times\ (E_p E_\ell E_\nu)^{-1}\, d^3p\, d^3k_\ell\, d^3k_\nu\ , \tag{3.4}$$

where $M_{1,2}$ stand for the invariant decay matrix elements of zero and one gluon loop corrections and of one gluon emission, normalized appropriately. The integral over d^3k_ℓ and d^3k_ν can be done analytically in (3.4). In fact, the inclusive decay distribution in the final quark momentum p is obtained from the previous works [18, 20, 21] on radiative corrections in quantum electrodynamics with a straightforward modification appropriate to QCD. It can be written for the V - A current-current interaction in the notations of [18, 20]:

$$d\Gamma/d^3p = (2\pi)^{-4}(M^2m^2/E_pE_p)\Big\{(\bar{g}_1^2 + \bar{g}_3^2)\cosh\theta\,(\cosh\omega - \cosh\theta) + \tfrac{1}{3}\sinh^2\theta$$
$$- (\bar{g}_1^2 - \bar{g}_3^2)(\cosh\omega - \cosh\theta)\Big\}, \tag{3.5}$$

where

$$\cosh\theta = (P\cdot p)/Mm\ , \qquad \omega = \ln(M/m)\ ,$$

$$\bar{g}_\sigma^2 = 1 + \frac{2\alpha_s}{3\pi}(a_\sigma + b_\sigma)\ , \qquad (\sigma = 1,\ 3), \tag{3.6}$$

$$a_{1,3} = 2S \pm \frac{2\theta}{\sinh\theta} + \frac{\omega\sinh\omega - (2\theta/3)\sinh\theta}{\cosh\omega - \cosh\theta + (\cosh\theta \pm 1)/3} - 4\ ,$$

$$b_{1,3} = 2D + 2C + \frac{Q + (\cosh\omega - \cosh\theta)^2\, Y}{3(\cosh\omega - \cosh\theta)(\cosh\theta \mp 1) + \sinh^2\theta}$$

$$S = (\theta - F_1)\coth\theta + (1 - \theta\coth\theta)(\omega - 2\omega_<),$$

$$F_1 = L\left(\frac{2\sinh\theta}{e^{\omega} - e^{-\theta}}\right) - L\left(\frac{2\sinh\theta}{e^{\theta} - e^{-\omega}}\right) + (\omega - \theta)\,\ln\left(\frac{\sinh\frac{1}{2}(\omega-\theta)}{\sinh\frac{1}{2}(\omega+\theta)}\right)\ ,$$

$$L(x) = \int_0^x \ln|1 - t|\, dt/t\ ,$$

$$D = 2(\theta\coth\theta - 1)(\omega - \omega_< - 1 - \ln 2)$$
$$+\left\{L(e^{-\theta-\omega}) - L(e^{\theta-\omega})\right\}\coth\theta - 1 - e^{-\omega}/\sinh\theta$$
$$+\left\{2\theta\coth\theta + \sinh\omega/\sinh\theta - 1\right\}\ln(1 - e^{-\theta-\omega})$$
$$+\left\{2\theta\coth\theta - \sinh\omega/\sinh\theta - 1\right\}\ln(1 - e^{\theta-\omega})\ ,$$

$$C = 2\ln 2\,(\theta\coth\theta - 1) + 1 + \theta\coth\theta\left\{1 - \ln 2 - \ln(\cosh\theta)\right\}$$
$$+\left\{L(\tanh\theta) - L(-\tanh\theta)\right\}\coth\theta$$
$$+\left\{L(\tfrac{1}{2}(1 - \tanh\theta)) - L(\tfrac{1}{2}(1 + \tanh\theta))\right\}\coth\theta\ ,$$

$$Q = 4(\theta\coth\theta - 1)\sinh^2\omega\ ,$$

$$Y = \frac{10}{3}(\theta\coth\theta - 1) + (\tfrac{5}{3}\cosh\omega \mp 1)(\theta/\sinh\theta),$$

$$\omega_< = \ln(\lambda_{min}/m). \tag{3.7}$$

The $\pm$ signs in (3.7) refer to $\sigma = 1$ (vector) and to $\sigma = 3$ (axial vector) and λ_{min} in the definition of $\omega_<$ stands for an infrared cutoff in gluon momenta, which disappears in $d\Gamma/d^3p$ after one sums up a_σ and b_σ in (3.6). We have written the complete expression partly because of the important correction term C which had been missed in the original article [20] and later pointed out in [18, 21].

In the following section we will integrate (3.5) over d^3p to obtain the α_s correction to the semileptonic decay rate. We will also plot the decay distribution in p, which can be compared with experiment if one detects a strange (charmed) particle in coincidence with a lepton in the charmed (bottom) hadron decay. A more direct prediction involving no final-state interaction of hadrons is made on the inclusive lepton energy distribution. Analytic computation for this distribution is more complicated than that of the p distribution (3.5) - (3.7). Going back to (3.4), we will make a crude numerical estimate for it.

4. Numerical estimate

The coupling α_s in (3.6) should be understood as the effective coupling of the gluon,

$$4\pi/\alpha_s = (11 - \tfrac{2}{3} N_f)\ \ell n(M/\Lambda)^2 , \qquad (4.1)$$

where N_f is the number of relevant light quarks at the mass scale M and Λ is a scale parameter to be determined from experiment such as deep inelastic lepton-hadron processes [22-24]. We have chosen α_s = 0.7 and 0.35 as representative values for c and b decays, respectively.

The correction $\Delta\Gamma$ of $O(\alpha_s)$ to the total semileptonic decay rate is plotted in Fig. 2 in the ratio to the uncorrected rate for the two values of α_s in the range of M/m from 2.6 to 3.8. With M/m = 30 and α_s = 0.35 corresponding to the $b \to u$ decay, we have found that $\Delta\Gamma/\Gamma_0$ = - 0.26. For completeness, the uncorrected

rate ($m \neq 0$) is plotted in the unit of the uncorrected rate in which m = 0. The striking result is that the semileptonic decay rate decreases by as much as 35 % for the charmed quark because of the destructive interference of the one-gluon loop diagrams with the main term of no gluon. Physically, the damping effect of the hadronic form factor wins over the opening of new channels. The same effect is only 0.4 % for the $\mu \to e\nu\bar{\nu}$ decay since $\alpha = 1/137$.

The form factor and the gluon emission together modify the momentum distribution of the final quark as it is shown in Fig. 3a and 3b. The p distribution is severely suppressed near the high momentum end. It turns out that for M/m = 5 and α_s = 0.7 corresponding to the $c \to u$ decay, the differential rate $d\Gamma/dp$ is negative at the high p end in the one-gluon approximation and therefore it is necessary to include multigluon diagrams for such a decay. It is not clear whether the p distribution calculated here is to be compared with the inclusive s (c) measurement in the semileptonic c (b) decay since soft gluon final interactions may further modify the distribution. Our p distribution takes into account one soft gluon emission and therefore part of fragmentation effects. In the QCD calculation the notion of quark fragmentation of parton model involves an ambiguity in separating soft gluons from hard gluons. But it would be a reasonable guess that the momentum distribution of the final leading hadrons is no harder than the distribution plotted in Fig. 3a and 3b.

Unlike the p distribution, the energy distribution of the final leptons is not modified by soft gluon interactions in final states. Emission of gluons from a heavy quark prior to weak decay is suppressed

by the OZI rule. It is more involved technically to do an analytic computation of the inclusive lepton distribution. We have to go back to (3.4) and perform a crude numerical estimate of the multiple integral. The result is given in Fig. 4. The form factor damping tends to deplete the high energy end of the k_ℓ distribution and the gluon emission makes the center of the distribution bulge. Consequently, the k_ℓ distribution is shifted a little towards the low energy direction.

5. Implication and discussion

The dimuon production by neutrinos and antineutrinos and the $D\bar{D}$ production in e^+e^- annihilation lead us to the semileptonic branching ratio of the D mesons as

$$B = \begin{cases} 8 \pm 2\ \%, & [11] \\ 11 \pm 3\ \%, & [12] \\ 8.2 \pm 1.9\ \%, & [13] \\ 15\ \%, & [8] \\ 12\ \%. & [9] \end{cases} \qquad (5.1)$$

This is the weighted average for the semileptonic branching ratio of the D mesons of $J^P = 0^-$ and the values of B determined from the dimuon production [8, 9] involve an unknown systematic error due to the fragmentation function of the charmed quark. Though our present calculation gives only a theoretical value for the "free" quark decay, we expect that it will be a good approximation to the real charmed hadrons which are bound loosely. Without gluon corrections the quark model predicts B = 20 % if one sets $m_u = m_d = 0$ and a considerably

larger value if one uses their constituent quark masses. The observed values for B smaller than 20 % has normally been atributed to the enhancement of nonleptonic decays. A numerical estimate of the short distance enhancement of nonleptonic heavy quark decays shows that the enhancement is rather insignificant for heavy quarks and that the observed values for B are not easily attained even if one takes 20 % as an uncorrected value. We have suspected that the rest may be accounted for by the long distance enhancement that we can not estimate reliably. We have found here that the problem might be solved by the semileptonic decay rate being largely suppressed. The semileptonic suppression is less for the bottom quark of $m_b = 4.5$ GeV, but it is still nonnegligible. The semileptonic suppression can be tested by measuring the decay rate instead of just the branching ratio, but it may not be so easy to test accurately because the values of the quark masses involve some ambiguity and the decay rate is quite sensitive to the quark masses. We could perform a similar calculation for nonleptonic decays of heavy quarks, but there is a reason to suspect that long distance enhancement through higher order gluon corrections to light quarks in final states may be more important than in semileptonic decays.

The inclusive quark momentum distribution may be compared with the inclusive leading hadron spectra (K and K^* in $c \to s + \ell + \nu + X$ and D, D^* etc in $b \to c + \ell + \nu + X$) in e^+e^- annihilation. It is not quite consistent to fold the distribution given in Fig. 3a and 3b with the socalled fragmentation functions of parton model in order to obtain the physical hadron distribution, since the one soft gluon emission has already been taken into account. The p distribution in Fig. 3a and 3b shows that even with one gluon emission the leading final quark is

softened significantly for the charmed quark ($\alpha_s = 0.7$). The trend that the softening is less severe for heavier quarks concides with the prediction of more intuitive argument that the fragmentation functions of the heavy quarks are weighted at higher values of the fragmentation variable [25-29].

We are ready to compare our result on the inclusive lepton spectrum in e^+e^- annihilation. Ignoring the Cabibbo suppressed $\Delta S = 0$ decays, we can use the curve of $M/m = m_c/m_s = 3$ and $\alpha_s = 0.7$ for the inclusive e (μ) distribution in the semileptonic charmed hadron decay. The lepton spectrum not corrected for gluons ($\alpha_s = 0$ curve) peaks at a value higher than the observed one. The gluon corrections shift the peak downwards to improve the agreement with experiment. Usually, the observed lepton spectrum is explained by adding the contributions of $D \to K \ell \nu$ and $D \to K^* \ell \nu$ [14,15]. The gluon corrections provide for the same effect as $D \to K^* \ell \nu$ does. For even heavier quarks such as the bottom quark of 4.5 GeV, the gluon corrections lead to less softening so that the lepton spectrum will be closer to the prediction of the four-fermion interaction without gluons. The extreme opposite is a statistical model where each of final hadrons and leptons carries nearly the same amount of energy. In such a model the lepton spectrum tends to shift toward the lower region in Fig. 4 as the initial quark mass increases with the ratio M/m kept fixed. It will be a crucial test for the present quantum chromodynamic picture against models like a statistical one to measure the inclusive lepton spectrum for the bottom quark and to compare it with that for the charmed quark.

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Figure captions

Fig. 1. One gluon corrections to semileptonic decay of a heavy quark. (a) Gluon emission. (b) Gluon loop correction.

Fig. 2. Correction to the semileptonic decay rate in the unit of the uncorrected rate Γ_0, $\Gamma = \Gamma_0 + \Delta\Gamma$. The uncorrected decay rate Γ_0 with the quark mass $m \neq 0$ is plotted in a broken curve in the unit of the uncorrected rate $\bar{\Gamma}_0$ with $m = 0$. M and m are the initial and final quark masses, respectively.

Fig. 3. Distribution of final quark momentum for (a) $M/m = 3$ and $\alpha_s = 0.7$ and 0.35 ($c \to s$ and $b \to c$) and (b) $M/m = 15$ and $\alpha_s = 0.35$ ($b \to u$).

Fig. 4. Distribution of final lepton momentum for $M/m = 3$. The inclusive energy spectrum for $e^+ + e^- \to e^\pm + \geq 2$ charged tracks + ≥ 0 photons measured by the DELCO Collaboration 12 at the center-of-mass energy 3.75 - 3.80 GeV is plotted to compare with our calculation of $M/m = 3$. $M (= m_c) = 1.8$ GeV is chosen to plot the data in our Figure.

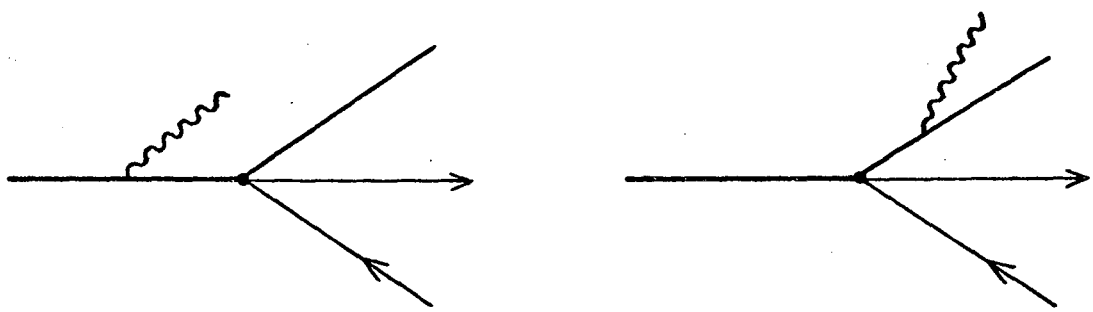


Fig. 1a

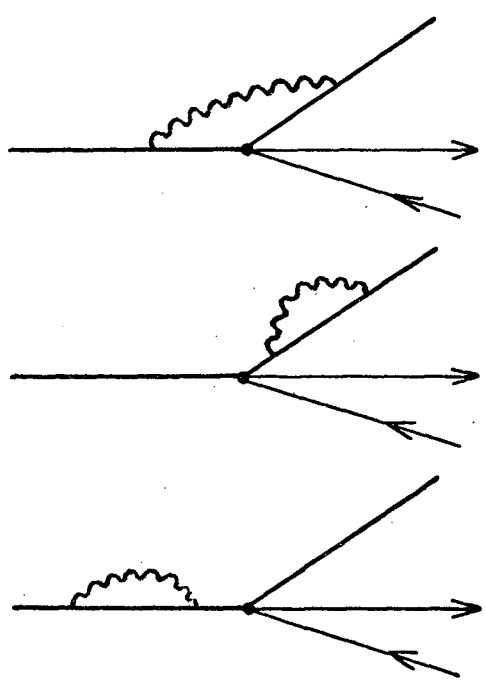
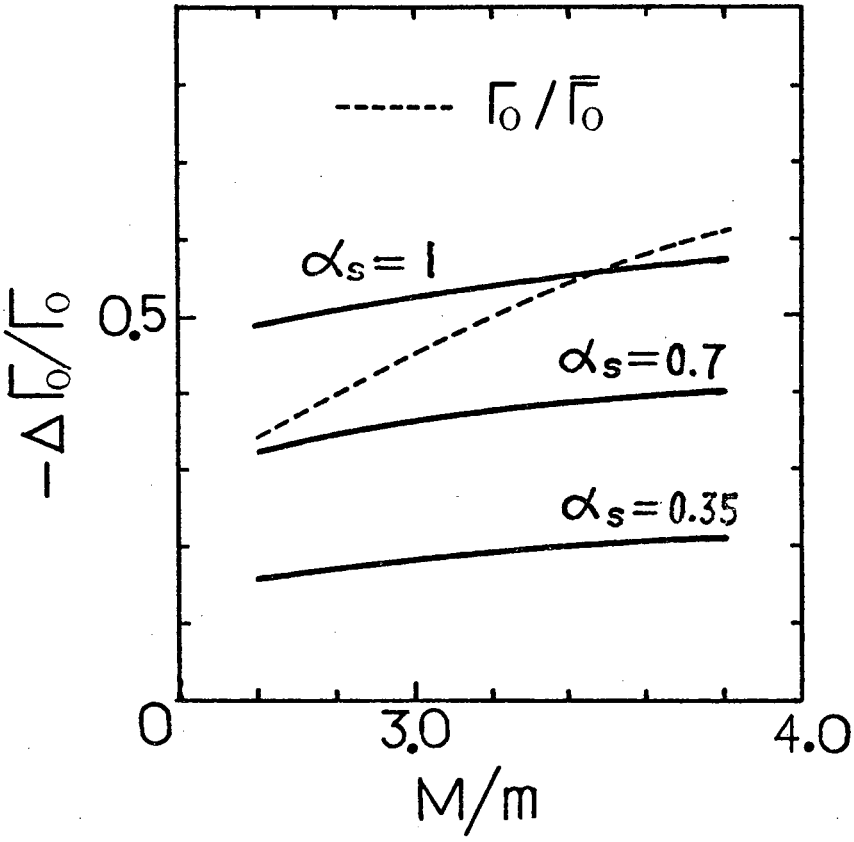


Fig. 1b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$\alpha_s = 1$
$\alpha_s = 0.7$
$\alpha_s = 0.35$
$-\Delta\Gamma_0/\Gamma_0$
0.5
0
3.0
4.0
M/m

Fig. 2

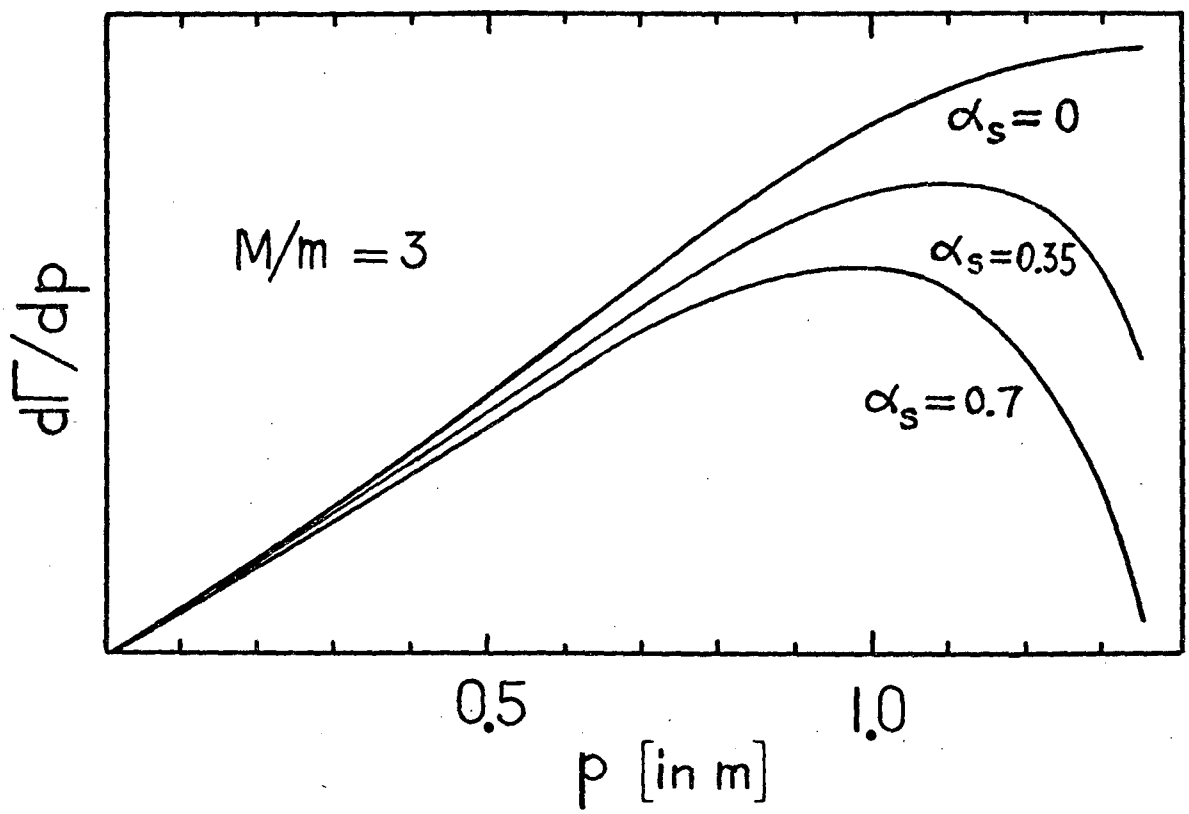

Fig. 3a.

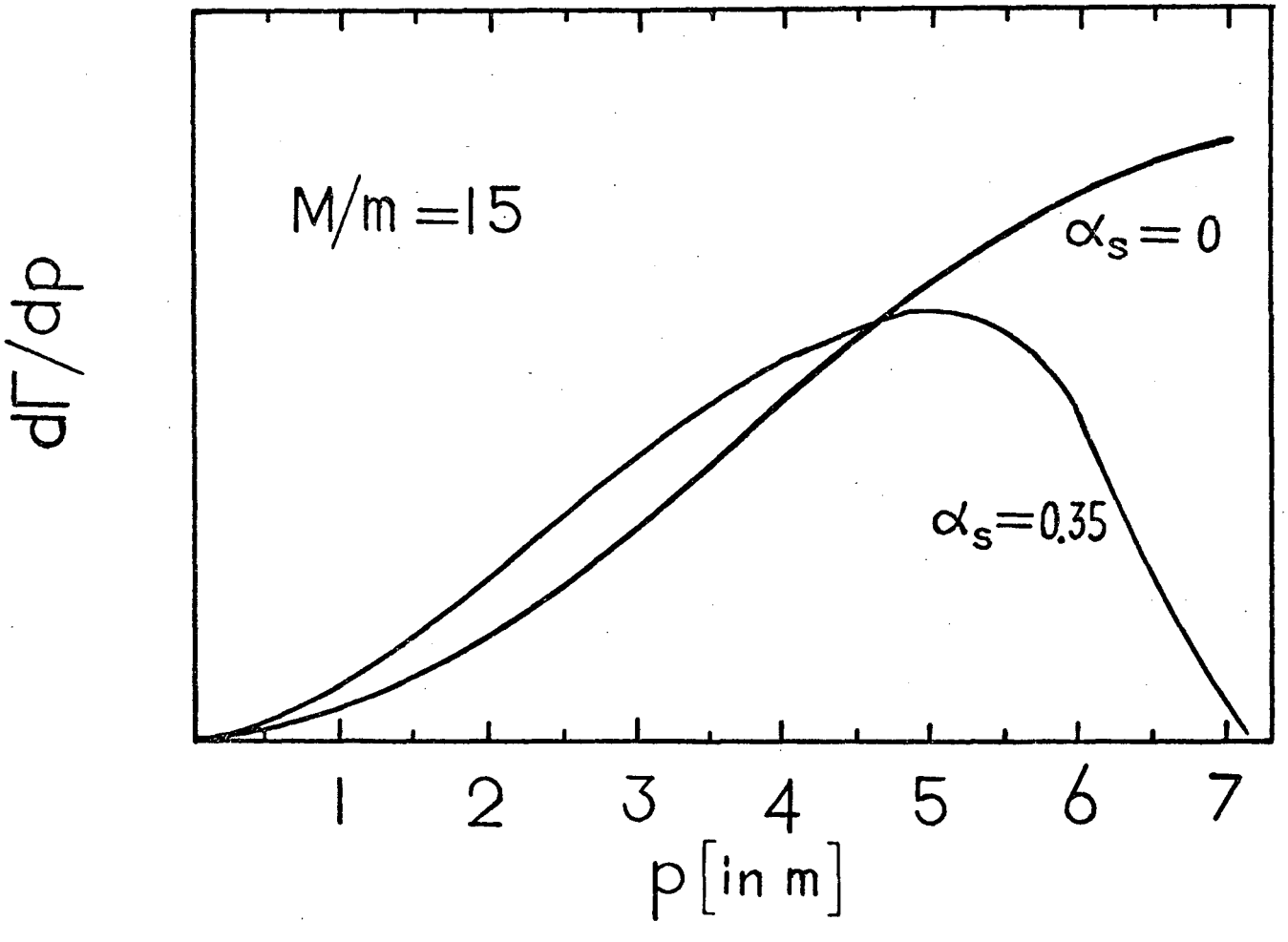

Fig. 3b

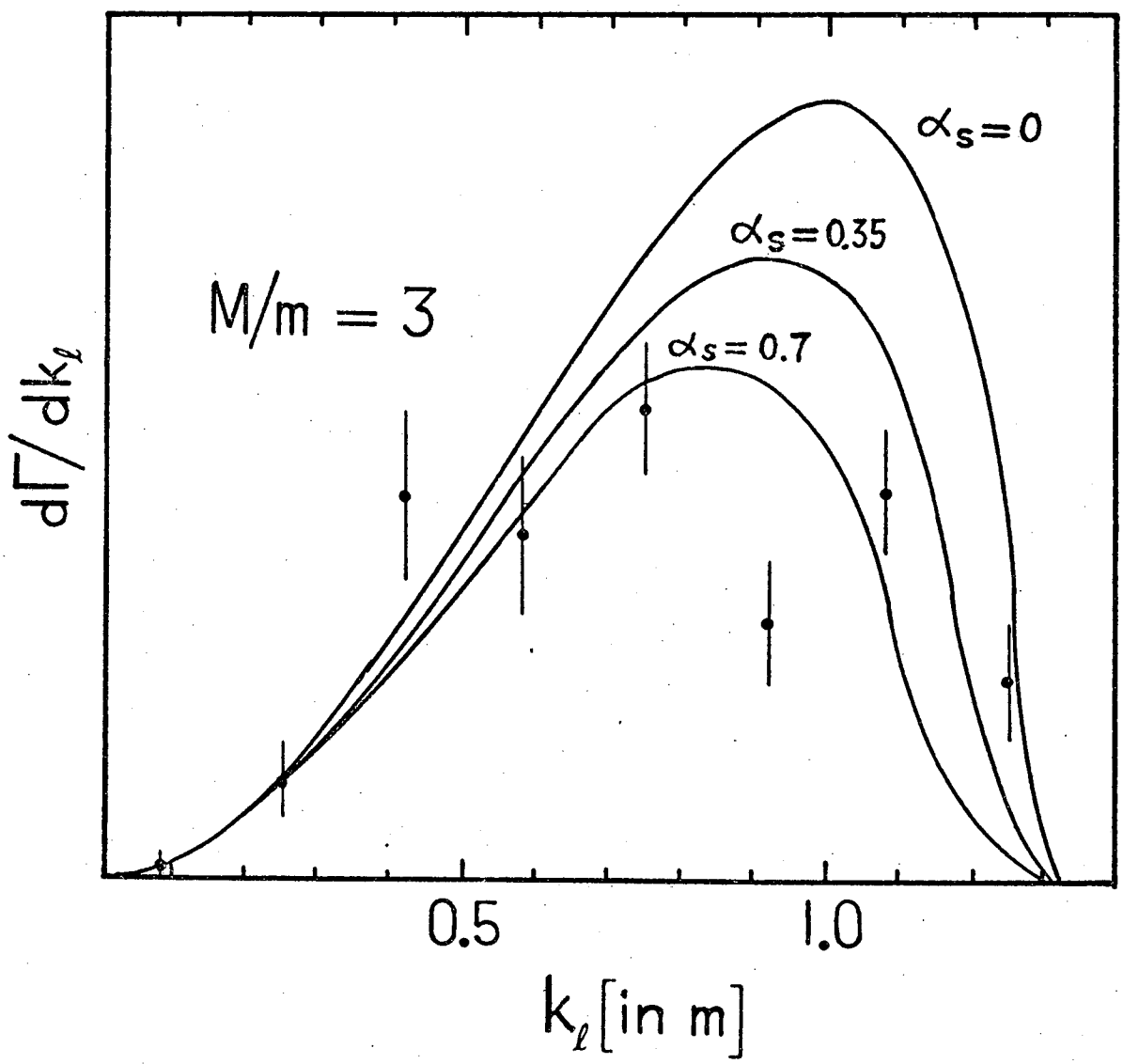

Fig. 4

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