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MARKOVIAN AND NONMARKOVIAN EFFECTS IN HEAVY-ION REACTIONS AT 10-200 MEV/amu*

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ABSTRACT

We have derived simplified formulas for the transition probability from an entrance channel $|i\rangle$ to an exit channel $|f\rangle$ that corresponds to a heavy ion reaction. It is shown that a rearrangement of the perturbation series leads to an expansion of the cross section in terms of the ratio y between the duration of a nucleon-nucleon collision and the elapsed time between two successive interactions. The cross section splits into two contributions, the markovian part independent of y and the nonmarkovian one containing explicit y-dependence. Such a decomposition allows us to demonstrate that a transition from the markovian to the nonmarkovian behavior is to be expected around 20MeV/amu in the initial channel. This transition manifests itself in a change in the slope of the width of the mass distribution as a function of the incident energy. This fact seems to be in agreement with existing data.

Key-Word Abstract Heavy-ion reactions, Gaussian random matrix ensembles, perturbative expansion, ratio between characteristic times. Markovian and nonmarkovian contributions to the cross section.

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1. INTRODUCTION

Heavy-ion reactions at intermediate energies (10-200 MeV per nucleon of the projectile) have recently received considerable attention.[1-3] The mass distributions of the fragments observed in the exit channel are essentially Gaussian shaped and their widths may contain valuable information on the reaction mechanism.[2] In addition, the measurement of charge distributions is being considered.[4]

The present experimental situation[1-5] indicates that in the domain of incident energies below 20 MeV/amu, these widths are increasing functions of the bombarding energy. Recent measurements[1,4] from ^{16}O and ^{40}Ar induced reactions on several targets show that the widths of the mass (and probably those of the charge) distributions become essentially constant when the energy increases beyond 20 MeV/amu. Although the available experimental information is restricted to few data points[1-4] between 20 and 200 MeV/amu, it is clear that these widths deviate from the predictions of the simple model of the evaporating nucleus. This fact configurates a challenging situation for whose interpretation we could resort to a model of partial equilibration at high energies[2] or to the abrasion-ablation model.[1,6]

The aim of this work is to offer a novel and rather different possibility. First, we observe that the notion of an evaporating nucleus assumes the existence of a microcanonical equilibrium. Secondly, it is well known that when the duration t_{coll} of a two-nucleon collision is small compared to the time t_{MFP} between

subsequent interactions (hereafter, MFP stands for mean free path) a series of such independent, uncorrelated collisions leads to the microcanonical equilibrium. In fact, only when the ratio parameter

$$y = \frac{t_{coll}}{t_{MFP}} \tag{1}$$

is much smaller than unity we can speak of an individual two-body interaction. In this case, the relaxation towards equilibrium is described by a Pauli-like master equation[7] and the chain of well-defined collisions is usually referred to as a markovian process.

In Fig. 1 we display results from an estimate of the parameter y as a function of the energy per incident nucleon. The details of the calculation are given in the Appendix. We realize that y is small at low energies ($\lesssim$10 MeV/amu) but it becomes comparable to unity at higher energies ($\gtrsim$ 30 MeV/amu). In this latter regime, the frequency of collisions is so high that the concept of an individual two-body collision becomes meaningless. This effect in the high energy domain has a strong influence on the energy dependence of the phase space populated in the reaction and introduces aspects that we could call "the nonmarkovian features of the process."

We are going to demonstrate that one of these features is, precisely, the fact that the predicted widths exhibit a transition, namely, they no longer increase with increasing energy. Furthermore, they tend to decrease at higher energies.

In Section 2 we provide the formal definition of the markovian and nonmarkovian parts of a physical quantity, as we shall use them in this paper. Section 3 deals with the derivation of analytical expressions for the markovian and nonmarkovian contributions to the transition probability, by means of perturbation theory in the framework of the Gaussian random matrix model. The main results of this section will be discussed and illustrated in Section 4. Conclusions and future perspectives are presented in Section 5.

2. FORMAL DEFINITION OF MARKOVIAN AND NONMARKOVIAN CONTRIBUTIONS

In the following we will refer to experiments in which the initial channel $|i\rangle$ contains two heavy nuclei and the final channel $|f\rangle$ contains several fragments. Since in the determination of the widths the experimentalist performs a highly inclusive (i.e., one body) measurement, it is meaningful to introduce the notation $|f\rangle \equiv |A_f, Z_f, E_f; \alpha_f\rangle$ where A_f, Z_f and E_f are the mass, charge and kinetic energy of a given fragment and α_f summarizes the quantum numbers of all other fragments.

The expectation value of an observable A, namely $\langle A\rangle = \mathrm{Tr}(\rho A)$ where ρ is the density operator of the system under consideration, can be formally expanded as a series of powers of the parameter y and thus we can always write

$$\langle A\rangle = \langle A\rangle_M + \langle A\rangle_{NM} \quad , \tag{2}$$

where the markovian part $\langle A\rangle_M$ is independent of y and the non-markovian $\langle A\rangle_{NM}$ contains all powers of y.

The width of the distribution of any observable x present in the exit channel is given by

$$\Gamma_x^2 = \langle x^2\rangle - \langle x\rangle^2 =$$

$$= (\int dx\, x^2 f(x))/\sigma_T - (\int dx\, xf(x))^2/\sigma_T^2 \quad , \tag{3}$$

where $f(x) = d\sigma_T/dx$ is the inclusive cross section for the variable x and $\sigma_T = \int dx\, f(x)$ is the total cross section. According to (2) we can write

$$f(x) = f_M(x) + f_{NM}(x) \tag{4a}$$

$$\sigma_T = \sigma_M + \sigma_{NM} \quad . \tag{4b}$$

Thus Eq. (3) becomes

$$\Gamma_x^2 = \Gamma_M^2(1 - s) + \Gamma_{NM}^2 s + [\langle x\rangle_M - \langle x\rangle_{NM}]\, s(1 - s) \quad , \tag{5}$$

where $s = \sigma_{NM}/\sigma_T$ varies between zero and unity.

The markovian and nonmarkovian contributions to the width are consistently defined according to the corresponding terms in the cross section, for example,

$$\Gamma_M^2 = (\int dx\, x^2 f_M(x))/\sigma_M - (\int dx\, x f_M(x))^2/\sigma_M^2 \quad . \tag{6}$$

It is useful to notice that the third term on the right hand side of Eq. (5) remains small, in general. This is obvious for the extreme values of s, $s \ll 1$ or $s \sim 1$, while for intermediate values of s, $\sigma_M \approx \sigma_{NM}$ and thus we can expect $\langle x\rangle_M - \langle x\rangle_{NM} \sim 0$.

It is clear from Eq. (5) that the width Γ_x^2 will differ significantly from the predictions of the evaporating nucleus model, $\Gamma_M^2 \propto \sqrt{\varepsilon}$ (where ε represents the excitation energy per particle) when s is not negligible, i.e., when the nonmarkovian term σ_{NM} dominates the total cross section. As this happens, the total width becomes closer to the value Γ_{NM}^2. We shall see that this quantity is expected to decrease with increasing excitation energy, consequently inducing a decrease in the total width Γ_x^2.

3. ANALYTICAL CALCULATION BY MEANS OF PERTURBATION THEORY

3.1. General Formulation

We recall that the cross section is proportional to $|\langle f|T|i\rangle|^2$, where the reaction matrix T satisfies the Lippmann-Schwinger equation. The corresponding series expansion reads,

$$T = V \sum_{n=0}^{\infty} (G_o V)^n \quad , \tag{6}$$

where H_o is the unperturbed Hamiltonian of the system whose resolvent is $G_o = (E - H_o)^{-1}$ and V is the two-body potential. We assum that the reaction mainly consists in the formation of a dinuclear system and its decay into several channels, whose details are determined by the dynamics of the intermediate complex. In this work, we are considering the properties of this intermediate system; accordingly, we confine ourselves to the situation in which H_o exhibits only a discrete spectrum with eigenstates $|\alpha\rangle$, such that $H_o|\alpha\rangle = E_\alpha|\alpha\rangle$.

If we insert these eigenstates as an intermediate decoupling basis in Eq. (6), we obtain, for the transition probability P_{fi},

$$P_{fi} = |\langle f|T|i\rangle|^2 =$$

$$= \sum_{n,n'=1}^{\infty} \sum_{\substack{\alpha_1,\ldots\alpha_n \\ \alpha'_1,\ldots\alpha'_n}} \left[\langle f|V|\alpha_1\rangle \frac{1}{E - E_{\alpha_1}} \langle\alpha_1|V|\alpha_2\rangle \ldots \frac{1}{E - E_{\alpha_n}} \langle\alpha_n|V|i\rangle\right]$$

$$\left[\langle f|V|\alpha'_1\rangle \frac{1}{E - E_{\alpha'_1}} \langle\alpha'_1|V|\alpha'_2\rangle \ldots \frac{1}{E - E_{\alpha'_n}} \langle\alpha'_n|V|i\rangle\right]^* \quad . \tag{7}$$

The measured cross sections, from which our information is derived, correspond to inclusive one-body spectra. Due to limited experimental resolution, all measurements represent averages over an energy interal. Therefore, the actual quantities of interest are the average matrix elements, whose form is

$$\overline{|\langle i|T|f\rangle|^2} =$$

$$\frac{1}{(\Delta E)^2}\int dE_f \int dE_i \int d\alpha_f \int d\alpha_i \, |\langle A_i Z_i E_i;\alpha_i|T|A_f Z_f E_f;\alpha_f\rangle|^2 \quad , \tag{8}$$

where ΔE is the energy resolution. This limitation, combined with inclusivity, the large number of degrees of freedom and the short-range character of the force, possibly justify the use of "random-phase" type assumptions in order to provide an averaged description of this complex many body physics. We actually consider an average procedure over the ensemble of two-body matrix elements $\langle\alpha_1|V|\alpha_2\rangle = V(\alpha_1,\alpha_2)$ under the assumption that these matrix elements are random numbers and posess a Gaussian distribution,[7]

$$\overline{\langle\alpha_1|V|\alpha_2\rangle} = 0 \quad , \quad \alpha_1 \neq \alpha_2 \tag{9a}$$

$$\overline{\langle\alpha_1|V|\alpha_2\rangle\langle\alpha_3|V|\alpha_4\rangle} =$$

$$\overline{\langle\alpha_1|V|\alpha_2\rangle^2}\,(\delta_{\alpha_1\alpha_3}\delta_{\alpha_2\alpha_4} + \delta_{\alpha_1\alpha_4}\delta_{\alpha_2\alpha_3}) \quad . \tag{9b}$$

Here the bars denote the ensemble averages.

One of the practical benefits of the assumed Gaussian distribution is the fact that the average of an even number of matrix elements can be calculated as a sum of all possible contributions of products of averages over pairs.[7] These distributions of matrix elements have been widely employed and tested in nuclear spectroscopy,[8] in nucleon induced multistep reactions[7] and deep-inelastic heavy-ion collisions.[9,10] Its validity seems to rely on the short-range of the force as compared with the linear dimensions of the system. Therefore, our results should not be generalized to systems that interact through long-range forces (i.e., a plasma).

We now proceed to outline the evaluation of the average of expression (7). The general n-th order term in this expansion can be split into markovian and nonmarkovian parts, according to the way in which the matrix elements are contracted. (Hereafter, the words "contraction" and "average" of pairs will be used as synonyms). First, we have to consider averages that involve two interactions inside the came bracket (ef. Eq. (7)). Among the several possibilities, we can recognize the following.

3.2. Uncorrelated Terms (UC)

We define as uncorrelated terms the averages of the form,

$$T_{UC}^{(n)} = \langle f | \overline{VG_oVG_o\overline{VG_oV}} \; . \; . \; . \; \overline{VG_oV} | i \rangle \quad , \tag{10}$$

where n is the number of interactions inside the bracket. These terms are the only contributions considered in the markovian limit at very low excitation energies.[7]

3.3. Negligible Terms

In this case, the terms contain at least one pair of crossing contraction lines,

$$T^{(n)}_{neg} = |f\rangle \; . \; . \; . \; \overline{VG_o\overline{VG_o\overline{VG_o}V}} \; . \; . \; . \; |i\rangle \tag{11}$$

and can be neglected provided that the Poincaré time[7] for the system is large as compared with t_{MFP} (see Section 1). This is the case for systems with many degrees of freedom and (or) continuous energy spectra.

3.4. Totally Correlated Terms (TC)

The general term of this type is built as a product of averages in the following way,

$$T^{(n)}_{TC} = \langle f|\overline{VG_o\overline{VG_o\overline{VG_o}V}} \; . \; . \; . \; \overline{VG_o\overline{VG_o\overline{VG_o}V}}|i\rangle \quad . \tag{12}$$

We want to establish that $T^{(n)}_{TC}$ is proportional to $y^{n/4}$. We will show this fact for n = 4; the general case can be treated by repeated application of the same procedure. We have

$$T_{TC}^{(4)} = \langle f|\overline{VG_o\overline{VG_o\overline{VG_o V}}}|i\rangle$$

$$\approx \delta_{if}\pi\rho(E_i)\int dE_{\alpha_1}\rho(E_{\alpha_1})\frac{\overline{(\langle i|V|\alpha_1\rangle^2)^2}}{(E - E_{\alpha_1})^2} \tag{13}$$

In Eq. (13), we have converted the discrete sum over eigenstates into an integral, introducing the level density $\rho(E_{\alpha_1})$ of the states $|\alpha_1\rangle$. Let us now define ΔE as the energy interval over which the quantity $\rho(E_{\alpha_1})\ \overline{(\langle i|V|\alpha_1\rangle^2)}^2$ changes significantly as a function of E_{α_1}. The integral on the r.h.s. of Eq. (13) can be approximated as

$$\int dE_{\alpha_1}\rho(E_{\alpha_1})\frac{\overline{(\langle i|V|\alpha_1\rangle^2)^2}}{(E - E_{\alpha_1})^2} \approx \frac{1}{\Delta E}\left[\rho(E_{\alpha_1})\overline{(\langle i|V|\alpha_1\rangle^2)^2}\right]_{E_{\alpha_1}=E} . \tag{14}$$

We notice that the time $\hbar/\Delta E$ is roughly the duration of a two-body collision in the system. Comparing the (correlated) term in Eq. (13)

with the corresponding uncorrelated expression $\langle f|\overline{VG_oVG_o\overline{VG_oV}}|i\rangle$ we can see that their ratio is essentially $\Gamma_i/\Delta E$, where

$$\Gamma_i = 2\pi \sum_{\alpha_1} \overline{\langle i|V|\alpha_1\rangle^2}\, \rho(E_{\alpha_1}) \tag{15}$$

is the decay width for the state $|i\rangle$ (cf. Ref. 7). Since the time scale $t_{MFP} = \hbar/\Gamma_i$ corresponds to the time between two subsequent collisions, the ratio of the correlated to the uncorrelated term is $y = t_{coll}/t_{MFP}$. This completes our argument.

3.5. Partially Correlated Terms (PC).

These terms contain mixed products of correlated and uncorrelated averages,

$$T_{PC}^{(n)} = \langle f|\overline{VG_oVG_o\overline{VG_oV}} \; . \; . \; . \; \overline{VG_o\overline{VG_oVG_oV}}|i\rangle \quad . \tag{16}$$

The preceding discussion concerning the TC case allows us to assert that this term is proportional to $y^{n_c/4}$, where n_c is the number of interactions that participate in correlated averages ($n_c < n$). The contribution from TC and PC terms is responsible for the nonmarkovian mechanisms in the behaviour of the nuclear system.

Our next step is to cast the discussed correlated (TC,PC) and uncorrelated (UC) contributions into more transparent closed form equations. For this purpose we would like to introduce a renormalized Green function,

$$\tilde{G} = G_M + G_{NM} \tag{17}$$

whose markovian part is given by

$$G_M = G_o + G_o \overline{VG_o VG_o} + G_o \overline{VG_o VG_o} \overline{VG_o VG_o} + \ldots \tag{18}$$

i.e., it contains all uncorrelated graphs. Following Ref. 7 we have $\langle\alpha|G_M|\alpha'\rangle = \delta_{\alpha\alpha'}(E - E_\alpha + i\Gamma_\alpha/2)^{-1}$. The additional nonmarkovian contribution to the Green function is the sum of all TC and PC terms and in a symbolic manner it can be written as,

$$G_{NM} = G_{PC} + G_{TC} \quad . \tag{19}$$

The totally correlated part of the nonmarkovian Green function admits a compact representation,

$$\langle\alpha|G_{TC}|\alpha'\rangle = \langle\alpha|G_o T_{TC} G_o|\alpha'\rangle =$$

$$= \frac{\delta_{\alpha\alpha'}\pi\rho(E_\alpha)}{(E - E_\alpha)^2} \frac{\partial}{\partial E_{\alpha_1}} \left[\overline{V^2(\alpha,\alpha_1)}\ \rho(E_{\alpha_1})\ \chi(\alpha_1,\alpha)\right]_{E_{\alpha_1}=E} , \qquad (20)$$

where the off-shell kernel $\chi(\alpha_1,\alpha)$ satisfies the equation

$$\chi(\alpha_1,\alpha) = \overline{V^2(\alpha_1,\alpha)} + \frac{\partial}{\partial E_{\alpha_2}} \left[\rho(E_{\alpha_2})\ \overline{V^2(\alpha,\alpha_2)}\ \chi(\alpha_1,\alpha_2)\right]_{E_{\alpha_2}=E_{\alpha_1}} . \qquad (21)$$

Eq. (21) can be easily obtained as one observes that its iterative solution leads exactly to a series expansion containing the terms that have been classified as TC.

Having defined the renormalized Green function by Eq. (15), we can see that the average of expression (7) for P_{fi} can be written as

$$\pi_{fi} \equiv \overline{|\langle f|T|i\rangle|^2} =$$

$$= \overline{\langle f|V\tilde{G}V \; . \; . \; . \; \tilde{G}V|i\rangle\langle i|V\tilde{G}V \; . \; . \; . \; \tilde{G}V|f\rangle} \quad , \tag{22}$$

where the average bar denotes countractions of interacting pairs with one component in the matrix element $\langle f|T|i\rangle$ and the other one in the complex conjugate $\langle i|T|f\rangle^*$. All other averages have been considered in the renormalized Green function.[7,9]

The main class of contractions that we will take into account in Eq. (22) is represented by the pattern

$$\langle f|V\tilde{G}V \; . \; . \; . \; V\tilde{G}V|i\rangle\langle i|V\tilde{G}V \; . \; . \; . \; V\tilde{G}V|f\rangle \quad , \tag{23}$$

where there is no line-crossing. We are not going to consider terms that contain crossing lines, as in $\langle f|V\tilde{G}V|i\rangle\langle i|V\tilde{G}V|f\rangle$. We have investigated the structure of these terms but for the sake of a more transparent presentation we will skip their discussion. The interested reader can verify that their inclusion does not affect the qualitative conclusions of this paper.

The sum of the averages of the type (23) can be case into a simple expression[9]

$$\pi_{fi} = \delta_{fi} + \sum_{\alpha} \langle f|\overline{\tilde{V}G\tilde{V}|\alpha\rangle\, \pi_{\alpha i}\langle\alpha|\tilde{V}G\tilde{V}}|f\rangle \quad . \tag{24}$$

Once again, the iterative solution of Eq. (24) produces the sum of the terms given by Eq. (23).

In the markovian limit ($y \ll 1$) the renormalized Green function is simply G_M in Eq. (18) and Eq. (24) reduces to the time-independent version of the master--(Pauli) equation,

$$\pi_{Mfi} = \delta_{fi} + \sum_{j}(\pi_{Mji}w_{jf} - \pi_{Mfi}w_{fi}) \tag{25}$$

where w_{jf} is the transition rate between states having the same energy $E_j = E_f = E$,

$$w_{jf} = \frac{2\pi}{\hbar}\,\overline{V^2(j,f)}\;\rho(E) \quad . \tag{26}$$

The transition rates w_{jf} are linear increasing functions of energy,[7] since the level density $\rho(E)$ and the average matrix elements $\overline{V^2}$ are roughly proportional to E^2 (two-body selection rule!) and $1/E$, respectively. Thus, as the excitation energy increases, the probability w_{jf} that a complex configuration j is populated as an intermediate state becomes larger. As we establish a chain of such transitions, according to Eq. (25), the volume of the phase space that

contains all available exit channels grows as well with increasing energy. This enlarging of the phase space is the obvious reason why the evaporation (compound nucleus) model predicts widths that are increasing functions of excitation energy.

However, this picture breaks down in the presence of a strong non-markovian component. In order to demonstrate this fact, let us consider explicitly the correction to the transition probability arising from the TC part of the Green function, as given by Eq. (20). Insertion of this contribution into Eq. (24) allows us to write down a closed-form equation for π_{fi} that we can solve iteratively. The first non-trivial term reads,

$$\pi^{(1)}_{TCfi} = \delta_{fi} + \sum_{\alpha\alpha'} \overline{\langle f|V|\alpha\rangle^2}\ \langle\alpha|G_{TC}|\alpha'\rangle^2 \overline{\langle\alpha'|V|i\rangle^2} \quad . \tag{27}$$

The TC contribution to $\tilde{G}$, namely G_{TC}, is given by Eq. (20) and can be written as $\langle\alpha|G_{TC}|\alpha'\rangle = \delta_{\alpha\alpha'}A(E)/(E - E_\alpha)^2$, with $A(E)$ a linearly increasing function of E. Then, we get

$$\pi^{(1)}_{TCfi} = \delta_{fi} + \int dE_\alpha \rho(E_\alpha)\ \frac{A^2(E)}{(E - E_\alpha)^4}\ \overline{V^2(f,\alpha)}\ \overline{V^2(\alpha,i)} \quad . \tag{28}$$

We observe that the average off-shell matrix elements $\langle\alpha|V|\alpha'\rangle^2$ $(E_\alpha \neq E_{\alpha'})$ should correspond to on-shell quantities $(E_\alpha = E_{\alpha'})$ that decrease with energy as E_α^{-1}. Using this property we conclude from

Eq. (27) that the TC transition probability in first (non-trivial) iterative order decreases with excitation energy, at least as E^{-1}. This feature is common to all higher-order iterations as of Eq. (24) in the TC limit. We should realize that the presence of partially correlated graphs in the Green function (Eq. (16)) will smooth out this dramatic decrease. Nevertheless, we can state that the volume of the phase space populated by the nonmarkovian transition probability is smaller, at higher energies, than the volume associated with the markovian process. As a consequence, we can tentatively assert that an observable realization of this fact would be a saturation or a decrease in the widths of the mass and charge distributions, as they become dominated by the nonmarkovian contributions (see Eq. (5)).

3.6 Interpretation

We can learn more about the meaning of UC, PC, and TC contributions to the transition probability with the help of a somewhat crude graphical representation in the time-dependent scheme. In Fig. 2 we illustrate a situation that corresponds to $y \ll 1$, i.e., where two subsequent collisions are well separated. Two independent particles in states (i,j) collide at time t_1. At a later instant $t_1 + t_{coll}$ they are in the single-particle orbitals (i',j') and they are able to start a new collision at time $t' + t_{coll} + t_{MFP}$. This clean time-scale separation ensures us that the locus of (i",j"), namely the phase space that can be reached through two consecutive collisions, is the product of the phase spaces populated by each separate collision. This is a well-established fact associated with the markovian

character of the microscopic chain of events (see, for example, the abundant work by Prigogine and co-workers).[11] In this way, after a few cascades the populated phase space has increased enormously and resembles the microcanonical equilibrium state. It is also worthwhile to recall that the information on the two-body interaction transmitted in a Markov process is the same as in a two-body scattering experiment, i.e., on-shell properties (see Eq. (26)). This is not valid for a nonmarkovian chain (see Eq. (20)).

In the opposite extreme of fully overlapping collisions ($y \gtrsim 1$), the process shown in Fig. 2 becomes very rare while the most likely situation is illustrated in Fig. 3. In this case, since the duration of a two-body collision is much larger than the time between two consequtive interactions, we represent two pairs of particles in states (i_1, j_1), (i_2, j_2) as undergoing collision simultaneously. In this strongly correlated nonmarkovian limit the phase space for the combined pairs (i_1', j_1'), (i_2', j_2') is no longer the product of the individual phase spaces. Destructive interference among the single particle wave functions involved in the event that takes place inside the box in Fig. 3 reduces the final phase volume. This observation contributes to understand the reason why this volume is a decreasing function of energy in the nonmarkovian limit.

4. QUALITATIVE PREDICTIONS OF THE MODEL

As we already stated in the Introduction, the fact that the width of the mass distribution increase with energy is a characteristic of a Markovian system. This increase is experimentally well-established at lower energies, below 20 MeV/amu.[1,5] Since the parameter y is an increasing function of the incident energy (Fig. 1) the relative importance of the markovian component diminishes. We could say, as the energy increases the system progressively behaves in a non-markovian manner. In the limit $y \gtrsim 1$, the dominating graphs contribute to a decreasing width.

Our next step is to express the parameter $s = \sigma_{NM}/\sigma_T$, which has been introduced in Section 2, as a function of y. Following the procedure outlined in Section 3 we may write

$$\sigma_T = \sigma_M + \sigma_1 y + \sigma_2 y^2 + \ldots \quad , \tag{29}$$

where σ_M is the markovian cross section and $\sigma_1, \sigma_2, \ldots$ are coefficients that contain the sum of all totally and partially correlated graphs. The nonmarkovian cross section is thus given by $\sigma_{NM} = \sigma_1 y + \sigma_2 y^2 + \ldots$ and in the linear approximation in which $\sigma_{NM} \approx \sigma_1 y$ we obtain $s = \alpha y$, with $\alpha = \sigma_1/\sigma_M$. A closer examination of the structure of σ_1 (see Section 3a) convinces us that α is approximately unity. Thus, we can write, in this linear limit,

$$\Gamma_x^2 \approx (1 - \alpha y)\ \Gamma_M^2 + \alpha y \Gamma_{NM}^2 \quad . \tag{30}$$

It is thus clear that for very small y the markovian width dominates, while an increasing y induces a relative cancellation of the term containing Γ_M^2 and a larger presence of the nonmarkovian contribution. Of course, for increasing y the linear approximation is no longer valid. It must be borne in mind, however, that a quantitative calculation of the nonmarkovian coefficients $\sigma_1, \sigma_2, \ldots$ of the cross section and the partial width Γ_{NM}^2 would rely on a particular model for the off-shell matrix elements of the nuclear potential. Such a calculation would be possible, in principle. However, we have preferred to exhaust the discussion on an analytical basis, and to state the results that should be independent of particular model assumptions. Our main requirement has been, that these matrix elements must be compatible with a short-range, regular interaction.

We conclude that the transition in the widths of the mass distributions is an effect that can be globablly understood. Its roots lie on the short-range nature of the nuclear force, as compared with the linear dimensions of the nucleus itself, in combination with the high level densities of the nuclear fluid. It seems that due to this universality, this transition would be an effect of the basic nuclear dynamics.

5. CONCLUSIONS

We have shown that when the assumption of a nuclear system interacting under the conditions that allow a markovian description breaks down, there is a novel behavior. We have illustrated this original fact by demonstrating that the widths of the mass distributions experience a change in slope as a function of increasing energy of the projectile. This peculiarity provides an example of nonmarkovian effects that can be detected in macroscopic observables. As a matter of fact, our model predicts that there is a change in the energy dependence of the width of any observable that might be present in the exit channels. However, in the case of the angular momentum, for example, its measurement is difficult in reactions where the exit channel contains more than two fragments. The most satisfactory candidates are thus the widths of the mass, and probably those of the charge, distribution.

A difficult aspect of our presentation resides in the fact that the nonmarkovian effects are quantitatively connected with the off-shell matrix elements of the nuclear interaction. We believe that our present analysis encourages further work on the subject, aiming at identifying observable quantities whose properties can be traced to the presence of off-shell mechanisms.

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APPENDIX

1) The time between two subsequent collisions is given by

$$t_{MFP} = \frac{v}{\lambda} \quad , \tag{A1}$$

where v is the asymptotic relative velocity of the two colliding particles and λ the mean-free-path. We assume that the Fermi motion can be disregarded for an estimate of the mean relative velocity, since it affects both collision partners.

The first collision involves two particles whose relative energy is the laboratory energy of the projectile's particles above the Coulomb barrier. This process causes the available energy to be shared by the outgoing collision partners; in consequence, the relative energy available for the next two or three interactions, before the separation of the complex occurs, is lower than the original incident value. Thus, it is reasonable to assume that in the average, the collision takes place at a relative energy equal to half the incident energy per particle. This choice allows us to calculate the asymptotic relative velocity v and to estimate the MFP from the results displayed in Ref. 12.

2) The duration of a two-body collision can be estimated through the semiclassical formula,

$$t_{coll} = \frac{1}{\sqrt{v^2 + V_o/2m}} (2d + k^{-1}) \tag{2A}$$

where v is the asymptotic velocity in (A1), V_o and d the average values of the strength and range of the two-body force and k is the wave number of the incident nucleon.

The calculations displayed in Fig. 1 correspond to two sets of parameters V_o and d (Ref. 13) (square-well)

1) V_o = 14 MeV , d = 2.7 fm , (no hard-core)

and

2) V = 28 MeV , d = 1.9 fm , hard-core radius 0.4 fm .

These values are adjusted to produce a bound deuteron state and to fit the low energy triplet scattering data.[13]

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FIGURE CAPTIONS

Fig. 1. The parameter $y = t_{coll}/t_{MFP}$ as a function of the energy per incident nucleon. Curve 1 corresponds to a square well potential with V_o = 14 MeV, d = 2.7 fm and curve 2 corresponds to V_o = 28 MeV, d = 1.9 fm (see Appendix).

Fig. 2. Schematic representation of two consecutive nucleon-nucleon collisions taking place at times t_1 and $t_1 + t_{coll} + t_{MFP}$, respectively.

Fig. 3. The same as in Fig. 2., but the two different nucleon-nucleon collisions are now overlapping along the time axis.

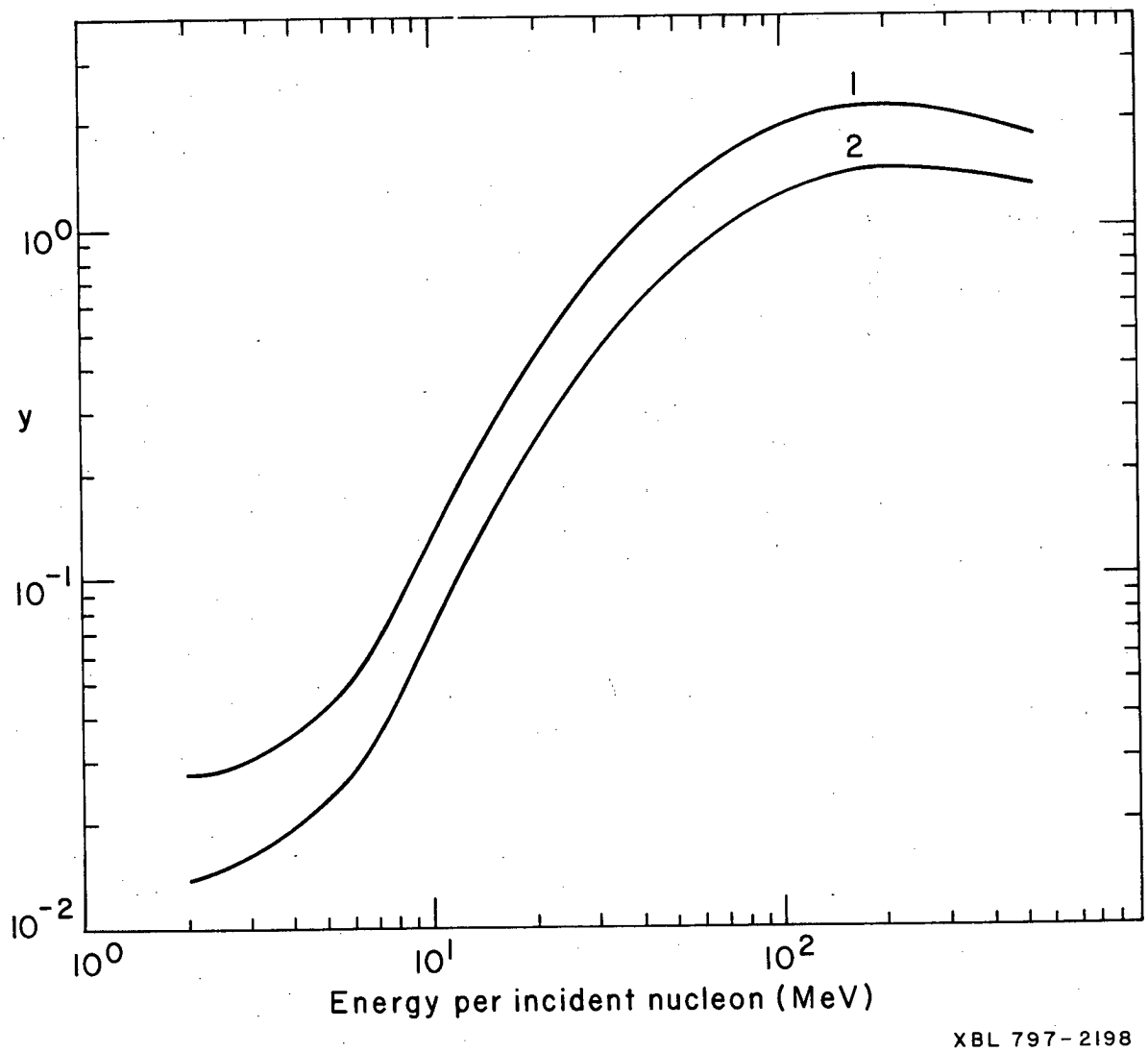
1
2
10^0
y
10^{-1}
10^{-2}
10^0
10^1
10^2
Energy per incident nucleon (MeV)
XBL 797-2198

Fig. 1

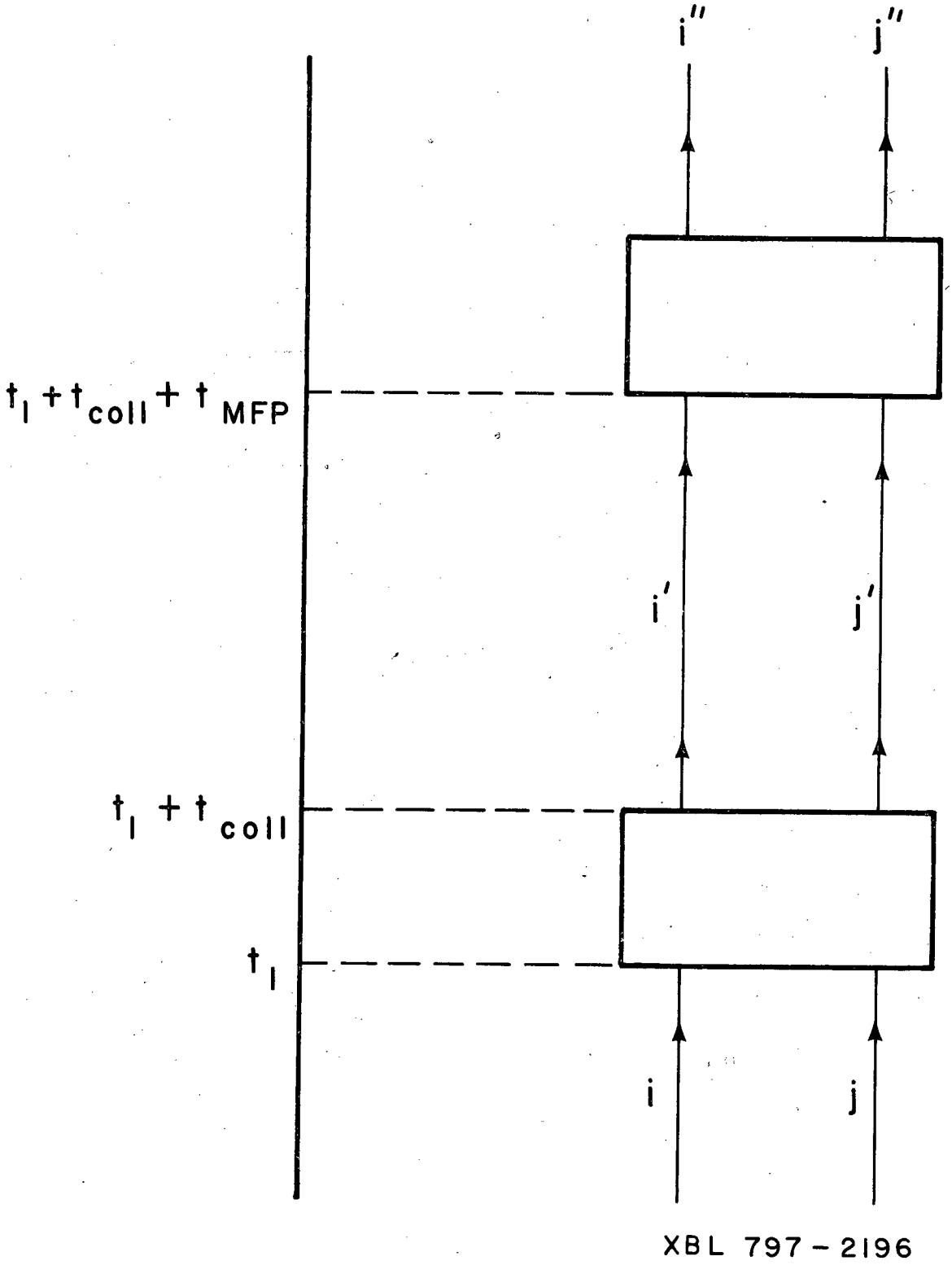

Fig. 2.

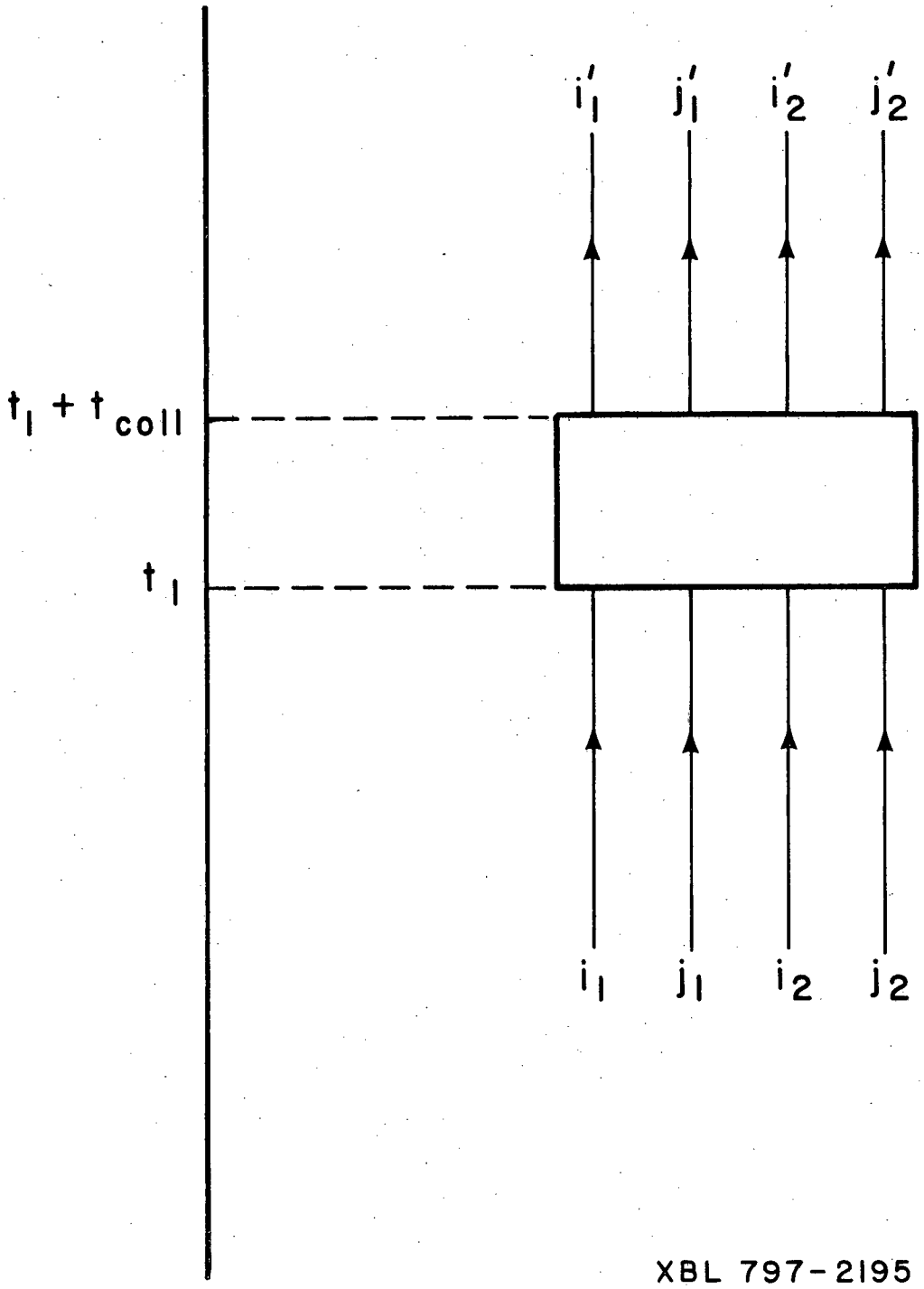

Fig. 3

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