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CALCULATION OF MUON FINAL STATE PROBABILITIES AFTER MUON-INDUCED FISSION

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Abstract

Experiments have shown that in mu-mesic uranium-238 fission the μ^- is bound mainly to the heavy fragment with a few percent of cases of binding to the light fragment. We use time-dependent perturbation theory to calculate the theoretical branching for a variety of fission asymmetries and fission fragment dynamical conditions. We go beyond earlier theoretical works in that we study extended nuclear charge distributions as well as point charges. Agreement with experiment is within limits of error.

I. Introduction

Following some muon-induced fission processes, the μ^- survives in final states bound to one of the fission fragments. Because the μ^- lifetime depends inversely on the mass of the binding nucleus, it is possible to determine the probability of the μ^- being bound to the heavy fragment, the light fragment, or ejected into the continuum.

The final state probabilities for the muon are associated with fission dynamics. If the motion of the nuclear system past the saddle point is infinitely slow, that is, by very viscous flow, the muon would always stay at the lowest energy level up to the scission point. Past the scission point the nuclear system is accelerated by the Coulomb force modified by the nuclear force and shape distortion at first, and the muon can be excited to a higher orbital and have a non-vanishing probability of ending up bound to the light fragment. On the other hand, if the motion of the nuclear system is not slow from saddle point to scission, that is, the motion is not so viscous, there should be already some fraction of muon excited-state population, even before scission. Thus, for non-viscous flow the greater percentage of muon binding to the light fragment should be obtained.

A number of experiments bearing on this problem have been done.[1-6]

Various aspects of the theory have also been treated. Leander and Möller[7] looked at the augmentation of the fission barrier in the presence of a negative muon in the lowest state. Karnaukhov proposed

that the complicated problem of final state probability of the muon could be simply solved if statistical thermal equilibrium between muon and internal nuclear degrees of freedom of the excited primary fission fragments is assumed.[8] Olanders, Nilsson, and Möller attacked the dynamical problem of final state probabilities by solving a time-dependent Schrödinger equation for the muon in the field of two separating point charges.[9] Since the point-charge model produces too large separation of the energy levels and too small transition matrix elements when compared with a more realistic model, the results for finding the muon on the light fragment are certainly underestimated. Experiments[6] and theory[8,9] indicate a few percent probability of the muon being captured onto the light fragment in the most probable asymmetric fission. The probability certainly deviates from an assumption, albeit not critical, in Ref. 2 analysis that the muon final state probability is proportional to the atomic numbers of the fission fragments. Of course, for completely symmetric fission the probabilities would have to be equal for the two fission fragments.

We felt it worthwhile to make new theoretical calculations, going beyond Ref. 9 in that we would examine extended charge distributions as well as point charges, study alternative forms of variational wave functions, and run a sufficient number of cases to establish the systematics with respect to fission asymmetry and nuclear viscosity from saddle to scission.

As a weakly interacting particle,the muon may be treated as having only electromagnetic interactions with the nucleus. However, the muon

is so much more massive than the electron that muon 1s wave functions for fission fragments and heavier nuclei are significantly within the nuclear volume, and binding energies and wave functions are considerably altered from the point nucleus values. A convenient reference for muon 1s binding energies is the theoretical paper of Ford and Wills.[10] For example, in barium (Z = 56) their calculated 1s binding energy is 6.167 MeV, to be compared with the point-nucleus Bohr formula of 8.47. Thus, it seemed to us important to explore the effects of using extended charge distributions. To use the familiar Woods-Saxon or Gaussian charge distributions was early seen to involve considerable calculational difficulties. The use of exponential charge distributions, however, seemed tractable, and we have for the most part used these, with the range constant chosen to match the experimental root-mean-square charge distribution of the nucleus.

With the extended charge distribution the potential no longer has the singularity of the point charge and is indeed of harmonic form at the nuclear center. For the heaviest nuclei, where the muon 1s wave function is appreciably inside the nucleus (~54% for uranium), the wave function approaches the Gaussian form of the 3-dimensional isotropic harmonic oscillator wave function. Thus, it is not obvious for fission fragments whether the better LCAO (linear combination of atomic orbitals) variational wave functions are of exponential hydrogenic form, as for point nuclei, or Gaussian harmonic oscillator form. Accordingly, we have investigated both forms of variational wave functions.

II. Static Two-Center Muonic Eigenvalues

We have first studied the static, symmetric problem. For point charges exact solutions exist based on separation in a prolate spheroidal co-ordinate system.[11] Figure 1 plots the lowest two eigenvalues as a function of separation distance between the two charge centers. The topmost curve (long dashes) in both families is for an extended charge distribution, while the other three curves are for nuclear point changes. The lowest solution is the exact solution (solid line). The next lowest solution (dashed line) is for a variational wave function based on two Gaussians, with the width parameter r_0 interpolated between values that minimize the energy in the zero-separation and infinite-separation limits. Above that is a curve (dash-dot) for a two-Gaussian variational wave function with the width parameter r_0 held constant at its infinite separation value. The topmost curve (dot, long-dash) for the lower family (symmetric wave function) gives the energy for an extended (exponential) charge distribution. In Appendix A we derive the Hamiltonian for the extended charge. The corresponding energies for the antisymmetric wave function are seen in the upper four curves.

Several features are to be noted. Although the eigenvalues for the variational wave functions lie above the exact eigenvalues by 0.15-0.2 atomic units over the range of interest ($R \gtrsim 2$), the energy difference between 1s σ_g symmetric (ground) and 1s σ_u antisymmetric (first excited state) is almost the same for exact and variational solutions. It is this energy difference that enters most

importantly in the time-dependent dynamic problem and not the absolute energies. The variational wave function with constant Gaussian widths is much poorer for $R \lesssim 1.2$. However, these short separation distances are not relevant to the dynamic muon fission problem, since at fission saddle the equivalent separation of charge centers is larger. Thus, to simplify calculations we have used the constant width Gaussians in the time-dependent problem.

We note the Bohr formula point nucleus energy for $Z = 46$ is -5.92 MeV. The Ford-Wills mu-binding energy is -4.57 MeV. Our point-charge Gaussian energy is -5.11 MeV. For unequal fission product charges we show examples in Figs. 2 and 3 of the dependence of eigenvalues on separation distance. In Fig. 2 the extended charge distribution and Gaussian wave functions were used. In Fig. 3 a point-charge and exponential wave functions were used, so in this case the wave functions are exact for infinite separation. Figure 3 may be directly compared with Ref. 9, and we find essential agreement on the energy levels.

III. The Time-Dependent Problem

We use the method of perturbed stationary states (PSS) for the problem of the μ^- in the field of moving nuclear charges. We neglect muon mass compared to the nuclear masses; that is, the separation motion of the nuclear centers is assumed not to be influenced by the muon motion.

In our two-state case we get two coupled time-dependent Schrödinger equations (see Appendix B for detailed algebra).

$$\dot{a}_u(t) = -a_g(t) \langle \psi_u(t) \left|\frac{\partial}{\partial t}\right| \psi_g(t)\rangle \exp\left\{\frac{i}{\hbar} \int_0^t \left[E_u(t') - E_g(t')\right] dt'\right\} \tag{1a}$$

$$\dot{a}_g(t) = -a_u(t) \langle \psi_g(t) \left|\frac{\partial}{\partial t}\right| \psi_u(t)\rangle \exp\left\{\frac{i}{\hbar} \int_0^t \left[E_g(t') - E_u(t')\right] dt'\right\} \tag{1a}$$

Since the amplitudes may be complex, we have four coupled equations in the real and imaginary parts.

After a series of transformation these equations can be reduced to two coupled equations in the amplitude and the difference of the phases.

$$\frac{d\eta}{dR} = -\cos\xi \langle \psi_u \left|\frac{\partial}{\partial R}\right| \psi_g \rangle \tag{2a}$$

$$\frac{d\xi}{dR} = \frac{E_u - E_g}{\hbar \dot{R}} + 2 \operatorname{ctg}\eta \sin\xi \langle \psi_u \left|\frac{\partial}{\partial R}\right| \psi_g \rangle \quad , \tag{2b}$$

where

$$a_g = \cos\eta e^{i\delta g} \quad , \quad a_u = \sin\eta e^{i\delta u}$$

$$\xi = \delta_u + \delta_g + \frac{1}{\hbar}\int_0^t (E_u - E_g)\, dt'$$

Using the modified Euler method we have integrated these equations on the NOVA 840 computer of the Atomic Energy Research Institute in Beijing. As t = 0 boundary conditions we assume the μ^- to be completely in the lowest state when the ^{238}U is at the second saddle point for fission. Möller and Nix[12] have given calculated center-to-center distances for forming fragments at second saddle for ^{240}Pu and ^{236}Th. Interpolating, we get for ^{238}U a value of $R_{\text{second saddle}} \approx$ 9.7 Fm. We use R = 10 Fm as starting distance.

To check against results of Olanders et al.[9] we have calculated the case of two point charges starting from rest at R = 11 fm with $Z_L = 38$, $A_L = 100$, $Z_H = 54$, and $A_H = 140$. We have used both of their alternative initial conditions of $|a_u(0)|^2 = 0$ and $|a_u(0)|^2 = 0.11$.

In Fig. 4 we plot the per cent population of the upper state ($|A_u|^2 \times 100$) as a function of separation distance, using the parameters of Olanders et al. While our results qualitatively match those of their Fig. 1, the agreement is not exact. For the initial condition $|A_u|^2 = 0$ their probability rises to a maximum of 0.006 at 24 Fm, dropping to a very small value asymptotically. Our

probability rises to a maximum of 0.016 at 23 Fm, falling to 0.005 asymptotically. For the other boundary condition our probability goes through somewhat wider excursions, though the final probability, after about 1-1/2 cycles of oscillation is near 0.12 for them and us.

We cannot explain the disagreement, though it may be related to their use of $(\nabla_{\vec{R}} H \cdot \vec{R}$ for the perturbing potential, whereas we use $\dot{R} \frac{\partial}{\partial R}$. These forms would be equivalent if the Schrödinger equation were exactly satisfied by the wave functions. This is not the case for LCAO variational wave functions, and we believe our use of $\dot{R}\frac{\partial}{\partial R}$ to be more correct.

Our next calculations are run from R = 9 or 11 Fm with and without an Igô (exponential) nuclear potential of diffuseness 0.65 Fm and strength adjusted to give a potential maximum at the initial distance. With the nuclear potential there is no acceleration at t = 0, and we choose an initial fission kinetic energy E_0 of 1 MeV at the saddle. This initial kinetic energy can be rationalized in terms of the size of the wave packet crossing the fission barrier.

In Fig. 5 we show the probability curves for a probable fission fragment asymmetry $Z_L = 40$, $Z_H = 52$. In most cases the extended charge distribution was used. We see a sensitivity to starting distance, the use of 9 Fm giving about 4% greater final occupation of the light fragment than the use of a starting distance of 11 Fm. The Gaussian and exponential variational wave functions give different results, with the Gaussians giving about 5-6% higher occupation. The addition of a nuclear potential, which makes initial acceleration

vanish, lowers the probability by only ~1.5 percent. This calculation, with nuclear potential and Gaussian wave functions, gives a final probability of ~14 percent on the light fragment, somewhat in excess of the experimental estimate of ~8 percent, but we need to do a weighted probability distribution over the fission mass-yield curve to compare with experiment.

It seems that using the Coulomb potential to accelerate the fission fragments from second saddle is inconsistent with fission data. In the calculation mentioned above the final kinetic energy of the nuclear system is about 300 MeV. That is much larger than the experimental value of 170 MeV. To match this final kinetic energy we choose the scission point of 17.6 Fm as starting distance. As for the excitation probability during the penetration of the fission barrier, we investigated two cases, $|a_u|^2 = 0$ and $|a_u|^2 \neq 0$. For the latter case we assumed that from the saddle point to the scission point the separation speed of the two parts of the fissioning nucleus is uniform with (a) $v = 10^9$ cm/sec or (b) $v = 3 \times 10^8$ cm/sec. At the scission point of 18 Fm the excitation probability is about 0.15 percent for case (a), ~0.017 percent for case (b), and the kinetic energy at 18 Fm is ~29.8 MeV for the former, ~2.68 MeV for the latter. The influence on the final excitation probability for case (b) can be neglected. Figure 6 shows the final excitation probability is ~6 percent for the case with $|a_u|^2 = 0$, $E_0 = 1$ MeV at 17.6 Fm and ~8 percent for the case with $|a_u|^2 = 0.15$ percent, $E_0 =$ 29.8 MeV at 18 Fm. These results are close to the experimental estimate.

From Figs. 7 and 8 we can see that the final occupation probability on the light fragment is a strong function of the fission asymmetry. The comparable (Gaussian and nuclear potential) calculation for a highly asymmetric split to $^{84}_{34}Se$ and $^{154}_{58}Ce$ gives a 1.5 percent probability (Fig. 7). The near-symmetric division to $^{116.4}_{45}Rh$ and $^{122.6}_{47}Ag$ gives 44 percent (Fig. 8). The The fractional mass numbers in the last case were used to strictly keep the center-of-mass and center-of-charge at the same point. We have taken care to avoid spurious center of mass motional terms by keeping the center-of-mass strictly fixed. We made some investigation of the effect of changing the masses at fixed charge values, thus varying a dipole term that can induce muon transitions. The effect is small for the range of charge-to-mass ratios found in fission fragments.

In order to make a more careful comparison with experiment we must make a weighted sum of muon probabilities over fission yields. For this purpose we wish to represent our foregoing calculations in a simple functional dependence on fission-charge asymmetry. It seems likely that for large asymmetries the probability of μ binding to the light fragment should decrease exponentially with the muon 1s binding energy difference for the two fragments. As we approach symmetric fission, we must approach a 50% occupation probability. The appropriate methematical function might be a Fermi function, and we approximate the binding energy dependence on Z as quadratic by the Bohr point-nucleus formula. Thus, for uranium (Z_i = 92)

$$P_\mu(Z) = \left\{1 + \exp[-b(Z^2 - (92 - Z)^2)]\right\}^{-1}$$
$$= \left\{1 + \exp[b'(46 - Z)]\right\}^{-1} \qquad (3)$$

The single adjustable constant b' can be determined from any one of our calculations. Let us here use the case of $Z_L = 40$, $Z_H = 52$, $|a_u|^2 = 0$ at R = 17.6 Fm, where the final probability is 6Δ. In this case the constant b' is 0.45.

For fission it is really the final mass chain yield distribution that is best known. In order to translate such data into primary charge yield we have to correct for neutron-evaporation and use some expression for primary charge distributions. We take the ^{239}Pu thermal neutron fission mass yields[13] as approximately those for ^{238}U radiationless muon induced fission. (The $2p \rightarrow 1s$ transition energy gives comparable excitation energy to the capture of the thermal neutron in ^{239}Pu and the mass difference of compound nuclei $^{238}U^*$ and $^{280}Pu^*$ approximately compensates for $\bar{\nu}$, the number of evaporated neutrons.) We assume constant charge-to-mass ratio in primary fragments, though this is known not to be quite so. Then the fission yield for the light fragment may be approximated by a Gaussian $y(Z_L) = B\exp[-(Z_L - 38.6)^2/\sigma^2]$ with $\sigma = 3.5$. The constant B is fixed by demanding the fission yield sum to unity. Thus, $B = (\sigma\pi^{1/2})^{-1}$. Taking the product of expressions (3) and (4) and approximating their sum over Z by an integral from $-\infty$ to $+\infty$ we get a light fragment muon probability of ~6.2%. This weighted sum is so nearly equal to the result for $Z_L = 40$, $Z_H = 52$ that we may use $Z_L = 40$, $Z_H = 52$ calculations to be representative of the full folded distribution.

It is satisfying to see that these simple calculations are in qualitative agreement with the experimental result of ~8% μ^- attachment to the light fragment. This result comes from using the natural boundary condition of muon population entirely in the lowest state while the system is moving most slowly at scission. There is no need to invoke special arguments about higher state population at saddle, as Olanders et al.[9] did for their alternative case. The case of muonic fission from the fission isomer, where there is barrier penetration, may require some such sudden-approximation expansion of muon wave function as in their alternative treatment.

There are obviously many possible refinements to be considered in future work, but we do not think they will alter the main conclusions here. For example, more realistic nuclear charge distributions than the single exponential might be used. Relativistic effects could be taken into account in muon motion through use of the Dirac equation or relativistic corrections to the Schrödinger equation. The effects on velocity of separation due to the low frequency quadrupole (and octupole) vibrations of the fission fragments might be taken into account. The time period for oscillation of the muon between light and heavy fragment (~10^{21} sec) is comparable to vibration periods. The coupling between muon motion and internal degrees of freedom of the excited fragments (central to Karnaukhov's Boltzmann factor arguments[8]) should be considered, particularly the beta vibrational mode. We believe these couplings to be weak after preliminary examination, but we have not made numerical calculations involving vibrational degrees of freedom.

Obviously the basis for the μ^- molecular wave functions should be expanded beyond the 2-state basis we used. The inclusion of the two 2p orbitals along with the 1s σ should not only improve the result for final 1s populations but also show the final 2p populations that in principle might be experimentally detectable via μ-mesic K X-rays characteristic of the fission fragments. The inclusion of 2p σ orbitals might be especially significant for fission asymmetries so large that the 2p level in the heavy fragment is close in energy to the 1s level in the light fragment. By the Bohr formula ($E = \text{const.}\ Z^2/n^2$) this special degeneracy would only occur for extreme asymmetry with $Z_H = 2\ Z_L$ or $Z_H \sim 62$, $Z_L \sim 31$. The fission yields are vanishingly small for such large asymmetries, so we think inclusion of 2p σ wave functions in the basis will not appreciably change results. Of course, the cylindrical symmetry of the problem excludes coupling to 2p π states or any other than σ states, so they need not be considered.

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Appendix A. Coulomb Potentials for Extended Charge Distributions and Solutions of the Two-Center Problem

The use of exponential charge distributions seems practical. We could make the charge distributions closer to the Woods-Saxon form by taking linear combinations of exponentials with different ranges, but we have felt the one term exponential charge distribution sufficiently accurate for our purposes in the mu-fission problems.

We take

$$\rho(r) = \rho_0 \exp\left(-\frac{r}{r_i}\right) \tag{A.1}$$

The two constants ρ_0 and r_i are fixed by the total charge condition and matching of the root mean square radii with uniform charge distribution.

$$\rho_0 = \frac{Z}{8\pi r_i^3} \tag{A.2}$$

$$\langle r^2 \rangle = \frac{\int r^2 \rho d\tau}{\int \rho d\tau} = 12\ r_i^2 = (3/5)^{1/2}\ r_c \tag{A.3}$$

We took $r_c = 1.2 \times A^{1/3}$ Fm. The potential of the extended charge distribution is

$$V(r) = -\frac{Ze^2}{r}\left[1 - \exp\left(-\frac{r}{r_i}\right) - \frac{r}{2r_i}\exp\left(-\frac{r}{r_i}\right)\right] \tag{A.4}$$

We have tried both Gaussian and exponential trial wave functions for the μ-atomic orbitals of each separated center.

$$\phi_G = \frac{1}{(\pi r_o^2)^{3/4}} \exp\left(-\frac{r^2}{2r_o^2}\right) \tag{A.5}$$

or

$$\phi_e = \frac{1}{\pi^{1/2} b^{3/2}} e^{-r/b} \tag{A.6}$$

The variational equation fixing r_o (or b) for the one-center asymptotic state is

$$-\xi^5 + \sqrt{\pi}\xi^4\left(\frac{1}{2} + \xi^2\right) e^{\xi^2}[1 - \Phi(\xi)] - \frac{4Ze^2 m_\mu}{3\hbar^2 \sqrt{\pi}\, r_i} = 0 \tag{A.7}$$

$$\xi = \frac{r_o}{2r_i} \quad , \quad \Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

or

$$\xi^4(\xi + 8) - \frac{\hbar^2}{Ze^2 m_\mu r_i} (\xi + 2)^4 = 0 \tag{A.8}$$

$$\xi = b/r_i$$

For the two-center variational problem we follow the standard LCAO-MO (Linear Combination of Atomic Orbitals-Molecular Orbitals) approach. The Hamiltonian for the 2-center muon motion problem with fixed mass-center is as follows:

$$H = -\frac{\hbar^2}{2m_\mu}\nabla_\mu^2 - \frac{Z_L e^2}{|\vec{r} - \vec{R}_L|}\left[1 - \left(1 + \frac{|\vec{r} - \vec{R}_L|}{2r_i}\right)\exp\left(-\frac{|\vec{r} - \vec{R}_L|}{r_i}\right)\right]$$

$$- \text{similar term for heavy fragment} \qquad (A.9)$$

where $\vec{r}$ is the vector for the muon location with respect to the center-of-mass of the whole system, and $\vec{R}_L$ and $\vec{R}_H$ are the positions, respectively, of light and heavy charge centers. In the 2-state basis we get the variational wave functions and corresponding energies.

$$\psi_{u(g)} = A_{u(g)}\phi_L + B_{u(g)}\phi_H \qquad (A.10)$$

$$E_{u(g)} = \frac{\beta \pm \sqrt{\alpha}}{2(1 - S^2)} + H_{HH} \qquad (A.11)$$

where

$$A_{u(g)} = \left[\frac{1 \pm \beta/\sqrt{\alpha}}{2(1 - S^2)}\right]^{1/2}$$

$$B_{u(g)} = -A_{u(g)} \cdot S \mp \left[\frac{1 \mp \beta/\sqrt{\alpha}}{2}\right]^{1/2}$$

$$\alpha = (H_{LL} + H_{HH} - 2SH_{HL})^2 - 4(1 - S^2)(H_{LL}H_{HH} - H_{HL}^2)$$

$$\beta = H_{LL} - (1 - 2S^2)\, H_{HH} - 2S \quad H_{HL}$$

$$S = \langle \phi_H | \phi_L \rangle$$

$$H_{HH} = \langle \phi_H | H | \phi_H \rangle$$

$$H_{HL} = \langle \phi_H | H | \phi_L \rangle$$

$$H_{LL} = \langle \phi_L | H | \phi_L \rangle$$

The H_{HH}, H_{HL}, H_{LL} and S integrals can be evaluated analytically and do not require numerical integration. For the case of the Gaussian trial wave function they are expressed in terms of error functions. The two variational wave functions ψ_u and ψ_g are normalized and orthogonal.

Appendix B. The muon dynamical equations in the 2-state model

We solve the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} = H\psi \tag{B.1}$$

in the 2-state model. Let

$$\psi = a_u(t)\, e^{-\frac{i}{\hbar}\int^t E_u(t')\, dt'} \psi_u + a_g(t)\, e^{-\frac{i}{\hbar}\int^t E_g(t')\, dt} \psi_g \tag{B.2}$$

Substituting into Eq. (B.1), we get two coupled equations:

$$\begin{aligned} \dot{a}_u(t) &= -a_g(t) \langle \psi_u(t) | \tfrac{d}{dt} | \psi_g(t) \rangle \exp\left[(i/\hbar) \int^t (E_u(t') - E_g(t'))\, dt'\right] \\ \dot{a}_g(t) &= -a_u(t) \langle \psi_g(t) | \tfrac{d}{dt} | \psi_u(t) \rangle \exp\left[(i/\hbar) \int^t (E_g(t') - E_u(t'))\, dt'\right] \end{aligned} \tag{B.3}$$

The time differential can be expressed as

$$d/dt = \dot{R} \cdot \vec{\nabla}_R$$

We assume no orbital angular momentum for the fission fragments. We can eliminate time from the equations by replacing dt with $dR/\dot{R}$. Hence,

$$\frac{da_u}{dR} = -a_g \langle \psi_u | \frac{\partial}{\partial R} | \psi_g \rangle \exp\left[(i/\hbar) \int_{Ro}^{R} \frac{E_u - E_g}{\dot{R}'} dR' \right]$$

$$\frac{da_g}{dR} = +a_u \langle \psi_u | \frac{\partial}{\partial R} | \psi_g \rangle \exp\left[(i/\hbar) \int_{Ro}^{R} \frac{E_g - E_u}{\dot{R}'} dR' \right] \tag{B.4}$$

Let

$$a_u = \sin\eta e^{i\delta_u}$$

$$a_g = \cos\eta e^{i\delta_g} \tag{B.5}$$

Substituting into Eq. (B.4) we get the coupled equations for η, δ_u, δ_g

$$\frac{d\eta}{dR} = -\cos\left[\delta_u - \delta_g - \int^{R} \frac{(E_u - E_g)}{\hbar \dot{R}'} dR' \right] \langle \psi_u | \frac{\partial}{\partial R} | \psi_g \rangle$$

$$\frac{d\delta_u}{dR} = -\sin\left[\delta_u - \delta_g - \int^{R} \frac{(E_u - E_g)}{\hbar \dot{R}'} dR' \right] \langle \psi_u | \frac{\partial}{\partial R} | \psi_g \rangle$$

$$\frac{d\delta_g}{dR} = \sin\left[\delta_u - \delta_g - \int^{R} \frac{(E_u - E_g)}{\hbar \dot{R}'} dR' \right] \langle \psi_u | \frac{\partial}{\partial R} | \psi_g \rangle \tag{B.6}$$

After further transformation we only need two coupled equations for the excitation probability.

$$\frac{d\eta}{dR} = -\cos\xi \langle \psi_u | \frac{\partial}{\partial R} | \psi_g \rangle$$

$$\frac{d\xi}{dR} = \frac{E_u - E_g}{\hbar \dot{R}} + 2\cot 2\eta \sin\xi \langle \psi_u | \frac{\partial}{\partial R} | \psi_g \rangle$$

$$\xi = -\delta_u + \delta_g + \int^{R} \frac{E_u - E_g}{\hbar \dot{R}'} dR' \tag{B.7}$$

In the zero initial condition ($\xi_o = 0$, $\eta_o = 0$) we have to make use of l'Hospital's rule for the second equation to determine the initial value of $d\xi/dR$

$$\left.\frac{d\xi}{dR}\right|_{\substack{\xi_o=0 \\ \eta_o=0}} = \frac{E_u - E_g}{\hbar \dot{R}}$$

The transition matrix element can be derived as follows:

$$\langle \psi_u | \frac{\partial}{\partial R} | \psi_g \rangle = \frac{1}{A_u} \frac{\partial A_g}{R} + \frac{Bg}{A_u(1 - S^2)} \frac{\partial S}{\partial R} - \frac{1}{\sqrt{1 - S^2}} \langle \phi_H | \frac{\partial}{\partial R} | \phi_L \rangle$$

The matrix elements $\langle \phi_H | \frac{\partial}{\partial R} | \phi_L \rangle$ can be derived analytically. They differ for Gaussian and exponentional functions, but no numerical integration is required.

Since ψ_u and ψ_g are orthogonal, the probability of the muon being in the upper level is $p = \sin^2 \eta$.

References

1. D. Ganzorig et al., Phys. Lett. 77B (1978) 257 and 78B (1978) 41.

2. G. S. Blanpied et al., Phys. Rev. C18 (1978) 1436.

3. W. D. Fromm et al., Nucl. Phys. A278 (1977) 387.

4. C. S. Wu, et al., Phys. Rev. Lett. 24 (1970) 604 and Nucl. Phys. A247 (1975) 452.

5. S. Kaplan, Nucl. Phys. 40 (1963) 54.

6. S. Polikanov, Intern. Atomic Energy Agency preprint IAEA-SM/23-EI (1979).

7. G. Leander and P. Möller, Phys. Lett. 57B (1975) 245.

8. V. A. Karnaukhov, Yadernaya Fizika 28 (1978) 1204 (in Russian).

9. P. Olanders, S. G. Nilsson, and P. Möller, Lund Institute of Technology preprint (1979).

10. K. Ford and Wills, Nucl. Phys. 35 (1962) 295.

11. J. C. Slater, "Quantum Theory of Molecules and Solids," V. 1, McGraw-Hill, New York (1963).

12. P. Möller J. R. Nix, International Atomic Energy Agency Report IAEA-SM-174/202, Symposium Physics and Chemistry of Fission, Rochester, NY, August 1973.

13. E. K. Hyde, "The Nuclear Properties of the Heavy Elements, v. III, Fission Phenomena," Prentice-Hall, Englewood Cliffs, New Jersey (1964), p. 109.

Figure Captions

Fig. 1. Comparison of μ^- binding energy calculations as a function of separation distance between two Z = 46 nuclei. The lower family of curves is for the 1s σ_g (symmetric) orbital and the upper for the 1s σ_u (antisymmetric). The lowest (solid) curve in each family is the exact solution, point charges. The middle two curves are Gaussian variational wave functions for point charges (see text). The top (long dashed) curve in each family is a variational calculation for extended exponential nuclear charge distributions.

Fig. 2 and Fig. 3. Mu-minus binding energies, variational wave functions, for unequal fission fragment charges.

Fig. 4. Dynamic muon fractional probability bound to light fragment as a function of fission fragment separation distance. Boundary conditions and other parameters of Olanders et al.[9] are used as a test case.

Fig. 5. Muon light fragment probability curves for the most probable charge division in U^{238} fission.

Fig. 6. Probability curves for more realistic viscous descent from saddle to scission point for fissioning system.

Fig. 7. Probability curves for very asymmetric fission.

Fig. 8. Probability curves for near-symmetric fission.

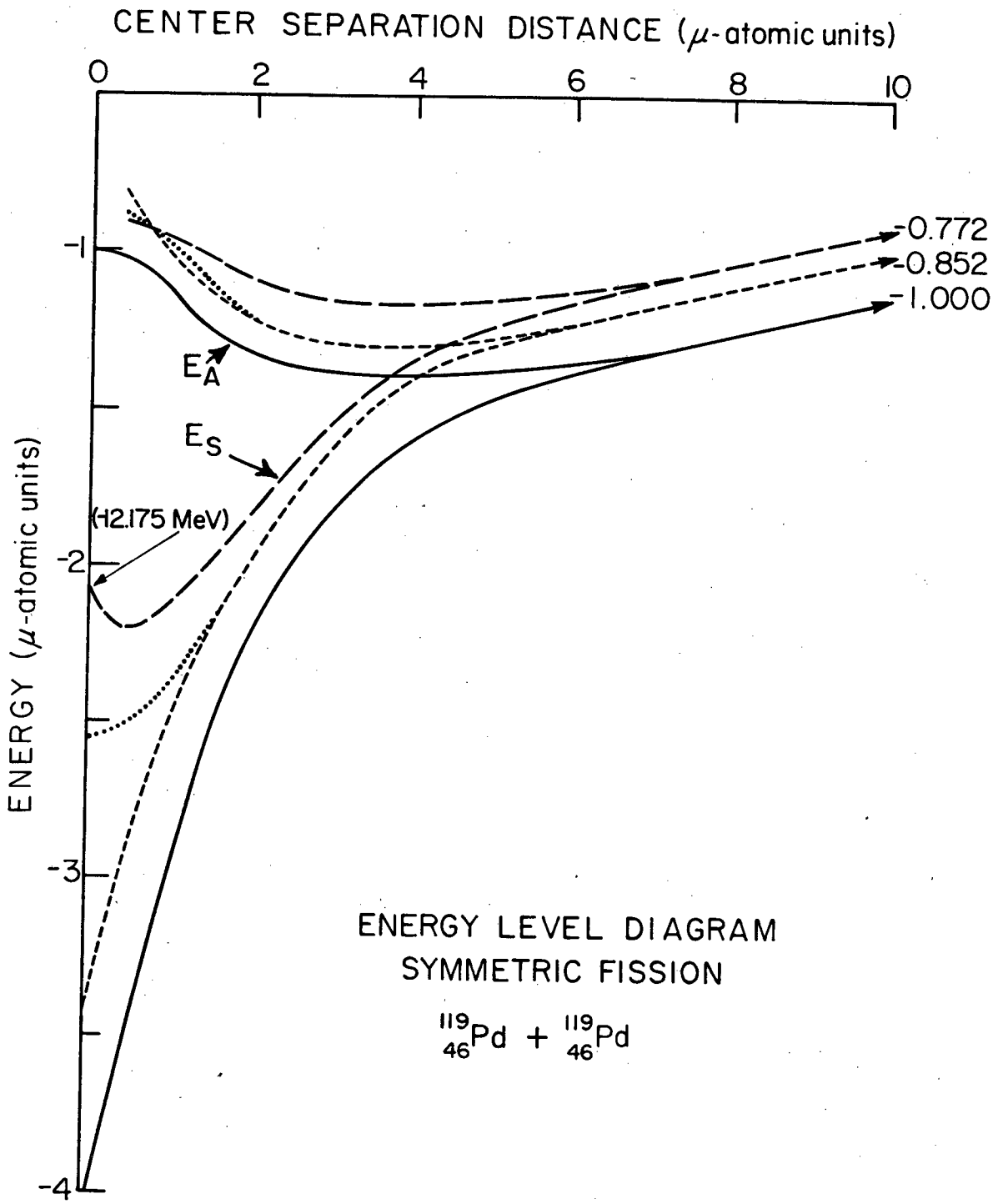
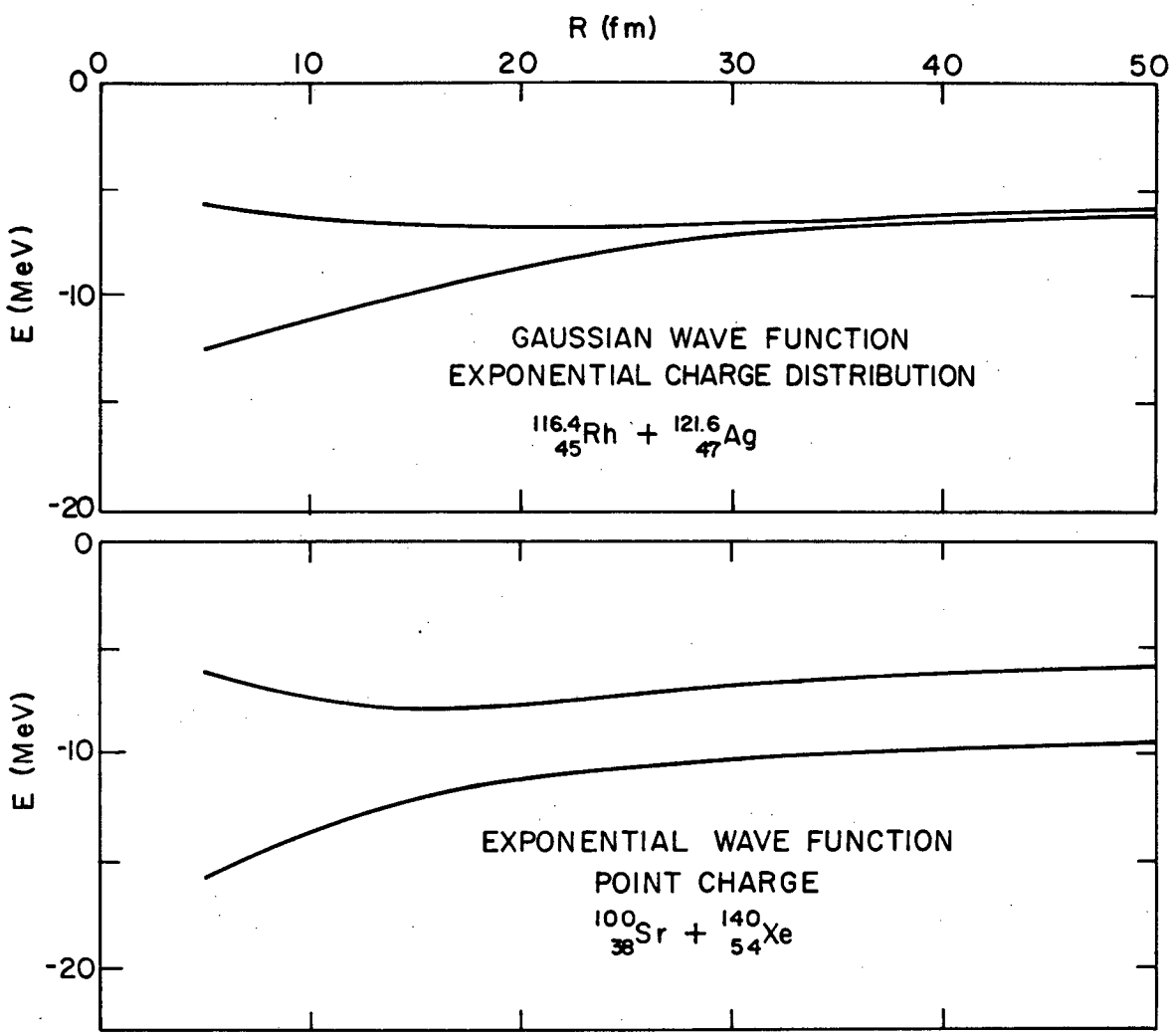

CENTER SEPARATION DISTANCE (μ-atomic units)
0
2
4
6
8
10
-1
-2
-3
-4
ENERGY (μ-atomic units)
-0.772
-0.852
-1.000
E_A
E_S
(12.175 MeV)
ENERGY LEVEL DIAGRAM
SYMMETRIC FISSION
$^{119}_{46}Pd + ^{119}_{46}Pd$
XBL 799-7039

Fig. 1



XBL 799-7034

Figs. 2, 3

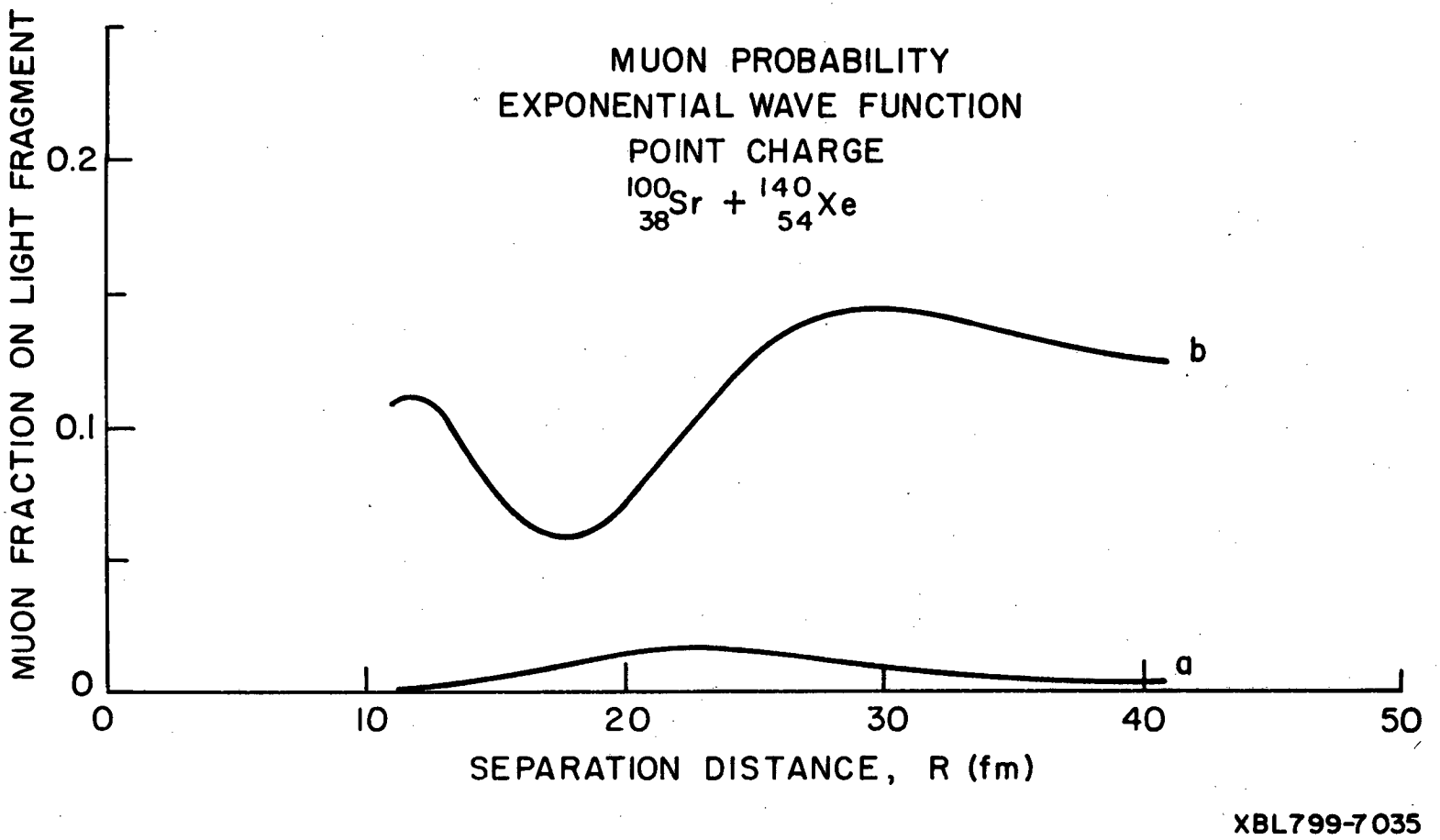
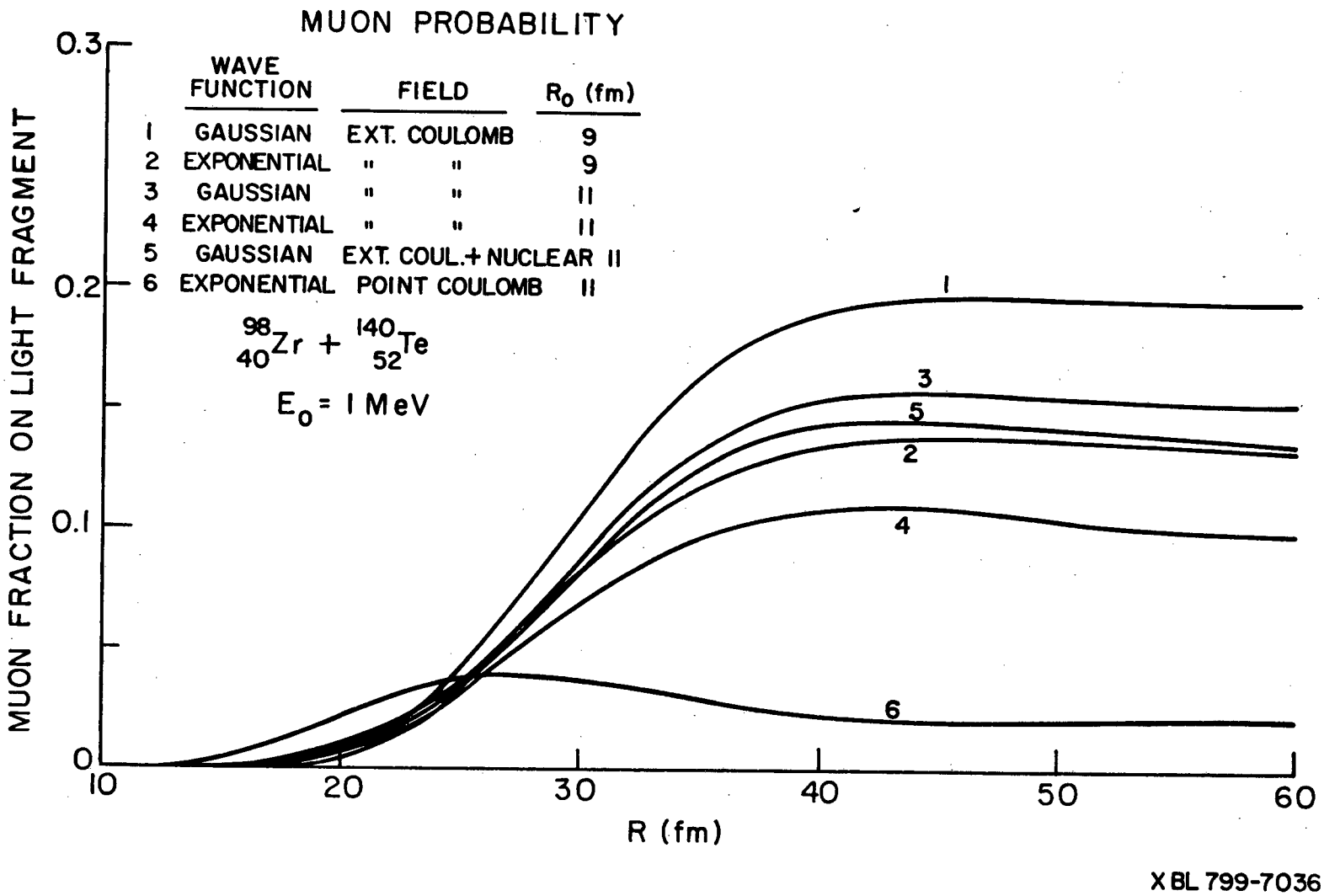

MUON PROBABILITY
EXPONENTIAL WAVE FUNCTION
POINT CHARGE
$^{100}_{38}Sr + ^{140}_{54}Xe$
MUON FRACTION ON LIGHT FRAGMENT
0.2
0.1
0
b
a
0
10
20
30
40
50
SEPARATION DISTANCE, R (fm)
XBL799-7035

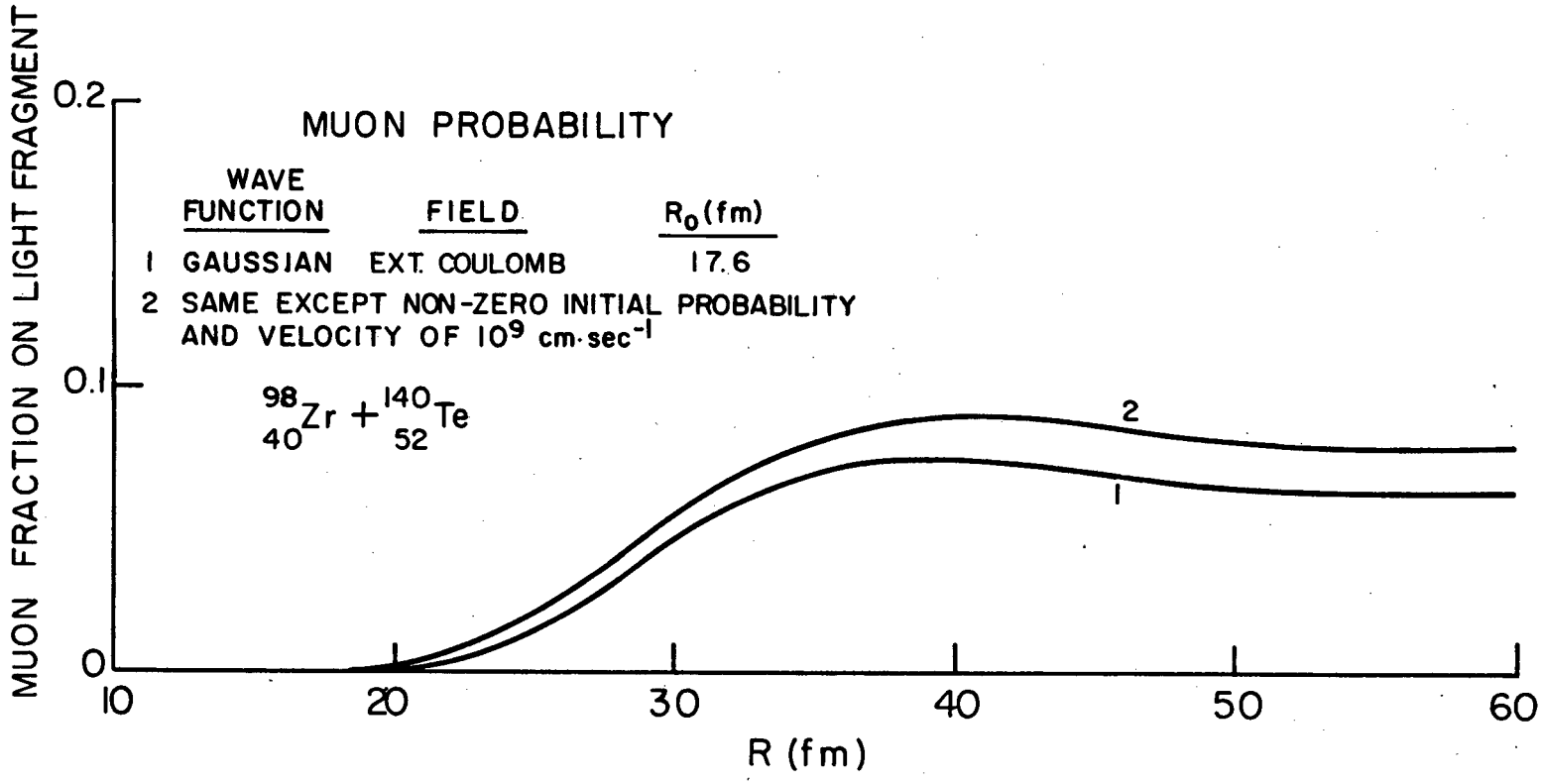
Fig. 4



XBL 799-7036

Fig. 5

0 0 0 5 0 0 4 7 4



XBL 799-7112

Fig. 6

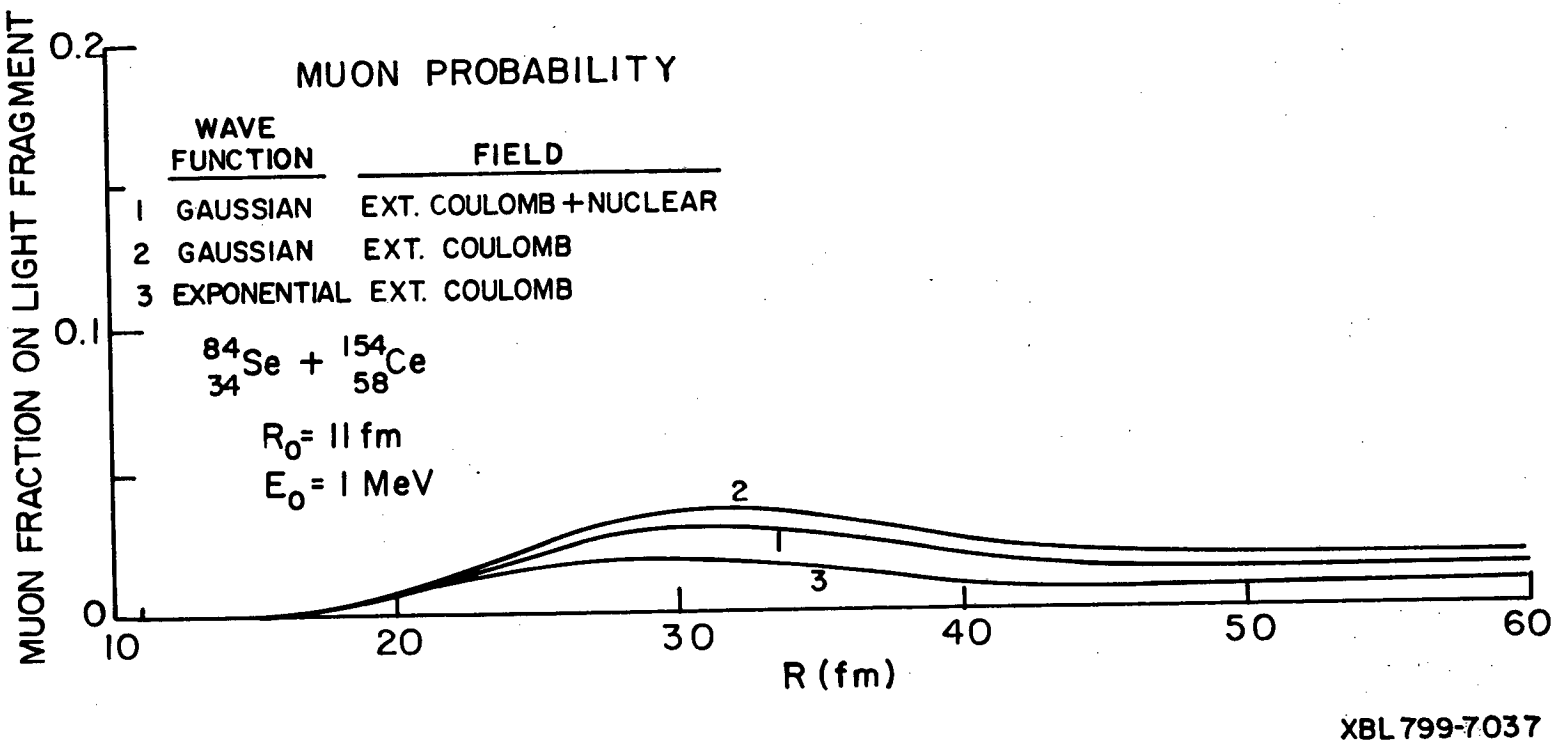

MUON PROBABILITY
WAVE FUNCTION
FIELD
1 GAUSSIAN EXT. COULOMB + NUCLEAR
2 GAUSSIAN EXT. COULOMB
3 EXPONENTIAL EXT. COULOMB
$^{84}_{34}Se + ^{154}_{58}Ce$
R_0 = 11 fm
E_0 = 1 MeV
MUON FRACTION ON LIGHT FRAGMENT
0.2
0.1
0
10
20
30
40
50
60
R (fm)
2
1
3
XBL 799-7037

Fig. 7

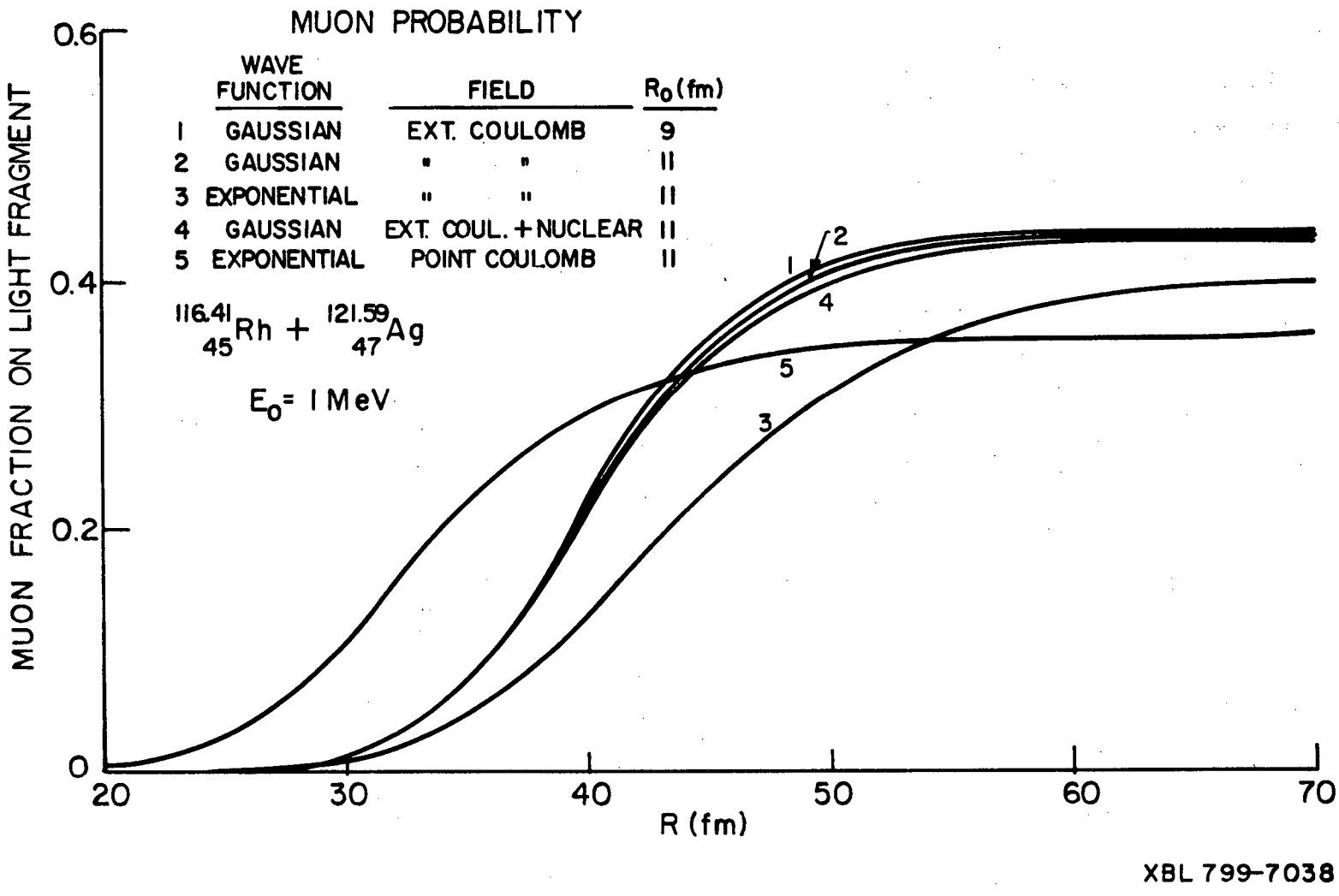

MUON PROBABILITY
WAVE FUNCTION
FIELD
R_0(fm)
1 GAUSSIAN EXT. COULOMB 9
2 GAUSSIAN " " 11
3 EXPONENTIAL " " 11
4 GAUSSIAN EXT. COUL. + NUCLEAR 11
5 EXPONENTIAL POINT COULOMB 11
$^{116.41}_{45}Rh + ^{121.59}_{47}Ag$
E_0 = 1 MeV
MUON FRACTION ON LIGHT FRAGMENT
0.6
0.4
0.2
0
20
30
40
50
60
70
R (fm)
1
2
3
4
5
XBL 799-7038

Fig. 8

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