MULTIPION PRODUCTION IN NUCLEAR COLLISIONS*

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ABSTRACT

We discuss two aspects of multipion production in nuclear collisions: (1) the negative pion multiplicity distribution, and (2) the $\pi^-\pi^-$ correlation function. The emphasis is on how these observables could be used to search for signals of collective phenomena in nuclear collisions.

INTRODUCTION

One of the prominent features of relativistic ($E_{lab} \sim 1$ GeV/nucleon) nuclear collisions is multipion production. In a typical reaction[1] such as Ar + Pb_3O_4 at 1.8 GeV/nucleon, up to 14 negative pions (π^-) alone are observed in certain events. The topic of my talk is what such multipion final states can teach us about the dynamics in nuclear collisions. Some aspects of what single pion inclusive cross sections can teach us are discussed elsewhere in these proceedings by R. Landau.[2]

The primary motivation for colliding relativistic nuclei has been the expectation that through such collisions the properties of dense, highly excited nuclear systems can be studied. Of special interest has been the search for collective phenomena. Several theoretical studies[3] have suggested in particular that pionic instabilities could possibly occur when nuclear matter is compressed to $\rho \sim (2\text{-}4)\rho_0$, $\rho_0 = 0.17\ \text{fm}^{-3}$, at temperatures $T \sim 50$ MeV. The elusive problem, however, has been to identify specific experimental signatures of such novel phenomena.

One natural direction to search for evidence of collective phenomena has been multi-pion production. In this talk, I will review our work on two aspects of multipion production. The first is the nature of the π^- multiplicity distribution, $P(n_\pi)$, and the second is the dynamical and geometrical information contained in the $\pi^-\pi^-$ correlation function[5,6] $R(\tilde{k}_1,\tilde{k}_2)$. For a more complete review of other aspects of relativistic nuclear collisions see, e.g., Ref. 7.

Multiplicity Distribution[4] (with S. K. Kauffmann)

Our first expectation is that the nuclear dynamics in this energy range is governed simply by intranuclear cascading,[8] i.e., multiple nucleon-nucleon collisions. In that case pions are produced mostly one at a time [via $\Delta_{33}(1232)$ decay] in separate

inelastic nucleon-nucleon collisions. To compute the multiplicity distribution in such a cascade picture, let $\gamma_{\pm}(t)$ denote the time-dependent π production and absorbtion rates during the nuclear collision. Then the pion multiplicity distribution $P(n,t)$ can be shown[4] to be a Poisson,

$$P(n,t) = e^{-\langle n_\pi(t)\rangle} \frac{\langle n_\pi(t)\rangle^n}{n!} \tag{1}$$

with a time-dependent mean satisfying

$$\frac{d}{dt}\langle n_\pi(t)\rangle = \gamma_+(t) - \gamma_-(t)\,\langle n_\pi(t)\rangle \quad . \tag{2}$$

If the interaction time were long enough for the nuclear system to come to chemical and thermal equilibrium with the pions, then $\langle n_\pi(t)\rangle$ would approach an equilibrium value $\langle n_\pi\rangle_{eq}$ which is independent of the detailed pion production and absorbtion history, $\gamma_{\pm}(t)$, and given by[4]

$$\langle n_\pi\rangle_{eq} = \frac{\gamma_+(\infty)}{\gamma_-(\infty)} = \int \frac{V d^3k}{(2\pi)^3} \frac{1}{e^{\sqrt{k^2+m_\pi^2}/T} - 1} \quad . \tag{3}$$

In the fireball model,[9] V and T follow from geometrical, kinematical, and equilibrium considerations.[4] The crucial parameter in this model is the freezeout nuclear density $\rho_c \approx 1/3\,\rho_0$, below which no further scattering is assumed to take place.

The results of this thermodynamic calculation[4] for Ar + Pb_3O_4 are shown in Fig. 1. While the features of the impact parameter averaged multiplicity distribution data[1] are indeed well accounted for, we stress that this agreement does not prove that chemical or thermal equilibrium have been reached. The only necessary condition is that the final average pion multiplicities, $\langle n_\pi(b)\rangle$, are well approximated. An example of a completely different dynamical model that also reproduces the impact-parameter averaged multiplicity data is given in Ref. 10.

What makes P(n) insensitive to dynamical assumptions about $\gamma_{\pm}(t)$ is that $\langle n_\pi(b,t_f)\rangle$ at the final (freezeout) time, t_f, can be the same for very many dynamical paths, $\gamma_{\pm}(t)$, in Eq. (2). However, it should be recalled that there is one major (though reasonable) assumption leading to Eq. (2). That assumption, which we call the cascade assumption, is that pion production and absorbtion involve only <u>one</u> pion at a time.

Consider now what happens if the cascade assumption were relaxed. If there were unusual collective phenomena, then pions could possibly be produced also in correlated groups or clusters.. For example, there may be a rate $\gamma_{+k}(t)$ for producing correlated groups of k-negative pions at a time. The effect of such unusual dynamics has been invesitgated in Ref. 4. The result is that if some $\gamma_{+k}(t) \neq 0$ for $k \geq 2$, then the resulting multiplicity distribution for a given impact parameter becomes a convoluted multiple Poisson[4]

$$P(n,t) = P(0,t) \sum_{n_1=0}^{n} \cdots \sum_{n_p=0}^{[n/p]} \cdots \sum_{n_n=0}^{1} \prod_{k=1}^{n} \left\{ e^{-C(k,t)} \frac{(C(k,t))^{n_k}}{n_k!} \right\} \delta(n - \sum_{r=1}^{n} r n_r) \tag{4}$$

with combinants[4] $C(k,t)$ satisfying the system of equations

$$\frac{d}{dt} C(k,t) = \gamma_{+k}(t) - \gamma_{-}(t)\left(k\, C(k,t) - (k+1)\, C(k+1,t)\right) \quad . \tag{5}$$

In the case $C(1,t_f) \gg C(k \geq 2,t_f)$, then Eq. (4) of course reduces to Eq. (1), with $C(1,t_f) \approx \langle n_\pi(t_f) \rangle$.

The signature of unusual pion production dynamics would thus be a non-Poisson form of the multiplicity distribution in central collisions. Preliminary streamer chamber data[11] taken in a central trigger mode (with high associated nucleon multiplicity) are as yet inconclusive as to whether the high ($n_\pi \gtrsim 10$) tail of the π^- multiplicity distribution is Poisson or not. (We note[4] that the low ($n_\pi \lesssim 5$) portion of $P(n_\pi)$ depends sensitively on the trigger bias.) Should any significant deviation be observed from Poisson behavior for large n_π, then these deviations would reflect on the inadequacy of the cascade assumption and thus provide us with a hint that more exotic pion production mechanisms are at work in nuclear collisions.

We also want to call attention to the results of Ref. 10 which show that even the impact parameter averaged P(n) can be sensitive to certain non-linear phenomena. The non-linearity studied in Ref. 10 involved the assumption that $\langle n_\pi(b) \rangle \propto [N(b)]^\alpha$ with $\alpha > 1$ and N(b) being the number of nucleons in the interaction region for a given b. A simple multiple scattering model would give $\alpha = 1$. However, non-linearity ($\alpha > 1$) could arise if pions are produced coherently. Analysis of the data of Ref. 1 suggests[10] that, in fact, $\alpha \approx 1$. A nonlinear growth of $\langle n_\pi(b) \rangle$ as a function of N(b) seems to be inconsistent with those data.

In summary, existing multiplicity distribution data can be well accounted for by simple multiple collision or thermal models. Thus far, no evidence for unusual pion dynamics is suggested by these data. However, whether the form of the high n_π tail is Poisson or not still needs future experimental study.

Pion Interferometry[5,6] (with S. K. Kauffmann and Lance W. Wilson)

While the multiplicity distribution P(n) offers some insight into the dynamics of multipion production, it is clear that more detailed information is contained in multipion correlation data. We consider here specifically the $\pi^-\pi^-$ correlation function defined as

$$R(\tilde{k}_1,\tilde{k}_2) = \frac{\langle n_\pi\rangle^2}{\langle n_\pi(n_\pi-1)\rangle}\,\sigma_\pi\,\frac{d^6\sigma(\pi^-\pi^-)/d^3k_1d^3k_2}{(d^3\sigma(\pi^-)/d^3k_1)(d^3\sigma(\pi^-)/d^3k_2)}, \qquad (6)$$

where $\langle n_\pi\rangle$ and $\langle n_\pi(n_\pi-1)\rangle$ are the first and second binomial moments of P(n), $d^3\sigma(\pi^-)$ and $d^6\sigma(\pi^-\pi^-)$ are the single and double pion inclusive cross sections, and σ_π is the total negative pion cross section.

The information that $R(\tilde{k}_1,\tilde{k}_2)$ is expected[12,13] ideally to provide is twofold: (1) R can reveal the space-time structure of the pion production source, and (2) R can determine the degree of coherence of the pion field. The first property is the result of the well known[12] Bose-Einstein interference between two identical pions (the Hanbury-Brown and Twiss effect) also known as the Goldhaber effect. The second property of R has only recently been emphasized in Ref. 13.

To understand the content of $R(\tilde{k}_1,\tilde{k}_2)$, we derived[5] the form of R for partially coherent fields in a simple dynamical model. The basis of the calculation was the approximate solution of the pion field equation

$$(\partial_\mu\partial^\mu + m_\pi^2)\,\phi(x) = J(x) \quad . \qquad (7)$$

To solve Eq. (7), the pion source current operator, J(x), was first approximated by its expectation value, thereby decoupling the pion and nuclear field equations. The physical motivation for this approximation is again the expectation that the nuclear dynamics is dominated by multiple nucleon-nucleon interactions. In effect we assume that the number of produced pions is small enough that pion-nucleon rescattering would not significantly affect the source current J(x). This physical picture of pion production is thus equivalent to that of bremsstrahlung radiation.[14]

The pion final state produced by a classical source current J in Eq. (7) is well known[14] to be a coherent state $|J\rangle$ given by

$$|J\rangle = e^{-\bar{n}/2}\exp\{i\int d^3k\; J(\tilde{k})\; a^+(k)\}\;|0\rangle \qquad (8)$$

where $J(\tilde{k})$ is the on-shell Fourier transform[5] of $J(\tilde{x},t)$ and $\bar{n} = \int d^3k\,|J(\tilde{k})|^2$.

The m-pion inclusive distribution is then given by[14]

$$P_m(\tilde{k}_1,\ldots,\tilde{k}_m) = \frac{1}{\sigma_\pi}\frac{d^{3m}\sigma(m\times\pi^-)}{d^3\tilde{k}_1\ldots d^3\tilde{k}_n}$$

$$= \langle J|a^+(\tilde{k}_1)\ldots a^+(\tilde{k}_m)\ a(\tilde{k}_m)\ldots a(\tilde{k}_1)|J\rangle$$

$$= |J(\tilde{k}_1)|^2\ldots|J(\tilde{k}_m)|^2 \quad . \tag{9}$$

Also, the multiplicity distribution is rigorously Poisson.[14] Thus, the correlation function in Eq. (6) is just

$$R(\tilde{k}_1,\tilde{k}_2) = \frac{P_2(\tilde{k}_1,\tilde{k}_2)}{P_1(\tilde{k}_1)\ P_1(\tilde{k}_2)} \quad . \tag{10}$$

From Eqs. (9,10) we see that for a coherent pion field, $R\equiv 1$, and there are no momentum space correlations of pions produced via this bremsstrahlung mechanism.

However, it is clear physically that no fixed form of $J(\tilde{x},t)$ can represent the pion source in all nuclear collisions. For example, the number of inelastic scatterings, N, will differ from event to event as will the impact parameter b. Therefore, the source current $J_\alpha(x)$ must depend on some set of parameters $\alpha = \{N,\tilde{b},\ldots\}$ that can vary from event to event. The observed inclusive distribution thus samples an <u>ensemble</u> of final states $|J_\alpha\rangle$. We can include such an ensemble average in Eq. (9) by

$$P_m(\tilde{k}_1,\ldots,\tilde{k}_m) = \sum_\alpha p(\alpha)\ |J_\alpha(\tilde{k}_1)|^2\ldots|J_\alpha(\tilde{k}_m)|^2 \quad , \tag{11}$$

where $p(\alpha)$ is the ensemble distribution of the parameters α. As a result of the ensemble average, it is clear that $R(\tilde{k}_1,\tilde{k}_2)\neq 1$ in general.

As an example of Eq. (11), we first consider a space-time parametrization of the pion production dynamics in the spirit of the intranuclear cascade model.[8] In this space-time picture, pions are produced in N separate inelastic nucleon-nucleon collisions at different space-time points x_i. If $J_\pi(x)$ is taken to parametrize the source current for an inelastic nucleon-nucleon collision at the origin, then the total source current can be written as

$$J_\alpha(x) = \sum_{i=1}^{N} J_\pi(x-x_i) \quad . \tag{12}$$

If we now assume that the "inelastic scattering centers," x_i, are distributed in a space-time reaction volume specified by a density distribution $\rho(x)$, and the number of inelastic collisions, N, is distributed according to $P_s(N)$, then from Eq. (11),

$$P_m(\tilde{k}_1,\ldots,\tilde{k}_m) = \sum_N P_s(N) \int d^4x_1\rho(x_1)\ldots d^4x_N\rho(x_N)\{|J_\alpha(\tilde{k}_1)|^2\ldots|J_\alpha(\tilde{k}_m)|^2\} \tag{13}$$

with $J_\alpha(\tilde{k})$ given by the on-shell Fourier transform of Eq. (12), $\alpha = \{N,x_i\}$. An impact parameter average can also be easily included in Eq. (13). Evaluating Eqs. (10,13), we obtain finally[5]

$$R(\tilde{k}_1,\tilde{k}_2) \approx 1 + \left\{1 - \frac{\langle N\rangle}{\langle N^2\rangle}\right\} |\rho(\tilde{k}_1-\tilde{k}_2,\ \omega_1-\omega_2)|^2 \quad , \tag{14}$$

in the limit when the interaction time $T \approx [\int d^4x\ x_0^2\rho(x)]^{\frac{1}{2}} >> m_\pi^{-1}$. In particular, we have neglected terms of order

$$O[\langle N\rangle|\rho(\tilde{q}=0,\ q_0=m_\pi)|^2] << 1 \ .$$

This limit is satisfied in all but the most peripheral nuclear collisions.

If the average number of inelastic collisions is large, then Eq. (14) reduces to the well known form of the correlation function in Ref. 12. Furthermore, we have also shown[5] that in the limit $\langle N\rangle \to \infty$, the multiplicity distribution for a given mode $\tilde{k}$ becomes a Bose-Einstein distribution with a mean $\langle n_\pi(\tilde{k})\rangle = P_1(\tilde{k})((2\pi)^3/V)$. Therefore, we are justified in calling the $\langle N\rangle \to \infty$ limit as the <u>chaotic field limit</u>.

For chaotic pion fields, Eq. (14) shows then that R-1 measures the space-time Fourier transform of the pion source region, $\rho(\tilde{x},t)$. Therefore, $R(\tilde{k}_1,\tilde{k}_2)$ provides geometrical information about nuclear collisions.

Next we consider the question of partially coherent fields. We suppose that in addition to pion production from separate inelastic nucleon-nucleon collisions, pions can also be produced coherently as a result of the collective action of some group of nucleons. Let $J_0(x)$ denote that coherent source current. The total pion source current is then

$$J_\alpha(\tilde{k}) = J_0(\tilde{k}) + J_\pi(\tilde{k})\left(\sum_{i=1}^{N} e^{i(\omega_k t_i - \tilde{k}\cdot\tilde{x}_i)}\right) \ . \tag{15}$$

Observe that the chaotic component involves random phases distributed according to $\rho(\tilde{x}_i,t_i)$.

Evaluating[5] the ensemble average, the single pion inclusive distribution is now found to be a sum of a coherent and chaotic part,

$$P_1(\tilde{k}) = |J_0(\tilde{k})|^2 + \langle N\rangle\ |J_\pi(\tilde{k})|^2 \equiv n_0(\tilde{k}) + n_{ch}(\tilde{k}) \quad , \tag{16}$$

where $n_0(\tilde{k})$ and $n_{ch}(\tilde{k})$ are the average number of pions in mode $\tilde{k}$ in the coherent and chaotic component respectively. From Eq. (16) we obtain therefore a natural definition of the <u>degree of coherence</u> of the pion field in mode $\tilde{k}$ as

$$D(\tilde{k}) = n_o(\tilde{k}) / P_1(\tilde{k}) \tag{17}$$

As we now show, $D(\tilde{k})$ can be measured from $R(\tilde{k},\tilde{k})$. Evaluating the double pion inclusive distribution, the correlation function is found to be[5]

$$R(\tilde{k}_1,\tilde{k}_2) \approx 1 + (1-D(\tilde{k}_1))(1-D(\tilde{k}_2)) \; |\rho(k_1-k_2)|^2 + 2\left(D(\tilde{k}_1)\;(1-D(\tilde{k}_1))D(\tilde{k}_2)(1-D(\tilde{k}_2))\right)^{\frac{1}{2}} \mathrm{Re}\,\rho(k_1-k_2) \; . \tag{18}$$

In deriving Eq. (18), we have taken $\langle N \rangle \gg 1$ in the chaotic component but have again used the long interaction time condition, $\langle N \rangle |\rho(\tilde{0},m_\pi)|^2 \ll 1$.

Equation (18) is our main result showing that the degree of coherence of the pion field can be measured <u>mode by mode</u> from $R(\tilde{k}_1,\tilde{k}_2)$ via the relation

$$R(\tilde{k},\tilde{k}) = 2 - D^2(\tilde{k}) \quad . \tag{19}$$

Therefore, pion interferometry (the analysis of $R(\tilde{k}_1,\tilde{k}_2)$) provides both geometrical and dynamical information about the pion source in nuclear collisions.

The first data[15] on pion interferometry in nuclear collisions are now available and are shown in Fig. 2. At the present, only impact parameter-averaged and momentum-averaged correlation data are available. Therefore, detailed study of $D(\tilde{k})$ is not yet possible. Nevertheless, these data seem to indicate that at least the average (over $\tilde{k}_1+\tilde{k}_2$ and b) degree of coherence is small. Planned correlation experiments at the Bevalac with much higher statistics will be able to determine whether there is coherence in at least some modes for central collisions.

While the ideal form of the $\pi^-\pi^-$ correlation function in Eq. (18) demonstrates how both dynamical and geometrical information can be extracted from $R(\tilde{k}_1,\tilde{k}_2)$, it is important to recognize that effects of final state interactions must also be taken into account.[6] In nuclear collisions, the nuclear remnant has, for example, a residual nuclear charge $Z_{eff} \sim 100$ which can distort the π^- wavefunctions and lead to distortions of $R(\tilde{k}_1,\tilde{k}_2)$ for $|\tilde{k}_1-\tilde{k}_2| \sim (Z\alpha m_\pi/R)^{\frac{1}{2}} \sim 50$ MeV/c. To incorporate distortion due to some effective optical potential, $V(\tilde{x})$, into the calculation of $P_m(\tilde{k}_1 \ldots \tilde{k}_m)$, we need to know the form of the incoming Møller scattering wavefunction, $\psi^{(-)}_{\tilde{k}}(x)$, for that potential. The final result[5] for partially coherent fields has the same structure as Eq. (18) except that the Fourier transform, $\rho(k_1-k_2)$, is replaced by the <u>distorted</u> transform

$$\rho(k_1-k_2) \rightarrow \bar{\rho}_v(\tilde{k}_1,\tilde{k}_2) = \rho_v(\tilde{k}_1,\tilde{k}_2)(\rho_v(\tilde{k}_1,\tilde{k}_1)\rho_v(\tilde{k}_2,\tilde{k}_2))^{-\frac{1}{2}} \tag{20}$$

where

$$\rho_v(\tilde{k}_1,\tilde{k}_2) = \int dt d^3x \; \rho(\tilde{x},t) \; e^{i(\omega_1-\omega_2)t} \; \psi^{(-)*}_{\tilde{k}_1}(\tilde{x}) \; \psi^{(-)}_{\tilde{k}_2}(\tilde{x}) \tag{21}$$

Also, the degree of coherence is distorted[5] ($D \to D_V$) via $\psi_k^{(-)}(k')$. In Ref. 6 we have evaluated Eqs. (20) and (21) using Coulomb wavefunctions. We find significant distortions of the shape of $R(\tilde{k}_1,\tilde{k}_2)$ in the region $20 \lesssim |\tilde{k}_1-\tilde{k}_2| \lesssim 100$ MeV/c. These distortions are also sensitive to the orientation of the relative momentum. However, we find[6] that distortions are minimized (~20%) for large pion momenta ($\gtrsim m_\pi$) especially for that orientation of $\tilde{k}_1-\tilde{k}_2$ corresponding to $|\tilde{k}_1| \approx |\tilde{k}_2|$. Experimentally, it will be important to concentrate on this equal energy configuration at high momenta to get the cleanest determination of the pion source geometry.

Finally, we note a potential problem in determining $D(\tilde{k})$ due to relative $\pi^-\pi^-$ Coulomb interaction. An apparent non-zero degree of coherence can result from a Coulomb hole[6] in $R(\tilde{k}_1,\tilde{k}_2)$ for $|\tilde{k}_1-\tilde{k}_2| \lesssim \sqrt{\alpha}\, m_\pi \lesssim 10$ MeV/c. It there were no strong external fields, then such a distortion could be easily taken into account[5,6] and unfolded from the data via a Gamov penetration factor,

$$|\psi^{(-)}_{(\tilde{k}_1-\tilde{k}_2)/2}(\tilde{x}=0)|^2 .$$

In the presence of an external potential though, this Coulomb hole tends to be filled up since the typical momentum transfer provided by the external optical potential $V(\tilde{x})$ is much larger than that provided by the relative $\pi^-\pi^-$ potential, $U(\tilde{x}-\tilde{y})$. The structure of the correlation function in this case is the same as that given in Eqs. (18, 20, 21), except that the correlation function is multiplied by a penetration factor $\mathcal{P}(\tilde{k}_1,\tilde{k}_2)$. In the limit $V \to 0$, $\mathcal{P}$ reduces to the familiar Gamov factor; for $U \to 0$, $\mathcal{P} \to 1$. However, for $V \neq 0$, $\mathcal{P}$ is much more difficult to evaluate. The formula for $\mathcal{P}$ given in Ref. 5 involves a 12-dimensional integral over products of Coulomb wavefunctions. Since the effect of $\mathcal{P}$ is mainly to distort $D(\tilde{k})$, we feel at this time that only if there were data indicating a significant degree of coherence would the effort to evaluate $\mathcal{P}$ be worth it. Here we only emphasize that $R(\tilde{k},\tilde{k}) \neq 2$ could in principal arise from relative $\pi^-\pi^-$ interactions. We note that for chaotic fields $R(\tilde{k},\tilde{k}) = 2$ if $U=0$, regardless of the form of V.

In summary, two-pion correlation data can provide both geometrical and dynamical information that can be used in the search for collective phenomena in nuclear collisions. However, final state interactions must be first unfolded from the raw data. In the case of chaotic fields ($D(k) \equiv 0$) this unfolding will be much easier than if partially coherent fields are produced. Nevertheless, we are still excited to have found at least one observable, $R(\tilde{k}_1,\tilde{k}_2)$, that is sensitive to possible pion field coherence.

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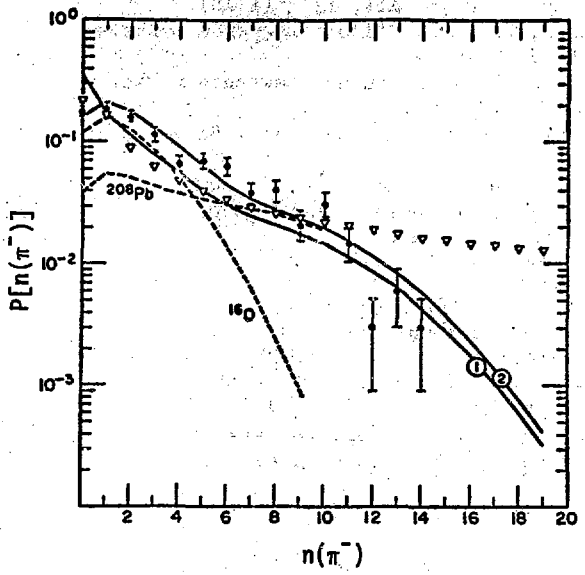

Fig. 1. π^- multiplicity distribution for Ar + Pb_3O_4 at 1.8 GeV/nucleon : dots are data,[1] curve 2 is the final result[4] of fireball model taking trigger bias into account.

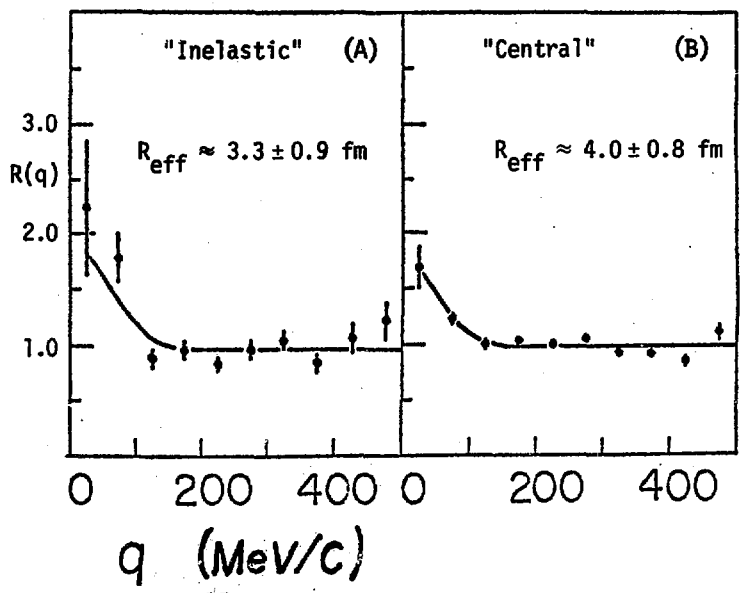

Fig. 2. First data[15] on the averaged $\pi^-\pi^-$ correlation function $R(q) \equiv \langle R(\tilde{k}, \tilde{k}+\tilde{q})\rangle_k$ for the same reaction as for Fig. 1. Solid curve is fit[15] of data based on pion interferometry[12] assuming a chaotic field in Eq. (18). The effective pion source radius is indicated in two trigger modes.