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SWEDISH-AMERICAN COOPERATIVE PROGRAM ON RADIOACTIVE WASTE STORAGE IN MINED CAVERNS IN CRYSTALLINE ROCK

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DETERMINATION OF IN-SITU THERMAL PROPERTIES OF STRIPA GRANITE FROM TEMPERATURE MEASUREMENTS IN THE FULL-SCALE HEATER EXPERIMENTS

METHOD AND PRELIMINARY RESULTS

J. A. Jeffry, T. Chan, N.G.W. Cook, and P. A. Witherspoon
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Berkeley, California 94720

May 1979

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PREFACE

This report is one of a series documenting the results of the Swedish-American cooperative research program in which the cooperating scientists explore the geological, geophysical, hydrological, geochemical, and structural effects anticipated from the use of a large crystalline rock mass as a geologic repository for nuclear waste. This program has been sponsored by the Swedish Nuclear Power Utilities through the Swedish Nuclear Fuel Supply Company (SKBF), and the U.S. Department of Energy (DOE) through the Lawrence Berkeley Laboratory (LBL).

The principal investigators are L. B. Nilsson and O. Degerman for SKBF, and N. G. W. Cook, P. A. Witherspoon, and J. E. Gale for LBL. Other participants will appear as authors of the individual reports.

Previous technical reports in this series are listed below.

1. Swedish-American Cooperative Program on Radioactive Waste Storage in Mined Caverns by P. A. Witherspoon and O. Degerman. (LBL-7049, SAC-01).

2. Large Scale Permeability Test of the Granite in the Stripa Mine and Thermal Conductivity Test by Lars Lundstrom and Haken Stille. (LBL-7052, SAC-02).

3. The Mechanical Properties of the Stripa Granite by Graham Swan. (LBL-7074, SAC-03).

4. Stress Measurements in the Stripa Granite by H. Carlsson. (LBL-7078, SAC-04).

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9. Theoretical Temperature Fields for the Stripa Heater Project by Tin Chan, Neville G. W. Cook, and Chin-Fu Tsang. (LBL-7082, SAC-09).

10. Mechanical and Thermal Design Considerations for Radioactive Waste Repositories in Hard Rock. Part I: An Appraisal of Hard Rock for Potential Underground Repositories of Radioactive Wastes by Neville G. W. Cook; Part II: In Situ Heating Experiments in Hard Rock: Their Objectives and Design by Neville G. W. Cook and Paul A. Witherspoon. (LBL-7073, SAC-10).

11. Full-Scale and Time-Scale Heating Experiments at Stripa: Preliminary Results by Neville G.W. Cook and Michael Hood. (LBL-7072, SAC-11).

12. Geochemistry and Isotope Hydrology of Groundwaters in the Stripa Granite: Results and Preliminary Interpretation by P. Fritz, J.F. Barker, and J.E. Gale. (LBL-8285, SAC-12).

13. Electrical Heaters for Thermo-mechanical Tests at the Stripa Mine by R. H. Burleigh, E. P. Binnall, A. O. DuBois, D. U. Norgren, and A. R. Oritz. (LBL-7063, SAC-13).

14. Data Acquisition, Handling, and Display for the Heater Experiments at Stripa by Maurice B. McEvoy. (LBL-7062, SAC-14).

15. An Approach to the Fracture Hydrology at Stripa: Preliminary Results by J. E. Gale and P. A. Witherspoon. (LBL-7079, SAC-15).

16. Preliminary Report on Geophysical and Mechanical Borehole Measurements at Stripa by P. Nelson, B. Paulsson, R. Rachiele, L. Andersson, T. Schrauf, W. Hustrulid, O. Duran, and K. A. Magnusson. (LBL-8280, SAC-16).

17. Observations of a Potential Size-Effect in Experimental Determination of the Hydraulic Properties of Fractures by P. A. Witherspoon, C. H. Amick, J. E. Gale, and K. Iwai. (LBL-8571, SAC-17).

18. Rock Mass Characterization for Storage of Nuclear Waste in Granite by P. A. Witherspoon, P. Nelson, T. Doe, R. Thorpe, B. Paulsson, J. Gale, and C. Forster (LBL-8570, SAC-18).

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21. Geology and Fracture System at Stripa by A. Olkiewicz, J. E. Gale, R. Thorpe, and B. Paulsson. (LBL-8907, SAC-21).

22. Calculated Thermally Induced Displacements and Stress for Heater Experiments at Stripa by T. Chan and N. G. W. Cook. (LBL-7061, SAC-22).

23. Validity of Cubic Law for Fluid Flow in a Deformable Rock Fracture by P. A. Witherspoon, J. S. Y. Wang, K. Iwai, and J. E. Gale. (LBL-9557, SAC-23).

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ABSTRACT

The in-situ thermal conductivity and thermal diffusivity of a granite rock mass at the Stripa mine, Sweden, have been extracted from the first 70 days of temperature data for the 5 kW full-scale heater experiment by means of least-squares fit to a finite-line source solution. Thermal conductivity and thermal diffusivity have been determined to be 3.69 W/(m-°C) and 1.84 x 10^{-6} m^2/s, respectively, at an average rock temperature of 23°C (the average value of the actual temperature data used). These values are only slightly higher than the corresponding laboratory values, i.e., there is no significant "size effect" in the thermal properties of this rock mass. Since the size and shape of the heater canister used are similar to those considered for nuclear waste canisters and a substantial volume of rock is heated, the thermal properties obtained in this study are representative of in-situ rock mass properties under actual nuclear repository operating conditions.

1. INTRODUCTION

A possible solution to the problem of radioactive waste isolation is to bury the wastes deep underground in a stable rock formation. In-situ electrical heater experiments are being conducted at the Stripa mine in Sweden to study the possible thermal and thermomechanical effects of the heat energy released by radioactive decay of the wastes after the waste canisters have been emplaced in granite. These experiments are part of the Swedish-American Cooperative Program on Radioactive Waste Storage in Mined Caverns in Crystalline Rock (Witherspoon and Degerman 1978). The temperature data from these experiments can also be used to determine the in-situ thermal properties of the rock mass: its thermal conductivity and thermal diffusivity. Due to the configuration of the experiments, the classical "probe" method for determining the properties, which requires a length-to-diameter ratio over 25:1 (Blackwell 1956), is not possible. Instead, a least-squares computer program has been written to determine the thermal properties. The least-squares method is in common use for statistical inversion of experimental data in the physical sciences (Hamilton 1964). Here we mention only two previous studies relevant to the present work. McEdwards and Tsang (1977) have developed a least-squares technique to study well test data. An infinite line source was used to model the well in their analysis. Toews, Larocque, and Wong (1977) have, as we have, studied both the estimation of in-situ thermal properties using a least-squares scheme and the stablity of the estimation process. In their closed form solution, they used as their model an infinite-line heat source, which is only adequate for short times and for points very near the heater. In order to model the finite length heater, they used a finite element solution and

tabulated the results, to be looked up during the least-squares iteration which utilizes dimensional analysis. They analyzed only hypothetical data because they had no field data at the time their report was written.

Initially we have attempted to use an infinite line source model in our analysis, but the least-squares procedure did not converge. Instead, we have used the closed-form, finite-line source solution developed by Chan, Cook, and Tsang (1978) in our least-squares scheme, which analyzes the temperature data from the in-situ heater experiment and extracts the thermal conductivity and thermal diffusivity of the granitic rock mass.

2. DESCRIPTION OF THE EXPERIMENT

The Stripa project consists of three experiments, two full-scale experiments and one time-scaled experiment, which are underway at the Stripa mine in Sweden. The experiments are located in separate drifts branching off from a main tunnel, at depths ranging from 338 m to 360 m below the surface (see Fig. 1). The waste canisters are simulated by cylindrical electric heaters of variable power output; thermocouples, extensometers, and stress-measuring gauges have been placed in the surrounding rock.

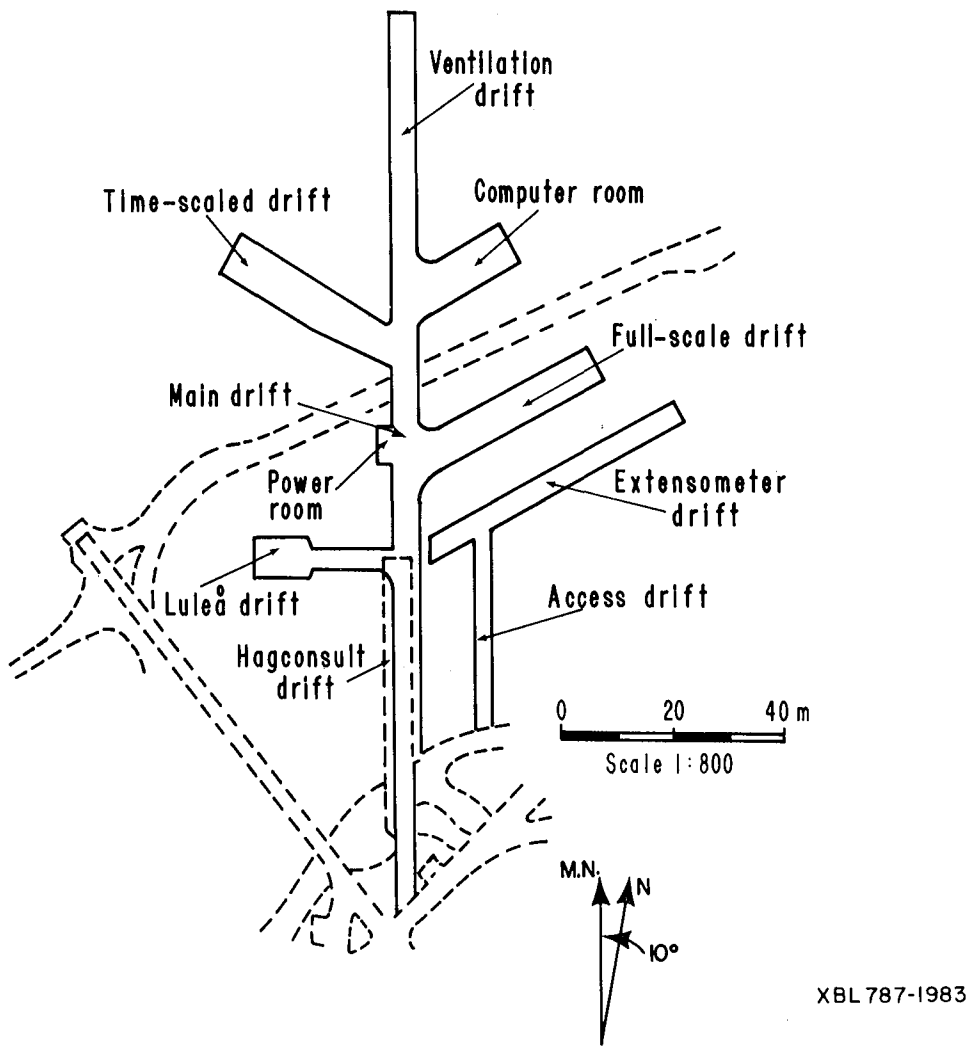

Fig. 1. General plan of test site of Stripa project (after Kurfurst et al. 1978).

The experiment we are analyzing is full-scale experiment 2 of the Stripa project (see Fig. 2). It consists of a central heater canister with radius 0.16 m, length 2.59 m, and constant power output of 5 kW. The hot section of the heater element is encased in a stainless canister, and is 15 cm shorter than the canister itself. The heater midplane is 4.25 m below the drift floor. A ring of peripheral heaters surrounds the central heater, but these heaters had not yet been activated at the time of our analysis. Thermocouples and other gauges are located at various positions about the heaters. On July 3, 1978, the central heater was turned on and data began to be recorded from the gauges.

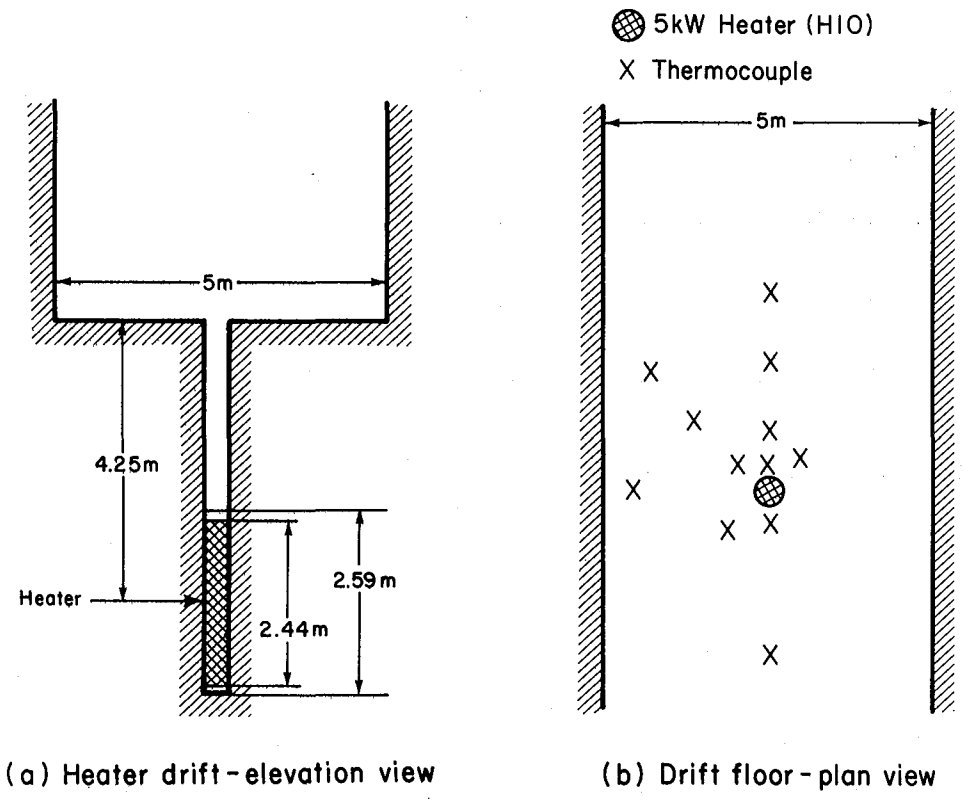

Fig. 2. Full-scale experiment 2: location of (a) central heater, side view, and (b) thermocouples used in least-squares analysis, top view.

3. DESCRIPTION OF THE MODEL

For our model of the experiment, we assume that the rock is homogeneous, isotropic, and linear; i.e., that the thermal conductivity and diffusivity of the rock are independent of position, orientation, and temperature. We have no evidence to indicate that the rock is homogeneous and isotropic, but lack of detailed information on in-situ conditions dictates that we make such simplifying assumptions as a first approximation. However, the laboratory work done by Terra Tek of Salt Lake City, Utah indicates that heat conduction in Stripa granite is not truly linear. Pratt et al. (1977) reported that the Stripa granite thermal conductivity, k, depends on the temperature of the rock as given by the following equation:*

$$k = 3.60 - .3745 \times 10^{-2}\ T \text{ (in W/m-°C)} \qquad (1)$$

Thus, our results will yield a value of k which is an average over the temperatures obtained from our experiment.

We have also assumed that the heater is in perfect thermal contact with the rock, and that heat transfer occurs by conduction only.

To calculate the temperature rise resulting from the heat source, we must choose between three models: a finite cylinder source (the actual geometry of the heater, but quite difficult to compute); a finite line source; and an infinite line source (the simplest to compute). The equations

---

*According to solid state theory, in the temperature range under consideration (Debye 1914), the thermal conductivity of (electrical) insulators, such as the minerals forming granite, will be directly proportional to the phonon mean free path, which varies inversely with the absolute temperature. However, for weak temperature dependence a linear approximation, e.g., Eq. (1), may be sufficiently accurate.

for constant power sources of the three mentioned geometries are given below in cylindrical coordinates; however, because the experiment is radially symmetric, there is no angular dependence. As the temperatures will later be considered as functions of the thermal properties also, k and κ will be included in the functional notation as T (r, z, t, k, κ).

Finite cylinder source (Mufti 1971):

$$T(r,z,t) = \frac{Q}{8bk} \int_0^t \frac{1}{\mu} \left[\mathrm{erf}\left\{ \frac{z+b}{2(\kappa\mu)^{1/2}} \right\} - \mathrm{erf}\left\{ \frac{z-b}{2(\kappa\mu)^{1/2}} \right\} \right] \int_0^a \exp\left\{ -\frac{(r^2+r'^2)}{4\kappa\mu} \right\} I_0\left(\frac{rr'}{2\kappa\mu}\right) r'dr'd\mu \tag{2}$$

Finite line source (Chan, Cook, and Tsang 1978):

$$T(r,z,t) = \frac{Q}{8\pi bk} \int_{-b}^{b} \frac{\mathrm{erfc}\sqrt{\frac{r^2+(z-z')^2}{4\kappa t}}}{\sqrt{r^2+(z-z')^2}} dz' \tag{3}$$

Infinite line source (Carslaw and Jaeger, 1959):

$$T(r,t) = \frac{Q}{8\pi bk} \int_{r^2/4\kappa t}^{\infty} \frac{e^{-u}}{u} du \tag{4}$$

where Q = heater power level per unit length (in W/m),

b = half-length of heater (in m),

a = radius of heater (in m)

k = thermal conductivity (in W/m-°C)

κ = thermal diffusivity in (m^2/s)

I_0 = modified Bessel function of the first kind, zeroth order

erf = error function

erfc = complimentary error function.

The finite line source approximates the finite cylinder source quite well. [Refer to Chan, Cook, and Tsang (1978) for table showing excellent agreement between the two models near the rock edge at the heater midplane, the location where the largest discrepancy is expected.] Hence we can quite satisfactorily use the finite line source [Eq. (3)], which requires only one spatial integral, to model the finite cylinder source [Eq. (2)], which requires one spatial and one temporal integral. Can we instead use the infinite line source model [Eq. (4)], which is even easier to compute.

As mentioned in Section 1 above, the problem of extracting the rock's thermal properties from temperature data has already been solved for an infinite line source. However, when the actual heater geometry is a finite cylinder, the results obtained using this simple model are unsatisfactory. The reason for this is made clear in Fig. 3. Except at very short times and very short distances from the heat source, the difference between the two models is substantial. Thus the infinite line model, though it is easy to compute, is not a satisfactory model for our experiment, and we must use the finite line source model.

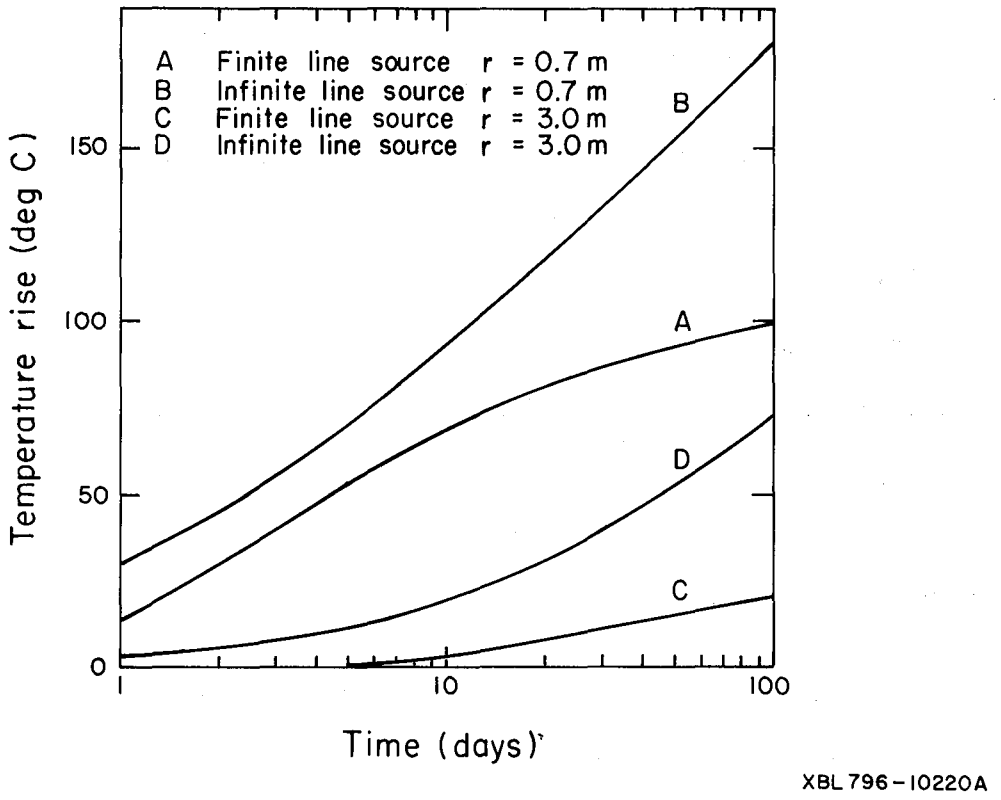

Fig. 3. Comparison of finite- and infinite-line source solutions for r = .7m and r = 3.0 m.

4. BOUNDARY CONDITIONS FOR THE MODEL

Our model uses one of three boundary conditions: an infinite medium, a semi-infinite medium with an isothermal boundary, or a semi-infinite medium with an adiabatic boundary. The experiment takes place beneath a drift, and the temperatures in the rock are affected by the drift floor boundary. If we assume the ventilation in the drift to be very efficient,then the drift floor would remain at a constant temperature. This yields a semi-infinite medium with an isothermal boundary as an approximating model.

If instead we assume that the air neither circulates nor conducts the heat, then we would require the drift floor to be an insulated boundary.

This yields a semi-infinite medium with adiabatic boundary as the model.

The above two boundary conditions can be modeled quite easily using the method of images (Chan, Cook, and Tsang 1978). Consider the presence of an identical additional "image" heater at the same distance from the (adiabatic or isothermal) boundary as the original heater, but above the drift boundary instead of below it. If this "image" heater absorbs energy (releases negative power) at the same rate that the original heater releases (positive) energy, then the rock at the boundary will have no net energy increase. Thus it is an isothermal boundary.

If instead the "image" heater releases positive energy, then at the boundary the amount of energy received from the two sources are equal, and there is no net flux of energy across the boundary surface.. Thus it is an adiabatic boundary. From these solutions and the symmetry of the experiment, we obtain the following equations for T_b, the temperature resulting from a finite line source in a semi-infinite medium with an isothermal or adiabatic boundary:

$$\begin{aligned} T_b\,(r,\,z,\,t) &= T(r,\,z,\,t) \mp T_{im}\,(r,\,z,\,t) \\ &= T(r,\,z,\,t) \mp T(r,\,2d - z,\,t) \end{aligned} \tag{5}$$

where T is the temperature due to one source at the location of the actual heater [Eq. (3)], T_{im} is the temperature due to the image heater, and d is the depth of the heater midplane in meters. The minus sign corresponds to an isothermal boundary, and the plus sign corresponds to an adiabatic boundary. In the absence of any knowledge of the effect of the drift, the best model to use is the infinite medium. It is the simplest and is accurate for relatively short times, regardless of the actual boundary

conditions. This is because the boundary effects take time to be felt at locations away from the boundary. Our calculations indicate that for this experiment, the boundary conditions will not influence the temperature by a detectable amount in the region of the thermocouple gauges for at least 70 days. After we have obtained data for a longer period of time, we will in fact be able to gain information about the effect of the drift by determining which boundary conditions best fit the data.

5. DETERMINATION OF THE THERMAL PROPERTIES

The classical "probe" method of determining the in-situ thermal properties (Blackwell 1956) involves the immersion of a long cylindrical heat source (the probe) of known dimensions and thermal properties into a rock formation of unknown thermal properties. From a record of the probe temperature as a function of time, the in-situ thermal properties of the rock can be deduced. However, for the approximating equations to be valid, the ratio of length to diameter of the probe must be at least 25:1.

Because the the thermal effects of radioactive waste burial are dependent on the thermal loading (power output of the canisters per square meter) and the geometry of the waste canisters, the design of the experimental heaters must be dictated by the geometries being considered for actual waste disposal. The length-to-diameter ratio of the canisters being studied is

2.59 m/.406 m* = 6.38, which is much less than 25. Thus we cannot treat the heater as the probe in the probe method for determining the in-situ thermal properties of the granite formation.

We have instead used the least squares method. Our program determines the values of the thermal conductivity and diffusivity which give the best fit between the temperature measurements and our calculations at the corresponding positions and times.

More precisely, using our finite-line source model, we have obtained an equation for the temperature in the rock as a function of position, time, and the thermal properties [Eq. (3)]. We seek those values of k and κ which minimize the following sum of squares of residuals:

$$\sum_{i=1}^{N} \mathrm{Res}^2 = \sum_{i=1}^{N} \left(T_{th}^{i}(k, \kappa) - T_m^i\right)^2 \qquad (6)$$

where $T_{th}(k, \kappa)$ is the theoretical value $T(r_i, z_i, k, \kappa)$ as given in Eq. (3) above, T_m is the temperature measurement at the position (r_i, z_i) at the time t_i, and N is the number of data points.

Thus k and κ are chosen to minimize the difference between the measured and the calculated values of temperature at the positions and times of the N data points.

---

* The actual diameter of the heater canister is 0.32 m while the diameter of the drillhole into which the canister is emplaced is 0.406 m. In an analytic conduction model, the existence of the air gap between the canister and the rock is ignored.

6. THE LEAST SQUARES ALGORITHM

The least squares problem is to minimize the following function with respect to k and κ:

$$\phi(k, \kappa) = \sum_{i=1}^{N} \left(T_{th}^{i} \ (k, \kappa) - T_{m}^{i}\right)^2. \tag{7}$$

This is equivalent to solving the equation $\nabla\phi = 0$, where $\nabla\phi$ designates the gradient in k-κ space. Unfortunately, ϕ is not a linear function of k and κ, and the equation cannot be solved directly. Two approaches can be used to find a minimum of ϕ: the method of steepest descent, and the Gauss-Newton algorithm.

Given the derivatives of T with respect to k and κ , we can calculate the gradient of ϕ using estimates of the rock parameters. We know that taking a sufficiently small step in the direction of the gradient of ϕ will decrease the value of ϕ. Thus by repeatedly adjusting our estimates of k and κ by these steps in the direction of steepest descent, we can minimize the value of ϕ. However, it is difficult to choose the best step size, and the method often converges rather slowly.

An alternative approach to the least-squares problem is to treat the temperature as a linear function of k and κ, yielding the Newton-Gauss algorithm. This approximation is always valid in a small region. In this case, the minimizing values of k and κ can be determined exactly from the normal equations. These equations follow from setting the gradient of φ equal to 0, and assuming the derivatives of T independent of k and κ.

The algorithm that we have used is Marquardt's algorithm (Jefferson 1974; Marquardt 1963), which incorporates the above two approaches into a single algorithm. It has been implemented in the FORTRAN subroutine TJMAR1 by T. H. Jefferson, and is available through the Sandia Library. We have included this subroutine in our least-squares program.

7. FURTHER EQUATIONS

As shown above, the partial derivatives of the temperature function with respect to k and κ are needed to determine the thermal properties. These derivatives can be written readily, as in the following equations, where T is as given in Eq. (3) and T_b is as given in Eq. (5):

Infinite medium:

$$\frac{\partial T}{\partial k} = \frac{-T}{k} \tag{8}$$

$$\frac{\partial T}{\partial \kappa} = \frac{Qe^{\frac{-r^2}{4\kappa t}}}{16\pi b k \kappa} \left[\operatorname{erf}\left(\frac{b + z}{\sqrt{4\kappa t}}\right) + \operatorname{erf}\left(\frac{b - z}{\sqrt{4\kappa t}}\right) \right] \tag{9}$$

Semi-infinite medium with isothermal/adiabatic boundary:

$$\frac{\partial T_b}{\partial k} = \frac{-T_b}{k} \tag{10}$$

$$\frac{\partial T_b}{\partial \kappa} = \frac{\partial}{\partial \kappa} \left[T(r, z, t) \mp T_{im}(r, z, t) \right]$$

$$= \frac{\partial T}{\partial \kappa}(r, z, t) \mp \frac{\partial T}{\partial \kappa}(r, 2d - z, t) \tag{11}$$

where the negative sign corresponds to an isothermal boundary, while the positive sign corresponds to an adiabatic boundary.

8. OUR PROGRAM

We have written the FORTRAN program LEAST to extract the in-situ thermal properties from our temperature data. The inputs to LEAST are the N data points (r_i, z_i, t_i, $T_m{}^i$), the original estimates of k and κ, and the desired boundary conditions. The user can choose between the finite line source [Eq. (3)] and the infinite line source [Eq. (4)] for his model. In addition, if the values of ρ (the rock density) and c (the specific heat) are assumed to be known, the thermal diffusivity is completely determined in terms of the thermal conductivity by the equation $\kappa = k/\rho c$. In this case, the problem reduces to the determination of the one parameter k. The user can specify this option.

A Romberg integration scheme is used to evaluate the temperature as a function of r, z, and time.

The output of the program is the minimizing values of k and κ, and the minimized value of φ, the sum of squares of residuals. The quantity $S = \sqrt{\Phi/N}$ is the average difference between the actual temperature values and the calculated values. Thus this minimum value of ϕ indicates the goodness-of-fit of the data to our model with the obtained values of k and κ.

9. RESULTS

The results of our analysis are given in Table 1. In these calculations, the infinite medium model was used since our preliminary calculations, referred to in Section 4 above has demonstrated that the boundary effects are unimportant for the first 70 days of the experiment which is the period covered by the analysis reported here. We used a total of 756 temperature measurements recorded during the first 70 days of the experiment from 28 different thermocouples (see Appendix). The temperatures range from 11°C to 70°C, and the average temperature is 23°C. Much of the data used is at low temperature because the thermocouples often became corroded at higher temperatures.

Table 1. In-situ thermal conductivity and thermal diffusivity of Stripa granite from least-squares fitting.

	k (W/m-°C)	κ(m²/s)	$\phi = \sum_{i=1}^{N} \left(T_{th}^{i} - T_{m}^{i} \right)^2$	$S = \sqrt{\phi/N}$ rms difference
In-situ values	3.69	1.84×10^{-6}	1681	1.49°C
Lab values (Pratt et al 1977)	3.51	1.61×10^{-6}	1864	1.57°C
% difference	5%	13%	10%	5%

The parameters φ and S indicate "goodness-of-fit."

We obtained the values 3.69 W/(m-°C) (or 8.82×10^{-3} cal/(cm-s-°C) for the thermal conductivity and 1.84×10^{-6} m^2/s (or 0.159 m^2/day) for the thermal diffusivity. These values do not differ radically from the laboratory results [Eq. (1)] which indicated that k decreases linearly with temperature, having the value 3.6 (W/m-°C) at 0 °C and the value 3.2W/m-°C at 100°C. However, we can also see from the reduced values of ϕ and S that the extracted values of k and φ fit the data more closely than the laboratory values do. The excellent agreement between the observed data and the calculated temperatures, using the extracted values of k and κ is displayed graphically in Figs. 4 and 5, which represent the best and worst cases, respectively. It was also found that using the external height of the heater canister (2.59 m) as the length of the line source yields a better fit than using the actual length of the hot section of the heater elements (2.44 m).

In Table 1, the quoted laboratory values estimates (Pratt et al. 1977) are for 23°C. The "laboratory value" for thermal diffusivity κ was deduced from the measured values of thermal conductivity (k = 3.51 W/m-°C), density (ρ = 2,600 kg/m^3), and specific heat (c = 0.200 cal/g-°C = 837 J/kg-°C) using the relationship $\kappa = k/\rho c$. Actually, thermal conductivity was determined for two samples as a function of temperature in the range 46° to 243°C. A straight line was then fitted through the data to obtain the temperature dependence quoted in Eq. (1). No information on the goodness of fit was given by Pratt et al. However, from their plot of thermal conductivity versus temperature it is clear that an error of at least a few percent may be introduced by linear extrapolation of the fitted line to obtain the thermal conductivity at 23°C.

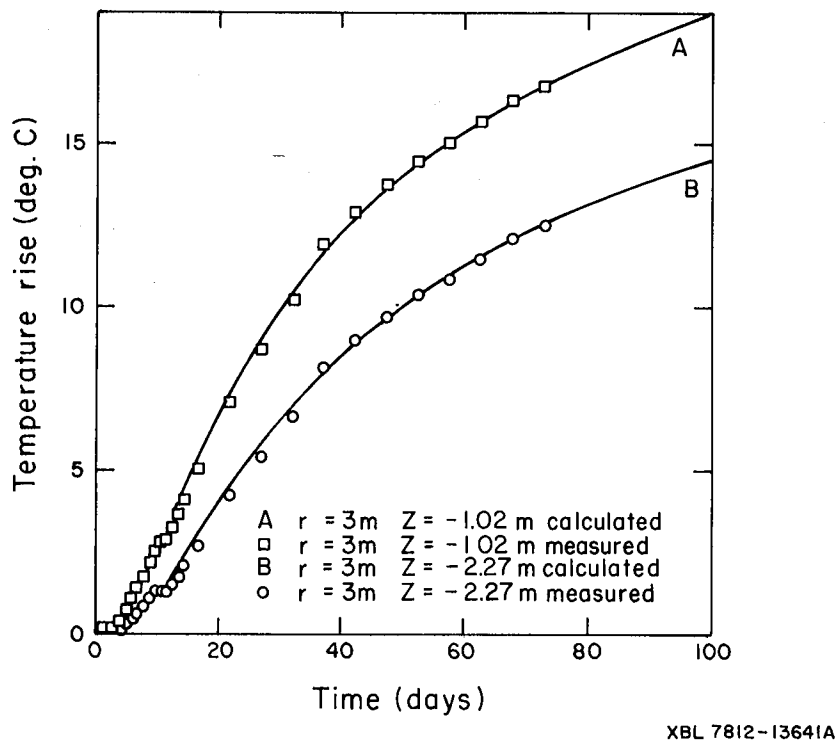

Fig. 4. Comparison of predicted and observed temperatures for thermocouples in hole E13, best case.

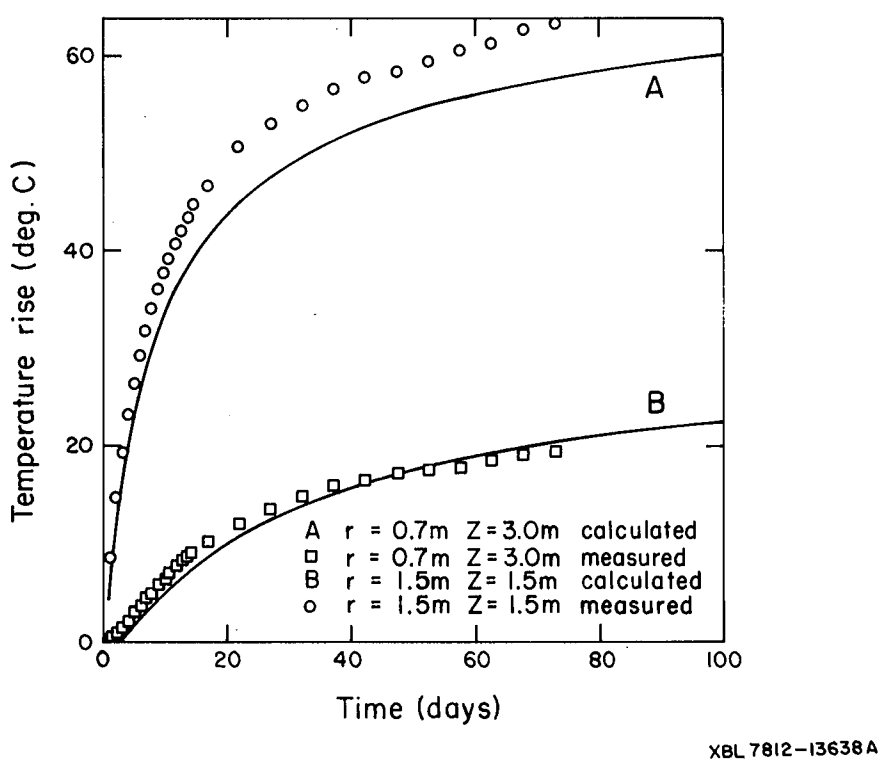

Fig. 5. Comparison of predicted and observed temperatures for thermocouples in hole T22, worst case.

The specific heat was determined by calorimetry on one sample over three temperature ranges, 113° to 31°C, 157° to 35°C, 230° to 43°C to be, respectively, 0.197, 1.197, and 0.200 cal/g°C. The authors concluded that there was no temperature dependence and recommended a value of 0.2 cal/g-°C. We have accepted that recommendation in quoting this laboratory value for comparison. The reader should recognize that this value may be a few percent off from the mean specific heat of Stripa granite at 23°C, if a large number of specimens were tested at the specified temperature.

As an internal consistency check, a separate least-squares fit was performed assuming the in-situ density and specific heat to be the same as the laboratory values, i.e., ρ = 2600 kg/m^3, c = 0.200 cal/g-°C = 837 J/kg-°C. As explained in Section 8, the problem reduces to a one-parameter fit. In this case the best-fit value for the thermal conductivity is 3.55 W/m-°C which falls in between the laboratory value of 3.51 W/m-°C and the in situ value of 3.69 W/m-°C obtained by means of a two-parameter fit. The root-mean-square deviation S, as defined in Table 1, for the one-parameter fit has a value of 1.58°C, slightly higher than that for the two-parameter fit.

10. CONFIRMATION OF RESULTS

We have seen that the obtained values for the rock parameters can predict temperatures quite accurately for the 5-kW experiment from which they were extracted, but we wish to test whether our results are actually representative of the surrounding rock mass.

We have substantiated the results of our experiment, located in the full-scale drift shown in Fig. 1, with the data from a pilot heater test carried out at the start of the Stripa project by Carlsson (1978) in the Luleå drift. Because this experiment (originally intended to monitor the rock during two months of heating by a cylindrical electric heater, and during the following three months of cooling) had severe problems due to a fluctuating power source, we have concentrated on modeling the cool-down period.* The graph in Fig. 6 displays the agreement between the pilot heater test data and temperatures predicted using our extracted thermal rock parameters.

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* Details of the cool-down modeling will be published in a separate LBL report (Chan, Carlsson, and Jeffry 1979).

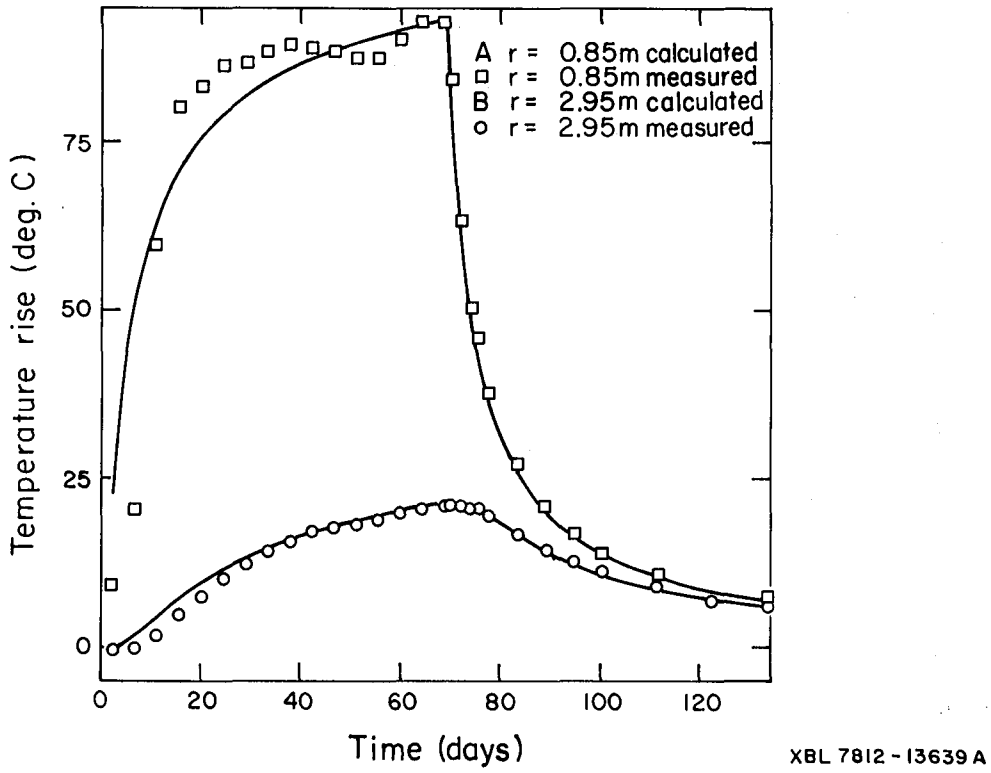

Fig. 6. Comparison of predicted and observed temperatures for the Luleå University pilot heater experiment (Carlsson, 1978) for r = .85 m and r = 2.95 m.

11. DISCUSSION

In conclusion, we have developed a least squares scheme using a finite line source model for determining the in-situ thermal properties of a rock mass, and the resulting values successfully predict independent temperature measurements. The in-situ thermal conductivity and thermal diffusivity of Stripa granite have been found to be 3.69 W/(m-°C) and 1.84 x 10^{-6} m^2/s, respectively. These values are only slightly higher than the corresponding laboratory values (Pratt et al. 1977) of 3.51 W/(m-°C) determined using two 51 x 12 mm core samples, and 1.61 x 10^{-6} m^2/s based on the thermal conductivity value from two samples and specific heat measurement on one 51 x 51 mm core sample at the same average temperature (23°C) as the actual field data used for the least-squares fit. In fact, the extracted in-situ values

probably fall within the scatter of the laboratory results if the thermal properties were measured on a large number of rock samples in the laboratory.

Another possible reason that the thermal conductivity measured by Pratt et al. in dry Stripa granite samples is slightly lower than in-situ values is that the rock mass is saturated with groundwater. It is common knowledge that the thermal conductivity of saturated rock can be as much as 10% higher than that of the dry sample, even for low-porosity rocks.

Note that the in-situ thermal conductivity deduced by Murphy and Lawton (1977) by means of type curve fitting for a Precambrian granitic rock in the hot dry rock geothermal project in the Jemez Mountains of northern New Mexico is also in close agreement with laboratory measurements on core samples (Sibbitt, Dodson, and Tester 1979).

Thus, the present results, taken together with those obtained in the hot dry rock geothermal project, appear to indicate that discontinuities in the rock mass do not significantly alter the bulk thermal conductivity, although they may lead to local heterogeneities.

Many authors have reported dramatic size effects on the mechanical (see Cook 1978, and Jaeger and Cook 1976 for reviews) as well as hydrological properties of rocks (Witherspoon et al. 1979). In contrast, the present work indicates that there is hardly any size effect at all on the thermal properties of granitic rocks. The thermocouples, which recorded the field data used in the least-squares analysis, span a volume of rock several meters in each linear dimension. Furthermore, the duration of the experiment is long enough so that a large volume of rock is heated. Consequently, any

significant size effect would have resulted in large discrepancies between in-situ and laboratory values for the thermal properties.

12. ACKNOWLEDGMENTS

We would like to acknowledge the kindness of Hans Carlsson for providing us with the temperature data from the Luleå University pilot heater experiment.

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14. APPENDIX - Data Used in Least-Squares Analysis

In this appendix, we describe the actual data we have used in our analysis. Table A1 contains the coordinates of the thermocouples whose measurements we used.

Because of the noise in the data, we first smoothed the raw data for each thermocouple using the cubic spline smoothing routine ICSMOU, from the International Mathematical and Statistical Library (IMSL) available at the LBL Computer Center. We then subtracted the initial ambient temperature of the rock at the corresponding location. There were 280 data points for each thermocouple. Because the temperature data were recorded at short intervals, in the least-squares analysis we have used only every tenth data point; i.e., 28 data points for each thermocouple.

Table A1. Locations of thermocouples used in least-squares analysis

Label	Sensor number*	r**	Θ	z
E12A	442	1.98	0	2.36
E12B	443	1.98	0	0.11
E12C	444	1.98	0	-0.89
E12D	445	1.98	0	-2.14
E13A	446	3.0	0	-1.02
E13B	447	3.0	0	-2.27
E14A	450	2.49	180	2.28
E14B	451	2.49	180	0.03
E14C	452	2.49	180	-2.22
E15A	454	2.0	90	2.25
E15B	455	2.0	90	0.0
E15C	456	2.0	90	-2.25
E16A	458	1.51	45	2.29
E16B	459	1.51	45	0.04
E16C	460	1.51	45	-0.96
E16D	461	1.51	45	-2.21
E17A	462	2.51	45	2.25
E17B	463	2.51	45	0.0
E17C	464	2.51	45	-1.0
E17D	465	2.51	45	-2.25
T19A	510	0.4	0	3.0
T20	515	0.9	0	3.0
T21	520	0.7	315	3.0
T21	521	0.7	315	1.5
T22	525	0.5	180	3.0
T23	530	0.8	135	3.0
T24	535	0.6	45	3.0

* Sensor numbers are used in the computer analysis of the data.

** The coordinates are given in a cylindrical system: the origin is at the heater's center; positive z is directed upwards. r and z are measured in meters, Θ in degrees, measured counter-clockwise from the major axis of the heater drift. The angle Θ does not enter into the calculation here.

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