Submitted to Nuclear Physics B

LBL-8270 c.2
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October 1978

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

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WEAK INTERACTIONS OF ULTRA HEAVY FERMIONS. II*

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October 3, 1978

ABSTRACT

We discuss the weak interactions of ultra heavy fermions at low and high energy in the $SU(2)_L \times U(1)$ gauge theory. Using partial wave unitarity at high energy we establish critical fermion mass values beyond which strong couplings occur in the theory and the perturbation expansion fails. The critical masses are $(500/\sqrt{N})$ GeV for quarks and $(1.0/\sqrt{N})$TeV for leptons, where N is the number of nearly degenerate $SU(2)_L$ doublets of quarks and leptons respectively. At low energies, far below their production threshold, we show that ultra heavy fermions would induce large, observable one loop radiative corrections. One of these corrections (obtained also by Veltman) implies, using present experimental data, an upper limit of ~ 500 GeV for a heavy lepton in an $SU(2)_L$ doublet with a massless neutrino.

* This work has been supported by the High Energy Physics Division of the U.S. Department of Energy.

I. INTRODUCTION

The recent discoveries of new quarks and leptons underscore our ignorance of some of the deepest questions we might ask. What is the nature and origin of the quark and lepton spectrum? How many are there? To what mass scale does the spectrum extend? For the moment we have no idea. The purpose of this paper is to explore the nature of the weak interactions if the spectrum extends significantly beyond the expected 100 GeV mass scale of the intermediate vector bosons.* The central point is that for such ultra heavy fermions the weak interactions of the $SU(2)_L \times U(1)$ gauge theory would actually become strong and the perturbation expansion would fail. Besides being interesting in its own right, this phenomenon means that low energy experiments can be sensitive to the existence of ultra heavy quarks and leptons. At energies far below their production threshold the ultra heavy fermions would induce large observable radiative corrections.

We work primarily in the context of the usual $SU(2)_L \times U(1)$ model with only one Higgs doublet[2]. But the essential features of our results are independent of the model. We will generalize our results for the $SU(2)_L \times U(1)$ model to allow for any experimentally acceptable assignment of Higgs representations.

As discussed below, in the weak interaction gauge theories the interactions of fermions must become strong as the fermion mass increases beyond gauge boson masses. The questions which we will answer fall into two distinct categories. First, what is the fermion

* A brief discussion of some of these results may be found in ref. [1].

mass scale at which strong coupling begins? We answer this question by applying partial wave unitarity to the scattering of ultra heavy fermions at very high energy. Second, what low energy measurements can probe for the existence of ultra heavy fermions? We present a complete classification of the observably large radiative corrections induced by ultra heavy fermions and explicitly compute those which are most able to be studied experimentally. The corresponding questions for a heavy Higgs particle were discussed by Lee, Quigg, and Thacker[3] in the former case and by Veltman[4] in the latter.

The basic point is very simple. In the spontaneously broken gauge theories a fermion F of mass m_F couples to the Higgs particle H and to the longitudinal modes of the W and Z bosons with a strength proportional to

$$\sqrt{G_F\, m_F^{\,2}} = \frac{g\, m_F}{2^{\frac{1}{4}} \cdot 2M_W} . \tag{1.1}$$

(G_F is the Fermi constant, g the $SU(2)$ gauge coupling constant, and M_W is the W mass.) So for some $m_F \gg M_W$ the interaction becomes strong and perturbation theory fails. To determine this critical mass scale we consider the tree approximation amplitudes for $F\bar{F} \to F\bar{F}$, W^+W^-, ZZ, HH, ZH. Each of these contains a term proportional to $G_F\, m_F^{\,2}$, which dominates for large enough m_F. As m_F increases these amplitudes saturate the bounds imposed by partial wave unitarity. For this critical value of m_F perturbation theory breaks down and the fermion is strongly interacting, with itself and with W, Z, and H.

As in ref.[3] we find the lowest critical value of m_F by computing the largest eigenvalue of the coupled channel T matrix in

the low partial waves. For quarks the $J = 0$ matrix yields the smallest critical mass. If the quarks in an $SU(2)_L$ doublet are nearly degenerate and there are N nearly degenerate doublets (i.e., mass splittings much less than masses), then the critical value is $\sim 550\ \mathrm{GeV}/\sqrt{N}$. For leptons the smallest critical value comes from the $J = 1$ matrix. It is $1.0\ \mathrm{TeV}/\sqrt{N}$ for N nearly degenerate doublets of ultra heavy charged leptons partnered with massless neutrinos and $880\ \mathrm{GeV}/\sqrt{N}$ for N nearly degenerate doublets of ultra heavy charged leptons with nearly degenerate ultra heavy neutral leptons. Because, as explained in Section II, our bounds are conservative and especially if $N \gtrsim 3$, ultra heavy quarks could be within the range of the $\bar{p}p$ and pp colliding beam facilities which will be built in the next few years.

If ultra heavy fermions exist then the theory acquires a "sthenon" sector in the sense of Appelquist and Bjorken[5]. The ultra heavy fermions couple strongly to one another and to W, Z, and H, but all remain weakly coupled to the light (i.e., lighter than M_W) fermions. If in addition H is ultra heavy ($m_H \sim O(1\ \mathrm{TeV})$ --see ref. [3]) then W, Z, and H also couple strongly among themselves with a strength characterized by $\sqrt{G_F\, m_H^{\,2}}$. Our analysis is independent of whether H is ultra heavy.*

If ultra heavy quarks exist, it is amusing to realize that their short distance dynamics might be dominated by the weak interaction gauge theory. If QCD is still asymptotically free at the

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* If H is ultra heavy our unitarity bounds could only improve.

ultra heavy mass scale--that is, if too many new flavors do not populate the intervening mass scales--then at short distances QCD would only contribute small corrections to the dominant weak interaction (broken gauge theory) dynamics. For instance, ultra heavy Quarkonium, $Q\bar{Q}$, would be bound primarily by W, Z, and H bosons with only small corrections from QCD gluons.* And if QCD is not asymptotically free at the ultra heavy quark mass scale, weak bosons and QCD gluons are likely to be of comparable importance in their short distance dynamics. In either case QCD gluons would dominate the dynamics at large distances because of the finite range of the weak quanta.

If ultra heavy leptons exist, they would also form strongly bound Leptonium states, bound by the weak quanta. The spectrum of ultra heavy Leptonium would closely resemble that of ultra heavy Quarkonium provided that QCD remains asymptotically free.

Ordinarily, because of the decoupling theorem[7], we expect heavy particles to have a negligibly small effect on the physics at energies far below their production threshold. But in the broken gauge theories the appearance of the parameter $G_F m_F^2$ means that ultra heavy fermions may produce important effects at low energies. If the parameter occurs in a measurable amplitude in one loop order, then ultra heavy fermions would cause a change of leading order in the tree approximation prediction

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* We assume, as argued in ref. [6], that the characteristic momentum scale of the $Q\bar{Q}$ bound state is $p \sim \alpha_s(p)\, m_Q$, where $\alpha_s(p)$ is the usual running coupling constant of QCD.

$$g^2\left(1 + \mathcal{O}(G_F m_F^2)\right) = g^2\left(1 + \mathcal{O}(1)\right) \tag{1.2}$$

giving rise to a large observable effect.

In the standard $SU(2)_L \times U(1)$ model with one Higgs doublet, Veltman[4] showed that the parameter $G_F m_H^2$ does not occur in one loop order, so that an ultra heavy Higgs boson would be "screened" from the physics at low energies. But ultra heavy fermions are not screened. One loop corrections proportional to $G_F m_F^2$ occur in the W, Z, and H self energies and in all amplitudes formed from them by adding arbitrary numbers of external Higgs bosons. In practice the most important consequence is that the following tree approximation relationships would get $\mathcal{O}(1)$ corrections if ultra heavy fermions exist:

$$\lambda_{Hff} = \frac{g\, m_f}{2\, M_W} \tag{1.3}$$

$$\lambda_{HWW} = g\, M_W \tag{1.4}$$

$$\lambda_{HZZ} = \frac{g\, M_Z}{\cos\theta_W} \tag{1.5}$$

$$M_W = M_Z \cos\theta_W\,. \tag{1.6}$$

Here λ_{Hff}, λ_{HWW}, and λ_{HZZ} are respectively the couplings of the Higgs boson to a (light) fermion f, to the W, and to the Z, and θ_W is the weak-electromagnetic mixing angle. Equations (1.3) to (1.5) are of present experimental interest because they underlie all strategies to search for the Higgs boson. The correction to eq. (1.6), obtained also by Veltman[8], can be used together with data on charged

and neutral current cross sections to put an upper bound on the mass of a heavy lepton in a doublet with a massless neutrino. With the most recent data the result is $M_L \lesssim 500$ GeV.

We have used dimensional regularization[9] to compute the one loop corrections. The calculations are straightforward except for an apparent ambiguity in how γ_5 is defined in n dimensions. We have resolved this ambiguity by examining the Ward identities which must be satisfied by the amplitudes being computed. It turns out to be correct for the graphs studied here (and for a much larger class of graphs) to assume that γ_5 anticommutes with all the Dirac γ^μ in n dimensions. A complete analysis is presented in a separate paper[10].

The rest of the paper is organized as follows. In Section II we use partial wave unitarity at high energy to establish critical fermion mass values for the onset of strong coupling. We also generalize the effective Lagrangian of ref. [3] to simplify the calculation of the contributions to tree amplitudes which are proportional to $G_F m_F^2$. In Section III we compute the one loop radiative corrections to low energy processes which are proportional to $G_F m_F^2$. In Section IV we discuss the generality of our results and explicitly compute the critical fermion masses for the $SU(2) \times U(1)$ model with a generalized Higgs sector. Section V contains some concluding remarks. In Appendices A and B we compute the relevant amplitudes for the $J = 0$ and $J = 1$ coupled channel matrices in the standard model. Appendix C contains a detailed discussion of the $J = 0$ coupled channel matrix in the model with a generalized Higgs sector.

II. BOUNDS FROM PARTIAL WAVE UNITARITY

In this section we use partial wave unitarity to study the scattering of the ultra heavy fermions at energies far above their thresholds. In analogy with ref. [3], we use the saturation of the unitarity constraints for the tree approximation amplitudes to determine critical values for the fermion masses beyond which strong coupling appears and the perturbation expansion fails. Consider for example the $J = 0$ partial wave a_o. Unitarity and Schwartz's inequality demand

$$|a_o| \geqslant \mathrm{Im}\, a_o \geqslant |a_o|^2 .$$

Now suppose we expand a_o in powers of the coupling constant g,

$$a_o = c_1 g^2 + c_2 g^4 + \cdots . \tag{2.1}$$

Since the Born term $c_1 g^2$ is purely real at very high energies, we obtain

$$\mathrm{Im}\, a_o = g^4\, \mathrm{Im}\, c_2 \leqslant g^4 |c_2|$$

and, therefore, to order g^4,

$$|c_1|^2 g^4 \leqslant |c_2|\, g^4 . \tag{2.2}$$

Now if the expansion (2.1) is to be reliable, we should also require

$$|c_2 g^4/c_1 g^2| \ll 1$$

and therefore, combining this with (2.2), we obtain

$$|a_o|_{\mathrm{Born}} \equiv g^2 |c_1| \leqslant |c_2 g^4/c_1 g^2| \ll 1 . \tag{2.3}$$

We shall see immediately that c_1 is proportional to the square of the

mass of the ultra heavy fermion, m_F^2, so that (2.3) puts an upper bound on m_F if the perturbation expansion is to be reliable. We define a critical value for m_F by the equation

$$|a_o|_{Born} = 1 . \tag{2.4}$$

If $m_F \gtrsim (m_F)_{crit}$, the fermions are strongly coupled, contributions to the scattering amplitude from higher order terms must be of the same order of magnitude as the lowest order terms, and perturbation theory has broken down. In practice strong coupling will set in at values of m_F lower than the critical value due to (2.3), so we are being conservative in defining $(m_F)_{crit}$ by saturating the unitarity bound (2.4).

(a) An Example: J = 0 Fermion Scattering

We assume the standard spontaneously broken $SU(2) \otimes U(1)$ model[2] to which we add a doublet of left-handed fermions F_1, F_2 and the corresponding right-handed singlets. We shall specify later whether the fermions are leptons or quarks, and for the moment assume that there is no Cabbibo mixing of this doublet with any others. In addition we assume that their masses are much larger than those of the gauge bosons. We shall compute T-matrix elements, where T is related to the S matrix by $S = 1 + iT$. Only neutral channels will be considered since this is sufficient for our purposes. The channels $F_i\bar{F}_i \to$ bosons do not contribute to $J = 0$ partial waves; they will be considered later.

We quantize the theory in the unitary gauge, so the W and Z propagators have the form

$$-i \frac{g^{\mu\nu} - k^\mu k^\nu/M^2}{k^2 - M^2 + i\epsilon} . \tag{2.5}$$

Since we are interested only in contributions proportional to $m_i m_j$, we neglect Feynman graphs with photon exchange and the $g^{\mu\nu}$ piece of the propagator in W and Z exchange graphs. It is easy to see on dimensional grounds that in the high energy limit these contributions are proportional to g^2 but not to $g^2 m_i m_j/M^2$.

The contributions to the amplitudes from each graph are displayed in Appendix A. We discuss the unitarity bounds only in the limit $s \gg m_i m_j$, so the amplitudes simplify considerably. If we denote by + or - the helicities of the fermions (written in the order $F\bar{F} \to F\bar{F}$) then only the channels $++ \to ++$, $-- \to --$, and $++ \longleftrightarrow --$ contribute to the $J = 0$ partial wave amplitude, which is given by

$$a_o = \frac{1}{32\pi} \int_{-1}^{1} d(\cos\theta) T .$$

As a simple example we consider the elastic amplitude $F_{i+}\bar{F}_{i+} \to F_{i+}\bar{F}_{i+}$; only the Z and H s-channel exchanges contribute in the limit of interest and we find, using the results of Appendix A,

$$a_o = -\frac{G_F m_i^2}{4\sqrt{2}\,\pi} = -\frac{g^2 m_i^2}{32\pi M_W^2} . \tag{2.6}$$

Partial wave unitarity implies that $|a_o| < 1$, so the validity of the perturbation expansion requires

$$m_i^2 < \frac{4\sqrt{2}\,\pi}{G_F} = (1.2 \text{ TeV})^2 . \tag{2.7}$$

For $m_i^2 \geq 4\sqrt{2}\,\pi/G_F$, as we have already discussed, the $\mathcal{O}(g^4)$ terms must make a larger contribution to a_o than does the Born term

given in (2.6). As we pointed out in (2.3), the inequality (2.7) is conservative: the perturbation expansion begins to fail for appreciably smaller values of m_i .

(b) Coupled Channels: $J = 0$

We may improve the bound (2.7) by considering the scattering of different helicity and flavor combinations. As in ref. [2], the most restrictive bound is obtained from the largest eigenvalue of the coupled channel T matrix.

We consider first one doublet of heavy fermions $\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$. The $J = 0$ coupled channel matrix is 4×4 ; the relevant states are $F_{i+}\bar{F}_{i+}$ and $F_{i-}\bar{F}_{i-}$ where $i = 1,2$. The diagonal matrix elements are due to s-channel Z and H exchanges (see Appendix A) and are given by (2.6). The matrix elements which are off-diagonal in both helicity and flavor $F_{i+}\bar{F}_{i+} \longleftrightarrow F_{j-}\bar{F}_{j-}$ with $i \neq j$, receive contributions from both s-channel Z and H exchanges and from t-channel W exchange. If there is no Cabbibo-like mixing these off-diagonal contributions cancel. All other off-diagonal matrix elements also vanish, so that the coupled channel matrix for $J = 0$ is already diagonal, and there is no improvement in the bound (2.7).

Now we assume that there is mixing between two doublets, parametrized by Cabbibo angle θ_c ,

$$\begin{pmatrix} F_1 \\ F_2 \cos\theta_c + F_3 \sin\theta_c \end{pmatrix} \qquad \begin{pmatrix} F_4 \\ -F_2 \sin\theta_c + F_3 \cos\theta_c \end{pmatrix} \tag{2.8}$$

We consider the coupled channel problem only for $F_1\bar{F}_1$ and $F_2\bar{F}_2$; if F_3 and F_4 are also ultra heavy, our bound could be improved by

including $F_3\bar{F}_3$ and $F_4\bar{F}_4$ in the coupled channel matrix.* The neutral currents remain unchanged by this modification but the charged current is multiplied by $\cos\theta_c$. Therefore the W exchange amplitudes are multiplied by $\cos^2\theta_c$ and the previously mentioned cancellation does not occur. If we write the channels in the order $F_{1+}\bar{F}_{1+}$, $F_{2+}\bar{F}_{2+}$, $F_{1-}\bar{F}_{1-}$, $F_{2-}\bar{F}_{2-}$, the matrix is

$$a_o = -\frac{G_F}{4\sqrt{2}\,\pi} \begin{bmatrix} m_1^2 & 0 & 0 & -m_1 m_2 x^2 \\ 0 & m_2^2 & -m_1 m_2 x^2 & 0 \\ 0 & -m_1 m_2 x^2 & m_1^2 & 0 \\ -m_1 m_2 x^2 & 0 & 0 & m_2^2 \end{bmatrix} \tag{2.9}$$

where $x^2 \equiv \sin^2\theta_c$. The absolute value of the largest eigenvalue is

$$|\lambda| = \frac{G_F}{8\sqrt{2}\,\pi}\left[m_1^2 + m_2^2 + \sqrt{(m_1^2 - m_2^2)^2 + 4 m_1^2 m_2^2 \sin^2\theta_c} \right]$$

and unitarity requires $|\lambda| < 1$. The critical curve $|\lambda| = 1$ defines the approximate limit of validity of the perturbation expansion. If we assume $m_1 = m_2 \equiv m$, we find that the bound (2.7) is improved by a factor $1 + \sin^2\theta_c$, i.e.,

$$m^2 < \frac{4\sqrt{2}\,\pi}{G_F} \frac{1}{1 + \sin^2\theta_c} . \tag{2.10}$$

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* If F_1 and F_2 are ultra heavy and F_3 and F_4 are light, then it is shown in ref. [11] that the flavor conservation of the neutral current to order αG_F implies that θ_c must be very small.

If there are N approximately degenerate doublets (i.e., mass differences much less than masses), the resulting $4N \times 4N$ matrix is approximated by a direct product of matrix (2.9) and the $N \times N$ matrix with all elements equal to one. Diagonalizing for $\theta_c = 0$ we have, therefore,

$$m^2 < \frac{4\sqrt{2}\,\pi}{G_F}\,\frac{1}{N} .$$

The improvement of the bound (2.10) can be understood by noting that $\theta_c = \pi/2$ corresponds to $N \geqslant 2$.

Next we consider the case in which the fermions are quarks, and again consider only the channels $F_1\bar{F}_1$ and $F_2\bar{F}_2$. Each quark comes in three colors denoted by R, B, Y.

Although the quark scattering amplitudes are not directly observable if quarks are confined, it is nevertheless correct for our purposes to insist that they satisfy partial wave unitarity. We are interested in whether the perturbation expansion for the weak interactions of the quarks is valid or not. Order by order in g, the perturbation expansion of this field theory is consistent with unitarity, and when the partial wave bounds are saturated by the Born terms, the expansion fails and the weak theory has become strongly interacting. This is all we are using the unitarity constraints to demonstrate.

For a quark doublet the coupled channel matrix is 36×36 because of the nine color channels, $R\bar{R}$, $R\bar{B}$, $\cdots$. But only the color-neutral channels, $R\bar{R}$, $B\bar{B}$, $Y\bar{Y}$ are important, so the matrix of interest is 12×12. The key difference from the lepton matrix

is that there is no t-channel W exchange in color off-diagonal matrix elements such as $R\bar{R} \to B\bar{B}$, so that the cancellation of flavor-helicity off-diagonal elements does not occur. The matrix is written in block form,

$$a_o = \begin{bmatrix} A & B & B \\ B & A & B \\ B & B & A \end{bmatrix} \tag{2.11}$$

where the matrices A and B are 4×4 . The matrix A contains only the color-diagonal amplitudes $R\bar{R} \to R\bar{R}$, $\cdots$ and is identical to the lepton matrix (2.9), while B represents the color off-diagonal amplitudes $R\bar{R} \to B\bar{B}$, $\cdots$ and is given by

$$B = -\frac{G_F}{4\sqrt{2}\,\pi}\begin{bmatrix} m_1^2 & 0 & 0 & -m_1m_2 \\ 0 & m_2^2 & -m_1m_2 & 0 \\ 0 & -m_1m_2 & m_1^2 & 0 \\ -m_1m_2 & 0 & 0 & m_2^2 \end{bmatrix} .$$

The matrix (2.11) is diagonalized in Appendix C. The absolute value of the largest eigenvalue is

$$|\lambda| = \frac{G_F}{8\sqrt{2}\,\pi}\left[3(m_1^2 + m_2^2) + \sqrt{9(m_1^2 - m_2^2)^2 + 4m_1^2m_2^2(2 + \sin^2\theta_c)^2}\,\right] . \tag{2.12}$$

For $m_1 = m_2 \equiv m$, inequality (2.7) is improved by a factor $5 + \sin^2\theta_c$,

$$m^2 < \frac{4\sqrt{2}\,\pi}{G_F}\,\frac{1}{5 + \sin^2\theta_c} . \tag{2.13}$$

For N nearly degenerate doublets the right-hand side is decreased by a factor of $\sim 1/N$. For $\theta_c = 0$ the bound in (2.13) implies that $m < 550$ GeV. For $\theta_c \neq 0$ and/or $m_1 \neq m_2$, the critical curve, obtained by setting the right-hand side of (2.12) to unity, approximately divides the (m_1, m_2) plane into weak and strong coupling regions.

(c) Coupled Channels: $J = 1$

In the case of higher partial waves $(J > 0)$, the amplitudes for $F_i F_i \to W^+W^-$, ZH, HH and ZZ also contribute to the coupled channel matrix. Their values in the limit $s \gg m_i^2 \gg M_W^2, M_H^2, M_Z^2$ are given in Appendix B. Only states where the vector mesons are longitudinally polarized are relevant. If the Higgs-particle were also ultra heavy then channels like $HH \to ZZ$ would be non-negligible[3]. The resulting matrix would be larger than the one discussed below and the bound would be improved. The $J = 1$ partial wave amplitudes are given by

$$a^1_{\mu\mu'} = \frac{1}{32\pi} \int_{-1}^{1} d(\cos\theta)\, d^1_{\mu\mu'}(\theta)\, T_{\mu\mu'} .$$

Here $d^1_{\mu\mu'}(\theta)$ is the well-known Wigner d-function appearing in the Jacob-Wick expansion. For fermions μ and μ' are defined by $\mu = (\lambda - \bar{\lambda})/2$ and $\mu' = (\lambda - \bar{\lambda}')/2$, where the λ's refer to the helicities of the fermions: λ $(\bar{\lambda})$ for the initial fermion (anti-fermion) and λ' $(\bar{\lambda}')$ for the final fermion (anti-fermion), and $\mu = 0$ for boson states which are either scalar H's or longitudinal W's and Z's. For $J = 1$ we can have $\mu, \mu' = 0, \pm 1$, and the possible channels are twelve

$\mu = +1$: $F_{1+}\bar{F}_{1-}$, $F_{2+}\bar{F}_{2-}$

$\mu = 0$: $F_{1+}\bar{F}_{1+}$, $F_{1-}\bar{F}_{1-}$, $F_{2+}\bar{F}_{2+}$, $F_{2-}\bar{F}_{2-}$, W^+W^-, $\frac{1}{\sqrt{2}}$ZZ, $\frac{1}{\sqrt{2}}$HH, ZH

$\mu = -1$: $F_{1-}\bar{F}_{1+}$, $F_{2-}\bar{F}_{2+}$.

However, when the partial waves are calculated, half of the channels decouple. We are therefore left with a 6×6 matrix. Writing the channels in the order $F_{1+}\bar{F}_{1-}$, $\bar{F}_{2+}F_{2-}$, W^+W^-, ZH, $F_{1-}\bar{F}_{1+}$, $F_{2-}\bar{F}_{2+}$, the $J = 1$ matrix is

$$a_1 = -\frac{G_F}{8\pi} \begin{bmatrix} 0 & 0 & -m_1^2 & m_1^2 & m_1^2/\sqrt{2} & m_1^2/\sqrt{2} \\ 0 & 0 & -m_2^2 & -m_2^2 & m_2^2/\sqrt{2} & m_2^2/\sqrt{2} \\ -m_1^2 & -m_2^2 & 0 & 0 & m_2^2 & m_1^2 \\ m_1^2 & -m_2^2 & 0 & 0 & m_1^2 & -m_2^2 \\ m_1^2/\sqrt{2} & m_2^2/\sqrt{2} & m_2^2 & m_1^2 & 0 & 0 \\ m_1^2/\sqrt{2} & m_2^2/\sqrt{2} & m_1^2 & -m_2^2 & 0 & 0 \end{bmatrix}$$

We have diagonalized this matrix analytically for the cases $m_1 = m_2$, and m_1 or m_2 vanishing. If $m_1 = m_2 \equiv m$, then the requirement that the absolute value of the largest eigenvalue be less than one yields the bound

$$m < 880 \text{ GeV}$$

while if either m_1 or m_2 is zero, the bound on the other is

$$m < 1.0 \text{ TeV}.$$

These are the best bounds we have found for leptons.

For quarks the $J = 1$ coupled channel matrix is 14×14 (color neutral channels only). However it yields higher values than the $J = 0$ matrix for the critical mass. For equal masses we find

$$m < 715 \text{ GeV}$$

which is larger than the value of 550 GeV obtained from the $J = 0$ matrix. We have also checked that $J = 2$ partial waves give less efficient bounds than $J = 0$ and $J = 1$.

(d) Effective Lagrangian

The leading high energy behavior of amplitudes which have at least one longitudinally polarized vector boson in the in or out states can be found very simply by studying the Higgs-Goldstone system which underlies the full broken theory. A simple effective Lagrangian was derived in ref. [3] for the case of scattering amplitudes involving only $W^{\pm}$, Z and H as external particles. Here we wish to extend this derivation to amplitudes involving fermions.

Before the gauge couplings are turned on, the fermion and Higgs sector of the $SU(2) \otimes U(1)$ model is described by the Lagrangian

$$\mathcal{L} = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 + \sum_{i=1}^{2} \bar{F}_{iR}\, i\not{\partial}\, F_{iR} + \bar{F}_L\, i\not{\partial}\, F_L$$

$$- G_1 \left[\bar{F}_L \tilde{\Phi} F_{1R} + \bar{F}_{1R} \tilde{\Phi}^\dagger F_L \right] - G_2 \left[\bar{F}_L \Phi F_{2R} + \bar{F}_{2R} \Phi^\dagger F_L \right] \tag{2.14}$$

where Φ and $\tilde{\Phi}$ are the complex Higgs scalar doublets

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_o \end{pmatrix} \qquad \tilde{\Phi} = \begin{pmatrix} \phi_o^\dagger \\ -\phi_- \end{pmatrix} .$$

Here ϕ_+ creates a negatively charged particle, and F_L and F_{iR} are the fermion doublet and singlets

$$F_L = \frac{1}{2}(1 - \gamma_5) \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \qquad F_{iR} = \frac{1}{2}(1 + \gamma_5) F_i .$$

The couplings G_1 and G_2 are to be determined. Shifting the origin of the ϕ_o field in the usual way by an amount $v/\sqrt{2} = \mu/\sqrt{2\lambda}$ and performing an infinitesimal gauge transformation away from the new vacuum, we have

$$\Phi = \exp\left[i \frac{\vec{\xi} \cdot \vec{\tau}}{v} \right] \begin{pmatrix} 0 \\ \frac{v + h}{\sqrt{2}} \end{pmatrix}$$

$$\Phi \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} i(\xi_1 - i\xi_2) \\ v + h - i\xi_3 \end{pmatrix} \qquad \tilde{\Phi} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} v + h + i\xi_3 \\ i(\xi_1 + i\xi_2) \end{pmatrix} .$$

The Lagrangian (2.14) then describes a theory with three massless Goldstone bosons $(w_\pm = (\xi_1 \mp i\xi_2)/\sqrt{2}, \quad z = \xi_3)$, a massive physical Higgs scalar h and massive fermions with masses $m_i = vG_i/\sqrt{2}$. When the gauge couplings are turned on, the Goldstone bosons acquire a gauge-dependent mass because of mixing with the longitudinal degrees of freedom of the vector bosons. If, however, we choose the 't Hooft-Feynman gauge, the masses of the $w_\pm$ and z are equal to those of $W^\pm$ and Z respectively. We then identify

$$\frac{1}{v} = \sqrt{\sqrt{2}\, G_F} = \frac{g}{2M_W}$$

and then the full interaction part of $\mathcal{L}$ is

$$\mathcal{L}_I = -\frac{1}{2} M_H^2 \sqrt{\sqrt{2}\, G_F}\, h(h^2 + z^2 + 2w_+w_-) - \frac{M_H^2 G_F}{4\sqrt{2}} (h^2 + z^2 + 2w_+w_-)^2$$

$$- \sqrt{\sqrt{2}\, G_F}\, h(m_1\bar{F}_1F_1 + m_2\bar{F}_2F_2)$$

$$- \sqrt{\sqrt{2}\, G_F}\, z(m_1\bar{F}_1 i\gamma_5 F_1 - m_2\bar{F}_2 i\gamma_5 F_2)$$

$$+ \sqrt{\frac{G_F}{\sqrt{2}}} \Big[i w_+ \bar{F}_1 (m_1 - m_2 - (m_1+m_2)\gamma_5) F_2$$

$$- i w_- \bar{F}_2 (m_1 - m_2 + (m_1+m_2)\gamma_5) F_1 \Big] \quad . \qquad (2.15)$$

It is straightforward to derive the Feynman rules from this effective Lagrangian and to compute the leading high-energy behavior of physical amplitudes. These calculations are considerably simpler than those in the original theory. As an example, consider $F_1\bar{F}_1 \to w_+w_-$. There are only two graphs, s-channel h exchange and t-channel F_2 exchange. The first one is $\mathcal{O}(1/\sqrt{s})$ in the limit of large s . The second one is

$$T_{\lambda\bar{\lambda}} = + \frac{G_F}{\sqrt{2}} \frac{M_{\lambda\bar{\lambda}}}{t - m_2^2}$$

where $M_{\lambda\bar{\lambda}}$ is the matrix element (with $m_\pm \equiv m_1 \pm m_2$)

$$M_{\lambda\bar{\lambda}} = \bar{v}^{(\bar{\lambda})}(m_- - m_+\gamma_5)(\not{q} + m_2)(m_- + m_+\gamma_5)u^{(\lambda)}$$

where q is the four-momentum transfer. Since we are interested only in the limit $s \to \infty$, we need keep only the $\mathcal{O}(s)$ term in $M_{\lambda\bar{\lambda}}$. This is

$$M_{\lambda\bar{\lambda}} = -2\bar{v}\Big[(m_1^2 + m_2^2 - (m_1^2 - m_2^2)\gamma_5)\not{p}_+\Big]u$$

where p_+ is the four-momentum of the outgoing w_+ . This is straightforward to compute and we get

$$M_{\lambda\bar{\lambda}} = s \sin\theta \Big[m_1^2 + m_2^2 + \lambda(m_1^2 - m_2^2)\Big]\, \delta_{\lambda,-\bar{\lambda}} \, .$$

Using $t - m_2^2 = -s\sin^2\theta/2 + \mathcal{O}(1/s)$ we obtain

$$T_{\lambda\bar{\lambda}} = -\sqrt{2}\, G_F \cot\frac{\theta}{2} \Big[m_1^2 + m_2^2 + \lambda(m_1^2 - m_2^2)\Big]\, \delta_{\lambda,-\bar{\lambda}}$$

which is identical to the exact result quoted in Appendix A.

It can be proved that the effective Lagrangian (2.15) gives the correct high energy behavior of amplitudes to leading order in the large fermion mass limit provided that there is at least one external vector meson which is longitudinally polarized. The proof is similar to that given in the appendix of ref. [3]. (The addition of fermions does not significantly affect the argument.)

In order to see that (2.15) also yields the leading high energy behavior when there are no external vector bosons, we note that when we computed with the full theory in the unitary gauge, we retained only the $k_\mu k_\nu$ piece of the vector boson propagator. This led to contributions to the amplitudes proportional to $g^2 m_F^2/M_W^2$. This part of the propagator corresponds to the longitudinal degree of freedom of the vector mesons. These degrees of freedom are precisely those given by $w_\pm$ and z in (2.15) so that the effective Lagrangian yields the correct amplitudes in the limit $m_F^2 \gg M_W^2$. The $g^{\mu\nu}$ piece of the full propagator gives a contribution to the amplitude which is constant

in the limit $s \to \infty$, and this contribution is not given by (2.15). However it is not proportional to m_F^2 and is therefore negligible.

III. LOW ENERGY EFFECTS

In this section we search for low energy manifestations of ultra heavy fermions. A similar search in the case of ultra heavy Higgs particles produced no interesting physical results[4]. We need renormalization effects which produce corrections to tree graphs of the form

$$A = A_{tree}\left[1 + \mathcal{O}\left(\frac{g^2 m_F^2}{M_W^2}\right)\right]$$

where A is some physical amplitude and A_{tree} is its value at tree level. Since for ultra heavy fermions (of mass scales given in the previous section) gm_F/M_W can be of order one, such effects should be observable. Effects proportional to $g^2 \log m_F$ of the form

$$A = A_{tree}\left[1 + \mathcal{O}\left(g^2 \log\left(\frac{m_F}{M_W}\right)\right)\right]$$

would be too small to detect in presently feasible experiments, as would effects which first appear in two loops,

$$A = A_{tree}\left[1 + \mathcal{O}(g^2) + \mathcal{O}(g^4 m_F^2/M_W^2)\right] .$$

In order to generate an effect proportional to m_F^2 from a loop containing a heavy fermion, we need either Higgs-fermion vertices, which are proportional to gm_F/M_W or a quadratically divergent loop integral. Consider the self-energy of the vector mesons W and Z,

$$\Pi^{\mu\nu}(p) = -\frac{g^2}{8M_W^2}\,\xi \int \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\Big[\gamma^\mu(C_V - C_A\gamma_5)(\not{k} - m_i)^{-1}$$

$$\times\, \gamma^\nu(C_V - C_A\gamma_5)(\not{p} + \not{k} - m_j)^{-1}\Big]$$

where for W

$$C_V = C_A = M_W$$

and for Z

$$C_V = M_Z(-T_3 + 2Q\sin^2\theta_W)$$

$$C_A = -T_3 M_Z .$$

In the Z case there are two graphs corresponding to $i = j = 1,2$, and for the W there is only one graph with $i = 1$, $j = 2$. T_3 is the weak isospin of the left-handed part of the fermion in the loop, and Q is its charge in units of the proton charge. ξ is a color factor; it has the value one for leptons and three for quarks. In order to obtain a factor of m_F^2 one must look at the quadratic divergence from the integral.

We evaluate the integral by dimensional regularization[9] replacing d^4k by d^nk and subtracting the pole at $n = 4$. We stipulate that $\{\gamma^\mu, \gamma_5\} = 0$ for all γ^μ in n dimensions. This prescription ensures that all relevant Ward identities are satisfied without the addition of extra counter terms[10]. Then, defining

$$\Pi^{\mu\nu}_{W,Z}(0) \equiv g^{\mu\nu}\,\Pi_{W,Z}$$

we get

$$\Pi_W = -\frac{i\,g^2}{32\pi^2}\left[m_1^2 \log\left(\frac{m_1^2}{\mu^2}\right) + m_2^2 \log\left(\frac{m_2^2}{\mu^2}\right) + \frac{m_1^2 m_2^2}{m_1^2 - m_2^2}\log\left(\frac{m_1^2}{m_2^2}\right) - \frac{1}{2}(m_1^2 + m_2^2)\right]$$

and

$$\Pi_Z = -\frac{i\,g^2}{32\pi^2 \cos^2\theta_W}\sum_{i=1}^{2} m_i^2 \log\left(\frac{m_i^2}{\mu^2}\right) . \qquad (3.1)$$

μ is an arbitrary regularization parameter with the dimension of mass. It must cancel out from all physical predictions. This self energy contributes to the mass renormalization of the W and Z

$$M_{W,Z}^2 = (M_{W,Z}^2)_0 \left(1 - i\frac{\Pi_{W,Z}}{M_{W,Z}^2}\right) . \qquad (3.2)$$

Here and in the following, quantities with a subscript zero are unrenormalized. There is a wave function renormalization of the W or Z from this graph which is of the form

$$1 + \frac{i}{4}\frac{d}{dp^2}\Pi_\mu^{\ \mu}(p^2)\Big|_{p^2=0} .$$

It involves only a logarithmic divergence in the integral over k and is therefore proportional to $g^2 \log m_F^2$ so we shall not consider it further. We can use (3.1) and (3.2) to calculate the one loop corrections to the formula $M_W/M_Z = \cos\theta_W$,

$$\frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 + \xi\frac{G_F}{8\sqrt{2}\,\pi^2} \times \left[\frac{2\,m_1^2 m_2^2}{m_1^2 - m_2^2}\log\left(\frac{m_2^2}{m_1^2}\right) + m_1^2 + m_2^2\right] . \qquad (3.3)$$

Notice that μ has cancelled out. There is no renormalization of $\cos\theta_W$ to order $m_F^2 g^2$; its renormalization, produced by the wave function renormalization of W and Z, is of order $g^2 \log m_F^2$. Notice that when $m_1 = m_2$, $M_W/M_Z\cos\theta_W = 1$ is not renormalized. For a lepton of mass m in a doublet with a massless neutrino we have

$$M_W^2 = M_Z^2 \cos^2\theta_W\left(1 + \frac{G_F m^2}{8\pi^2\sqrt{2}}\right) .$$

Using a compendium of all existing data, Sehgal[12] has concluded that

$$\frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 0.98 \pm 0.05 .$$

From this we can conclude that $m < 500$ GeV, i.e., there cannot exist a heavy lepton (with massless neutrino partner) of mass greater than 500 GeV.

All other graphs which produce terms of order $g^2 m_F^2$ involve Higgs particles. The Higgs particle self energy is

$$\Pi_H(p^2) = -\sum_{i=1}^{2}\frac{g^2 m_i^2}{4 M_W^2}\,\xi\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[(\not k - m_i)^{-1}(\not k + \not p - m_i)^{-1}\right] .$$

We compute

$$\Pi_H(0) = \frac{i\,g^2}{16\pi^2\,M_W^2}\,\xi \sum_{i=1}^{2} m_i^4\left(3\log\left(\frac{m_i^2}{\mu^2}\right) - 1\right)$$

and

$$\Pi_H'(0) \equiv \frac{d}{dp^2}\,\Pi_H(p^2)\Big|_{p^2=0} = -\frac{i\,g^2}{48\pi^2\,M_W^2}\,\xi \sum_{i=1}^{2} m_i^2\left(3\log\left(\frac{m_i^2}{\mu^2}\right) + 2\right).$$

To one loop the Higgs particle propagator becomes, when $p^2 \to 0$,

$$\frac{i\left(1 + i\,\Pi_H'(0)\right)}{p^2 - M_{HO}^2 + i\,\Pi_H(0)}$$

so the renormalized Higgs particle mass is

$$M_H^2 = M_{HO}^2 - \frac{g^2\,\xi^2}{48\pi^2\,M_W^2} \sum_{i=1}^{2} m_i^4\left(3\log\left(\frac{m_i^2}{\mu^2}\right) + 2\right). \qquad (3.4)$$

All other one loop graphs producing terms proportional to $g^2\,m_F^2$ can be obtained by attaching an arbitrary number of external Higgs particle legs to the fermion loops in the Higgs or vector boson self energies. We gain one power of m_F due to the coupling and lose one power since the loop becomes more convergent.

The HWW and HZZ proper vertices receive contributions

$$\Gamma^{HZZ}(p_1 = p_2 = 0) = \frac{-i\,g^3}{32\pi^2\cos^2\theta_W\,M_W} \sum_{i=1,2} m_i^2\left(\log\left(\frac{m_i^2}{\mu^2}\right) + 1\right)$$

$$\Gamma^{HWW}(p_1 = p_2 = 0) =$$

$$= \frac{-i\,g^3}{32\pi^2\,M_W}\left[\frac{m_1^2\,m_2^2}{m_1^2 - m_2^2}\log\left(\frac{m_1^2}{m_2^2}\right) + \sum_{i=1,2} m_i^2\left(\log\left(\frac{m_i^2}{\mu^2}\right) + 1\right)\right].$$

Using the result above we now calculate the large renormalizations of the Higgs coupling to light fermions and gauge bosons. Consider the coupling of the Higgs particle to a light fermion f. At tree level it is

$$\left(\lambda_{Hff}\right)_0 = g\,m_f/2M_{WO}\,.$$

At one loop

$$\lambda_{Hff} = \frac{g\,m_f}{2M_W}\left(1 - i\,\frac{\Pi_W}{2M_W^2} + \frac{i}{2}\,\Pi_H'(0)\right)$$

$$= \frac{g\,m_f}{2M_W}\left[1 + \xi\,\frac{G_F}{8\sqrt{2}\,\pi^2}\left\{\frac{m_1^2\,m_2^2}{m_1^2 - m_2^2}\log\left(\frac{m_2^2}{m_1^2}\right) + \frac{7}{6}\,(m_1^2 + m_2^2)\right\}\right].$$

Also the couplings of the Higgs particle to vector bosons become

$$\lambda_{HWW} = g\,M_W\left(1 + \frac{\Gamma^{HWW}}{ig\,M_W} + i\,\frac{\Pi_W}{2M_W^2} + \frac{i}{2}\,\Pi_H'(0)\right)$$

$$\lambda_{HWW} = g\,M_W\left[1 - \xi\,\frac{G_F}{8\sqrt{2}\,\pi^2}\left\{\frac{m_1^2\,m_2^2}{m_1^2 - m_2^2}\log\left(\frac{m_1^2}{m_2^2}\right) + \frac{5}{6}\,(m_1^2 + m_2^2)\right\}\right]$$

$$\lambda_{HZZ} = \frac{g\,M_Z}{\cos\theta_W}\left[1 - \xi\,\frac{G_F}{6\sqrt{2}\,\pi^2}\,(m_1^2 + m_2^2)\right].$$

Notice that the effect of the ultra heavy fermions is to decrease the coupling of the Higgs particle to vector bosons and to increase its coupling to light fermions. For quarks of equal mass which saturate the bound (2.13) the effect is of order 50% in λ^2_{HWW} and 25% in λ_{Hff}. For a lepton with a massless neutrino saturating the bound of Section IIc, the effect is of order 25% in λ^2_{HWW} and 20% in λ^2_{Hff}. Of course if fermions of larger masses exist then we will have no guide as to the values of these couplings due to the breakdown of perturbation theory.

There will also be large corrections to the three and four point Higgs particle self-couplings. These will be of the form

$$g^2 m_F^4/M_H^2 M_W^2 .$$

If M_H is not ultra heavy then the corrections are so large that perturbation theory is meaningless for them.

For two fermion doublets mixed by a Cabbibo angle θ_c as in (2.8) we have the following results

$$\frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 + \xi \frac{G_F}{8\sqrt{2}\,\pi^2}\left[2 \cos^2 \theta_c \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \log\left(\frac{m_2^2}{m_1^2}\right) + (m_1^2 + m_2^2)\right]$$

$$\lambda_{Hff} = \frac{g\, m_f}{2M_W}\left[1 + \xi \frac{G_F}{8\sqrt{2}\,\pi^2}\left(\cos^2 \theta_c \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \log\left(\frac{m_2^2}{m_1^2}\right) + \frac{7}{6}(m_1^2 + m_2^2)\right)\right]$$

$$\lambda_{HWW} = g\, M_W\left[1 - \xi \frac{G_F}{8\sqrt{2}\,\pi^2}\left(\cos^2 \theta_c \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \log\left(\frac{m_1^2}{m_2^2}\right) + \frac{5}{6}(m_1^2 + m_2^2)\right)\right]$$

$$\lambda_{HZZ} = \frac{g\, M_Z}{\cos \theta_W}\left[1 - \xi \frac{G_F}{6\sqrt{2}\,\pi^2}(m_1^2 + m_2^2)\right].$$

IV. GENERALIZATION OF THE STANDARD MODEL

In the computations of Sections II and III we worked within the context of the standard $SU(2) \times U(1)$ model with one Higgs doublet. We could vary the model by changing the gauge group or the representation content of the fermions and Higgs bosons. This would change our results quantitatively but not qualitatively, and in most models simple enough to merit consideration the quantitative changes would be less than an order of magnitude. The central feature is quite model independent: the Ward identities necessary for renormalizability guarantee that ultra heavy fermions must couple strongly to the longitudinally polarized gauge bosons and to the Higgs boson systems. The quantitative details depend on Clebsch-Gordan coefficients determined by the group and representation content of the model.

We do not propose to consider a laundry list of all possible models. Rather we will give attention to the experimentally least motivated feature of the standard model, which is the Higgs sector. So we continue to assume the $SU(2) \times U(1)$ gauge group with fermions in right-handed singlets and left-handed doublets, but we will allow

for any Higgs sector which is consistent with the present experimental results. The generalization of the low energy corrections computed in Section III is straightforward and in particular the correction to $M_W = M_Z \cos \theta_W$ is not affected at all. Here we will discuss explicitly the generalization of the determination of the critical mass values from partial wave unitarity in Section II.

The important experimental result which we use is the observation that the relation $M_W = M_Z \cos \theta_W$ is well satisfied in charged and neutral current neutrino scattering[12]. This means that the W and Z bosons receive their masses primarily from Higgs doublets. So we shall imagine n Higgs doublets Φ_i, $i = 1, \cdots, n$, each with a real neutral component ϕ_i, which acquires a vacuum expectation value $v_i \equiv \langle \phi_i \rangle_0$. Then the W boson mass is

$$M_W^2 = \frac{1}{4} g^2 \sum_{i=1}^{n} v_i^2$$

$$= \frac{1}{4} g^2 \vec{v}^2 \tag{4.1}$$

where $\vec{v} = (v_1, \cdots, v_n)$ is an n-dimensional vector that is real up to possible CP violating phases which are completely negligible for our purposes.

Next consider the spin zero bosons which can couple to the fermions. Here too only isodoublet representations are relevant: since $\bar{\psi}_L \psi_L = \bar{\psi}_R \psi_R = \bar{\psi}_L \gamma_5 \psi_L = \bar{\psi}_R \gamma_5 \psi_R = 0$ and since we take ψ_L to have $I = \frac{1}{2}$ and ψ_R to have $I = 0$, the only spin zero bosons which can couple to the fermions are isodoublets. The interaction is just the obvious generalization of the standard model, given explicitly in Appendix C, eq. (C.1). In particular the real neutral components ϕ_i

couple to a fermion doublet (F_1, F_2) by

$$\sum_{a=1,2} \sum_{i=1}^{n} \lambda_{ai} (\bar{\psi}_{La} \psi_{Ra} + \bar{\psi}_{Ra} \psi_{La}) \phi_i \tag{4.2}$$

so that the fermion masses are

$$m_a = \sum_{i=1}^{n} v_i \lambda_{ai} = \vec{v} \cdot \vec{\lambda}_a$$

$$= |\vec{v}| \cdot |\vec{\lambda}_a| \cos \theta_a \tag{4.3}$$

where $\vec{\lambda}_a$ is also a real n-dimensional vector making an angle θ_a with respect to $\vec{v}$.

Finally we must compute and diagonalize the $J = 0$ coupled channel matrix for this model. Using the effective Lagrangian of Section IIId, we see that the amplitudes are trivial generalizations of our previous results. In place of the four fields h, $w_\pm$, z in eq. (2.15) we now have 4n such fields with analogous couplings. The details are presented in Appencix C; here we simply state the results.

We assume that all Cabbibo angles are zero. Then for leptons the eigenvalues of the $J = 0$ matrix are

$$|a_o| = \frac{G_F}{4\sqrt{2}\,\pi} \frac{m_a^2}{\cos^2 \theta_a} . \tag{4.4}$$

Comparing with eq. (2.6) we see that the critical mass value is decreased by the factor $\cos^2 \theta_a$. For quarks we find

$$|a_o|_{max} = \frac{G_F}{8\sqrt{2}\,\pi}\left\{3\left(\frac{m_1^2}{\cos^2\theta_1} + \frac{m_2^2}{\cos^2\theta_2}\right)\right.$$

$$\left. + \sqrt{9\left(\frac{m_1^2}{\cos^2\theta_1} - \frac{m_2^2}{\cos^2\theta_2}\right)^2 + \frac{16\, m_1^2 m_2^2 \cos^2\theta_{12}}{\cos^2\theta_1 \cos^2\theta_2}}\right\} \tag{4.5}$$

where θ_{12} is the angle between $\vec{\lambda}_1$ and $\vec{\lambda}_2$. For $\theta_{12} = 0$, eq. (4.5) is greater or equal to the standard model result ($\theta_1 = \theta_2 = \theta_{12} = 0$) given in eq. (2.12) but in general it may be larger or smaller. However even in the worst case, $\theta_{12} = \frac{\pi}{2}$, the maximum eigenvalue is

$$|a_o|_{max} = \frac{3\, G_F}{4\sqrt{2}\,\pi}\left(\frac{m_a^2}{\cos^2\theta_a}\right)_{max} \tag{4.6}$$

so that the critical quark mass is less than or equal to $700(\cos\theta_a)_{min}$ GeV. As θ_1 or θ_2 approach $\pi/2$ strong coupling sets in for arbitrarily small fermion masses. (The phenomenological implications of this limit have been explored by Haber and Kane[13] in the two Higgs doublet model.)

V. CONCLUSION

We have considered the interactions and low energy renormalization effects of fermions much heavier than the W and Z bosons in the standard $SU(2) \times U(1)$ model of the weak interactions. By considering the scattering of such ultra heavy fermions we were able

to compute the critical mass values above which they become strongly interacting and the perturbation expansion fails. The values are of order $500\ \text{GeV}/\sqrt{N}$ for quarks and $1000\ \text{GeV}/\sqrt{N}$ for leptons, where N is the number of nearly degenerate doublets. Because, as we emphasized in the introduction to Section II, these critical mass estimates are conservatively large, and especially if $N \gtrsim 3$, it is possible that manifestations of such "strong weak interactions" could be seen at the coming generation of pp and $p\bar{p}$ colliding beam facilities. It is not inconceivable that strongly bound states of ultra heavy quarks could be observed with the weak interactions providing the binding force. In general the short distance dynamics of such quarks could be oxymoronic: while their weak interactions would be strong, their strong interactions would become weak due to asymptotic freedom if there are not too many quark flavors. Ultra heavy leptons would also form strongly bound states, but these could probably only be observed in e^+e^- collisions at the presently unimaginable TeV energy scale.

We have found large low energy renormalization effects which would change some of the predictions of the Weinberg-Salam model. The effect of most immediate interest is the correction to $M_W = M_Z \cos\theta_W$, obtained also by Veltman[8]. Using a compendium[12] of data on charged and neutral current scattering, this enables us to put an upper limit of ~ 500 GeV on the mass of a heavy lepton in a doublet with a massless neutrino. If there were a deep reason for the masslessness of the electron and muon neutrinos which would apply to all lepton flavors, then this bound would be very compelling. If in addition new quarks and leptons have masses of the same order of magnitude, as seems

to be true experimentally and is predicted in some super-unified models[14] of strong and weak interactions, then our bound would also imply a bound of the same order of magnitude on the quark mass spectrum. Though highly conjectural, this is the only clue we now have to an upper limit on the lepton and quark mass spectrum.

All the other large low energy renormalization effects involve Higgs mesons. In Section III we have catalogued all the amplitudes and Green's functions which get large one-loop corrections. We have explicitly computed the corrections to the predicted values of three physical quantities: the coupling constant of the Higgs boson to the W and Z bosons and the light fermions. The effect is to decrease the W and Z couplings and to increase the fermion couplings. Unlike the correction to $M_W = M_Z \cos \theta_W$, these corrections do not vanish if the ultra heavy fermion doublet is degenerate. If ultra heavy fermions exist, these corrections would make standard predictions for the production and decay of the Higgs particle unreliable.

We argued in Section IV that the essential qualitative features of our results are more general than the standard $SU(2) \times U(1)$ model assumed in Sections II and III. Because of the chiral Ward identities which must be satisfied by any broken gauge theory (except vector-like theories) in order to be renormalizable, ultra heavy fermions must quite generally be strongly coupled. This is even true if the gauge symmetry is broken dynamically and there are no elementary Higgs fields in the Lagrangian.*

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* We thank Ken Lane for an interesting discussion of this point.

ACKNOWLEDGMENTS

We wish to acknowledge the participation of Stanley Jones in some of the material in Section II. We are grateful for conversations with John Ellis, Howard Haber, Ken Lane, and Chris Quigg.

APPENDIX A

The helicity spinors are defined by

$$\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\, u^{(\lambda)}(\vec{p}) = \lambda\, u^{(\lambda)}(\vec{p}), \qquad \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\, v^{(\lambda)}(\vec{p}) = -\lambda\, v^{(\lambda)}(\vec{p})$$

where $\vec{\Sigma}$ denotes the spin operator, $\lambda = \pm 1$ indicates helicity $\pm\frac{1}{2}$ for both particle and antiparticle, and $\vec{p}$ is the momentum of either. As we explained in the text, we ignore photon exchange graphs and the $g^{\mu\nu}$-piece in the $W^{\pm}$ and Z propagators.

Defining λ, $\bar{\lambda}$ and λ', $\bar{\lambda}'$ to be the helicities of the incoming and outgoing fermion and anti-fermion respectively we obtain, in the center-of-mass frame, the following contributions to the helicity amplitudes for elastic scattering $F_i\bar{F}_i \to F_i\bar{F}_i$:

s-channel H-exchange

$$-4\sqrt{2}\, G_F\, m_i^{\,2}\, \frac{\vec{p}^{\,2}}{s - M_H^{\,2}}\, \lambda\lambda'\, \delta_{\lambda,\bar{\lambda}}\, \delta_{\lambda',\bar{\lambda}'}$$

t-channel H-exchange

$$-4\sqrt{2}\, G_F\, m_i^{\,2}\, \frac{1}{t - M_H^{\,2}} \left(E \sin\frac{\theta}{2}\, \lambda\, \delta_{\lambda,-\lambda'} - m_i \cos\frac{\theta}{2}\, \delta_{\lambda,\lambda'}\right) \times \left(E \sin\frac{\theta}{2}\, \bar{\lambda}\, \delta_{\bar{\lambda},-\bar{\lambda}'} - m_i \cos\frac{\theta}{2}\, \delta_{\bar{\lambda},\bar{\lambda}'}\right)$$

s-channel Z-exchange

$$-\sqrt{2}\, G_F\, m_i^{\,2}\, \frac{s}{s - M_Z^{\,2}}\, \delta_{\lambda,\bar{\lambda}}\, \delta_{\lambda',\bar{\lambda}'}$$

t-channel Z-exchange

$$4\sqrt{2}\, G_F\, m_i^{\,2}\, \frac{\vec{p}^{\,2} \sin^2\frac{\theta}{2}}{t - M_Z^{\,2}}\, \delta_{\bar{\lambda},-\bar{\lambda}'}\, \delta_{\lambda,-\lambda'} .$$

For the quantum number exchange scattering $F_1\bar{F}_1 \longleftrightarrow F_2\bar{F}_2$ we obtain:

s-channel H-exchange

$$-4\sqrt{2}\, G_F\, m_1 m_2\, \frac{\vec{p}^{\,2}}{s - M_H^{\,2}}\, \lambda\lambda'\, \delta_{\lambda,\bar{\lambda}}\, \delta_{\lambda',\bar{\lambda}'}$$

s-channel Z-exchange

$$\sqrt{2}\, G_F\, m_1 m_2\, \frac{s}{s - M_Z^{\,2}}\, \delta_{\lambda,\bar{\lambda}}\, \delta_{\lambda',\bar{\lambda}'}$$

t-channel W-exchange

$$2\sqrt{2}\, G_F\, \frac{1}{t - M_W^{\,2}} \left(E \sin\frac{\theta}{2} \left((1+\lambda)m_1 + (1-\lambda)m_2\right)\delta_{\lambda,-\lambda'} - \frac{1}{2}\cos\frac{\theta}{2}\,(m_1^{\,2} - m_2^{\,2})(1-\lambda)\delta_{\lambda,\lambda'}\right) \times \left(E \sin\frac{\theta}{2} \left((1-\bar{\lambda})m_1 + (1+\bar{\lambda})m_2\right)\delta_{\bar{\lambda},-\bar{\lambda}'} + \frac{1}{2}\cos\frac{\theta}{2}\,(m_1^{\,2} - m_2^{\,2})(1+\bar{\lambda})\delta_{\bar{\lambda},\bar{\lambda}'}\right) .$$

Here s and t are the usual Mandelstam variables, G_F is Fermi's constant, related to M_W and the gauge coupling constant g by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\, M_W^{\,2}}$$

and θ is the center-of-mass scattering angle.

In the limit $s \gg m_i^2$ the amplitudes simplify considerably, and the nonvanishing contributions from each graph are:

$\underline{F_i\bar{F}_i \to F_i\bar{F}_i}$

s-channel H

$$\left.\begin{array}{l} ++ \to ++ \\ -- \to -- \end{array}\right\} = -\sqrt{2}\, G_F\, m_i^2$$

$$\left.\begin{array}{l} ++ \to -- \\ -- \to ++ \end{array}\right\} = +\sqrt{2}\, G_F\, m_i^2$$

t-channel H

$$\left.\begin{array}{l} ++ \to -- \\ -- \to ++ \end{array}\right\} = +\sqrt{2}\, G_F\, m_i^2$$

$$\left.\begin{array}{l} +- \to -+ \\ -+ \to +- \end{array}\right\} = -\sqrt{2}\, G_F\, m_i^2$$

s-channel Z

$$\left.\begin{array}{l} ++ \to ++ \\ -- \to -- \\ ++ \to -- \\ -- \to ++ \end{array}\right\} = -\sqrt{2}\, G_F\, m_i^2$$

t-channel Z

$$\left.\begin{array}{l} ++ \to -- \\ -- \to ++ \\ +- \to -+ \\ -+ \to +- \end{array}\right\} = -\sqrt{2}\, G_F\, m_i^2$$

$\underline{F_1\bar{F}_1 \longleftrightarrow F_2\bar{F}_2}$

s-channel H

$$\left.\begin{array}{l} ++ \to ++ \\ -- \to -- \end{array}\right\} = -\sqrt{2}\, G_F\, m_1 m_2$$

$$\left.\begin{array}{l} ++ \to -- \\ -- \to ++ \end{array}\right\} = +\sqrt{2}\, G_F\, m_1 m_2$$

s-channel Z

$$\left.\begin{array}{l} ++ \to ++ \\ -- \to -- \\ ++ \to -- \\ -- \to ++ \end{array}\right\} = +\sqrt{2}\, G_F\, m_1 m_2$$

t-channel W

$$\left.\begin{array}{l} ++ \to -- \\ -- \to ++ \end{array}\right\} = -2\sqrt{2}\, G_F\, m_1 m_2$$

$$+- \to -+ \quad = -2\sqrt{2}\, G_F\, m_1^2$$

$$-+ \to +- \quad = -2\sqrt{2}\, G_F\, m_2^2 \,.$$

Adding up all graphs, the final result for the nonvanishing amplitudes for leptons or quarks in the color-diagonal channels $R\bar{R} \to R\bar{R}, \cdots$, are

$\underline{F_i\bar{F}_i \to F_i\bar{F}_i}$

$$\left.\begin{array}{l} ++ \to ++ \\ -- \to -- \\ +- \to -+ \\ -+ \to +- \end{array}\right\} = -2\sqrt{2}\, G_F\, m_i^2$$

$\underline{F_1\bar{F}_1 \longleftrightarrow F_2\bar{F}_2}$

$$+- \to -+ \quad = -2\sqrt{2}\, G_F\, m_1^2$$

$$-+ \to +- \quad = -2\sqrt{2}\, G_F\, m_2^2 .$$

In the color-changing channels $R\bar{R} \to B\bar{B}, \cdots,$ the nonvanishing amplitudes are

$$\underline{F_i\bar{F}_i \to F_i\bar{F}_i}$$

$$\left.\begin{array}{l} + + \to + + \\ - - \to - - \end{array}\right\} = -2\sqrt{2}\, G_F\, m_i^{\,2}$$

$$\underline{F_1\bar{F}_1 \leftrightarrow F_2\bar{F}_2}$$

$$\left.\begin{array}{l} + + \to - - \\ - - \to + + \end{array}\right\} = 2\sqrt{2}\, G_F\, m_1 m_2 \; .$$

APPENDIX B

We shall compute the amplitudes for the processes $F_i\bar{F}_i \to W^+W^-$, ZZ, HH and ZH, where the vector bosons are longitudinally polarized in all cases. We first compute in detail the process $F_2\bar{F}_2 \to W^-W^+$ in the CMS. The scattering angle θ is defined between the outgoing W^- and the incoming F_2. The couplings of the Z boson to the fermions has the general form

$$\mathcal{L}_I = \frac{M_Z}{2}\sqrt{\sqrt{2}\, G_F}\;\; \bar{F}_i\, \gamma^\mu (C_V + C_A\, \gamma_5) F_i\, Z_\mu \, , \qquad i = 1,2$$

where

$$C_{Vi} = (4\, Q \sin^2 \theta_W - 2\, T_3)_i$$

$$C_{Ai} = (2\, T_3)_i$$

and Q is the electric charge of F_i in units of the proton charge, and T_3 is the z-component of the weak isospin of the left-handed doublet ($T_3 = +1/2$ or $-1/2$ for F_1 or F_2, respectively). We compute the T-matrix elements $T_{\lambda\bar{\lambda}}(s,\theta)$ where λ and $\bar{\lambda}$ denote the helicities of F and $\bar{F}$, respectively.

We obtain the following nonvanishing contributions from each graph for $F_2\bar{F}_2 \to W^-W^+$:

s-channel photon

$$T_{++} = -T_{--} = +4\sqrt{2}\, G_F\, m_2\, Q_2 \sin^2 \theta_W \sqrt{s} \cos\theta\, \beta_W \left(1 + \frac{2\, M_W^{\,2}}{s}\right)$$

$$T_{+-} = T_{-+} = +2\sqrt{2}\, G_F\, Q_2 \sin^2 \theta_W\, s \sin\theta\, \beta_W \left(1 + \frac{2\, M_W^{\,2}}{s}\right) .$$

s-channel H

$$T_{++} = -T_{--} = -\sqrt{2}\, G_F\, m_2 \frac{s\sqrt{s}}{s - M_H^2}\, \beta \left(1 - \frac{2 M_W^2}{s}\right).$$

s-channel Z

$$T_{++} = -T_{--} = -\sqrt{2}\, G_F\, m_2\, C_{V2} \frac{s\sqrt{s}\cos\theta}{s - M_Z^2}\, \beta_W \left(1 + \frac{2 M_W^2}{s}\right)$$

$$T_{+-} = -\frac{1}{2}\sqrt{2}\, G_F (C_{V2} + C_{A2}\,\beta) \frac{s^2 \sin\theta}{s - M_Z^2}\, \beta_W \left(1 + \frac{2 M_W^2}{s}\right)$$

$$T_{-+} = -\frac{1}{2}\sqrt{2}\, G_F (C_{V2} - C_{A2}\,\beta) \frac{s^2 \sin\theta}{s - M_Z^2}\, \beta_W \left(1 + \frac{2 M_W^2}{s}\right)$$

t-channel F_1

$$T_{++} = -T_{--} = +\sqrt{2}\, G_F\, m_2 \sqrt{s}(1 + \cos\theta)\beta_W$$

$$- \frac{\sqrt{2}\, G_F\, m_2\, s\sqrt{s}}{4(t - m_1^2)} \left[-\beta_W (1 + \cos\theta)\left((\beta - \beta_W)^2 + \frac{4 m_1^2}{s}\right) + \frac{4 M_W^2}{s}(\beta_W \cos\theta - \beta \cos 2\theta)\right]$$

$$T_{+-} = +\frac{\sqrt{2}\, G_F\, m_2^2\, s \sin\theta}{(1+\beta)(t - m_1^2)} \left(\beta\beta_W + \beta\cos\theta - \frac{2 M_W^2}{s}\beta_W\right)$$

$$T_{-+} = +\frac{1}{2}\sqrt{2}\, G_F\, s \sin\theta\, \beta_W (1 + \beta)$$

$$- \frac{\sqrt{2}\, G_F\, s \sin\theta}{t - m_1^2} \left[- m_2^2 \beta_W + \frac{1}{2}\beta_W (1+\beta)(m_2^2 - m_1^2 + 2 M_W^2) - \beta(1+\beta) M_W^2 \cos\theta \right].$$

We have defined β and β_W to be the velocities of the fermions and W's, respectively. When one adds up all the graphs, all the pieces that grow with s or $\sqrt{s}$ cancel, as required by renormalizability. The amplitude for $F_1\bar{F}_1 \to W^+W^-$ is obtained from the above one by replacing $m_1 \leftrightarrow m_2$ and $Q_2, C_{V2}, C_{A2} \to Q_1, C_{V1}, C_{A1}$.

For the amplitude $F_i\bar{F}_i \to ZZ$ we simplify the calculation by neglecting all pieces proportional to $G_F M_Z^2$. Then for $i = 1$ or 2, we get:

s-channel H

$$T_{++} = -T_{--} = -\sqrt{2}\, G_F\, m_i\, \beta \frac{s\sqrt{s}}{s - M_H^2}$$

t-channel F_i + u-channel F_i

$$T_{++} = -T_{--} = +\sqrt{2}\, G_F\, m_i \sqrt{s}\, \beta \left(1 - \frac{4m_i^2}{s} \times \frac{\cos^2\theta}{\sin^2\theta + \frac{4m_i^2}{s}\cos^2\theta}\right)$$

$$T_{+-} = T_{-+} = -2\sqrt{2}\, G_F\, m_i^2\, \beta \frac{\sin\theta\cos\theta}{\sin^2\theta + \frac{4m_i^2}{s}\cos^2\theta}$$

For $F_i\bar{F}_i \to HH$ the exact answers are:

s-channel H

$$T_{++} = -T_{--} = +3\sqrt{2}\, G_F\, m_i\, \beta \frac{M_H^2\sqrt{s}}{s - M_H^2}$$

t-channel F_i + u-channel F_i

$$T_{++} = -T_{--} = \frac{-4\sqrt{2}\, G_F\, m_i^3}{\sqrt{s}} \left(\frac{\beta - \cos\theta}{1 + \beta^2 - 2\beta\cos\theta + \frac{4m_i^2}{s}} + (\cos\theta \to -\cos\theta) \right)$$

$$T_{+-} = T_{-+} = +2\sqrt{2}\, G_F\, m_i^2 \sin\theta$$

$$\times \left(\frac{1}{1 + \beta^2 - 2\beta\cos\theta + \frac{4m_i^2}{s}} - (\cos\theta \to -\cos\theta) \right).$$

Finally, for $F_i\bar{F}_i \to ZH$, neglecting again terms proportional to M_Z^2, we find:

s-channel Z

$$T_{++} = T_{--} = -\sqrt{2}\, G_F\, m_i\, C_{Ai} \sqrt{s}$$

t-channel F_i + u-channel F_i

$$T_{++} = T_{--} = \sqrt{2}\, G_F\, m_i\, C_{Ai} \sqrt{s} \left[1 - \frac{2m_i^2}{s} \left(\frac{1}{1 - \beta\cos\theta} + \frac{1}{1 + \beta\cos\theta} \right) \right]$$

$$T_{+-} = -T_{-+} = \sqrt{2}\, G_F\, m_i^2\, C_{Ai}\, \beta \sin\theta \left[\frac{1}{1 - \beta\cos\theta} + \frac{1}{1 + \beta\cos\theta} \right].$$

In the limit $s \gg m_i^2 \gg M_W^2$ the amplitudes simplify considerably, and the only nonvanishing amplitudes left over are:

$\underline{F_1\bar{F}_1 \to W^+W^-}$

$$T_{+-} = -2\sqrt{2}\, G_F\, m_1^2 \cot\frac{\theta}{2}$$

$$T_{-+} = -2\sqrt{2}\, G_F\, m_2^2 \cot\frac{\theta}{2}$$

$\underline{F_2\bar{F}_2 \to W^-W^+}$

$$T_{+-} = -2\sqrt{2}\, G_F\, m_2^2 \cot\frac{\theta}{2}$$

$$T_{-+} = -2\sqrt{2}\, G_F\, m_1^2 \cot\frac{\theta}{2}$$

$\underline{F_i\bar{F}_i \to HH}$

$$T_{+-} = T_{-+} = 2\sqrt{2}\, G_F\, m_i^2 \cot\theta$$

$\underline{F_i\bar{F}_i \to ZZ}$

$$T_{+-} = T_{-+} = -2\sqrt{2}\, G_F\, m_i^2 \cot\theta$$

$\underline{F_i\bar{F}_i \to ZH}$

$$T_{+-} = -T_{-+} = C_{Ai}\, 2\sqrt{2}\, G_F\, m_i^2 \csc\theta\,.$$

APPENDIX C

In this Appendix we derive the results which were stated in Section IV. As explained there, we consider the $SU(2) \times U(1)$ gauge theory with fermions in left-handed doublets and right-hand singlets and with an arbitrary number of Higgs doublets, $\Phi_1, \cdots, \Phi_n$. We will compute the $J = 0$ coupled channel matrix for a fermion doublet (F_1, F_2) in the annihilation channels $F_a\bar{F}_a \longleftrightarrow F_b\bar{F}_b$, $a,b = 1,2$, in the limit $\sqrt{s} \gg m_1, m_2 \gg M_W, M_Z$ and $\sqrt{s} \gg$ all physical Higgs boson masses.

Each Higgs doublet Φ_i has four components. Three of the $4n$ components are "swallowed" by the gauge bosons and the remaining $4n - 3$ components survive as physical particles. As remarked in the discussion of the effective Lagrangian, Section IId, it is precisely the three "swallowed" Higgs bosons which are responsible for the contribution of the gauge bosons in the limit of interest. In the effective Lagrangian we work in the 't Hooft-Feynman gauge, which amounts to computing as if the three unphysical Higgs bosons were not swallowed. We follow that approach now, so that the effective Lagrangian becomes a sum of n terms, each like eq. (2.15). We can easily use this generalized effective Lagrangian to compute to leading order in $G_F m_a^2$, without bothering to isolate the three unphysical Higgs bosons.

Since we are only computing $F_a\bar{F}_a \longleftrightarrow F_b\bar{F}_b$ in tree approximation, it is sufficient to give the boson-fermion interaction in the generalized effective Lagrangian:

$$\mathcal{L}_I = \sum_{i=1}^{n} -h_i(\lambda_{1i}\bar{F}_1F_1 + \lambda_{2i}\bar{F}_2F_2) - iz_i(\lambda_{1i}\bar{F}_1\gamma_5F_1 - \lambda_{2i}\bar{F}_2\gamma_5F_2)$$
$$+ \sqrt{2}\, i\, w_i^+\bar{F}_1\big(\lambda_{1i} - \lambda_{2i} - (\lambda_{1i} + \lambda_{2i})\gamma_5\big)F_2$$
$$- \sqrt{2}\, i\, w_i^-\bar{F}_2\big(\lambda_{1i} - \lambda_{2i} + (\lambda_{1i} + \lambda_{2i})\gamma_5\big)F_1 \quad . \tag{C.1}$$

Here h_i, z_i, and $w_i^\pm$ are the components of Φ_i, defined by

$$\Phi_i = \frac{1}{\sqrt{2}}\begin{pmatrix} i\sqrt{2}\, w_i^+ \\ v_i + h_i - i\, z_i \end{pmatrix}.$$

It is now very easy to generalize the s and t channel exchange diagrams compiled in Appendix A: in place of each diagram we get the sum of n terms with $gm_a/2M_W$ replaced by λ_{ai}. We will neglect possible Cabbibo angles. Adding all the diagrams we find that the lepton coupled channel matrix is already diagonal because of the same cancellations that occurred in Section II:

$$a_o = \frac{-G_F}{4\sqrt{2}\,\pi}\begin{pmatrix} \frac{m_1^2}{\cos^2\theta_1} & 0 & 0 & 0 \\ 0 & \frac{m_2^2}{\cos^2\theta_2} & 0 & 0 \\ 0 & 0 & \frac{m_1^2}{\cos^2\theta_1} & 0 \\ 0 & 0 & 0 & \frac{m_2^2}{\cos^2\theta_2} \end{pmatrix} \tag{C.2}$$

The rows and columns are labeled in the order $F_{1+}\bar{F}_{1+}$, $F_{2+}\bar{F}_{2+}$, $F_{1-}\bar{F}_{1-}$, $F_{2-}\bar{F}_{2-}$. Where $\vec{v} = (v_i)$ and $\vec{\lambda}_a = (\lambda_{ai})$ are real n dimensional vectors, as discussed in Section IV, we have used the relations

$$M_W^2 = \frac{1}{4} g^2 \vec{v}^2 \tag{4.1}$$

$$m_a = \vec{\lambda}_a \cdot \vec{v} \tag{4.3}$$

$$= |\vec{\lambda}_a| \cdot |\vec{v}| \cdot \cos\theta_a .$$

For the quark matrix we again find the same structure as we found in Section II for the standard model. We display only the color neutral channels since they contain the maximum eigenvalue. Where the rows and columns are labeled by the color channels $R\bar{R}$, $B\bar{B}$, and $Y\bar{Y}$, the $J = 0$ matrix is

$$a_o = \begin{pmatrix} A & B & B \\ B & A & B \\ B & B & A \end{pmatrix} . \tag{C.3}$$

A and B are 4×4 matrices labeled just like the lepton matrix (C.2). A is given by precisely the same expression as (C.2) and B is given by

$$B = \frac{-G_F}{4\sqrt{2}\,\pi} \begin{pmatrix} \frac{m_1^2}{\cos^2\theta_1} & 0 & 0 & x \\ 0 & \frac{m_2^2}{\cos^2\theta_2} & x & 0 \\ 0 & x & \frac{m_1^2}{\cos^2\theta_1} & 0 \\ x & 0 & 0 & \frac{m_2^2}{\cos^2\theta_2} \end{pmatrix} \tag{C.4}$$

where

$$x \equiv -\frac{m_1 m_2 \cos\theta_{12}}{\cos\theta_1 \cos\theta_2}$$

$$\cos\theta_{12} \equiv \frac{\vec{\lambda}_1 \cdot \vec{\lambda}_2}{|\vec{\lambda}_1| \cdot |\vec{\lambda}_2|} .$$

To diagonalize (C.3) we first diagonalize the color degrees of freedom. Since the matrix

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

has eigenvalues (2, -1, -1), after a unitarity transformation of the color degrees of freedom (C.3) becomes

$$\begin{pmatrix} A + 2B & 0 & 0 \\ 0 & A - B & 0 \\ 0 & 0 & A - B \end{pmatrix} . \tag{C.5}$$

The maximal eigenvalue is found in $A + 2B$, which can be put in 2×2 block diagonal form by a unitarity matrix which interchanges $F_{2+}\bar{F}_{2+} \longleftrightarrow F_{2-}\bar{F}_{2-}$:

$$U^\dagger (A + 2B) U = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix}$$

where

$$C = -\frac{G_F}{4\sqrt{2}\,\pi}\begin{pmatrix} \dfrac{3m_1^{\,2}}{\cos^2\theta_1} & -2x \\ -2x & \dfrac{3m_2^{\,2}}{\cos^2\theta_2} \end{pmatrix}. \qquad (C.7)$$

Diagonalizing the matrix C we obtain the maximal eigenvalue quoted in eq. (4.5).

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This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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