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# BOOTSTRAPPING QUARKS AND GLUONS*

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## ABSTRACT

Dual topological unitarization (DTU)--the approach to S-matrix causality and unitarity through combinatorial topology--is reviewed. The approach may alternatively be called "bootstrap quark theory" because quarks and gluons emerge as automatic aspects of a self-consistent S matrix. Amplitudes associated with triangulated spheres are shown to constitute the core of particle physics--all symmetries and conserved quantum numbers arising through requirements of unitarity and causality in a spherical Hilbert space. Each sphere is covered by triangulated disc faces corresponding to hadrons. The leading current candidate for the hadron-face triangulation pattern employs 3-triangle "basic subdiscs" whose orientations correspond to baryon number and "topological color." Additional peripheral triangles lie along the hadron-face perimeter. Certain combinations of peripheral triangles with a basic-disc triangle can be identified as "quarks," the flavor of a quark corresponding to the orientation of its edges that lie on the hadron-face perimeter. Both baryon number and flavor are additively conserved. "Quark helicity," which can be associated with triangle-interior orientation, is not uniformly conserved and interacts with particle momentum, whereas flavor does not. Three different "colors"

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attach to the 3 quarks associated with a single basic subdisc, but there is no additive physical conservation law associated with color. In fact, because topological color is defined by the pattern of triangle-interior orientations within a basic disc, there is interplay between color and quark helicity. In hadron faces with more than one basic subdisc, there may occur pairs of adjacent flavorless but colored triangles with net helicity $\pm 1$ that are identifiable as "gluons." Color-carrying quarks and gluons cannot individually be particle faces; their color becomes meaningless when they are separated from their neighbors within a particle face. The pattern in which quarks and gluons fit together to cover closed surfaces describes the momentum singularity structure of hadronic amplitudes, the dynamical equations of the bootstrap being dispersion relations building amplitudes from their singularities together with Landau (unitarity) formulas giving singularity-discontinuities as products of amplitudes.

Broken symmetry is an automatic feature of the bootstrap, nonspherical components of a topological expansion gradually dissipating the symmetries of the sphere. T, C and P symmetries, as well as up-down flavor symmetry, will persist on all orientable surfaces, allowing association of strong interactions with the orientable components of the topological expansion.

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## 1. INTRODUCTION

In the early sixties there was widespread discussion of the "bootstrap" conjecture that all particle properties are uniquely fixed by requirements of S-matrix causality and unitarity. Causality requires S-matrix connected parts (amplitudes) to be analytic functions of particle momenta except for isolated singularities. The bootstrap cycle is closed by two complementary assumptions: (1) The assumption, related to Regge asymptotic behavior and now universally accepted, that amplitudes may uniquely be constructed from the discontinuities of their singularities via Cauchy formulas that physicists call "dispersion relations." (2) The assumption that the location and discontinuity of any singularity in the Riemann surface is given by a nonlinear amplitude product associated with a Landau graph. The nonlinear nature of the bootstrap cycle made it possible to conjecture uniqueness for a causal and unitary S matrix, i.e., a unique set of particles and particle properties.

Two different developments caused the bootstrap idea to fade in popularity during the late sixties and early seventies. Firstly, after initial encouragement from models based on S-matrix poles, with the neglect or mistreatment of more complex singularities, it became apparent that a systematic theoretical approach was lacking. More and more particles and particle regularities were being experimentally discovered. The naked idea of pole dominance was not providing an adequate clue to the broad picture, and no path was visible for systematic improvement of this idea. Secondly, the quark concept and its evolution into QCD was enjoying prodigious success; quarks seemed intrinsically at odds with a bootstrap, being viewed as <u>a priori</u>

entities, not as products of S-matrix self consistency. Only philosophers and a handful of die-hard physicists remained attached to the bootstrap.

Then in 1974 it began to appear that combinatorial topology might provide a path through the maze of indefinitely proliferating S-matrix singularity complexity. A hint had been given in 1969 by the Harari-Rosner duality diagrams, but as originally introduced these diagrams described only poles. In 1974 Veneziano proposed an interpretation that allowed a certain category of Landau branch points as well as poles; this was the beginning of "dual topological unitarization" (DTU).

Let me remind you how Harari-Rosner diagrams assigned to quarks a topological, singularity-specifying role that did not allow quarks themselves to be particles. Scattering amplitudes were associated with the type of graph shown in Fig. 1(a), particles (in this case, mesons) being associated with a pair of oppositely-directed lines each carrying an index. One could speak of each line as a "quark" (or antiquark, depending on the direction) and identify the index as quark flavor, but the essential character of the graph was that of a continuous planar ring divided into segments. "Planar" means that the ring, when embedded in a plane, does not intersect itself. Physical particles were associated with points of access to the ring at the joining of two successive ring segments, poles of the amplitude corresponding to planar rings joined through those access points so as to form new planar rings (Fig. 1b). (We have here an example of a "connected sum" of topological objects, a notion tailored to the needs of a causal unitary S matrix; you will hear much more about connected sums in what follows.)

The individual ring segments are not interpretable as particles since they cannot separately propagate from one ring to another in such a way as to form a new ring. The connection with amplitude singularities is shown by Fig. 1(b) and further analogous graphs that can build the ring of Fig. 1(a) by "adding" together other planar rings. The only "resonating" channels are seen to be those corresponding to adjacent particles around the ring. Thus, in Fig. 1(a) channels AC, BE, AD, BD and EC do not have poles. The absence of singularities in nonadjacent-particle channels gives to Harari-Rosner ring amplitudes a variety of special regularities that makes them much easier to deal with than amplitudes with a full set of poles.

Veneziano's 1974 observation was that singularities more general than poles could be associated with the "planar" addition of rings to form new rings. Figure 2 show a planar triangle branch point of the amplitude represented by Fig. 1(a). The collection of ring amplitudes with planar singularities were said to obey "planar unitarity." They exhibited most of the special regularities, such as exchange degeneracy and OZI selection rules, as the original Harari-Rosner dual-model amplitudes that contained only poles.

It now became possible to develop a systematic topological expansion generating, from planar ring amplitudes, physical mesonic amplitudes with the most general singularities; and there occurred between 1974 and 1977 an impressive increment in the understanding of low transverse-momentum meson properties. Elucidation of the meaning of the pomeron is an example. In retrospect the most important achievement may have been recognition of the inevitability of broken symmetry in a unitary causal S matrix. The DTU story up to a Berkeley

summer workshop in 1977 has been reviewed by Carl Rosenzweig and me in a Physics Reports article[1] and in a more readable article by Fritjof Capra in the American Journal of Physics.[2] More mathematical details can be found in articles by Jean-Pierre Sursock[3] and by George Weissmann.[4]* I shall not pause here to explain what is meant by a "topological expansion" and by broken symmetry, since I must in any event give such an explanation below for a generalized version of the ideas just described.

The success of DTU for mesons made inevitable an attempted extension to all hadrons. Astonishingly, what has emerged over the last $1\frac{1}{2}$ years holds the promise of eventually also describing weak interactions and weakly-interacting particles such as leptons. Already one begins to understand why quarks have spin 1/2 and precisely 3 colors, as well as why there is "up-down" flavor symmetry and at least two flavors beyond up and down. Light is being shed on the meaning of gluons. The bootstrap idea is beginning to deliver on its promise to explain the origin of particle attributes that are seen as arbitrary input into constructive Lagrangian models.

Contributing to developments at Berkeley since the 1977 summer workshop have been Jerry Finkelstein and Henry Stapp, as well as Jean-Pierre Sursock and George Weissmann. Fritjof Capra also has played a significant part. Extremely recently, in Paris, contributions have been made by Basarab Nicolescu, Julian Uschersohn and R. Vinh-Mau as well as by topologist, V. Poenaru. In a separate report to this meeting Nicolescu will describe certain striking hadron spectrum

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* In all these articles, what I have here described as a "ring" amplitude is called an "ordered" amplitude.

predictions that follow from bootstrap quark theory. These more experimentally-oriented questions are also discussed in my lectures to the 1979 Winter Meeting on Fundamental Physics in Segovia. My report today will concentrate on theoretical foundations.

Those of you who will be seeing for the first time the application of combinatorial topology to the bootstrapping of quarks, gluons, leptons, etc., may feel stunned. You may come away from this talk believing that you understood nothing. My advice is to relax and to try to pick up the spirit of the approach. Don't worry about details. The third or fourth time that you hear a talk on this subject you will find that the rules are easy to grasp--much easier than those of a quantized gauge field theory.

Before beginning to explain what is meant by the topological expansion of the S matrix, let me warn you that none of the diagrams you will see are Feynman diagrams. All the particles in these diagrams are physical particles with physical masses. Landau graphs look like Feynman diagrams but they do not have the same meaning.* So watch out!

## 2. THE TOPOLOGICAL EXPANSION

In particle physics the term "topological expansion" seems first to have been used in connection with Feynman-graph summations in perturbative field theory. There the small expansion parameter is $1/N$, where $N$ characterizes some internal symmetry; in QCD $N$ might be the number of colors. Successive terms in the expansion carry successively-higher powers of $1/N$. Although like many other ideas

* Fifteen years ago my pedagogical problem would have been easier because all particle physicists of that epoch were familiar with dispersion relations.

that have proved important for the S matrix, the notion of topological expansion was spawned by Feynman diagrams, we shall see that convergence of the S-matrix topological expansion does not rest on the smallness of $1/N$. Indeed, as we begin a bootstrap investigation we do not know the value of $N$. "Internal" quantum numbers are supposed to emerge from the demands of unitarity and causality.

For the S matrix the significance of a topological expansion is to be understood in terms of singularity complexity. What does that mean? As for many ingredients of bootstrap theory one must close a consistent cycle of definitions before a clear meaning emerges. So be patient. Let us start by writing

$$M_{fi} = \sum_{\gamma} M_{fi}^{\gamma} , \tag{1}$$

where $M_{fi}$ is any connected part (amplitude) of the S matrix, describing a reaction leading from initial channel $i$ to final channel $f$. We know that $M_{fi}$ is an analytic function of initial and final particle momenta with a vast collection of singularities. The decomposition (1) rests on the notion that $M_{fi}$ can be built (through dispersion relations) from a linear superposition of components associated with different singularities. The index $\gamma$ somehow classifies these singularities according to complexity. By writing Eq. (1) we are implicitly expecting that "simple" singularities are more important than "complicated" singularities. Each $M_{fi}^{\gamma}$ is an analytic function of the $p_f$ and $p_i$ but with only a small subset of all the singularities that are present in the full amplitude $M_{fi}$. We hope that by dealing with the simpler amplitudes, $M_{fi}^{\gamma}$, one at a time, in order of increasing complication, a path can be found to the achievement

of a consistent S matrix.

Now comes the crucial guess, which can only finally be justified by its success--that is, by its self consistency. We assume that the complexity of a momentum singularity can be placed in correspondence with the complexity of two-dimensional surfaces. Why 2 dimensions?

The central reason is that S-matrix singularities are associated with graphs, and it is known that the complexity of graphs can be characterized by the complexity of two-dimensional surfaces on which the graphs are embedded. Textbooks on 2-dimensional combinatorial topology typically have alternating introductory chapters on graphs and surfaces. (See, for example, the books by A. J. White and by G. Ringel, listed as Ref. (5).) It seems impossible to say that the one concept is more fundamental than the other. The important point to appreciate is that one does not need to go beyond two-dimensional surfaces in order to represent the complexity of which the most general graph is capable, and, if one believes that all of particle physics can be described by an S matrix, then the complexity to be grasped is no more than that representable by graphs.

This latter fact, which may seem to you so obvious as to be empty, is profound. Let me point out that a constructive field theory such as QCD provides no guarantee of localizable events in space time and of the particles that correspond to causal relationships between these events. As physicists we inevitably interpret "reality" in graphical terms, that is, as a succession of causally-connected collisions between particles. The S-matrix takes for granted this graphical nature of physical reality and, indeed, Landau graphs in momentum space can be put in one-to-one correspondence with graphs such

as Fig. 3 representing a succession of causally-connected events in space-time.[6] But constructive field theory gives no assurance that the solutions of the continuum field equations will have this kind of discrete space-time structure. I believe that we have here the essential advantage of the S matrix which allows it to explain the origin of those internal quantum numbers and symmetries that must be accepted as arbitrary input into Lagrangian field theory. Reversing the logic, it seems to me that if constructive field theory is to have a chance of producing a satisfactory (i.e. graphical) description of physical reality, it must have precisely the symmetries and internal quantum numbers exhibited by physical particles. In particular, so long as the number of quark colors and flavors is regarded as arbitrary, I doubt that confinement can be proved.

Many of you will be stupefied by such statements and perhaps inclined to stop listening to anyone who believes in such an absurd scenario. But please be tolerant. It is unnecessary to share my expectations for the future in order to find interesting the consequences of the assumption that S-matrix singularity complexity corresponds to the complexity of two-dimensional surfaces.

Two vague arguments to support such a correspondence may be added to the basic argument just presented:

(1) Scattering amplitudes are complex analytic functions, and complex numbers correspond to a 2-dimensional space.

(2) Harari-Rosner diagrams are often usefully visualized as "rubber sheets" with quart-line boundaries. Planar diagrams are sheets that can be cut out of a plane, while more complicated diagrams are sheets cut from more complex surfaces. (See Fig. 4.) The Harari-Rosner

based topological expansion, described in Ref. (1), can thus be seen as an association of singularity complexity with surface complexity. What we are now to consider generalizes this expansion, which has proved so impressively relevant to mesons.

I have said that the index $\gamma$ characterizes the momentum singularity structure of the analytic function $M_{fi}^{\gamma}$ and at the same time corresponds to some 2-dimensional surface. Our generalization of Harari-Rosner is to assume this a closed triangulated surface, various combinations of triangles being associated with the individual particles in the channels $i$ and $f$. Calling such a combination a "particle face," the entire closed surface is covered by faces belonging to ingoing and outgoing particles. Suppose, for example, that in a reaction $AB \to CD$ each of the ingoing particles $A$ and $B$ is associated with a single-triangle face, while the outgoing particles $C$ and $D$ are each associated with a pair of adjacent triangles, as shown in Fig. 5(a). (Notice that a "triangle" consists of three vertices joined by "edges." It is not important to draw these edges as straight lines.) One way to cover a closed surface with these four particle faces is shown in Fig. 5(b). Here the triangular face $B$ must be understood as covering all of the sphere not covered by $A$, $C$ and $D$. This particular way of covering a particular surface corresponds to a value of the index $\gamma$. Were we to cover a sphere in a different pattern with these same four faces (e.g. by interchanging faces $A$ and $B$), that would correspond to a different value of $\gamma$. A torus covered with these four faces would be still another value of $\gamma$.

What is the connection between Harari-Rosner ring diagrams and spheres covered with particle faces? Unitarity turns out generally to

demand a ring structure for spheres in that each particle face can form a disc with precisely two other particle faces. A disc is characterized by a perimeter that touches itself only once. In Fig. 5, for example, face A can combine with C to form a disc and also with D, but not with B; D forms discs with A and B but not with C, and so on. A cyclic order, or ring, ADBC is thus defined. Later we shall see that "channel discs," combinations of particle faces that constitute discs, correspond to "resonating" channels where poles and other singularities occur in the same sense as for adjacent particles in Fig. 1. Spherical components of the topological expansion thus maintain the essential features of ring amplitudes associated with Harari-Rosner diagrams. The spherical generalization allows consideration of particle structure more complex than quark plus antiquark and also will reveal color, flavor, etc. to be topological aspects of quark structure that are not arbitrarily assignable.

Why do we use triangles to build hadron faces? Because the triangle is the simplex for two dimensions. It provides the most general way of dividing up a surface: any surface can be divided into triangles. No other polygon provides such generality. What I shall eventually call "quarks" and "gluons" will be triangle combinations from which hadron faces are built. Unitarity precludes particle faces from being as simple as in the illustration of Figure 5. A certain degree of structure is essential, and an important aspect of the bootstrap problem is to determine the minimum amount of structure that a particle face must possess. It will turn out that the triangle combinations we call "quarks" and "gluons" are too simple to be candidates for particle faces. Unitarity demands confinement!

A technical point that I shall not have time to explain properly is that unitarity requires the closed surfaces associated with momentum singularity structure to include singular points. We shall, for example, need to consider spheres on which two points have been identified. As shown in Fig. 6, such a singular surface has somewhat the character of a torus. Such singular points may occur at vertices of the triangulation pattern where particle faces make contact with each other, and we must include in the meaning of the index $\gamma$ an enumeration of such singular points together with a statement of where they occur in the triangulation pattern of the closed surface.

Let us now proceed to see how unitarity provides an algebra for the $\gamma$ index. The rules of this algebra provide a connection between momentum singularity structure and surface complexity and they will be seen to be vital to the convergence of the expansion.

## 3. UNITARITY ALGEBRA FOR THE $\gamma$ INDEX

I have several times referred to the possibility of expressing unitarity through Landau graphs. What does this mean? A simple and familiar example will remind you. [Those who wish to know the general rule should consult Iagolnitzer's book,[6] Chap. III.] Consider the reaction $AB \to CD$ and the variable $s = (p_A + p_B)^2 = (p_C + p_D)^2$ --the square of the center-of-mass energy. If there exists a channel EF which communicates with both AB and CD, then in the analytic reaction amplitude $M_{CD,AB}(s)$ there is a branch point at $s = (m_E + m_F)^2$. The discontinuity of the amplitude in passing once around the branch point is given by a formula conveniently represented by a graph. Ignoring normalization, the formula is:

$$\mathrm{disc}_s\, M_{CD,AB} = \int dp_{EF}\, \bar{M}_{CD,EF} \times M_{EF,AB}\,, \tag{2}$$

where the integration is over the phase space of the EF channel, and the associated graph is shown in Fig. 7. I am here using standard S-matrix "bubble" notation, the (+) bubble representing the amplitude and the (-) bubble its Hermitian conjugate. The dashed line on the left indicates the variable in which the singularity occurs. For what follows the distinction between (+) and (-) is not important; one should think of each bubble as simply the vertex of a graph.

Now suppose that we substitute the topological expansion (1) into the discontinuity formula (2). Recognizing that the channel AB corresponds to the index i and the channel CD to the index f, we find

$$\mathrm{disc}_s \sum_{\gamma} M^{\gamma}_{fi} = \int dp_n \sum_{\gamma''} \bar{M}^{\gamma''}_{fn} \sum_{\gamma'} M^{\gamma'}_{ni} \tag{3}$$

if the EF channel is designated by the index n. Let us now postulate that <u>each</u> individual product on the right-hand side of Formula (3) is a discontinuity of <u>one</u> term on the left, allowing us to write

$$\mathrm{disc}_s\, M^{\gamma}_{fi} = \sum_{\gamma'+\gamma''=\gamma} \int dp_n\, \bar{M}^{\gamma''}_{fn}\, M^{\gamma'}_{ni}\,, \tag{4}$$

where the notation $\gamma' + \gamma'' = \gamma$ implies that each combination of $\gamma'$ and $\gamma''$ corresponds to one value of $\gamma$. What is the rule by which we determine this value? Remembering that each value of $\gamma'$ and $\gamma''$ corresponds to a particular triangulated surface, a natural prescription for $\gamma$ is to make a <u>connected sum</u> of these surfaces by identifying

the perimeters of certain triangles on $\gamma'$ with the perimeters of triangles on $\gamma''$ and erasing the triangle interiors. Which triangles do we identify? The answer is given by the Landau graph of Fig. 7. We identify (and then erase) triangle subsets corresponding to the "intermediate" particles E and F. The new surface is then completely covered by triangles corresponding to the "external" particles A, B, C, D.

It should be apparent that any Landau graph, with an arbitrary combination of vertices (amplitudes) and connecting arcs (particles), can thus be interpreted as a connected sum of surfaces. If each amplitude vertex is assigned a definite value of $\gamma$, the sum of the various surfaces is always a surface covered by collections of triangles belonging to initial and final (external) particles; the sum therefore also corresponds to a definite value of $\gamma$. In this sense unitarity provides an algebra in topological index $\gamma$ for the component amplitudes in the topological expansion.

At the same time the Landau formulas determine the singularities in complex momentum space of the analytic functions $M_{fi}^{\gamma}$, so there is a connection between momentum-singularity structure and surface topology. According to Formula (4) and its generalization, each "topological" amplitude $M_{fi}^{\gamma}$ contains only singularities whose "structure" is of the type $\gamma$. It is not yet generally understood what "structure" in the sense of $\gamma$ means for the Riemann surface, but one guesses that "simplicity" of $\gamma$ surface means "simplicity" of Riemann surface. For example, the simplest Riemann singularities --pure poles in complex momentum--correspond to Landau tree graphs (no closed loops), which we shall see below yield the simplest type of

connected sum for $\gamma$ surfaces. Let us now turn to the general question of "complexity" versus "simplicity" for $\gamma$ surfaces. Herein lies the essential point of the topological approach to particle physics.

## 4. SIMPLICITY AND COMPLEXITY: ENTROPY AND BROKEN SYMMETRY

Physicists instinctively concentrate their efforts wherever they see simplicity and hope that complex problems can be deferred. The topological expansion provides formal justification for this time-honored approach. Suppose that to each value of $\gamma$ we associate a (hopefully finite) set of complexity indices $i_\gamma$, $j_\gamma$,...that do not depend on the triangulation pattern but only on topological invariants such as genus, number of singular points, and number of "almost separate" surfaces--in contact only at discrete points. A (single) sphere with no singular points (e.g., Fig. 5(b)) has all such indices equal to zero. A slightly more complex surface is that of Fig. 6; here the genus is zero (still a sphere) but there is a pair of singular points. Figure 8 shows two spheres in "contact" at two distinct points. The variety of surface complexity (i.e., the number of distinct complexity indices) to be considered is determined by unitarity through the connected surface sums implied by Landau graphs. That is, even if we start with Landau graphs where all amplitude vertices correspond to spheres, connected sums corresponding to sufficiently "complex" graphs will generate values of $\gamma$ for which a variety of complexity indices are not equal to zero. It is not known at the present time how many different categories of complexity must be recognized, but need for the complexity types illustrated in Figs. 6 and 8 seems certain, and highly probable is the familiar complexity of closed

orientable surfaces without singular points--described by genus illustrated in Fig. 9. An issue of great potential importance is whether nonorientable surfaces such as the Klein bottle are required by unitarity.

Now, for closed surfaces without singular points, a principle of "entropy" can be established for connected sums: If the surfaces to be summed have complexities $i_{\gamma'}$, $j_{\gamma'}$, $\cdots$, $i_{\gamma''}$, $j_{\gamma''}$ $\cdots$, then the complexity of the connected sum is always greater than or equal to the sum of the complexities of the constituent surfaces. That is,

$$\begin{aligned} i_\gamma &\geq i_{\gamma'} + i_{\gamma''} + \cdots \\ j_\gamma &\geq j_{\gamma'} + j_{\gamma''} + \cdots . \end{aligned} \tag{5}$$

For example, if we identify a single triangle on a surface with genus 1 with a single triangle on a surface with genus 2, we arrive at a surface with genus 3--an example of the equality in Formula (5). If we identify two separated triangles on the first surface with two separated triangles on the second, we produce a surface with genus 4 --an example of the inequality.

Suppose that an "entropy" condition of the form (5) holds for each of the complexity indices required by unitarity. It follows that in a general Landau formula

$$\text{disc } M^\gamma = \sum_{\gamma'+\gamma''+\cdots=\gamma} \int M^{\gamma'} \times M^{\gamma''} \times \cdots \tag{6}$$

(I here suppress channel indices, complex-conjugation and phase-space volume element.), no complexity on the right-hand side can be larger than that on the left and, furthermore, a complexity on the left can

appear no more than once on the right <u>except for complexity zero</u>. Such a circumstance would mean that the bootstrap problem has been reduced to the sphere! Why?

The possibility of generating particle symmetries and quantum numbers (and, indeed, particles) from nothing more than the requirements of causality and unitarity is conceivable because of the nonlinearity of Landau formulas for amplitude discontinuities. We now see that the set of amplitudes with all complexity indices zero are self-determining through nonlinear relations--without reference to other amplitudes. In contrast, any nonspherical component of the topological expansion may be computed from less complex components through linear relations of a familiar type which all our physical experience indicates should not generate new symmetries or quantum numbers.

The entire topological expansion thus emanates from the sphere. The "degrees of freedom," i.e. all symmetries and conserved quantum numbers, are fixed by the mutual consistency of nonlinear Landau relations for spherical amplitudes--relations whose collective content may appropriately be characterized as "spherical unitarity."

Implicit throughout all our reasoning is an assumption of convergence for the topological expansion when components are ordered according to complexity. A physical basis for convergence will be explained in a moment. If you are prepared to believe that the "simplest" components are the most important, then immediately you see the inevitability of broken but recognizable symmetry in particle physics. Spherical amplitudes will exhibit special regularities that cannot be sustained by surfaces with less symmetry. If amplitudes

belonging to symmetry-breaking surfaces are smaller than spherical amplitudes, then physicists will notice the special symmetries of the sphere even though these symmetries are broken.

It is furthermore likely that certain spherical regularities will be sustained by nonspherical surface bearing limited types of complexity. I shall make plausible below that time reversal, parity and charge conjugation symmetry are sustained by all orientable surfaces. It is then natural to conjecture that strong interactions correspond to the orientable components of the topological expansion. In this talk I shall deal only with strong interactions, but you will not be surprised to hear that attempts are underway to understand the origin of weak interactions through nonorientable surfaces.

## 5. CONVERGENCE OF THE TOPOLOGICAL EXPANSION; DISCS AND RESONANCES

Why are "simple" momentum singularities more important than "complex" singularities? Let us contrast the Landau product of Fig. 7 for two different arrangements, on the ingredient amplitude surfaces, of the triangles corresponding to the intermediate particles E and F. I have already noted that the face belonging to each particle in the spherical Hilbert space is a disc, a collection of adjacent triangles whose perimeter closes only once upon itself. Figure 10 gives examples of discs that might be associated with E and F, although in the following section we shall find unitarity requiring more structure for particle discs than that of Fig. 10. The two different arrangements to be contrasted are shown in Fig. 11(a) and (b). In Case (a) the two particle faces are adjacent so as to constitute a single disc. In Case (b) they are not adjacent and remain two distinct discs on the surface. (Other triangles, not shown, intervene.)

Now suppose that each of the ingredient surfaces is a sphere, with the particle faces E and F occurring on both surfaces in the <u>same</u> single disc configuration of Fig. 11(a). Identification of these faces in the connected sum is then equivalent to identifying the entire two-particle discs. The result is another sphere. On the other hand if on both spheres we have the separated 2-disc configuration of Fig. 11(b) then the connected sum will be torus. By our definition of singularity complexity, the singularity for Case (a) is simpler than that for Case (b) and therefore is supposed to be more important. How are we to understand this difference in terms of our physical experience with the way particles behave?

The physical distinction between the particle-face arrangements of Fig. 11(a) and Fig. 11(b) involves the <u>resonance</u> concept. If particle faces are discs, then the combination of two faces in Fig. 11(a) is in some sense equivalent to a single particle. We have here, in other words, a <u>resonating</u> configuration of the particles E and F. Figure 11(b), by contrast, is a nonresonating configuration. There is no way here to consider the combination as equivalent to a single particle. A discontinuity where the intermediate particles resonate is experimentally well known to be larger than when they do not.

In slightly more general terms one recognizes that single-disc identification on different surfaces leads to the equality in Formula (5), i.e. no increase of "disorder". Any Landau <u>tree</u> graph (no closed loops) possesses this feature. Since tree graphs correspond to pure poles in the Riemann surface, the phenomenon discussed here is related to pole dominance. We recognize, however, that singularities other than poles, if the intermediate-particle faces cluster into discs, have the same essential degree of complexity as poles. The spherical

components of the topological expansion do not, therefore, correspond precisely to the dual models of the late sixties, which contained only pole singularities in the Riemann surface. One may nevertheless attribute convergence of the topological expansion to the general fact that, the simpler the topology the more resonances there are in intermediate states. The more complex the topology the fewer the resonances.

Another way to express the same idea is in terms of phase space. A resonating two-particle channel is more important than a non-resonating channel because threshold factors cause 1-particle phase space to be larger than 2-particle phase space. Two-particle phase space is larger than 3-particle phase space, and so on. Phase-space (threshold) factors, which often are thought to be "purely kinematical" and not of profound significance are central to convergence of the topological expansion and thus to the bootstrap.

From the S-matrix bootstrap viewpoint there is no distinction between "kinematics" and "dynamics"; we are similarly going to find no sharp distinction between "internal" and "external" quantum numbers. If your minds have now been loosened to the point where you can contemplate such blurring of traditionally-distinct concepts, then you can perhaps believe that the convergence mechanism I am describing does not exclude a role for "1/N," where N somehow measures the multiplicity of the particle spectrum and relates to "internal" symmetry. There turns out to be such a role that is similar to that for perturbative field theory. Because "internal" quantum numbers are tied to topology, a larger multiplicity of intermediate channels is tolerated by simple topology than by complex topology.[1] In bootstrap quark

theory we cannot assume "large N" (small 1/N) as a basis for convergence of the topological expansion, because internal quantum numbers must emerge from the demands of spherical unitarity, but experimental knowledge that N is large (e.g., that there exist many different flavor and spin states of hadrons) may further encourage you to believe in convergence according to a hierarchy of simplicity.

## 6. SURFACE ORIENTATION; C, P, T, BARYON NUMBER AND "COLOR"

The S matrix transforms ingoing states into outgoing states, and analytically-continued Lorentz invariance implies antiparticles and crossing; i.e., negative-energy ingoing particles correspond to positive-energy outgoing antiparticles and vice-versa. Although the concepts of time-reversal (T) and charge-conjugation (C) are thus intrinsic to a causal S matrix, there is no general requirement of symmetry under these operations. By contrast, symmetry under PCT, where P is the parity operation, is implied by crossing. In making the topological expansion (1), it is tacitly assumed that each separate component $M^{\gamma}_{fi}$ obeys the crossing principle and admits a meaning for C, P and T. We now explore that meaning and will argue that the largest terms in the expansion exhibit not only PCT symmetry but separate C, P and T symmetries.

It is natural to suppose that the operations C, P and T, each of which has the character of a reflection (in that the operation, twice repeated, restores the original state), will somehow be associated with "surface inversion." Now the simplices for one and two dimensions <u>each</u> admit the notion of "opposite" orientations. If we think of a triangle as including its one-dimensional boundary as well as its two-dimensional interior, it is accordingly possible to define

two different kinds of inversion for surfaces built from oriented triangles. We may think of orienting "up" or "down" a triangle interior (without the boundary) and we may think of orienting the perimeter "clockwise" or "counterclockwise." Physicists are accustomed in 3-dimensional space to equating these two kinds of orientation--e.g., using a "right-hand rule"--but our S-matrix surfaces are not part of a 3-dimensional space.

We shall use the notation of Fig. 12 to indicate the four different orientations of a triangular disc. The interior orientation is given by the + or - sign, while the perimeter orientation is given by the arrow. Let us now attempt to find a set of rules associating the C, P, T operations with surface inversions.

We start with the connection between ingoing and outgoing faces for the same particle, which must fit together smoothly when identified on two different surfaces in Landau products. At the same time disc combinations of particle faces (channel discs) must fit together smoothly, since channel discs must follow the same topological rules as single-particle faces. Furthermore the sequence in which individual particle faces within the channel disc are identified and erased should not matter. All these requirements are met if we postulate that an outgoing channel disc is obtained from the corresponding ingoing disc by "turning over" the disc and then reversing (+ ↔ -) all triangle interior orientations. Examples are presented in Fig. 13. This in ↔ out rule reverses all perimeters, including the order of triangles along a disc perimeter, but leaves interior orientations unchanged. With this rule, that we tentatively identify with time reversal, the two discs match perfectly when one is placed against the other.

A closely-related question relates to "forward scattering": an amplitude where an outgoing particle carries all the same attributes, including spin and momentum, as an ingoing particle. One expects a close topological connection between such an amplitude and one where this ingoing-outgoing particle is absent, all other particles maintaining the same relative posture. (Physically, forward scattering and no scattering are indistinguishable.) The desired smooth connection is achievable if the corresponding ingoing-outgoing faces are adjacent on the surface, sharing one or more corresponding edges as in Fig. 14(a) so as to form a 2-face disc. Such 2-face discs can be removed smoothly from the surface without disturbing the topology of the remaining faces. That is, if we identify the boundaries of these two particle faces with each other, in exactly the same way as when forming a Landau product, then the face-antiface pair disappears--leaving a surface (covered by the remaining faces) that has the same complexity indices as the original surface. Once again multiparticle channels corresponding to discs behave in the same way as single particles (Fig. 14(b)). The notion of surface contraction, introduced here in connection with forward scattering, becomes heavily used when discussing the spherical bootstrap problem. Notice that two adjacent triangles are contractible when their boundary orientations are opposite and their interior orientations are the same.

By the crossing principle in $\leftrightarrow$ out is equivalent to replacing particle by antiparticle with the opposite helicity. That is, in Fig. 13(a) if the face on the left represents a particle with helicity $\lambda$, then the face on the right represents the corresponding antiparticle with helicity $-\lambda$. Now suppose we make the guess that charge conjugation corresponds to complete inversion of a particle face or collection

of faces (viewing the surface "from the other side"), as in Fig. 15. Since charge conjugation carries particle into antiparticle with helicity unchanged, it follows (e.g., from comparison of Figs. 15 and 13(b)) that inversion of triangle-interior orientation is associated with inversion of helicity. A more general conjecture can be made: If the in $\leftrightarrow$ out operation (Fig. 13) is T, then inversion of interior orientations is CT . Now on a single particle or a channel with spin J, the action of PCT gives a phase $(-1)^J$, so we have

$$\text{Interior inversion} \approx (CT) = (PCT)\,P^{-1} = (-1)^J\,P\,. \tag{6}$$

We are, in other words, conjecturing an association of interior inversion of bounded surfaces with natural parity (naturality). For an amplitude (closed surface) the total angular momentum of all particles is zero, so interior inversion simply corresponds to the parity operation.

The foregoing conjectures need further study and may require revision, but they illustrate the plausibility of associating surface inversions with C and T . Now, given any triangulated <u>orientable</u> surface (where the two sides are not connected) there are three other <u>different but equivalent</u> triangulations that can be reached by our two types of surface inversion. In other words, orientable surfaces necessarily exhibit C and T symmetry. No such statement can be made for nonorientable surfaces. I shall not here pursue the fascinating possibility that weak interactions are to be associated with nonorientable surfaces. I hope, nevertheless, that you have heard enough

to find reasonable an association of strong interactions with the orientable components of the topological expansion.

There are two alternative simple rules to guarantee orientability of a closed surface covered by triangles. One may insist that all triangles have the same perimeter orientation or one may insist that the perimeter orientation (clockwise or counterclockwise) of each triangular disc be opposite to that of its neighbors. The latter rule, for reasons I have inadequate time to develop here, turns out to be suited to the requirements of unitarity. This story is told in Ref.(7) and I here merely call attention to the connection with contractibility, which requires adjacent triangle pairs of opposite perimeter orientation. At the same time unitarity rules out a single triangle as a possible particle face because of its symmetry, which allows identification of two (oriented) triangles on different surfaces to be made in 3 possible ways. There must be no ambiguity in forming the connected sum of surfaces implied by Landau graphical products. The simplest triangular discs of well-defined boundary orientation and with no symmetry are shown in Fig. 16. These discs, which each consist of 3 triangles of the same boundary orientation, are being used as basic building blocks in current efforts to satisfy spherical unitarity. We shall refer to the four configurations of Fig. 16 as "basic discs."

Although in the following section I shall discuss a further complexification of particle faces that is interpretable in terms of "flavor," it is pedagogically useful here to discuss certain physical questions as if there were no need for flavor. Most of what I say here follows Ref. (7).

In covering closed surfaces with triangular discs one must employ an even total number, so fermions are to be associated with faces built from an odd number of basic discs and bosons with faces built from an even number. It then automatically follows that the total number of ingoing and outgoing fermions in any reaction is even. Furthermore, if we restrict consideration to orientable surfaces where all neighbors of a given basic disc have an opposing perimeter orientation, the total number of clockwise and counterclockwise basic discs must be equal on any closed surface. An additively-conserved quantum number immediately follows: For each particle face or channel disc this is the excess of clockwise over counterclockwise basic discs from which the disc is built; we identify this conserved quantity as baryon number. (Although conservation of baryon number is not automatic for nonorientable surfaces, it is possible to triangulate certain nonorientable surfaces so as to conserve baryon number.)

Each basic disc can be described as a combination of 3 "quarks" (clockwise orientation) or 3 "antiquarks" (counterclockwise), and within each basic disc every quark or antiquark can be assigned one of three distinct "colors." For example, if we designate 3 colors by the symbols $\alpha$, $\beta$, $\gamma$, we might assign color $\gamma$ to the triangle with the odd interior orientation and distinguish $\alpha$ from $\beta$ by the clockwise or counterclockwise sense of the perimeter. (See Fig. 17.) Notice that such a definition of color depends on the 3-triangle cluster pattern. There is no way to define the color of an independent triangle; its color depends on its relation to the other two constituents of its basic disc. From the S-matrix standpoint there is no need to define color, but the possibility of doing so may help in seeing

*connections between bootstrap theory and conventional quark theory.*

A remark should be added here that none of the bootstrap papers published or submitted for publication up to this point (even Refs. (8) and (9), the most recent) employ triangle orientation as described here. Baryon number and color in these papers are represented in a different fashion and no effort is made to consider C, P and T. The conclusion of these papers concerning baryon number and color nevertheless coincide with those described here, since basic 3-triangle unsymmetrical discs are employed in the same way.

## 7. FLAVOR AS EDGE ORIENTATIONS ALONG HADRON-FACE PERIMETERS

References (8) and (9) give a set of addition-contraction rules for the construction of hadron faces from basic discs. These rules reflect the demands of unitarity and are closely coupled to the rules by which spherical surfaces are covered by hadron faces. In the present report I shall not describe the addition-contraction rules; they are discussed at length in my lectures to the VIIth International Winter Meeting on Fundamental Physics in Segovia. Applications are given in Nicolescu's report to this Moriond meeting. I nevertheless do want here to tell you something about the concept of flavor from the bootstrap viewpoint.

Flavor resides in edge orientations along the perimeter of a hadron face, this perimeter being built from triangles that are in addition to basic discs. The extra triangles do not affect baryon number or color although they carry the same kinds of orientations as triangles within basic discs. Topologically speaking, flavor is more superficial than the particle properties attached to basic discs. We shall see, for example, that flavor disappears from closed surfaces

and thus is not represented through the topological index $\gamma$ . Flavor can nevertheless not be ignored. Its necessity is connected to the existence of mesons--the unique hadron species from whose faces basic discs are absent.[8] (The "quark" and "antiquark" within mesons have no color--only flavor and helicity.) But it remains for the future to develop a clean argument for the inevitability of flavor.

Consider a baryon face consisting of a single basic disc with three additional triangles attached around the perimenter, as shown in the example of Fig. 18(a). Each edge along the baryon face perimeter is individually oriented. Such a structure can still be described as 3 differently-colored quarks, as indicated in Fig. 18(b), but now each quark consists of two triangles, whose orientations correspond not only to color and baryon number but to helicity and flavor. For example the attributes of the $\alpha$-colored quark from Fig. 18, shown in Fig. 19, are as follows: The clockwise orientation of the right-hand color-carrying triangle tells us that this quark is part of a clockwise-oriented basic disc whose baryon number is +1. In other words, Fig. 19 represents a quark. (There is no harm in saying that the quark baryon number is 1/3, but nothing is gained because quarks always appear in basic-disc clusters of 3.) We have seen that interior orientations relate to helicity and so expect somehow to interpret the + and - in Fig. 19 in terms of "quark helicity." This interpretation we defer to the following section; a surprising and subtle interplay between helicity and color requires careful thought. Much easier to grasp is the association of flavor with the two arrows on the left-hand portion of the quark "perimeter" in Fig. 19

I use the term "perimeter" with quotation marks because the quark of Fig. 19 is not a disc in the sense that we have heretofore

been using the term. It has a meaning only as part of a hadron face. Already emphasized is the fact that quark color ($\alpha$ in Fig. 13) cannot be given topological meaning if the quark is isolated. I am now calling your attention to the additional fact that quark flavor does not have the same meaning as hadron flavor because the quark has an incomplete boundary. Only two of its four "perimeter" edges are oriented. The flavor-carrying edges of a quark "perimeter" nevertheless constitute a portion of a complete hadron-face perimeter, so flavor is a quark attribute that can be described as "additive." That is, the boundary of any hadron face is built by adding the flavor-carrying edges from quark "perimeters." The full "flavor content" of a hadron is characterizable by giving the succession of flavors attached to the quark-antiquark sequence that appears along the hadron perimeter.

Each quark of the "primary" 2-triangle type illustrated by Fig. 19 has four possible flavors, as shown in Fig. 20. In combining hadron faces to form amplitude surfaces we must obey two immediate rules of continuity. (1) As was discussed in the preceding section each basic disc has basic-disc neighbors of opposite boundary orientation. Translating into quark language, each quark along the perimeter of a hadron face must be adjacent to an antiquark from another hadron face (and vice-versa). (2) In joining hadron faces to build surfaces, boundary edges are matched and then erased (again a connected sum, this one involving bounded surfaces rather than closed surfaces). The notion of matching depends on the fact that face boundaries consist of a sequence of individually-oriented edges; matched edges must have the same direction. Translating into quark language, each quark has to be paired against an antiquark of the "same" flavor, the four antiquark

flavors that match those of Fig. 20 being shown in Fig. 21. If flavor content of type $i$ ($i = 1, 2, 3, 4$) for any hadron face is defined as the number of quarks with flavor $i$ minus the number of antiquarks with flavor $i$, then each flavor is separately conserved for all orientable surfaces.

A remarkable property of flavor is that it disappears from any closed surface, since hadron-face boundaries are erased in the process of building the surface. Triangle orientations survive, allowing combinations of triangles to be identified with particle faces, but the surface-topological index $\gamma$ makes no contact with flavor. Because there is then no immediate connection between flavor and momentum-singularity structure, and consequently none between flavor and physical space-time, it is legitimate to describe flavor as an "internal" quantum number. Flavor is an aspect of particles and channels (bounded surfaces) but not of amplitudes (closed surfaces).

There is no known reason to restrict baryon faces to the form of Fig. 18 or, correspondingly, to restrict quarks to the 2-triangle form of Fig. 19. The essential feature of a baryon is that it contains one clockwise basic disc. No known principles exclude additional "peripheral" triangles as in the example of Fig. 22. Here the $\alpha$-colored quark has the 3-triangle structure shown in Fig. 23, which involves the notion of a new flavor--beyond the four shown in Figs. 20 and 21. Sixteen different such "secondary" flavors (3-triangle quarks) may be counted, and there is no presently-visible limit to flavor proliferation.

Hadron mass plausibly tends to increase with complexity of associated face, so we expect to associate the four primary flavors of

Figs. 20 and 21 with "up", "down", "strange" and "charmed." A general symmetry suggests that the flavors numbered 1 and 2 in these figures correspond to "up" and "down." The symmetry operation is an edge inversion rather than the surface inversions of the previous section. Take the collection of separate hadron faces that are to be combined into an amplitude surface and for each face reverse the direction of each perimeter edge and also reverse the perimeter orientation of each triangle within the disc (do not "turn over" the disc or in any way alter the relative positions of the different constituent triangles). It will be seen that flavors #1 and #2 have been interchanged. At the same time quarks have become antiquarks, but for orientable surfaces with assured charge-conjugation symmetry we seem here to be looking at up-down flavor symmetry.

Although I shall not here discuss the general rules for hadron face structure, I show in Fig. 24 examples of meson faces with primary flavor. Notice how superficially similar are these meson faces to the quark of Fig. 19. Meson faces are complete discs, however, with a full boundary all of whose edges are directed. There is no color. Meson faces can be built from a quark-antiquark pair where the color-carrying triangles match in orientation so as to be contractible, as shown in Fig. 25, leaving only the pair of flavor-carrying triangles.

In my Segovia lectures and in Ref. (9) there is described the complementary situation where the flavor-carrying triangles of a quark-antiquark pair have been contracted, leaving a structure with two color indices that may be described as a "topological gluon." See Fig. 26. Such structures appear only in the interior of hadron faces, never on the perimeter. As is the case for quarks, the associated colors are

not topologically meaningful for an isolated gluon. The pair of colors depends on how the gluon is combined with neighboring triangles.

## 8. QUARK AND GLUON HELICITY

Figures 24 and 25 suggest identification of the usual notion of "quark helicity," $\pm 1/2$, with the interior orientation of the quark's flavor-carrying (peripheral) triangle. At the same time Fig. 26 suggests that gluons carry helicity associated with the interior orientation of the quark's color-carrying (basic-disc) triangle. We encounter here a manifestation of the paradoxical relationship between topological color and helicity. Typical of the paradox is the contraction shown in Fig. 27, which can occur when there is matching of the interior orientations of the quark's two triangles, in effect wiping out information about quark helicity. At the same time (see Fig. 17) the quark color becomes ambiguous, even when the interior orientations are known for the other two triangles within the basic disc.* Quark helicity in consequence is not conserved in the sense of quark flavor, although for mesons (which carry no color) Figs. 24 and 25 suggest a parallel status for helicity and flavor. We have earlier stressed that helicity will affect the momentum-singularity structure of amplitudes and thus relate to physical space-time in a fashion that flavor cannot.

The connection between quark helicity and particle helicity is not immediate--as it was for flavor; this story remains to be developed and will depend on the relation between the topological index $\gamma$ and the Riemann surface of the analytic functions $M^{\gamma}_{fi}$. The physical

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* The quark disappears except for its flavor, like the smile of the Cheshire cat in Alice in Wonderland.

significance of gluon helicity is even more obscure. It is interesting to note that contractions wipe out topological gluons unless the interior orientations of the two triangles are opposed--which means total gluon helicity $\pm 1$, as for zero-mass vector particles.

Topological color-spin interaction leads to tentative bootstrap predictions about hadron spectra that differ drastically from the expectations of models where quarks and gluons are regarded as particles.[9] Bootstrap theory is telling us to expect new hadron families whose ground states have low spin, relatively low mass and extremely great stability.

## CONCLUDING REMARKS

Many issues remain obscure in bootstrap quark theory, but I have done my best to persuade you that a variety of qualitative features experimentally observed for strong interactions, and accepted as arbitrary input into traditional constructive theory, promise to be explainable as necessary features of a causal and unitary S matrix. No doubt mistakes are currently being made in the use and interpretation of combinatorial topology, mistakes that will take time to ferret out, but we have seen indication that 2-dimensional surfaces provide an S-matrix capability for grasping the origin of baryon number, color, flavor and half-integral spin, as well as P, C, T and isospin symmetries. At the same time the meaning of "constituent" quarks and gluons is being clarified, with a probable outcome that topological color-spin interplay will lead to hadron spectrum predictions incomprehensible from the viewpoint of constructive quark models.[9]

Untouched so far by bootstrap theory are the experimental manifestations of pointlike "parton structure" for hadrons. The S

matrix is inherently awkward in its description of phenomena where the classical notion of a space-time continuum is useful. It is nevertheless well known that transverse particle "structure functions" can be given an approximate S-matrix meaning for large angular-momentum collisions. I take even more encouragement from the less-well-known S-matrix deduction of the Schrödinger equation for the interaction of two nucleons through a potential. It was shown in 1959 by Charap and Fubini,[10] following the double-dispersion relation discovery by Mandelstam,[11] that to the extent meson masses are small compared to the baryon mass, the Schrödinger equation can be derived as an approximate representation of the analytic S-matrix elements for low-energy nucleon-nucleon scattering. The local deuteron wave function $\psi(\vec{r}, t)$ has no a priori significance but is nevertheless meaningful on a scale of distance large compared to the nucleon Compton wavelength; this wave function is expressible through S-matrix discontinuities. Thus microscopic locality in space-time, a concept apparently absent from S-matrix theory, can be given an approximate meaning in semi-classical circumstances. So there can be hope for an eventual S-matrix bootstrap explanation of parton structure.

Mention of the Schrödinger equation reminds me that most audiences, when they hear about building particles from triangles, lose sight of the fact that triangles were introduced to represent singularities in momentum space. Combinatorial topology is an adjunct to the dispersion relations which provide dynamical content (like the Schrödinger equation) and allow calculation of particle masses and spins. Regge theory here will be important because one of the great simplifications believed to be achieved in reducing the bootstrap

to the sphere is that spherical amplitudes have no Regge branch points --only poles.

The spherical bootstrap, being nonlinear, is still difficult, but the special regularities of the sphere, such as the generalized OZI rule and the presence of only one double discontinuity in 4-point functions, vastly simplify the problem.[1] Even if precise mass calculations remain elusive, it may be possible to understand qualitative questions such as why there is approximate symmetry between strange and up-down flavors--a symmetry that does not extend to charm.

Noteworthy in bootstrap quark theory is the fact that "color," although absent from mesons and always buried more deeply in hadron-face interiors than flavor or quark helicity, is not an "internal" degree of freedom but, together with quark helicity, influences momentum singularity structure. Topological color, in contrast to flavor, is coupled to space-time, color dynamics relating to an absence of symmetry between the 3 colors $\alpha$, $\beta$ and $\gamma$. ($\gamma$ is "odd"; you may remember that the need to break triangular symmetry was the reason for topological color to appear in the first place.) So not only does bootstrap theory show power superior to QCD by explaining why the number of colors can only be 3, but the bootstrap promises to make <u>different</u> physical predictions from QCD--where there is complete color symmetry.

Some of you may be saying to yourselves that DTU is not really a bootstrap because there is a fundamental building block--the oriented triangle. Quarks have been demoted in importance, you may be thinking, only by replacing them with more fundamental units. The role of the triangle as a basic unit, however, is traceable to the Cartesian view

of reality on which physics is based. Physicists recognize reality as a collection of causally-related isolated events in space-time--in other words as a graph. The graphical complexity of physical reality, as we allow ourselves to perceive it, is therefore that of a 2-dimensional surface, and the simplex for any surface is the triangle. We physicists therefore inevitably encounter the triangle as our building block. The fact that reality is based on "threeness" was understood more than a hundred years ago by the eccentric and obscure American philosopher, Charles Sanders Peirce, but it has required the discovery of quarks to bring home the profundity of Peirce's reasoning.[12]

*A final remark relates to electromagnetism.* I have suggested that the S-matrix bootstrap may eventually encompass weak interactions through nonorientable surfaces, but no topological prospects have thus far been identified for bootstrapping the photon. The difficulty is profound and was discussed already in a 1971 paper.[13] Let me here merely point out that the notion of a unitary S matrix is incompatible with the long-range classical aspects of electromagnetism, a fact obscured by the success of perturbative QED. There should be no difficulty in perturbatively accommodating the S matrix described in this report to the existence of weakly-coupled photons (using, for example, the notion of vector dominance), but that is quite different from explaining the reason for the photon's existence and the value of the fine structure constant.

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## FIGURE CAPTIONS

Fig. 1. (a) A Harari-Rosner ring diagram for a 5-particle amplitude.

(b) Pole diagram for the same amplitude.

Fig. 2. Planar triangle branch-point diagram for the amplitude of Fig. 1(a).

Fig. 3. Landau graph corresponding to a sequence of four causally-connected particle collisions.

Fig. 4. Harari-Rosner branch-point diagram seen as a "rubber sheet" cut from the surface of a torus.

Fig. 5. (a) A set of four particle faces.

(b) An arrangement of the four faces on a sphere.

Fig. 6. A pseudotorus: sphere with two identified points.

Fig. 7. Bubble diagram for a 2-particle discontinuity of a 4-particle amplitude.

Fig. 8. Two spheres in contact at two distinct points.

Fig. 9. Two orientable surfaces without singular points.

Fig. 10. Disc-faces of intermediate particles E and F.

Fig. 11. Two possible arrangements of E and F on a surface.

Fig. 12. The four possible orientations of a triangle.

Fig. 13. The action of time reversal (a) on a particle face

(b) on a channel disc.

Fig. 14. Contractible discs (a) In-out particle pair

(b) In-out channel pair.

Fig. 15. The action of charge conjugation on a channel disc.

Fig. 16. The four basic discs.

Fig. 17. The attachment of color to the 3 quarks within a basic disc.

Fig. 18. A baryon face.

Fig. 19. The α-colored quark from the baryon of Fig. 18.

Fig. 20. Quarks with the four primary flavors.

Fig. 21. Antiquarks with the four primary flavors.

Fig. 22. Baryon face where the α-colored quark has a secondary flavor.

Fig. 23. The α-colored quark from the baryon of Fig. 22.

Fig. 24. Meson faces with primary flavor.

Fig. 25. Quark-antiquark contraction of color-triangles to form a meson.

Fig. 26. Quark-antiquark contraction of flavor-triangles to form a gluon.

Fig. 27. Collapse of a quark.

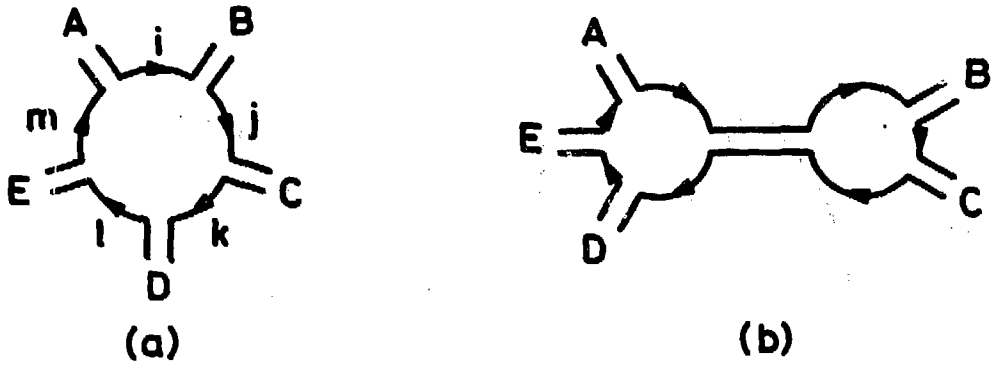


FIG. 1

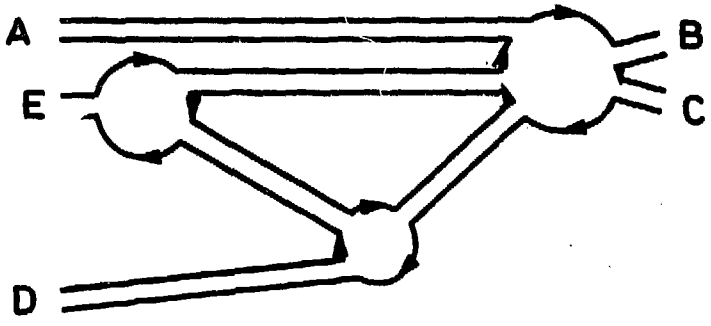


FIG. 2

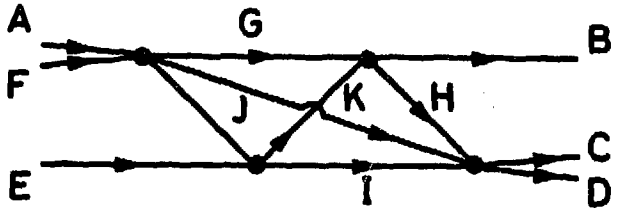


FIG. 3

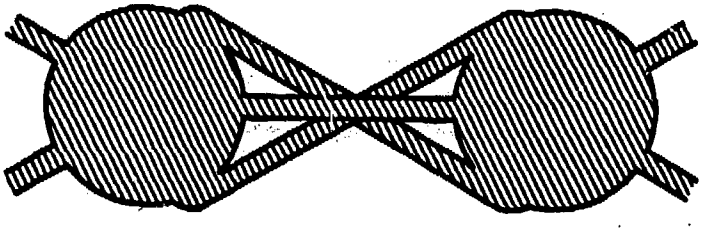

FIG. 4



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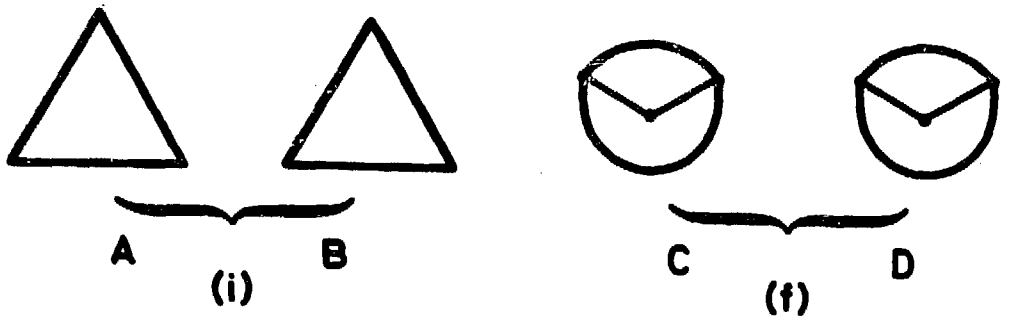
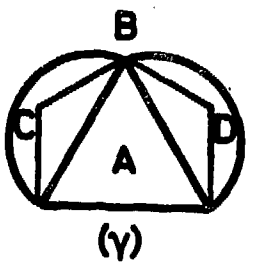



a)

FIG.5




b)

FIG.5

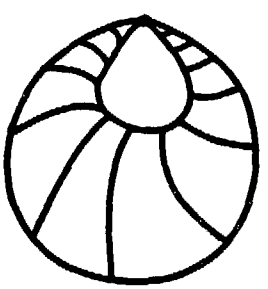


FIG.6

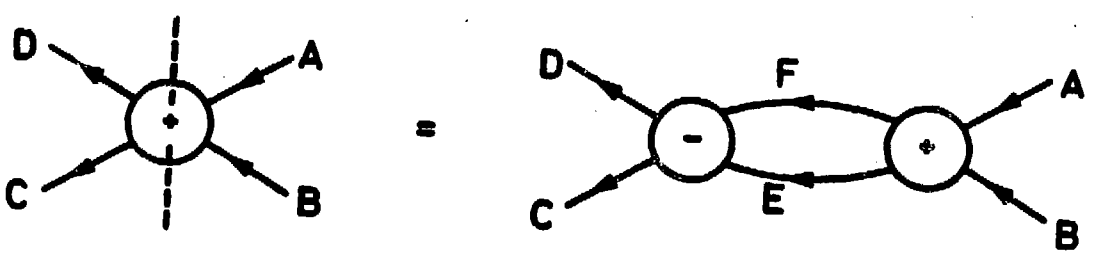



FIG.7

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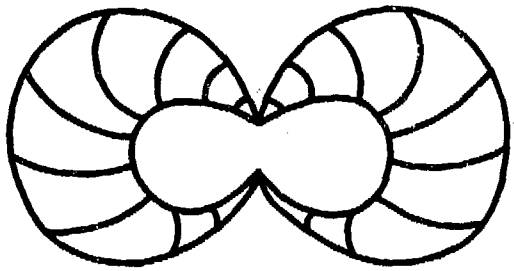

FIG. 8

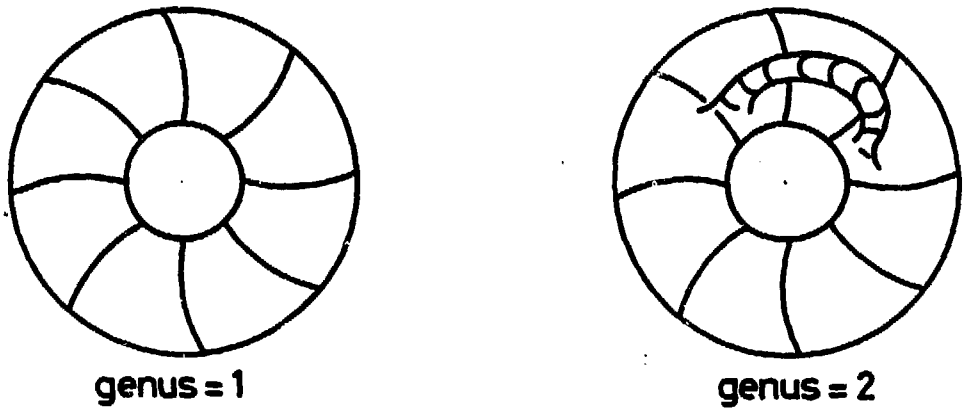


FIG.9

FIG.10

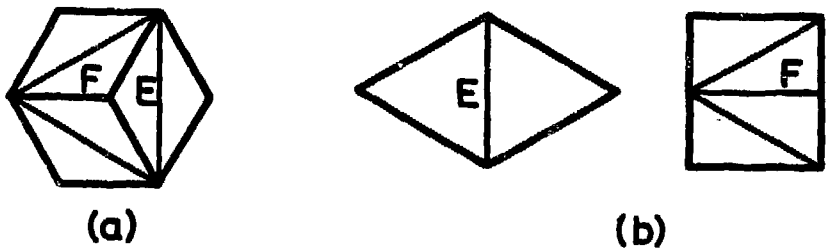


FIG.11



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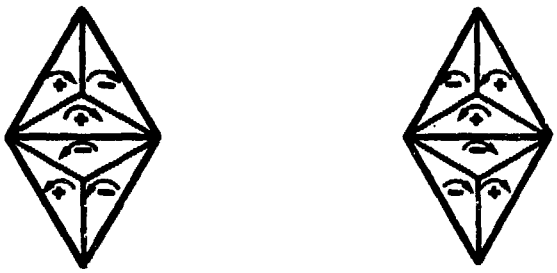
FIG. 12

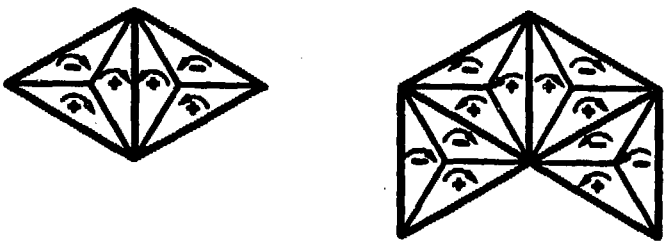
a)



b)

In Out

FIG. 13



a) b)

FIG. 14

C

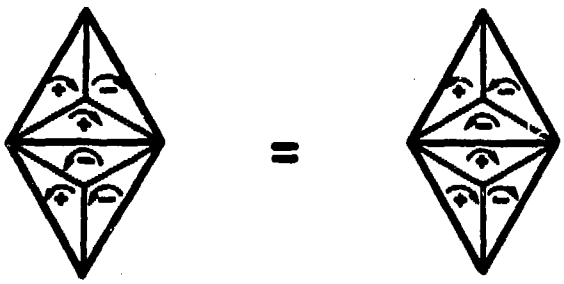


FIG. 15

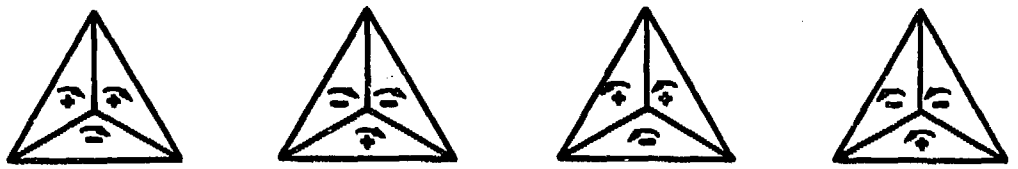


FIG. 16

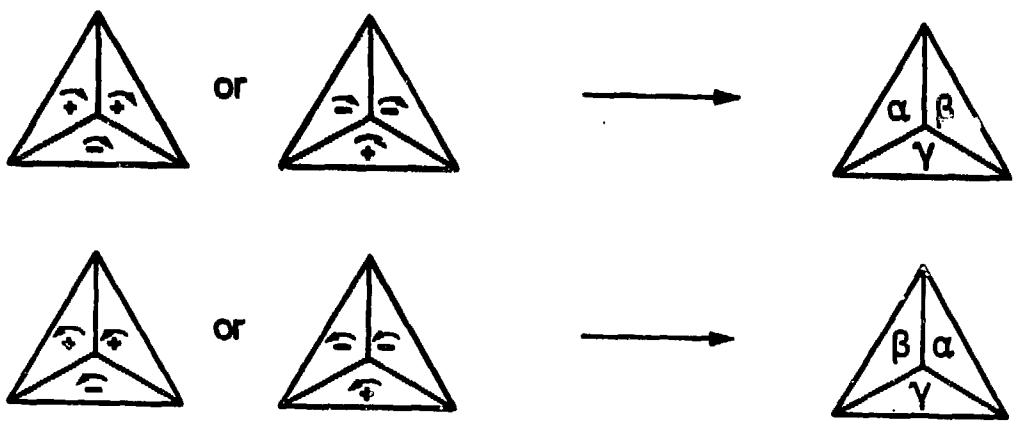



FIG. 17

FIG. 18

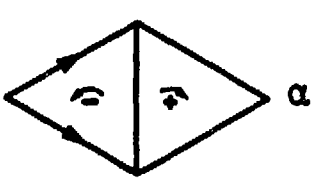



FIG. 19



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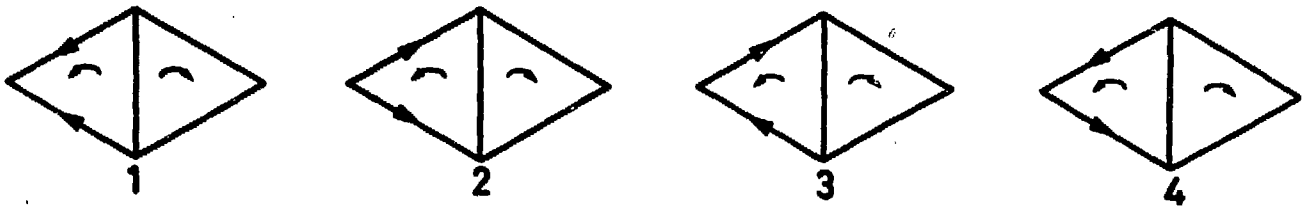


FIG. 20

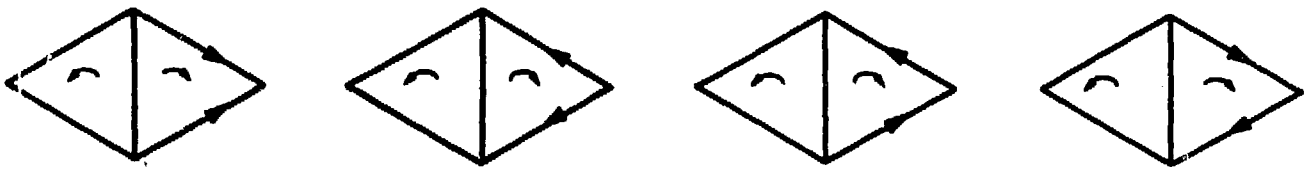

FIG. 21

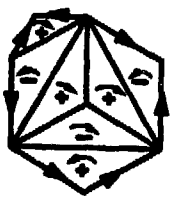

FIG. 22

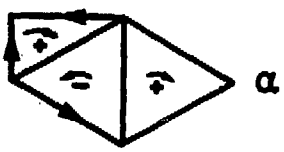


FIG. 23

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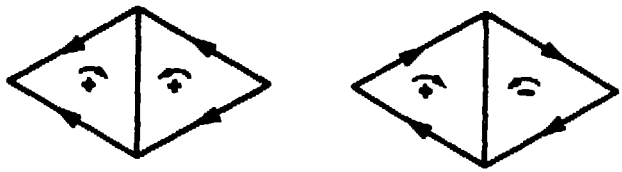

FIG.24

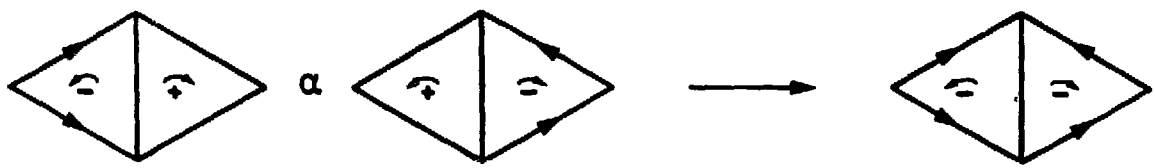


FIG. 25

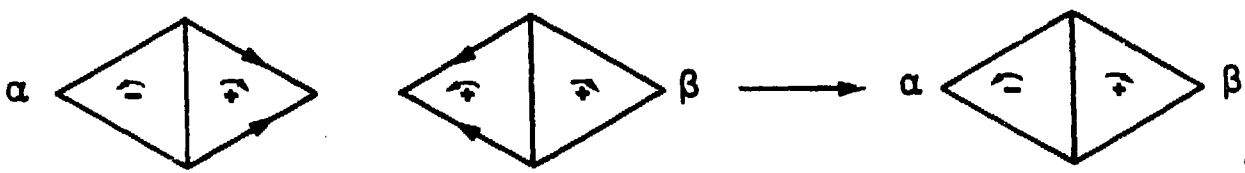


FIG. 26

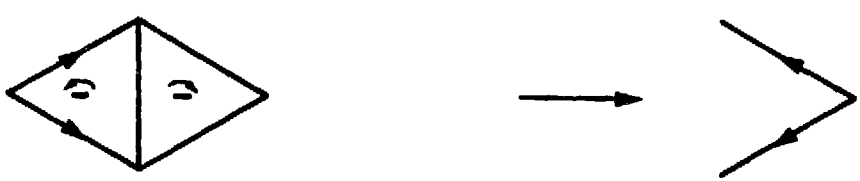

FIG.27

XBL 794-9483