Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA, BERKELEY, CA

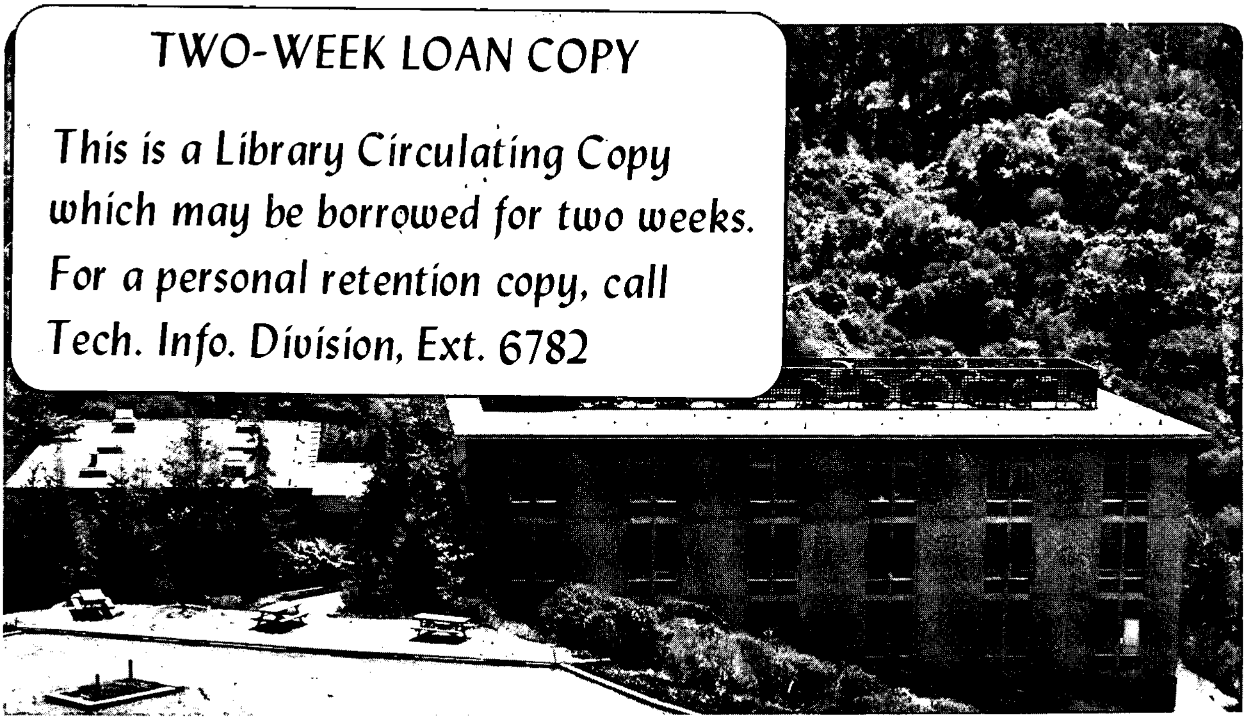
Materials & Molecular Research Division

Submitted to the Journal of Low Temperature Physics

DC SQUID: CURRENT NOISE

Claudia D. Tesche and John Clarke

May 1979



Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

DC SQUID: Current Noise

Claudia D. Tesche and John Clarke*

Materials and Molecular Research Division, Lawrence Berkeley Laboratory
and Department of Physics, University of California
Berkeley, California 94720

The computer model used by Tesche and Clarke to calculate the voltage noise across the dc SQUID is extended to calculate the circulating current noise around the SQUID loop, and the correlation between the circulating current noise and the voltage noise across the SQUID. The parameters chosen are $\beta = 2LI_0/\Phi_0 = 1$, $\Gamma = 2\pi k_B T/I_0\Phi_0 = 0.05$, and an applied flux of $\Phi_0/4$ (L is the SQUID inductance, I_0 is the critical current per junction, T is the temperature, and Φ_0 is the flux quantum). At frequencies well below the Josephson frequency and at the optimum current bias, the voltage power spectral density is approximately 16 $k_B TR$, the current power spectral density is approximately 11 $k_B T$, and the voltage-current correlation spectral density is approximately 12 $k_B T$, where R is the resistance per junction.

*Guggenheim Fellow

I. INTRODUCTION

The dc SQUID[1,2] (Superconducting Quantum Interference Device) can be used either directly as a magnetometer, or in conjunction with various input circuits as a magnetometer, gradiometer, or voltmeter. In a previous paper,[3] we developed a computer model for the isolated SQUID, and calculated the voltage noise at the SQUID output as a function of the device parameters. Thus, we determined the values of these parameters that optimized the energy resolution of the isolated SQUID. However, in addition to the voltage noise, the SQUID also generates a circulating current noise that induces noise into any input circuit coupled to it. To properly optimize the input circuit and SQUID, it is essential to include this additional noise. In this paper, we extend our computer model[3] to calculate the current noise as a function of the appropriate SQUID parameters. In the following paper,[4] the results will be applied to practical circuits used for magnetometers and voltmeters.

II. COMPUTER MODEL FOR CURRENT NOISE IN THE dc SQUID

Our model for the isolated dc SQUID consists of two resistively shunted Josephson tunnel junctions[5] mounted on a superconducting ring (Fig. 1). This model was discussed in detail in an earlier paper[3]; here, we restrict the discussion to a symmetric SQUID with a loop inductance L, and with critical current I_0 and shunt resistance R per junction. As before, we neglect the junction capacitance. The SQUID is biased at a constant current I and threaded by an applied flux Φ_a. The voltage across the SQUID is V, and the current around the SQUID is J. The phase differences across the junctions are δ_1 and δ_2. The Johnson noise generated in the shunt resistors is modeled by the noise voltages V_{N1} and V_{N2}. We use the following dimensionless units: Voltage, v, in units of I_0R; currents, i and j, in units of I_0; flux, ϕ, in units of the flux quantum; and time, θ, in units of $\Phi_0/2\pi I_0R$. We define $\beta = 2LI_0/\Phi_0$. It is straightforward to show that the dimensionless SQUID equations are[3]

$$j = (\delta_1 - \delta_2 - 2\pi\phi_a)/\pi\beta, \tag{1}$$

$$v = \frac{1}{2}\frac{d\delta_1}{d\theta} + \frac{1}{2}\frac{d\delta_2}{d\theta}, \tag{2}$$

$$\frac{d\delta_1}{d\theta} = \frac{i}{2} - j - \sin\delta_1 + v_{N1}, \tag{3}$$

and

$$\frac{d\delta_2}{d\theta} = \frac{i}{2} + j - \sin\delta_2 + v_{N2}. \tag{4}$$

We solve these equations numerically on a computer by integrating the phases in Eqs. (2) and (3) stepwise in time. For brevity we confine our calculations to the value $\beta = 1$ that was found[3] to optimize the energy resolution, and to the case $\Gamma = 2\pi k_B T/I_0\Phi_0 = 0.05$. The Johnson noise voltages, v_{N1} and v_{N2}, are modeled by two uncorrelated trains of voltage pulses of constant duration.[3] The random pulse heights are Gaussian distributed about a zero mean with voltage power spectral densities satisfying $S_v^N = 4\Gamma$. From the computed values of $\delta_1(\theta)$ and $\delta_2(\theta)$ we calculate $v(\theta)$ and $j(\theta)$ from Eqs. (1) and (2). The voltage power spectral density, $S_v(f)$, the circulating current power spectral density, $S_j(f)$, and the real part of the correlation power spectral density, $S_{vj}(f)$, are then computed as functions of the bias current, i, and applied flux, ϕ_a. The spectral densities have noise-broadened resonances at the Josephson frequency $f_J = \langle v \rangle/2\pi$ and its harmonics ($\langle v \rangle$ is the time-averaged voltage). At frequencies below the resonance at f_J (say, $f < f_J/10$), the spectral densities are white. We define S_v^0, S_j^0, and S_{vj}^0 to be the low frequency values of S_v, S_j, and S_{vj}.

Figures 2 to 4 show S_v^0, S_j^0, and S_{vj}^0 as functions of i for several values of ϕ_a. (Figure 2 duplicates Fig. 14(a) of the previous paper.[3]) The parameters $\beta = 1$ and $\Gamma = 0.05$ correspond to $L \simeq 0.3\text{nH}$, $I_0 \approx 3\mu\text{A}$, and $T \approx 4\text{K}$. All of the SQUID responses are periodic in ϕ_a with unity period. In addition, for the symmetric SQUID the spectral densities are unchanged if $\phi_a \to -\phi_a$ and $j \to -j$. For each value of ϕ_a, the spectral densities peak approximately at the corresponding noise-free critical current, i_c, which decreases as ϕ_a increases. For $i >> i_c$, where the junctions contribute negligibly to the noise spectral densities, S_v^0 and S_j^0 tend to the Johnson noise values: $S_v^0 \rightarrow 2\Gamma = 2\, k_B TR/(I_0 R\Phi_0/2\pi)$, and $S_j^0 \rightarrow 2\Gamma = (2k_B T/R)/(I_0\Phi_0/2\pi R)$. Furthermore, since the voltage and circulating current noises produced by the Johnson noise in two parallel resistors are uncorrelated, $S_{vj} \rightarrow 0$ for $i \gg i_c$. Near i_c, the effect of the junctions and loop is to increase both S_v^0 and S_j^0 aobve the value for the resistors, 2Γ, and, in addition, to corrrelate the voltage and current noises. The correlation can be understood qualitatively from the following argument. The total flux through the SQUID is $\phi_T = \phi_a + \beta j/2$. Thus, Johnson noise superimposed on j produces a noise in the total flux that is similar to an externally applied flux noise. The SQUID transfer function, $\partial\bar{v}/\partial\phi_a$, relates changes in the time averaged voltage, $\bar{v}$, to changes in ϕ_a, and thus relates the effective flux noise due to noise in j to the total voltage noise. For $i \sim i_c$, $\partial\bar{v}/\partial\phi \neq 0$ and the voltage noise is correlated with the current noise. For $i \gg i_c$, $\partial\bar{v}/\partial\phi_a \rightarrow 0$ and no correlation is introduced between the voltage and current noises. Note that in the special cases $\phi_a = 0$, 0.5, $\partial\bar{v}/\partial\phi_a = 0$, and $S_{vj}^0 = 0$ for all values of bias current.

As ϕ_a is increased from 0 to 0.5, S_v^0 decreases in a way that is consistent with the corresponding decrease in the dynamic resistance of the SQUID.[3] On the other hand, the maximum value of S_j^0 increases as ϕ_a is increased from 0 to 0.5. In addition, for $\phi_a = 0.5$, S_j^0 rises rapidly as i is lowered below i_c. This behavior can be understood by examining the stable configurations of the noise-free SQUID below i_c. For the case $\phi_a = 0.5$, two such states exist, one with $j > 0$ and the other with $j < 0$. The Johnson noise generated in the shunts not only produces small excursions of the circulating current about the noise-free value, but also induces transitions between the two states at random times. This switching behavior is illustrated in Fig. 5 for $i = 0.9$, $\phi_a = 0.5$, $\beta = 1.0$, and $\Gamma = 0.05$. As the current switches (Fig. 5a), a corresponding voltage pulse appears across the SQUID (Fig. 5b). Both S_v^0 and S_j^0 are dominated by the switching noise at bias currents well below i_c. In most applications, the SQUID is biased at $i \sim i_c$ and $\phi_a \sim 0.25$, and the large current noise at $i \ll i_c$, $\phi_a \sim 0.5$ is not observed.

III. SUMMARY

We have computed low frequency voltage power spectral densities, S_v^0, current power spectral densities, S_j^0, and correlation power spectral densities, S_{vj}^0, vs. bias current for the dc SQUID operated at $\beta = 1$ and $\Gamma = 0.05$. At the operating bias current, we find in dimensioned units $S_V^0 \approx 16k_BTR$, $S_J^0 \approx 11k_BT/R$, and $S_{VJ}^0 \approx 12k_BT$. These values are used in the following paper[4] to calculate the noise temperature of SQUID voltmeters, and the energy resolution of SQUID magnetometers.

This work was supported by the Division of Materials Sciences, Office of Basic Energy Sciences, U. S. Department of Energy.

REFERENCES

1. R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercereau, Phys. Rev. Lett. 12, 159 (1964); Phys. Rev. Lett. 14, 887 (1965); Phys. Rev. 140, A1628 (1965).
2. J. Clarke, W. M. Goubau, and M. B. Ketchen, J. Low Temp. Phys. 25, 99 (1976).
3. Claudia D. Tesche and John Clarke, J. Low Temp. Phys. 29, 301 (1977).
4. J. Clarke, C. D. Tesche, and R. P. Giffard, J. Low Temp. Phys.(following paper)
5. W. C. Stewart, Appl. Phys. Lett. 12, 277 (1968);
   D. E. McCumber, J. Appl. Phys. 39, 3113 (1968).

FIGURE CAPTIONS

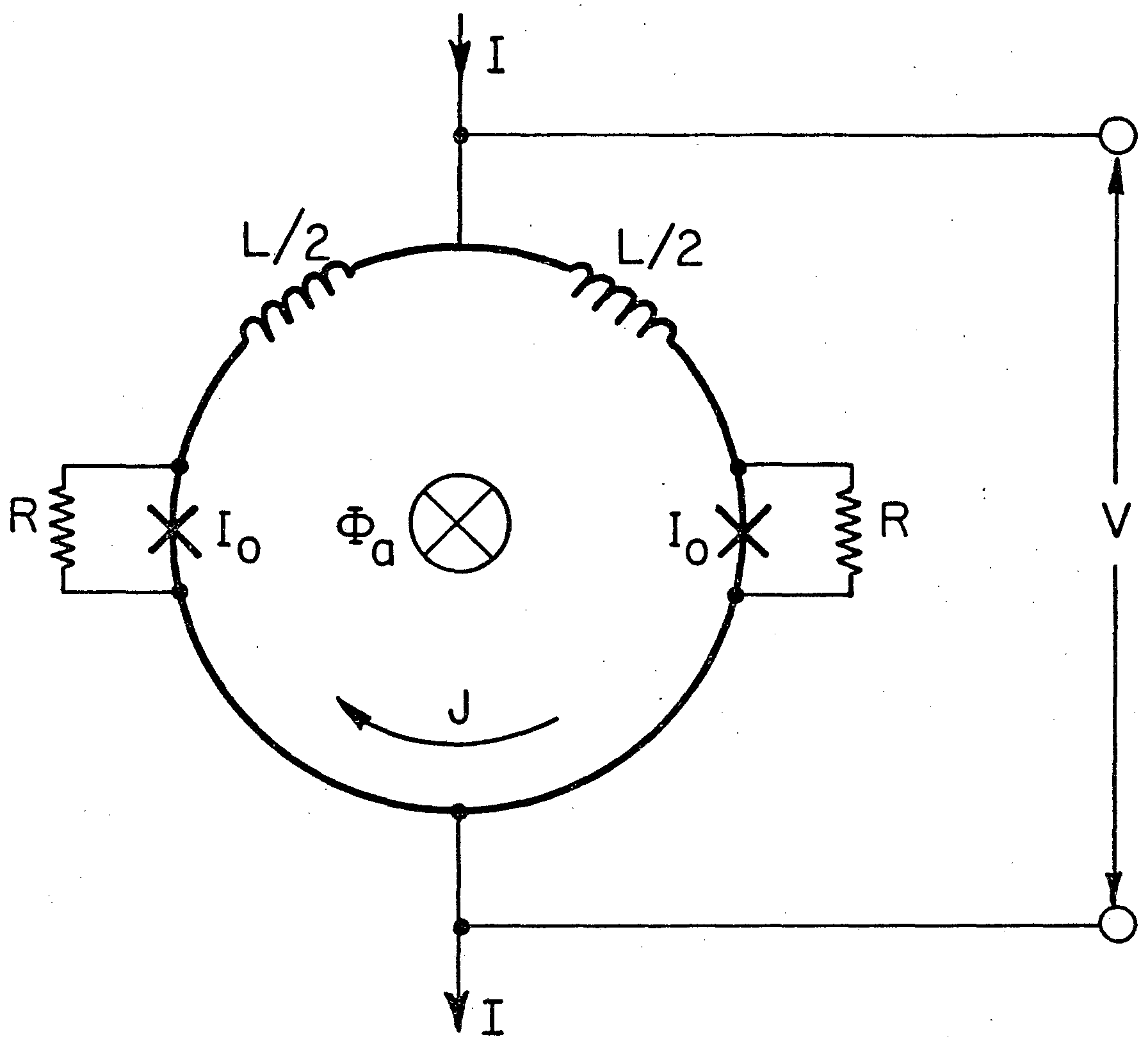
Fig. 1. Model for the isolated symmetric dc SQUID.

Fig. 2. Normalized low frequency voltage power spectral density, $S_v^0/2\Gamma$, vs. bias current, i, as a function of applied flux, ϕ_a.

Fig. 3. Normalized low frequency current power spectral density, $S_j^0/2\Gamma$, vs. bias current, i, as a function of applied flux, ϕ_a.

Fig. 4. Normalized low frequency correlation power spectral density $S_{vj}^0/2\Gamma$ vs. bias current, i, as a function of applied flux, ϕ_a.

Fig. 5. (a) Circulating current, j, and (b) voltage, v, as functions of time θ for bias current $i = 0.9$, and applied flux $\phi_a = 0.5$.



XBL 787-5386

Figure 1

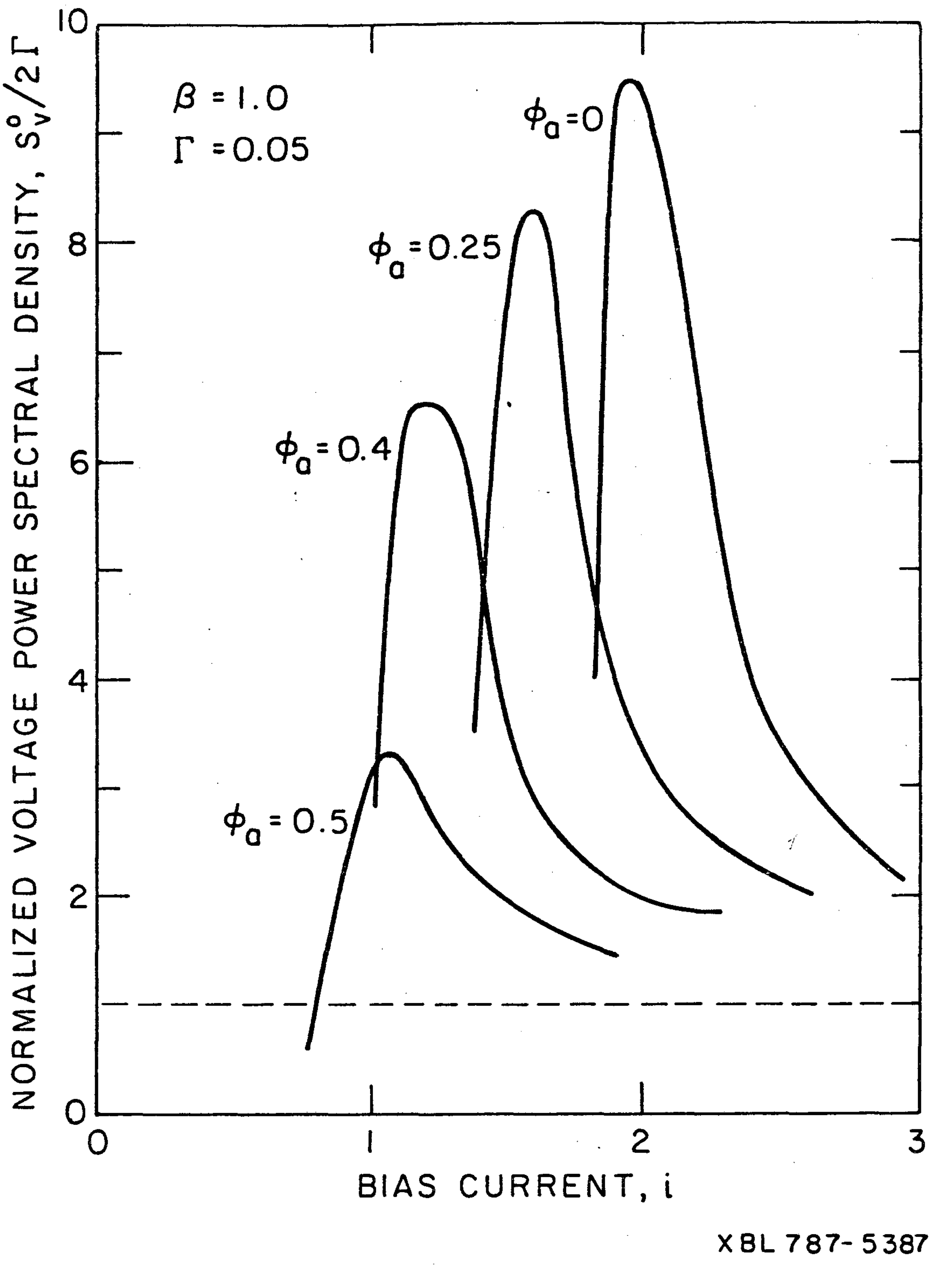

β = 1.0
Γ = 0.05
φa=0
φa=0.25
φa=0.4
φa=0.5
10
8
6
4
2
0
0
1
2
3
NORMALIZED VOLTAGE POWER SPECTRAL DENSITY, S°v/2Γ
BIAS CURRENT, i
XBL 787-5387

Figure 2

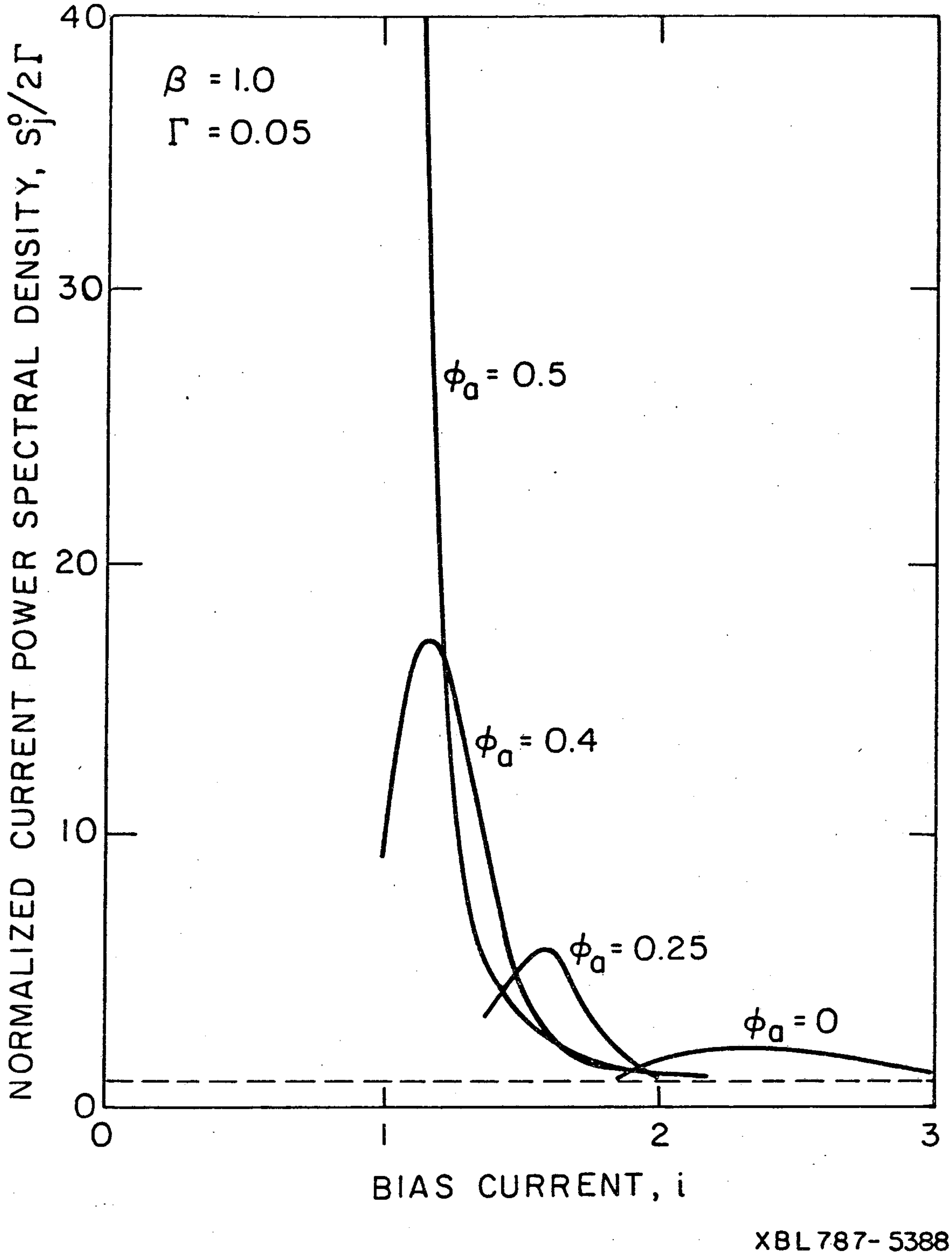
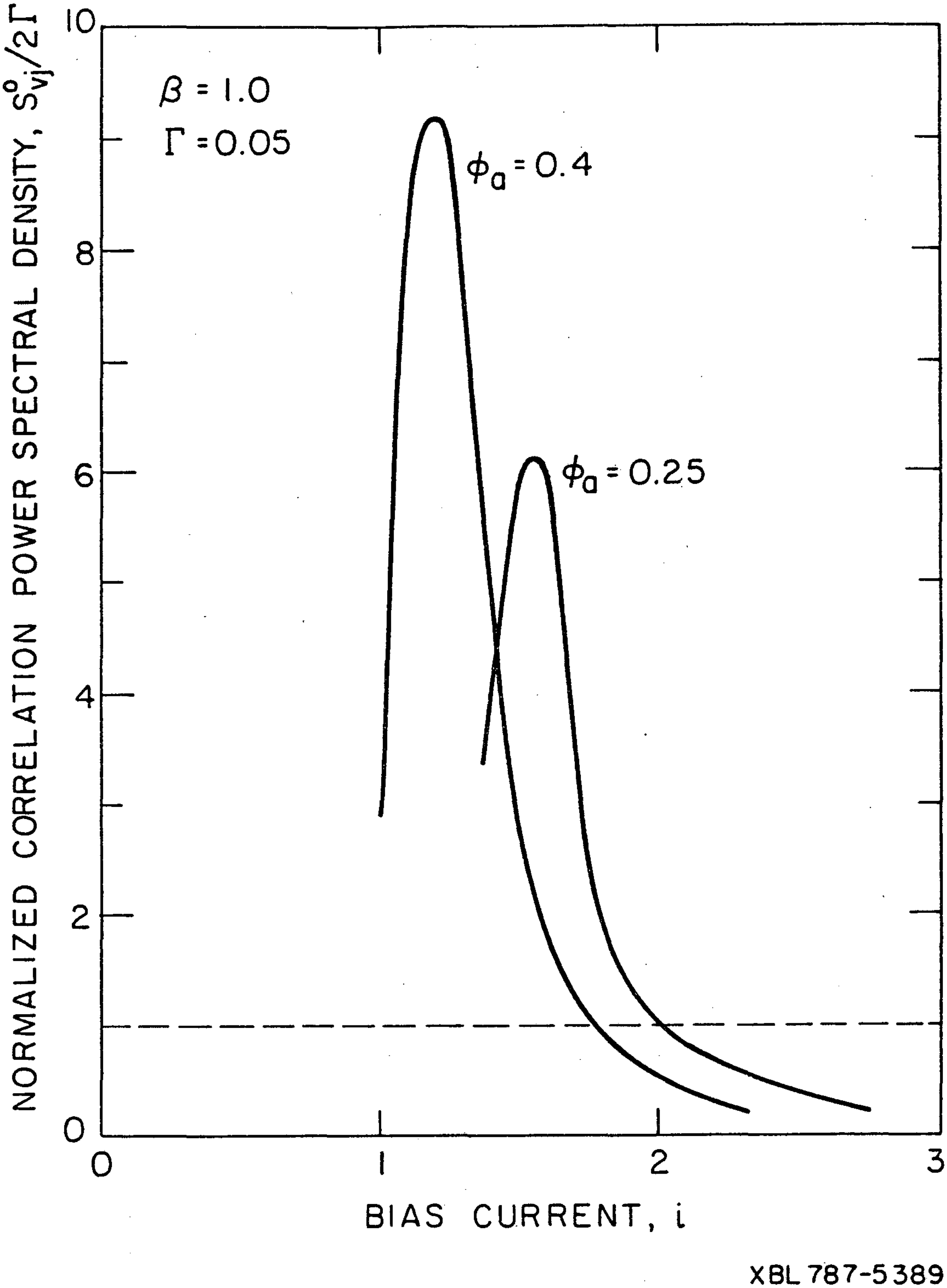

40
30
20
10
0
β = 1.0
Γ = 0.05
NORMALIZED CURRENT POWER SPECTRAL DENSITY, S°j/2Γ
φa = 0.5
φa = 0.4
φa = 0.25
φa = 0
0
1
2
3
BIAS CURRENT, i
XBL 787-5388

Figure 3



NORMALIZED CORRELATION POWER SPECTRAL DENSITY, $S^{0}_{vj}/2\Gamma$
β = 1.0
Γ = 0.05
ϕ_a = 0.4
ϕ_a = 0.25
0
2
4
6
8
10
0
1
2
3
BIAS CURRENT, i
XBL 787-5389

Figure 4

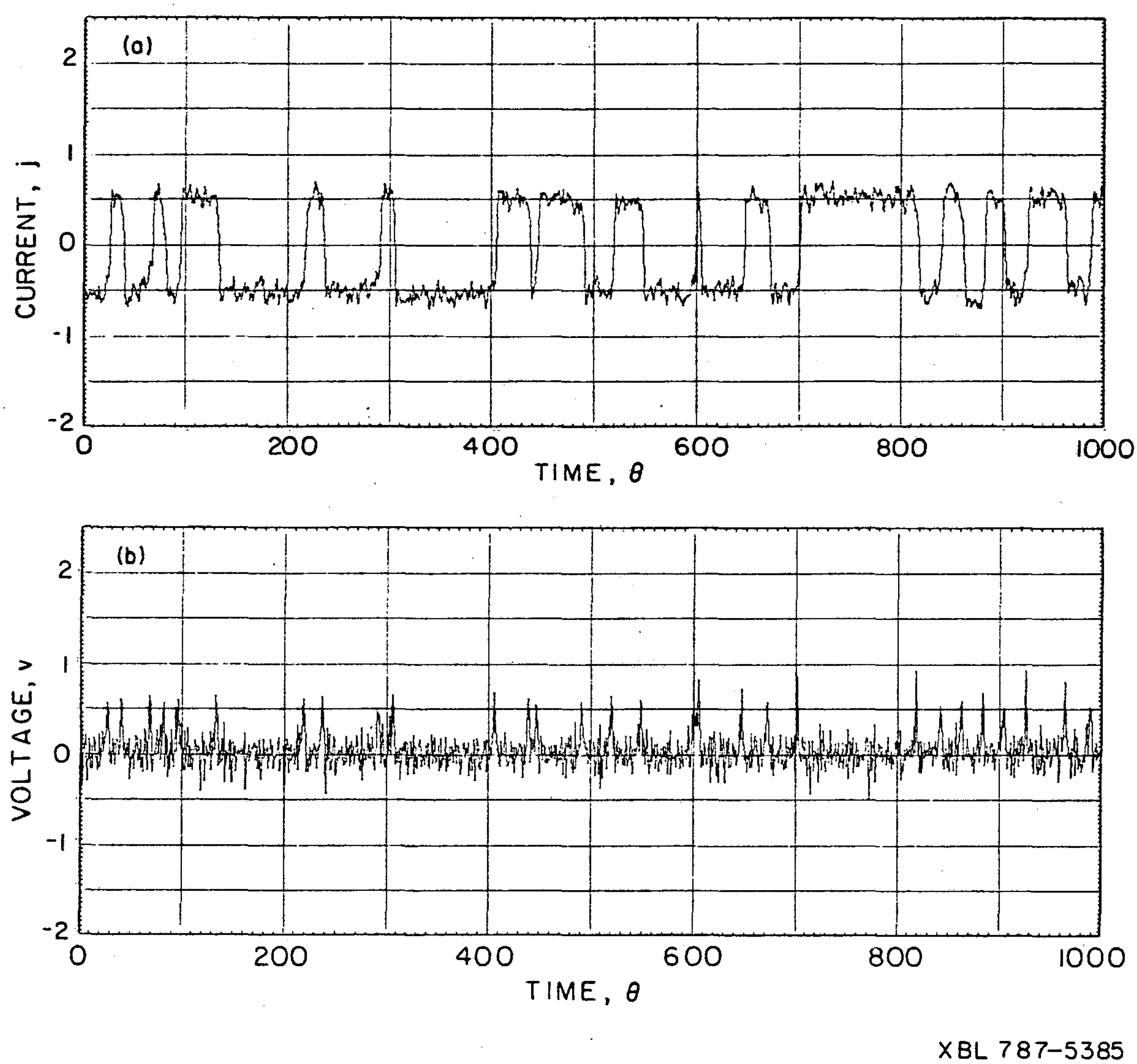


Figure 5

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720