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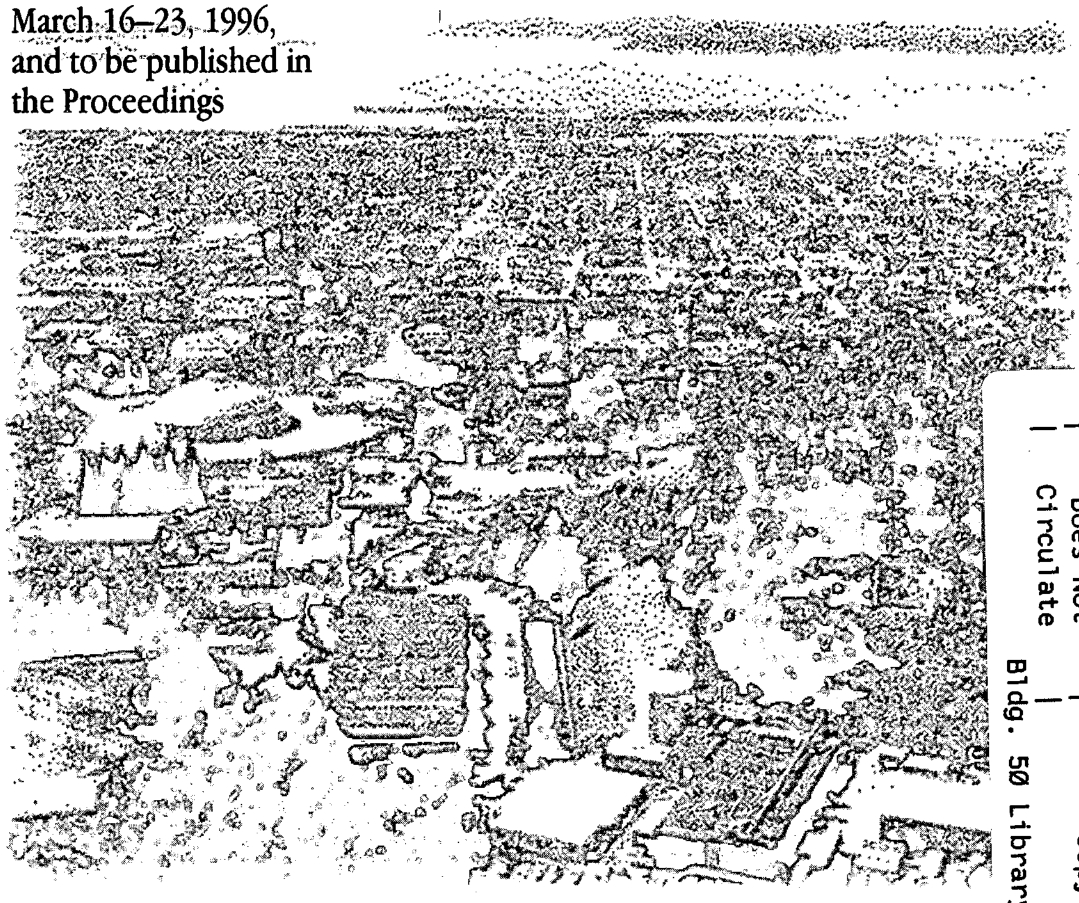

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Open universes from bubbles: an introduction and update

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Abstract

This is an introduction to models of open universes originating from bubbles, including a summary of recent theoretical results for the power spectrum.

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A flat ($\Omega_{tot} = 1$) universe has long been considered a generic prediction of inflationary [1] models. It has recently been demonstrated that inflation can also produce viable open ($\Omega < 1$) universes [2]. As the data on Ω has not yet converged, further investigation of these models is worthwhile. In the following, why and how bubbles give open universes is reviewed, the main features of the new models are outlined and current results and questions are summarized. Due to space limitations, this is necessarily just an overview, and readers are referred to the cited papers and their references for more in depth discussion and comprehensive referencing.

The basic reason bubbles give an open universe can be illustrated with empty Minkowski space, which does not have a unique coordinate system leading to a metric of the form

$$ds^2 = d\eta^2 - a^2(\eta)d\sigma^2 \ . \tag{1}$$

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In empty space ($\rho = 0$), with zero cosmological constant ($\Lambda = 0$), the curvature k obeys

$$H^2 = \left(\frac{\dot{a}(\eta)}{a(\eta)}\right)^2 = \frac{-k}{a^2(\eta)} \,. \tag{2}$$

Consider the two coordinate systems for Minkowski space shown in figure one. On the left, lines of constant time are shown for a coordinate system with metric ($x^2 = x_1^2 + x_2^2 + x_3^2$)

$$ds^2 = dt^2 - dx^2 \,, \tag{3}$$

so one can read off that $a(t) = 1$. As a result, for this coordinate system,

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = 0 = -k \,, \tag{4}$$

the universe has $k = 0$ and is therefore spatially flat.

A second coordinate system (T, χ) is shown on the right. Here, lines of constant time T are shown. In terms of (x, t), we have $T = \sqrt{t^2 - x^2}$, $\tanh \chi = x/t$. The metric becomes

$$ds^2 = dT^2 - T^2 d\chi^2 \,. \tag{5}$$

So in this case, we identify $a(T) = T$. Then,

$$\left(\frac{\dot{a}(T)}{a(T)}\right)^2 = \frac{1}{T^2} = -\frac{k}{T^2} \;\Rightarrow k = -1 \,, \tag{6}$$

a spatially open universe.

In empty space, there is no reason to prefer either coordinate system. However, if there is matter present, one or the other might be preferred. For example, if the background matter distribution is homogeneous in space only in a particular coordinate system, the metric has the form eqn.(1) only for that choice of coordinates. A bubble is said to create an open universe because inside it the (T, χ) coordinate system is preferred.

To get a bubble, start with a system (here described by a field ϕ) stuck in a false vacuum. A bubble forms when a region of space, the bubble interior,

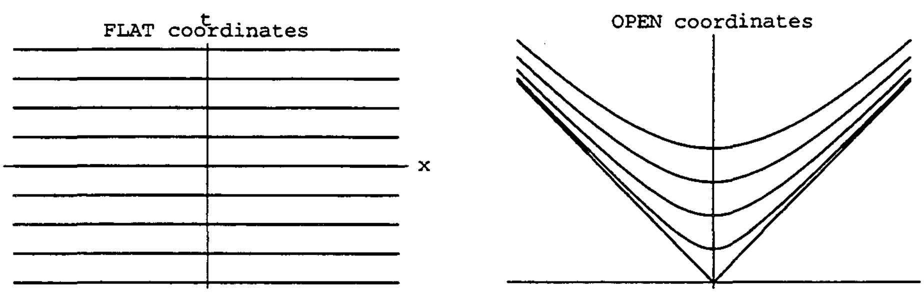

Figure 1: Lines of constant time in two coordinate systems for Minkowski space

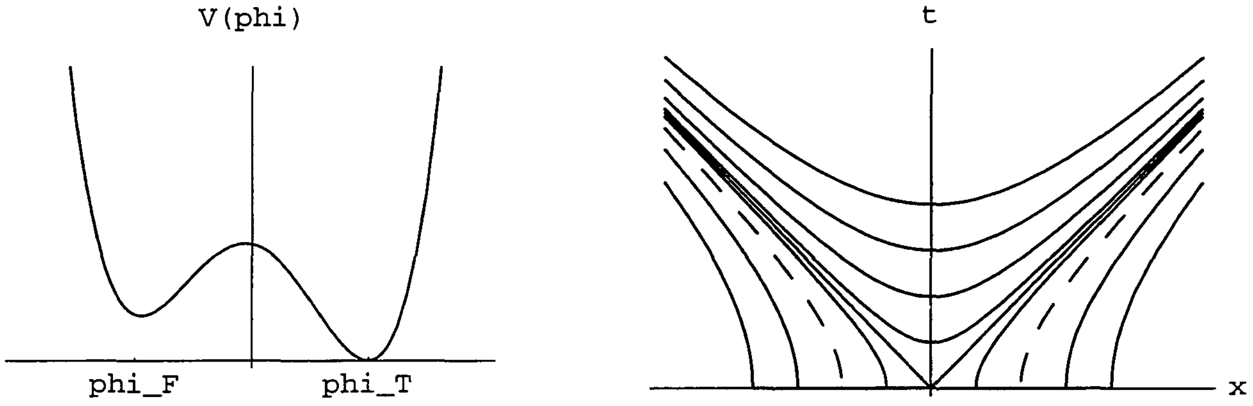

Figure 2: On the left, a potential with a false and true vacuum. On the right, lines of constant ϕ after the bubble nucleates.

tunnels to the true vacuum. An example of a potential where this could happen is shown on the left of figure two. After tunneling, there is a true vacuum interior, where $\phi = \phi_T$, a bubble (domain) wall, and a false vacuum exterior where $\phi = \phi_F$. The lowest energy solution for tunneling in Minkowski space [3] has spherical $O(4)$ symmetry, $\phi = \phi(x^2 - t^2)$. As the matter (the field ϕ) is constant on surfaces of constant $t^2 - x^2 = T^2$, it is natural to use the open coordinates [4], where the matter is homogeneous in space.

On the right side of figure two is a spacetime diagram showing lines of constant ϕ. The dashed line is the bubble wall, and the horizontal axis is the nucleation time. As a function of the flat coordinate time t, the bubble wall moves out. Classically the energy difference between the false and true vacua (the latent heat) goes into accelerating the wall outwards. Inside the future light cone of the center of the bubble is a patch with open coordinates. As the light cone is $T = 0$ in this coordinate system, and the bubble wall in this example is exterior to the light cone (approaching it at large times), the bubble wall is "before" $T = 0$.

This picture can be used to generate a candidate for the early universe by making some modifications. Gravity must be included, the field ϕ is taken to be part of the inflationary potential, and the bubble nucleates in a vacuum, the de Sitter vacuum, $\phi \sim const$. Nucleating a bubble in de Sitter space also means that the global structure of spacetime is more complicated–because the spacetime is expanding in the false vacuum, the future light cone of the bubble does not eventually cover the whole future of the space (see, e.g. [5]). Many of the calculations require normalizing on a Cauchy surface (a spacelike hypersurface which every non-spacelike curve intersects exactly once [6]) and thus use information exterior to the open universe.

The interior of the vacuum bubble has energy density depending on the true vacuum potential, $V(\phi_T)$. If $V(\phi_T) \approx 0$, then after ϕ has tunneled, the universe is effectively empty. All the energy density is in the bubble wall and not in the resulting open universe, which has $\Omega \sim 0$ and stays that way. Under our

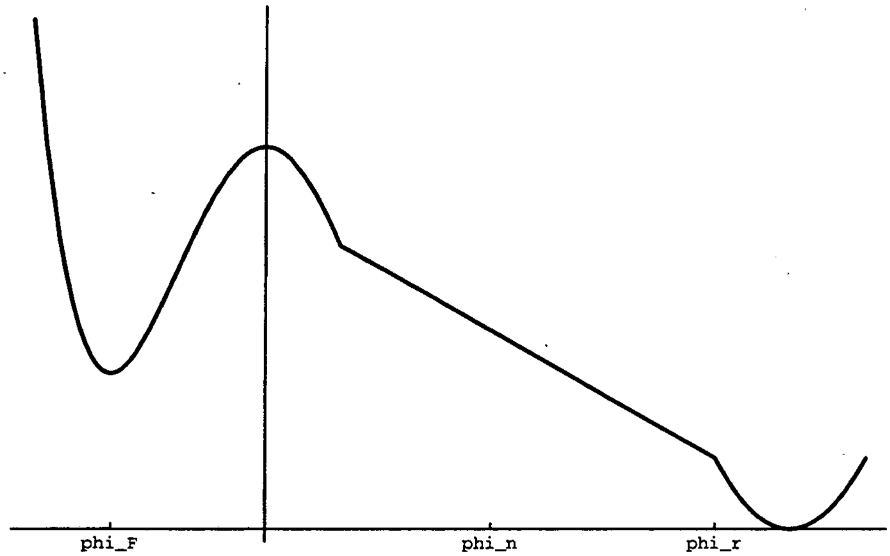

Figure 3: A potential with the features that give a viable open universe inflationary model.

assumptions, to get a universe with something in it means that $V(\phi_T) \neq 0$. If $V(\phi_T) \neq 0$, then the resulting vacuum is also an expanding de Sitter space, that is, it is still inflating. So one needs a bubble plus inflation before and after nucleation. If one has inflation but no bubble, the amount of inflation providing sufficient homogeneity to agree with observation drives $\Omega \rightarrow 1$ (for a nice summary in the context of bubbles, see [7]).

Bucher, Goldhaber and Turok [2] combined bubble formation with inflation to give an open universe. They use four stages, corresponding to potential features sketched in figure 3:

- First, on the far left, the system is trapped in a potential well (at $\phi = \phi_F$), a false vacuum. Inflation occurs, driving the system to the Bunch-Davies vacuum (the natural vacuum here, respecting the symmetries and going over at short distances to the Minkowski space vacuum) and wiping out inhomogeneities.
- Then ϕ tunnels to $\phi \sim \phi_n$, resetting $\Omega \sim 0$;
- Next, on the approximately linear part of the potential, from $\phi \sim \phi_n$ to $\phi \sim \phi_r$, slow roll inflation drives Ω up to a specific value less than or equal to one.
- Finally, on the far right, at $\phi \sim \phi_r$, ϕ decays, leading to the usual reheating after inflation.

The endpoint of the tunneling $\phi \sim \phi_n$ will be called the true vacuum (since $\phi \sim const.$), even though it is also the starting point of the slow roll inflationary stage.

There are constraints on the potential so that this picture is consistent. The potential $V(\phi)$ has to be such that the tunneling rate is rare, so that tunneling from $\phi \sim \phi_F$ only occurs after a long period of inflation has erased initial conditions. The false vacuum well cannot be too flat or ϕ will stochastically go over the barrier rather than tunneling through it (the Hawking-Moss instanton) [8], leading to density perturbations which are too large [9]. The potential $V(\phi)$ must be tuned to within a few percent so that at the end of the slow roll stage $0.1 \leq \Omega \leq 0.9$.

A simple polynomial potential (up to order ϕ^4) does not satisfy these conditions [10], although in principle an effective potential could arise in a supergravity theory with the right features. One can also use more fields, driving inflation by one field while the other one tunnels. Some of the two-field variants of the model lose the ability to predict a definite value of Ω [11], and lead to more questions which have been studied extensively but are outside the scope of this summary.

The density perturbations in these models are a modification of those in $\Omega = 1$ inflationary models. For $\Omega = 1$ inflationary models, inflation drives the universe to the vacuum, and the quantum fluctuations of the inflaton in this vacuum become seeds for structure formation once they re-enter the horizon. In these open universe bubble models, there are vacuum fluctuations (from the false vacuum before tunneling) outside the bubble wall. These evolve through the bubble wall, a time dependent background, to reach the open universe. There is a continuum of these modes and sometimes (depending on properties of the wall and the mass in the false vacuum) a discrete mode. The bubble wall itself also has fluctuations. The calculations incorporate methods for bubbles in Minkowski space [12, 13], and techniques for including gravity as in [14]; see these papers and references therein for background. Various choices for the true and false vacuum masses and the wall profile have been considered.

Calculations so far have been for vacuum fluctuations due to one field, in two field models the second field has been frozen out. Thus one solves

$$\left(\Box - \frac{\delta^2 V(\phi)}{\delta\phi^2}|_{\phi=\phi_{\mathrm{bkgd}}}\right)\delta\phi = 0 \tag{7}$$

The gauge invariant gravitational potential produced by the continuous modes has been found assuming a constant value of H in the resulting de Sitter space. The power spectrum has been calculated for a thin wall with arbitrary false vacuum mass $M^2 \geq 2H^2$ and zero radius [15] and nonzero radius [16, 17]. It has also been found for a large false vacuum mass and varying wall profile (with the restriction that the wall is completely exterior to the light cone of the bubble center, the open universe) [16]. In all of these cases, the contribution to the fluctuations is within an envelope between $\coth \pi p/2$ and $\tanh \pi p/2$ times a scale invariant spectrum. (The correspondence with wavelength is $p = k/(H_0\sqrt{1-\Omega_0})$.) Thus, except at very large scales, where cosmic variance interferes with their measurement, these spectra all coincide.

In open de Sitter spacetime, when there is no bubble present, the Bunch Davies vacuum fluctuations of a field of mass $M^2 < 2H^2$ has a continous spectrum of fluctuations plus a discrete mode [18]. This discrete mode was shown to be part of the complete basis of states [18] and is normalizable on a Cauchy surface, but not inside the open universe (which does not contain a Cauchy surface). In the presence of a bubble, this discrete 'supercurvature' mode with $-1 \leq p^2 \leq 0$ may remain, depending upon the false vacuum mass and the wall profile.

If the field providing density perturbations does not change its mass across the wall (as can happen in two field models), and has $M^2 < 2H^2$, this mode appears [19, 20]. Its contribution to density fluctuations has been found and used to constrain models when combined with the CMB for various cases [19, 20, 21, 22]. It does not seem to give a very strong constraint. However, it has been argued that this state's contribution can be enhanced significantly when the ratio of false to true vacuum energy is large, ruling out some models [22]. There is not yet complete agreement on this. If there is a mass change across the wall [16], an analogue of this state may persist. When matched across the bubble wall, the original vacuum discrete mode does not automatically remain normalizable, but a mode with some other value of $-1 \leq p^2 < 0$ may become normalizable instead. The effects of this new mode have been found in a two field model for a thin wall [16]. This mode disappears if the false vacuum mass is large enough (as argued in [16], the false vacuum mass is necessarily large for most one field models considered so far, so that it does not appear there).

There are also fluctuations of the wall itself, contributing mostly at large scales, and considered in [23, 10, 13, 24, 21, 16]. Requiring these fluctuations to be small constrains models. Schematically, the allowed models have a high barrier between the false and true vacuum, producing a large surface tension and making it energetically unfavorable for the wall to fluctuate.

In summary, the continuum modes reproduce a scale invariant spectrum starting at very large scales. The vacuum supercurvature mode or its analogue (when present) and the wall fluctuation (also a supercurvature mode) have been used so far to constrain possible potentials by requiring that their contribution be small.

Many fits to the data [25] have been made for open universes assuming a scale invariant [26] spectrum, and, except at large scales, the models considered so far also have this spectrum, and so carry over. Many things remain to be done, including adding gravity waves and tilt, and motivating models from particle theory.

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