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Notes for a lecture given at the Enrico Fermi Summer School of Nuclear Physics at Varenna, Italy, July 10-20, 1979

NUCLEAR PROXIMITY FORCES

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Nuclear Proximity Forces*

Notes for a lecture given at the Enrico Fermi Summer School of Nuclear Physics at Varenna, Italy, 10-20 July, 1979.

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ABSTRACT

This lecture gives a general introduction to nuclear proximity forces.

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1. Introduction

For certain physical systems such as homogeneous solids, fluids, or the heavier atomic nuclei, made up of elements interacting by short-range forces and possessing a surface region which is thin compared to the size of the object under consideration (leptodermous systems), the potential energy of the system may be decomposed into a bulk term and a surface-layer term. The surface-layer term is associated with the surface region and is, therefore, approximately proportional to the area of the surface bounding the object. For a simply-connected system the above decomposition is accurate if the principal radii of curvature of the surface are everywhere much larger than the thickness of the surface region. Moreover, when this condition is satisfied, corrections to the leading area-proportional term in the surface-layer energy (such as the curvature correction) may be derived by expansions in powers of the ratio of the thickness of the surface to the size of the system, thus making the expression for the potential energy even more accurate. Such a series expansion has been useful in discussing the average binding energies (masses) of atomic nuclei, and one might have thought that, apart from effects arising from special symmetries (shell effects) there was no more to the problem of average nuclear energies than the calculation of the above series expansions to a sufficiently high order. This is <u>not</u> the case. Thus, when the surface of the system becomes contorted into features whose characteristic dimensions are of the order of the thickness of the surface region itself, the above series expansions become of limited usefulness. This failing is by no means of merely academic interest; it is serious for a system with a thin neck, on the verge of dividing into two fragments (as in nuclear fission), or in the

case of two subsystems about to come into contact (as in collisions between heavy nuclei). In the latter case in particular, when the system is not simply connected, a calculation of the surface-layer energies of the two pieces, no matter how accurately they are corrected for the curvatures of the two surfaces, can never give rise to the (strong) attraction that in practice appears when the two surfaces approach to within a distance comparable with the surface thickness.

It is possible to derive expressions for the additional potential energy (or forces) associated with certain of the more important types of violently contorted surfaces, which should enable one to complement in a useful way the usual series expansions of the nuclear energy. These additional forces may be called "proximity forces" because they arise from the proximity of elements of the contorted surface, the contortion being such that different pieces of the surface actually face each other across a (small) gap or crevice. In particular it is possible to relate (approximately) the interaction between two finite nuclei to the interaction between two flat parallel slabs of semi-infinite nuclear matter, a problem that is simpler, and can be solved (in a suitable approximation) once and for all. An extended discussion of the proximity forces has been given in ref.1.

2. General Treatment

The starting point of our considerations is an expression for the proximity energy V_p associated with a curved gap or crevice of gently variable width D, which we shall write in the form

$$V_p = \iint e(D)d\sigma + \text{corrections.} \tag{1}$$

Here e(D) is the interaction energy per unit area of two parallel surfaces at the appropriate separation D. The integral is over the area of the gap or crevice and the "corrections" become negligible as the curvatures of the surfaces defining the gap become small.

The geometry of the gently variable gap may be specified by first choosing a mean gap surface Γ (a two-dimensional surface in space) and then considering normal displacements n_A, n_B, locating the gap, $n_A - n_b = D(u,v)$ being the distance between the two sides Γ_A, Γ_B of the gap. The gap width D(u,v) is a (slowly varying) function of position on the surface Γ, the position being specified by two coordinates u and v, say.

Since e(D) is, by definition, a function of only one variable, D, rather than of the two position variables u and v, the surface integral in Eq. (1) may be converted, at once, into a one-dimensional integral. Thus imagine that a family of (closed) curves (or set of curves) is constructed on the surface Γ corresponding to constant values of D. Denote by J(D)dD the area of the surface Γ that lies between two such curves (or sets of curves) defined by D and D + dD. Then we may write

$$V_p = \int e(D)\ J(D)\ dD + \cdots. \tag{2}$$

The function J(D) is characteristic of the geometry of the gap, and thus if the two sides of the gap are shifted, rotated, or deformed in some way so that the gap surface Γ and gap width D are changed, J will be a function (strictly speaking a functional) of these shifts, rotations, or deformations. We shall write J(gap,D) to exhibit explicitly the dependence of J on the set of degrees of freedom specifying the gap geometry. The function e(D) (the interaction energy per unit area of two parallel

surfaces) is, by definition, independent of the geometry of the gap. It does depend, however, on the nature of the surfaces. In particular, if the structure of the surface (e.g., the density falloff profile in the surface region) is considered as variable, and specified by a set of degrees of freedom, we may exhibit this by writing e(surf, D). Thus our basic equation for the energy associated with a gently variable gap or crevice is,

$$V_P(\text{ gap, surf}) = \int e(\text{surf, } D)\ J(\text{gap, } D)dD + \cdots, \tag{3}$$

In what follows we shall specialize at once to the case of surfaces with an invariable structure corresponding to that of a standard plane surface at equilibrium, characterized by an equilibrium density profile and a standard surface-energy coefficient. As regards the geometry of the gap we shall illustrate the applications of Eq.(3) by several assumptions about the function D(u,v) and the mean gap surface Γ.

2.1. Gap with Gently Variable Paraboloidal Width

Consider a mean gap surface Γ which is so gently curved that the coordinates u, v on the surface Γ may be taken as cartesian coordinates x, y, and the normal coordinate n used to specify the gap ($n_A - n_B = D$) may be taken as the cartesian coordinate z, with $z_A - z_B = D$. Consider now as an example a gap width D(x,y) which has a least value D = s at x = y = 0, say, and whose width in the vicinity of this point is given by the Taylor expansion

$$\begin{aligned} D(x, y) &= s + 1/2\ D_{xx}\ x^2 + 1/2\ D_{yy}\ y^2 + \cdots \\ &= s + 1/2\ (x^2/R_x) + 1/2\ (y^2/R_y) + \cdots. \end{aligned} \tag{4}$$

Here D_{xx} and D_{yy} are the second derivatives of D with respect to x and y

evaluated at the point of least gap width. In the second line these derivatives are written in terms of the principal radii of curvature, R_x and R_y, of the surface obtained by plotting the gap width D as a function of x and y. The directions of x and y are assumed to have been chosen along the principal axes of the quadratic form D(x,y) so there is no cross term in xy in Eq.(4).

Now change variables from x, y to ξ, η, defined by $\xi = x/(2R_x)^{1/2}$, $\eta = y/(2R_y)^{1/2}$, so that D may be written as $D = s + \rho^2$, with $\rho^2 = \xi^2 + \eta^2$. The proximity energy can then be transformed as follows:

$$V_p(s) = \int\int dx\, dy\, e(D) \tag{5}$$

$$= 2(R_x R_y)^{1/2} \int\int d\xi\, d\eta\, e(D)$$

$$= 2(R_x R_y)^{1/2} \int_0^\infty 2\pi\rho\, d\rho\, e(D)$$

$$= 2\pi\bar{R} \int_{D=s}^\infty dD\, e(D)$$

$$= 2\pi\bar{R}\,\mathcal{E}(s). \tag{6}$$

In the last few lines the integration has been extended to infinity. This assumes that the gap width grows to <u>beyond the range of</u> the interaction function e(D) and that e(D) approaches zero sufficiently rapidly for large values of D so that the integral becomes essentially independent of the upper limit. The quantity $\bar{R}$ is the geometric mean of the two principal radii of curvature R_x, R_y, characterizing the gap D. The reciprocal of $\bar{R}$ is $1/(R_x R_y)^{1/2}$, the square root of the invariant Gaussian curvature at x = y = 0 of the surface obtained by plotting D

versus x and y.

The negative of the partial derivative of $V_p(s)$ with respect to s gives the force between the two surfaces as a function of the separation degree of freedom

$$F(s) = -(\partial V_p/\partial s) = 2\pi\bar{R}e(s). \tag{7}$$

This leads to the following Proximity Force Theorem

<u>The force between two gently curved surfaces as a function of the separation degree of freedom s is proportional to the interaction potential per unit area, e(s), between two flat surfaces, the proportionality factor being 2π times the reciprocal of the square root of the Gaussian curvature of the gap width function at the point of closest approach.</u>

The usefulness of the Proximity Force Theorem lies in the circumstance that the principal feature of the function e(s) may be derived from experimentally known surface properties, such as the surface energy coefficient γ and the degree of diffuseness of the surface layer. Thus for s greater than the thickness of the surface (2 or 3 fermis in the case of nuclei) e(s) tends rapidly to zero. For smaller values of s, e(s) becomes negative, and for s = 0 it is approximately equal to minus twice the surface energy per unit area of the material of which the system is composed. This is because at s = 0 the two juxtaposed density distributions add up to an approximately constant bulk value, so that the net effect of bringing the surfaces together from infinity is to destroy the two surfaces. Thus $e(0) \approx -2\gamma$, where γ is the surface energy coefficient (about 1 MeV/fm^2 for nuclear matter and about 75 ergs/cm^2 for water).

If one were to countinue on to negative values of s, adding up the two density distributions without allowing them to get out of each other's way, the function e(s) would begin to increase, would go through zero, and would eventually grow without limit, reflecting the energy cost of doubling the density in the overlap region. It follows that e(s) exhibits a minimum, and this minimum occurs in fact near $s = 0$, where $e(0) \approx -2\gamma$. This is because it is at this separation that the total density is approximately equal to the standard bulk density, and the bulk energy of stable saturating systems (such as nuclei or ordinary matter) is a minimum with respect to deviations of the density from the standard value. From this circumstance follows an interesting result. The <u>maximum</u> attraction predicted by Eq.(7) occurs where e(s) is most negative, and this as we saw occurs at $s \approx 0$ where $e \approx -2\gamma$. Hence

$$\text{Maximum Attraction} \approx 2\pi\bar{R}e(0) \qquad (8)$$
$$\approx -4\pi\bar{R}\gamma.$$

This equation expresses the remarkable result that the maximum attraction (in the separation degree of freedom) between gently curved bodies may be written down approximately without any knowledge of the nature of the cohesive interactions between the particles constituting the bodies, provided only the surface energy coefficient is known.

As the two curved objects begin to overlap beyond the point of maximum attraction at $s \approx 0$, the attraction in the separation degree of freedom decreases and becomes zero at some point s_1 where the function $e(s_1)$ is zero. Since the zero of e(s) is, in the nature of things, independent of the curvatures of the two objects, we deduce a second noteworthy result. The equilibrium point (a minimum) in the separation degree of

freedom for two gently curved objects, such as two (uncharged) nuclei, occurs at one and the same overlap distance s_1 for all pairs of nuclei, independently of their sizes.

Under the conditions stated Eq.(8) would apply equally well to the contact force in the separation degree of freedom between ordinary solids (where the attraction is due to molecular forces), or to nuclei, where the attraction is due to nucleon-nucleon forces.

In order to calculate the force between two nuclei in its dependence on the separation s one has to use Eq.(7), whose right-hand side is a product of a geometrical factor $2\pi\bar{R}$ depending on the two nuclei in question, and a universal function of distance e(s), independent of the nuclei. The semiquantitative appearance of e(s) has been sketched out above, but in order to calculate e(s) in detail one needs to have a theory describing the structure of the nuclear surface region, so that one may take two flat nuclear surfaces and calculate their interaction energy per unit area as a function of the separation. Such a calculation of e(s) has been performed with a theory of the nuclear surface based on the Thomas-Fermi treatment. (See later).

2.2 Other Gap or Crevice Geometries

The proximity potential may be readily generalized to other equations besides the paraboloidal relation between D and x,y given by Eq.(4).

First note that Eq.(5) for the proximity potential is invariant with respect to an area-preserving stretching and compression of the transverse coordinates from x,y to x',y' where $x' = cx, y' = (1/c)y$. It follows that even if we specialize to systems for which D is assumed to

be only a function of the transverse radial distance r, where $r^2 = x^2 + y^2$, the results will hold, without any modifications, for "stretched" systems obtained by deforming the original circular contours of constant D into area-preserving ellipses. Conversely, in any problem where the contours of constant D are ellipses, the analysis may be simplified at once by considering the case of circular contours, where D is a function of r^2 only.

Consider now the relation between r^2 and D to be of the form

$$r^2 = \sum_{n=0}^{N} c_n D^n, \tag{9}$$

where c_n are arbitrary coefficients. Inserting Eq.(9) in Eq.(5) we find

$$\begin{aligned} V_p &= \int 2\pi r \, dr \; e(D) \\ &= \pi \int \sum_{1}^{N} n c_n \mathcal{E}_{n-1}, \end{aligned}$$

where the quantities

$$\mathcal{E}_n = \int_{s \text{ or } 0}^{\infty} D^n e(D) \, dD, \tag{10}$$

are moments of the universal function e(D). The lower limit "s or 0" distinguishes between what we shall call a "gap" and a "crevice." The former means that D has a least value (say s) at r = 0. (This value may be negative, corresponding to an overlapping of the two bodies which, for large negative s, would lead to a region of doubled density.) The latter means that D becomes zero at some finite value of r, say a neck or crevice radius r_{neck} (related to c_0 by $r_{neck}^2 = c_0$), and there is no overlap or density doubling. Thus a gap refers to two bodies (overlapping or not) and a crevice to a single indented body. The upper limit in Eq.(10) has been set equal to infinity under the same assumptions as before.

The case $N = 1$, with $c_1 = 2\bar{R}$ and $c_0 = -sc_1$ corresponds to the paraboloidal gap considered before. For a paraboloidal crevice (a crevice formed by portions of two intersecting paraboloids) we have

$$V_p = 2\pi\bar{R} \int_0^\infty e(D)\, dD.$$

Note that the proximity potential for a paraboloidal crevice with given $\bar{R}$ is independent of c_0 (or r_{neck}), i.e, is a constant independent of the degree of overlap.

It follows that the proximity potential along a sequence of configurations which for $s > 0$ consists of approaching gently curved surfaces (approximated by paraboloids) and for $s < 0$ turns into the corresponding crevice, exhibits a discontinuity in the first derivative at $s = 0$. This is because as s tends to zero from above, $\partial V_p/\partial s$ tends to about $4\pi\bar{R}\gamma$ (Eq.8), whereas for $s < 0$, $\partial V_p/\partial s \equiv 0$. As is readily verified, this $4\pi\bar{R}\gamma$ is precisely the negative of the discontinuity in the derivative of the <u>surface energy</u> that occurs when two gently curved surfaces characterized by a gap width curvature radius $\bar{R}$ turn into a crevice at contact. It follows that the addition of the proximity energy rounds off (approximately) the familiar kink that is present when only the surface energy is retained in the calculation of the potential for two fusing nuclei parametrized by separated or overlapping figures such as spheres or spheroids.

The case $N = 2$ corresponds to a relation between r^2 and D which defines a conic

$$r^2 = c_0 + c_1 D + c_2 D^2, \tag{11}$$

leading to

$$V_P = \pi c_1 \mathcal{E}_0 + 2\pi c_2 \mathcal{E}_1. \tag{12}$$

For gaps, $\mathcal{E}_0$ (the same as $\mathcal{E}$) and $\mathcal{E}_1$ are two universal functions of s, and for crevices two universal constants, characteristic of the material of which the surface is made. Several different geometrical arrangements are covered by Eqs.(11), (12).

In particular, for the case of a gap between two coaxial elliptic paraboloids with tip distance s, with radii of curvature A_1 and A_2 in the principal planes of curvature through the tip of paraboloid A and B_1, B_2 for paraboloid B, and an azimuthal angle ϕ between the principal planes of curvature of A and B we find $c_1 = 2\bar{R}$, $c_2 = 0$, where

$$\frac{1}{\bar{R}^2} = \frac{1}{A_1A_2} + \frac{1}{B_1B_2} + \left(\frac{1}{A_1B_1} + \frac{1}{A_2B_2}\right)\sin^2\phi + \left(\frac{1}{A_1B_2} + \frac{1}{A_2B_1}\right)\cos^2\phi \tag{13}$$

This formula could be useful for discussing the nuclear force between two nonspherical nuclei. In this case, in addition to an attraction along the line of least separation, there is a <u>torque</u> around it, trying to align the principal planes of curvature in the vicinity of the point of least separation. (For a special discussion of the proximity treatment of the interaction between deformed nuclei, see ref.2.)

Note that the above results are valid insofar as the juxtaposed surfaces are nearly parallel. Contributions from parts of the surfaces where this is not satisfied cannot be expected to be given accurately. Fortunately these contributions usually tend to zero, which is at least qualitatively correct in most cases.

3. Specialization to Nuclei

The above treatment is general. We now turn to the nuclear case. It is convenient to introduce the following dimensionless proximity function

$$\phi(\zeta) \equiv e(\zeta b)/2\gamma, \tag{14}$$

obtained from e(s) by measuring the separation s in units of the surface width b, $\zeta \equiv s/b$, and by measuring energy in units of twice the surface energy coefficient γ.

Similarly we define the dimensionless (incomplete) proximity moments by

$$\Phi_n(\zeta) = \int_\zeta^\infty \zeta^n \phi(\zeta)\, d\zeta = E_n(\zeta b)/2\gamma b^{n+1}. \tag{15}$$

(For the zeroth moment Φ_0 we shall simply write Φ.)

By dealing with these dimensionless functions one can make the predictions of the theory insensitive to certain quantitative shortcomings of the models of the nuclear surface that have to be used in calculating e(D) and its integrals. Thus a model of the surface could be somewhat inaccurate in reproducing the absolute values of the experimental surface energy and/or width (diffuseness) and, nevertheless, generate relatively accurate dimensionless functions ϕ and Φ. Combining these functions as predicted by the model with the values of γ and b (rather than with the possibly inaccurate values given by the model) will then result in the most reliable exploitation of the theory, where the model of the surface only enters in determining the functional form (but not the scale) of the proximity functions.

We have carried out a calculation of the universal nuclear proximity function ϕ and of the associated moment functions Φ_n using the nuclear Thomas-Fermi model with Seyler-Blanchard phenomenological nucleon-nucleon interactions. The resulting function $\phi(\zeta)$ is displayed in Fig. 1 and the integrated function $\Phi(\zeta)$ in Fig. 2.

For practical applications it is useful to have available a simple analytical representation of the function Φ which enters in the nucleus-nucleus proximity potential that follows from Eq. (6)

$$V_p = 4\pi\bar{R}b\cdot\Phi(\zeta). \tag{16}$$

One such approximation is given by the following "cubic-exponential" pocket formula

$$\Phi(\zeta \leq \zeta_1) = -(\zeta - \zeta_0)^2 - k(\zeta-\zeta_0)^3, \tag{17}$$

$$\Phi(\zeta \geq \zeta_1) = -3.437 \exp(-\zeta/0.75),$$

where $\zeta_1 = 1.2511 \approx 5/4$, $\zeta_0 = 2.54$, and $k = 0.0852 \approx 1/12$. This pocket formula should be adequate for many purposes. The cubic expression has been chosen to go smoothly to zero and so it could be used by itself in cases where the tail of the interaction is not an important feature. The accuracy of this approximation is illustrated in fig. 3. Approximate expressions for the higher moments Φ_n can be obtained by integrating Eq. (17). One can also obtain an approximation to ϕ by differentiating Eq. (17), but the result is only a rough representation, since a

discontinuity in the slope of ϕ appears at $\zeta = 1.2511$ (a result of the discontinuity in the curvature in Eq. (17) at that point).

In applications of the above formulas to nuclear problems the following choice of numerical parameters would be reasonable. For the surface energy coefficient γ one might use the Lysekil mass formula according to which

$$\gamma = 0.9517[1 - 1.7826I^2] \text{ Mev/fm}^2, \tag{18}$$

where $I = (N - Z)/A$, and N, Z, and A refer to the combined system of two interacting nuclei. In this way some allowance is made for the dependence of the proximity potential on the neutron excess, even though the universal functions ϕ and Φ were calculated for zero neutron excess. (A rough approximation to γ would be simply $(1 - 2I^2)$ MeV/fm^2.)

In calculating the value of $\bar{R}$ in Eq. (16) and the separation $s = r - C_1 - C_2$ for spherical nuclei the surfaces should be located using Süssmann's central radius C, related to the effective sharp radius by

$$C = R(1 - b^2/R^2 + \ldots) \tag{19}$$

For more general geometrics the proper location of the surface profile is more complicated (see ref. 1). For the effective sharp radius one may use the formula

$$R = 1.28\ A^{1/3} - 0.76 + 0.8\ A^{-1/3} \tag{20}$$

$$\approx\ 1.15\ A^{1/3}\ \text{fm.} \tag{21}$$

When used in conjunction with the Proximity Force Theorem the central radius C is preferable to the effective sharp radius R, which moves out relative to the density profile by an amount proportional to $A^{-1/3}$, and which is not, therefore, in an invariant relation to the location of the profile for finite nuclei with different radii. Note that the Proximity Force Theorem is based on Eq. (1), whose physical content is the approximate replacement of the interaction energy of two curved surfaces by an integral of the interaction energy per unit area of parallel flat surfaces for which the separation D(x,y) matches, point by point, the distance between pairs of elements of the actual curved surfaces. In order for the approximation to be accurate the flat surfaces should be located so that their profiles match the locations of the actual profiles as well as possible. This is ensured to a higher degree of accuracy by working with the central radius C rather than with the effective sharp radius R.

The width b in Eq. (16) has the approximate value of 1 fm. Thus for a Woods-Saxon surface profile the Süssmann width b is related to the "10-90% fall-off distance" by

$$b = [\ \pi/(2(3)^{1/2}\ln 9)\ ]t_{10-90}\ , \tag{21}$$

which for t_{10-90} = 2.4 fm gives 0.99 fm.

The above equations give only average estimates of nuclear sizes. Variations caused by shell effects including deformations, have to be taken

into account separately. In particular, when using the proximity potential for deformed nuclei, the distance of closest approach s, as well as the mean curvature radius $\overline{R}$ of the gap width function in the vicinity of the point of closest approach, have to be carefully related to the orientations and shapes of the nuclei.

The accuracy of the Proximity Theorem has been tested in several cases of interest in nuclear physics. Some examples are discussed in ref.1. A particularly detailed study has been made in ref.3. The accuracy of various additional approximations frequently employed when considering deformed nuclei is discussed in ref.2. The general conclusion from such studies is that the Proximity Theorem is quite accurate provided proper care is taken when matching the curved nuclear surfaces with the corresponding flat ones.

In the formulas given above there are no adjustable parameters in the predicted nucleus-nucleus interaction. For some purposes it might be useful to have adjustable parameters that could be varied, within reasonable limits, in correlating experimental data. For example, some slight variations in the nuclear radii from the nominal values given by Eqs. (20), (21) would be reasonable. Similarly the nominal values of the surface energy coefficient γ (Eq. (18)) and of the surface width b (= 1 fm) need not, of course, remain inviolate. Altogether the slight freedoms associated with adjustments in the values of the radius, surface energy, and surface width are together equivalent to slight freedoms representing a horizontal shift, and horizontal and vertical stretchings of the nucleus-nucleus potential $V_p(r)$. Such adjustments of the parameters should, however, be used with moderation and should not

be in conflict with what is known about nuclear radii and surface properties. In particular, if in the interpretation of nucleus-nucleus scattering experiments by means of the frozen idealization (in which all degrees of freedom except the separation between nuclei are frozen), unreasonable adjustments of the parameters are called for, the explanation is very likely to be the inadequacy of the frozen idealization. One would then be wiser to display clearly the discrepancies rather than to mask the expected failure of the frozen idealization by an abuse of the nuclear parameters.

With reference to some earlier misunderstandings, let us stress that the use of the proximity energy expressions (as generalized in Section 2 for various geometrical arrangements including necks and crevices) is not tied to the frozen idealization. Thus if a neck degree of freedom is added in the description of (e.g., deep inelastic) scattering, an appropriate proximity energy formula, taken for example from Section 2, may be used.

The proximity potential is often employed in analyses of nuclear collisions and appears to give reasonable estimates for the interaction energy between nuclei. Really unambiguous tests of the proximity expressions against experiment are not easy because the proximity theory is a theory of the interaction between nuclear surfaces as a function of their separation, but the precise locations of the relevant portions of the surfaces of two nuclei during a collision are, in general, not known. If the nuclei are assumed not to deform during the collision, then the surface separation can be deduced from the separation between the mass centers if such an undeformability assumption is made one gets

the impression that the theoretical potential is often too weak by up to a factor of two. But it may well be that the calculated potential is in fact approximately correct but that the surfaces have deformed, i.e., reached out towards each other by a few tenths of a fermi and thus increased the interation at the given center separation of the two nuclei. Such polarization or deformation effects are to be expected and should be pursued, experimentally as well as theoretically. Figure 4 gives a rough indication of the relation between estimated potential depths and the theoretical predictions. One might summarize by saying that a semiquantitative understanding of the nonlocal part of the nuclear potential energy has been achieved.

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Fig.1. The dimensionless proximity force function $\phi(\zeta)$ as a function of the dimensionless separation ζ. The minimum and point of inflexion in ϕ are indicated.

FIG.2. The dimensionless proximity potential function $\phi(\zeta)$ as a function of the dimensionless separation ζ. The minimum (at $\zeta = -1.3734$) and the point of inflexion in ϕ are indicated. The potential between two nuclei is a geometrical factor times ϕ. It follows that the equilibrium point (in the separation degree of freedom) for the nuclear interaction between any two nuclei occurs at the universal interpenetration distance of 1.3734 b or about 1.37 fm. The dashed rectangle on the right indicates the region where a comparison of the function ϕ is made with experimental values in Fig.4, using are expanded scale.

Fig.3. The universal nuclear function ϕ is shown by the solid line and the cubic-exponential approximation by circles. The dots show the continuation of the cubic beyond the dashed line (locating its junction with the exponential) to where it touches the ζ-axis at 2.54. The frozen Thomas-Fermi densities of the two semi-infinite distributions touch at $\zeta_1 = 2.74$.

Fig.4. The theoretical proximity interaction potential between nuclei is shown as a function of surface separation. The experimental depths are deduced from elastiec scattering and heavy-ion fusion data. The unit on the abscissa corresponds to about 1 fm. The range of potential depths displayed goes from a fraction of an MeV to several ten of MeV. (From ref. 4)

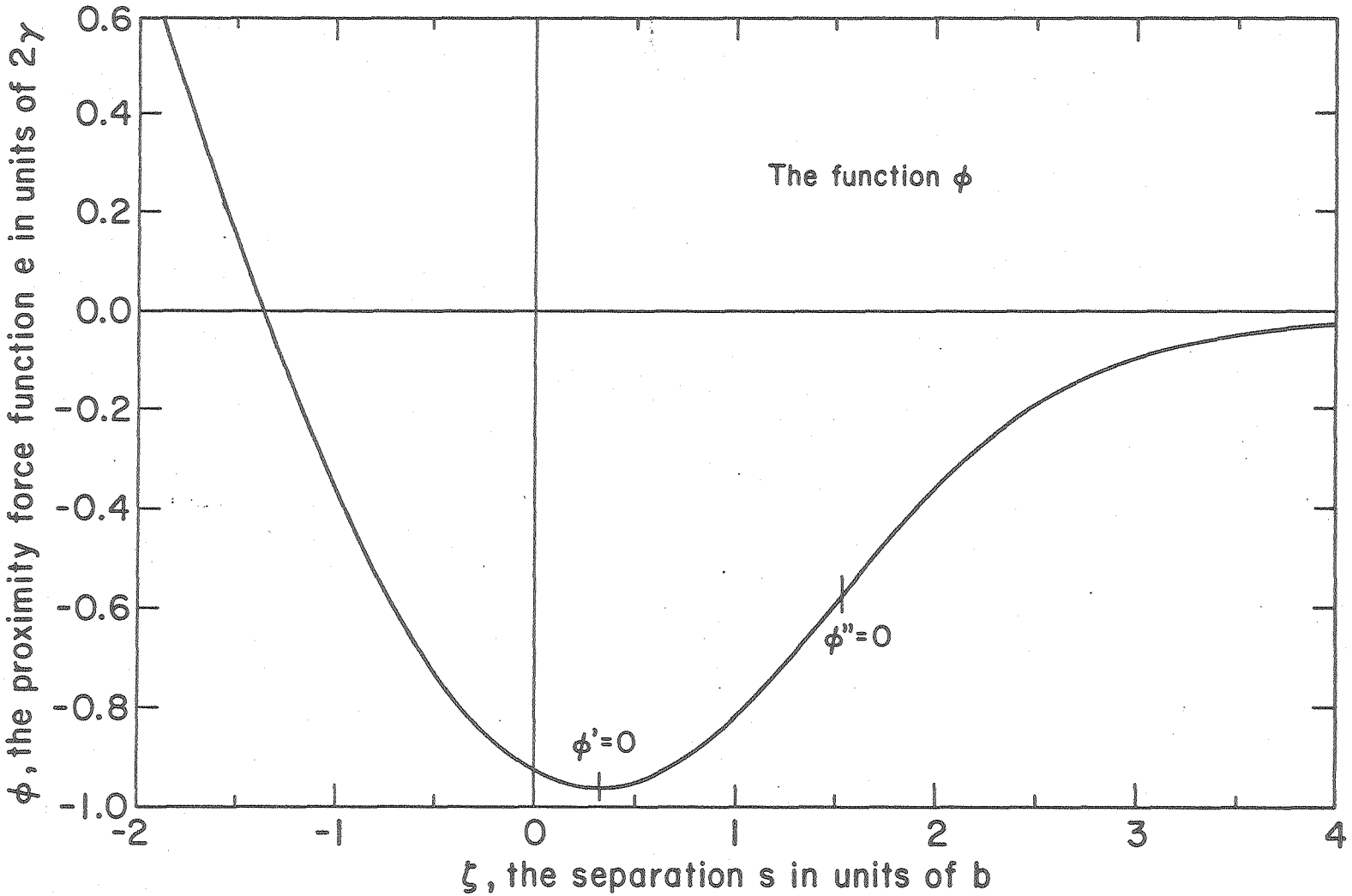

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Figure 1

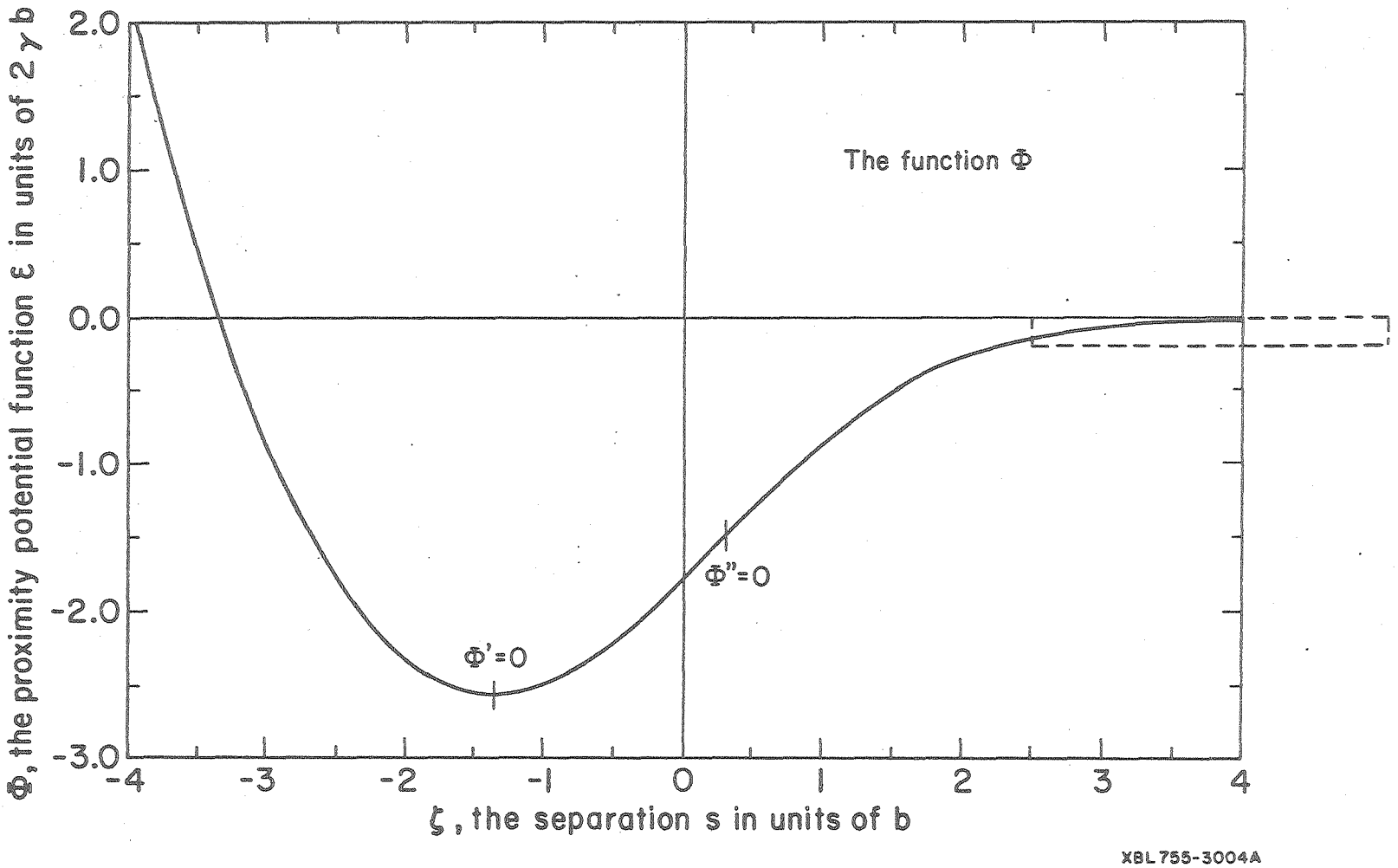
The function Φ
Φ', the proximity potential function ε in units of 2γb
ζ, the separation s in units of b
Φ'=0
Φ"=0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4
-3
-2
-1
0
1
2
3
4
XBL755-3004A

Figure 2

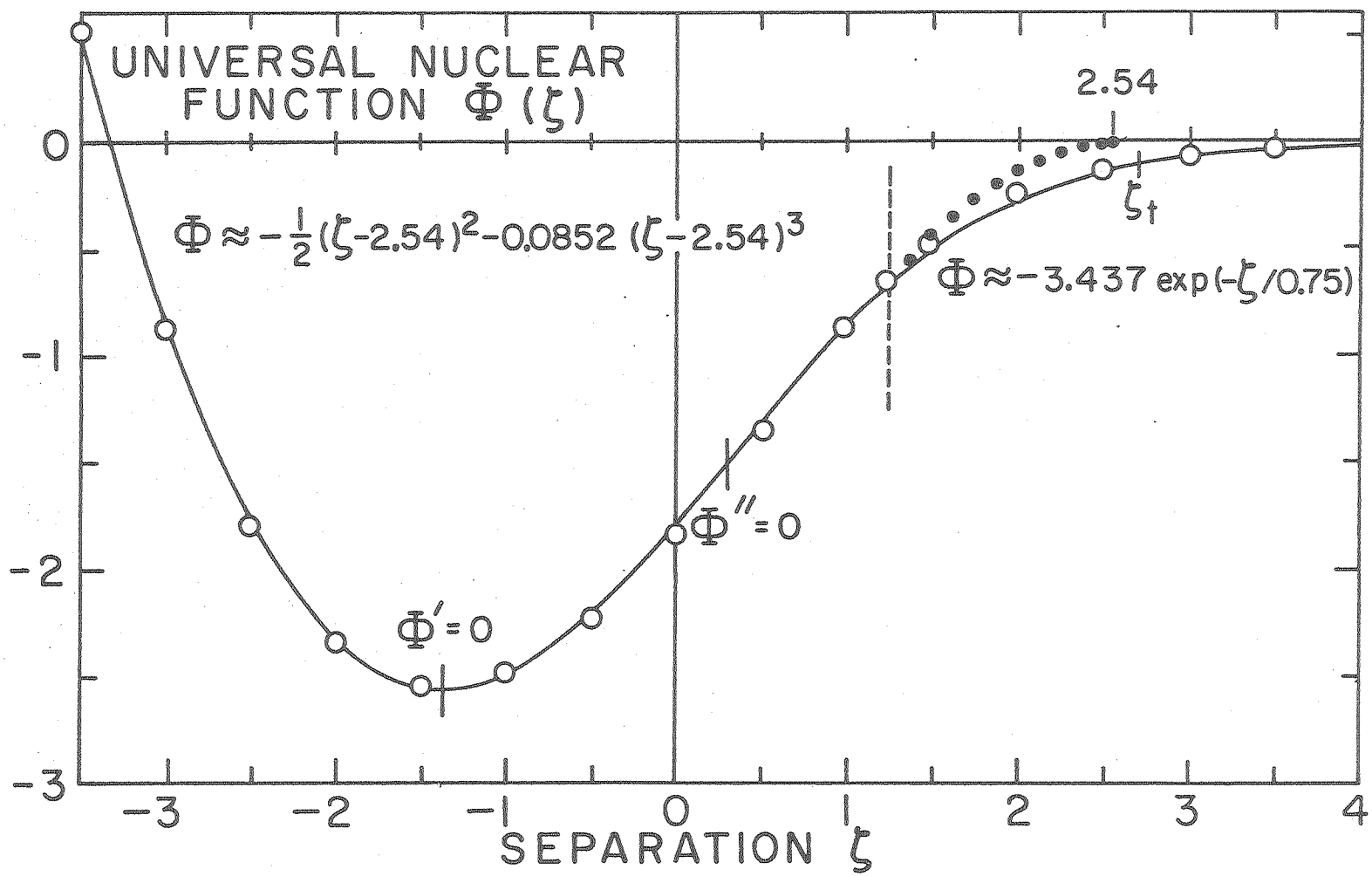

Figure 3

XBL759-4073

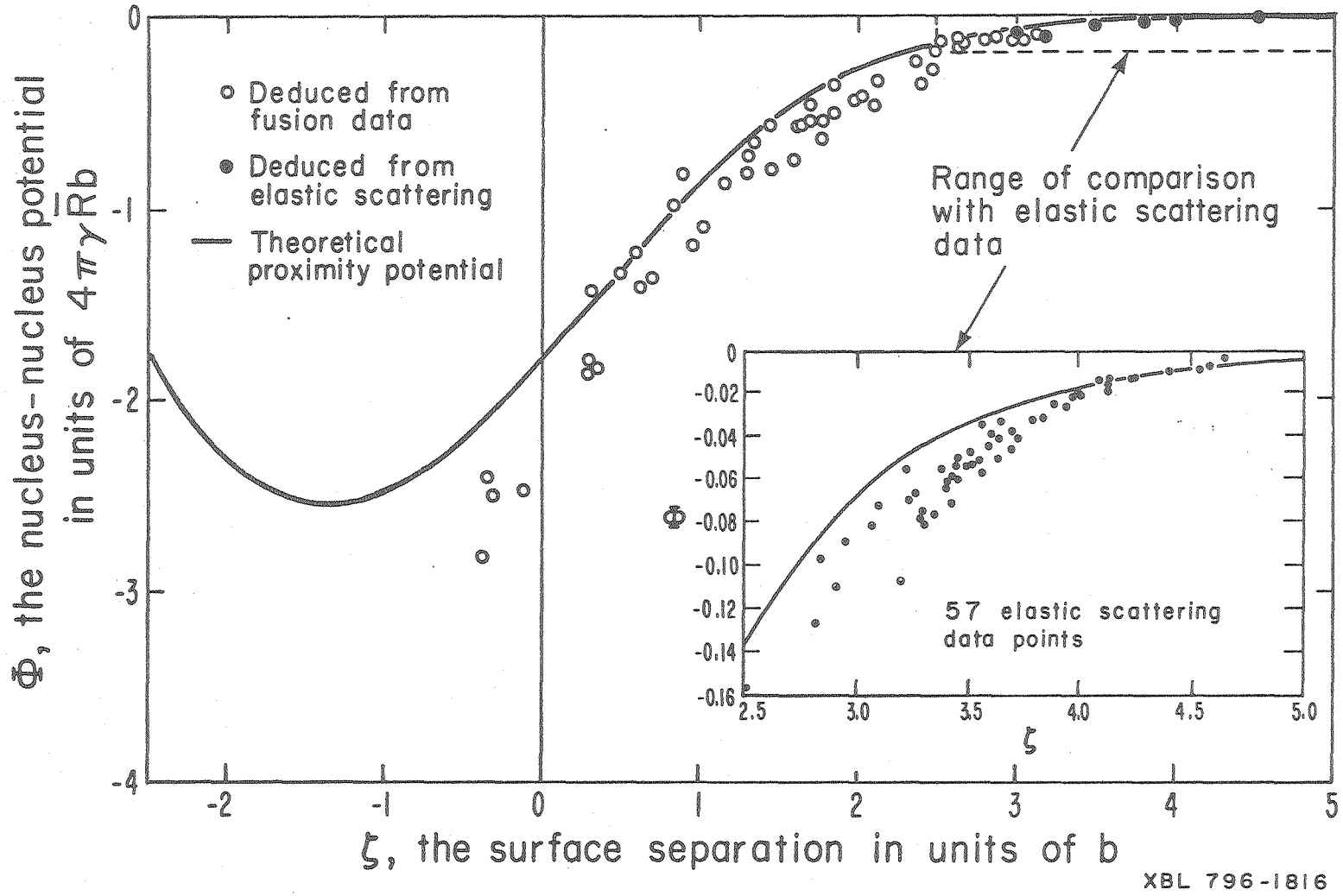
Φ, the nucleus-nucleus potential in units of $4\pi\gamma\bar{R}b$
ζ, the surface separation in units of b
Deduced from fusion data
Deduced from elastic scattering
Theoretical proximity potential
Range of comparison with elastic scattering data
57 elastic scattering data points
Φ
ζ
XBL 796-1816

Figure 4

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