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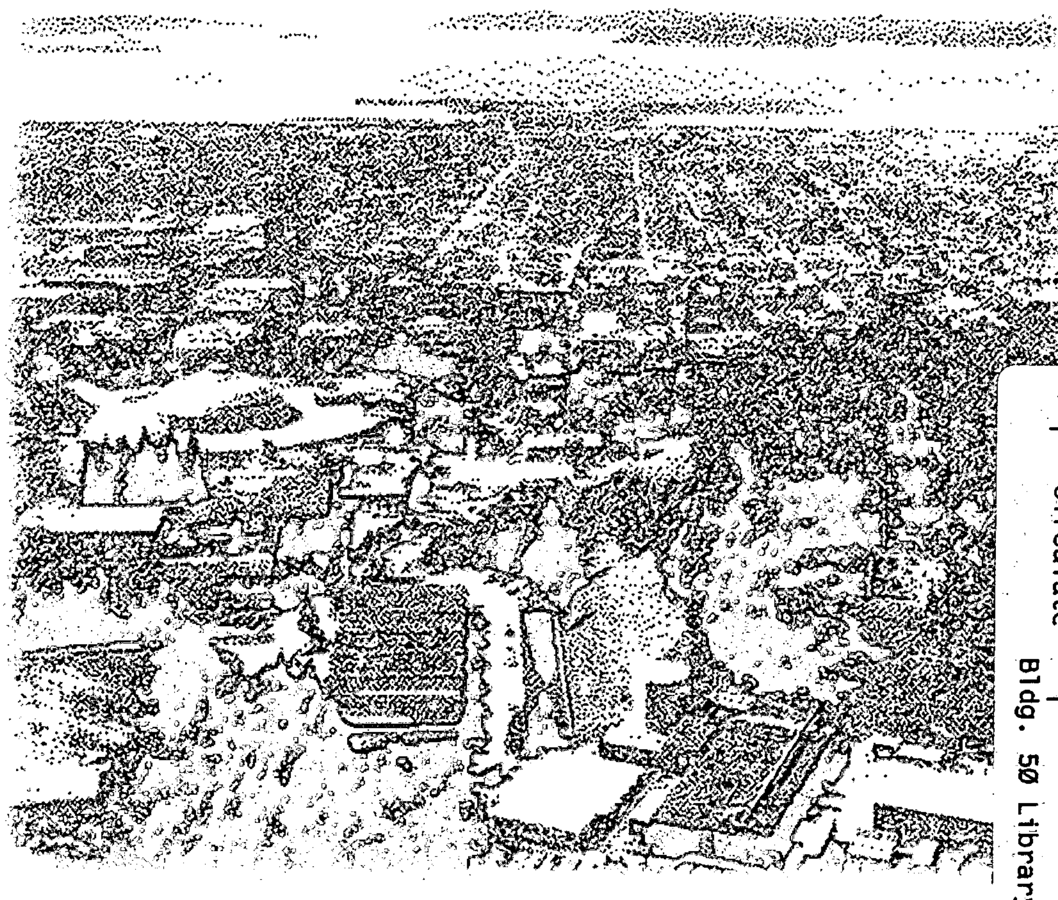

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Gauge invariant formulation of strong WW scattering [1]

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Abstract

Models of strong WW scattering in the s-wave can be represented in a gauge invariant fashion by defining an effective scalar propagator that represents the strong scattering dynamics. The $\sigma(qq \rightarrow qqWW)$ signal may then be computed in U-gauge from the complete set of tree amplitudes, just as in the standard model, without using the effective W approximation (EWA). The U-gauge "transcription" has a wider domain of validity than the EWA, and it provides complete distributions for the final state quanta, including experimentally important jet distributions that cannot be obtained from the EWA. Starting from the usual formulation in terms of unphysical Goldstone boson scattering amplitudes, the U-gauge transcription is verified by using BRS invariance to construct the complete set of gauge and Goldstone boson amplitudes in R_ξ gauge.

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Introduction

The traditional approach to strong WW scattering begins by assuming a model for the scattering of the corresponding unphysical Goldstone bosons, and then uses the equivalence theorem (ET) and the effective W approximation (EWA) to compute the cross section for strong production of the longitudinally polarized gauge bosons, $\sigma(qq \to qqW_LW_L)$.[1] In this paper I present a gauge invariant formulation for s-wave strong W_LW_L scattering amplitudes (L denotes longitudinal polarization) that allows the $\sigma(qq \to qqW_LW_L)$ signal to be computed directly from the complete set of tree amplitudes without using the EWA. The strong dynamics is carried by an effective scalar propagator, and the complete $qq \to qqWW$ tree amplitude can be computed in any covariant R_ξ gauge or in unitary gauge. This formulation is more accurate and provides more information than the traditional method using the EWA.

Both approaches begin with a model Goldstone boson scattering amplitude, $\mathcal{M}_R^X(ww \to ww)$, required to obey unitarity and the low energy theorems of chiral symmetry. (X labels the model, R denotes a covariant renormalizable gauge, most appropriately Landau gauge, and w_i is the unphysical Goldstone boson corresponding to gauge boson W_{iL}.) The equivalence theorem[2] asserts the equality of $\mathcal{M}_R^X(ww)$ at high energy to the corresponding amplitude of longitudinally polarized gauge bosons W_L,

$$\mathcal{M}^X(W_L(p_1)W_L(p_2)\ldots) = \mathcal{M}_R^X(w(p_1)w(p_2)\ldots)_R + \mathrm{O}\left(\frac{m_W}{E_i}\right). \qquad (1)$$

In the traditional approach the subprocess cross section $\sigma(W_LW_L \to W_LW_L)$ is convoluted with the effective W_LW_L luminosity[3] (which is a function of $z = s_{WW}/s_{qq}$) to obtain the cross section for the WW fusion subprocess,

$$\sigma(qq \to qqW_LW_L) = \int dz \frac{d\mathcal{L}}{dz}\sigma(W_LW_L \to W_LW_L). \qquad (2)$$

The traditional method is simple and effective but it neglects the transverse momentum of the final state q jets and the WW diboson. Knowledge of these transverse momentum distributions is important experimentally, e.g., to determine the efficiency of jet tag and veto detection strategies. In Higgs boson models the p_T distributions are readily obtained by computing the complete set of tree diagrams for $qq \to qqWW$, thus avoiding the EWA. It would be useful and interesting to compute strong WW scattering cross sections in the same way. A U-gauge "transcription" to accomplish this was presented previously and verified by explicit computation for specific examples[4]. Here an algorithm is presented for s-wave scattering models to construct the complete family of gauge and Goldstone boson amplitudes in R_ξ gauge ($WWWW$, $WWWw$, $WWww$, $Wwww$) which follow from the initial Goldstone boson model amplitude $\mathcal{M}_R^X(ww \to ww)$ by BRS invariance. The construction allows the gauge boson amplitude to be evaluated in any R_ξ gauge and in particular validates the U-gauge transcription.

In addition to providing information about the final state that is lost in the EWA, the new method is also more accurate since it correctly sums the Higgs sector signal and gauge sector background amplitudes coherently, while the EWA neglects the interference terms.[3] The transcription has other interesting consequences that will be discussed elsewhere: it reveals the "K-matrix" model, an *ad hoc* construction borrowed from nuclear physics to implement partial wave unitarity and chiral symmetry, as a (very!) nonstandard Higgs boson model, and allows a direct estimate of the effect of strong WW scattering on low energy radiative corrections.

The U-gauge transcription

The equivalence theorem, equation (1), already relates the gauge and Goldstone boson amplitudes, but not in a way that can be used to extract the WW fusion amplitude $\mathcal{M}(qq \to qqWW)$ for strong WW scattering. The first step is to observe that to leading order in the $SU(2)_L$ coupling g the on-shell WW amplitude is the sum of gauge sector and Higgs sector terms, e.g., in U-gauge

$$\mathcal{M}^X_{\text{Total}}(WW \to WW) = \mathcal{M}_{U,\text{Gauge}}(WW \to WW) + \mathcal{M}^X_{U,\text{H}}(WW \to WW). \quad (3)$$

and that at high energy, $E \gg m_W$, the gauge sector amplitude for longitudinal modes is dominated by its "bad high energy behavior", a term growing like E^2 which is at the same time the low energy theorem amplitude $\mathcal{M}_{\text{LET}}$ of a strongly coupled Higgs sector[5, 1]. Using the equivalence theorem the U-gauge Higgs sector amplitude for model X is then

$$\mathcal{M}^X_{U,\text{H}}(W_LW_L \to W_LW_L) = \mathcal{M}^X_R(ww \to ww) - \mathcal{M}_{\text{LET}} + \text{O}(g^2, \frac{m_W}{E}), \quad (4)$$

[3]The interference term is important when the gauge sector background is large, for instance, near Coulomb singularities.

which is just the Goldstone boson model amplitude $\mathcal{M}_R^X(ww)$ with its leading threshold behavior subtracted.

This is still not in a form that can be readily embedded in $\mathcal{M}(qq \to qqWW)$. To proceed we limit the discussion to s-wave amplitudes and define an effective "Higgs" propagator $P_X(s)$ by using the standard model Higgs sector amplitude for $W^+W^- \to ZZ$ as a "template" for the effective theory,

$$\mathcal{M}_{U,\mathrm{H}}^X(W_LW_L \to W_LW_L) = -g^2 m_W^2 \epsilon_1 \cdot \epsilon_2 \; \epsilon_3 \cdot \epsilon_4 \; P_X \tag{5}$$

where 1,2 denote the initial state bosons and 3,4 the final state. Here $W_LW_L \to W_LW_L$ represents generically the two channels with s-wave threshold behavior,[4] $W_L^+W_L^- \to Z_LZ_L$ and $W_L^+W_L^+ \to W_L^+W_L^+$. For W^+W^+ this is a big departure from the standard model since the s-channel exchange carries electric charge $Q = +2$, an effective "H^{++}" exchange.

For simplicity I assume the weak gauge group is just $SU(2)_L$ so that $m_W = m_Z = gv/2$. (I have verified that the conclusions do not depend on this assumption.) Then for $E \gg m_W$ and up to corrections of order g^2 the effective propagator is

$$P_X(s) = -\frac{v^2}{s^2}(\mathcal{M}_R^X(ww \to ww) - \mathcal{M}_{\mathrm{LET}}). \tag{6}$$

The low energy theorem[5, 1] amplitudes are

$$\mathcal{M}_{\mathrm{LET}} = \eta \, \frac{s}{v^2} \tag{7}$$

where $\eta = +1$ for $W^+W^- \to ZZ$ and $\eta = -1$ for $W^+W^+ \to W^+W^+$. Notice that $\mathcal{M}_{\mathrm{LET}}$ contributes $\pm 1/s$ to P_X, corresponding to a massless scalar pole, making explicit the connection between the spontaneously broken symmetry that implies $\mathcal{M}_{\mathrm{LET}}$ and the cancellation of the bad high energy behavior by Higgs boson exchange. The residual contribution to P_X from $\mathcal{M}_R^X$ carries the model dependent strong interaction dynamics.

With the effective propagator P_X we can formulate the U-gauge transcription in a way that allows us to embed $\mathcal{M}_{U,\mathrm{H}}^X(W_LW_L)$ into $\mathcal{M}(qq \to qqWW)$. The prescription is simple: compute the usual gauge sector tree diagrams for

[4]Within these channels the restriction to s-wave models is not very onerous, since in these channels the LHC and electron colliders with energy $\lesssim$3 TeV will only probe energies for which the $ff \to ffWW$ signals are dominantly in the WW s-wave.

$\mathcal{M}(qq \to qqWW)$ in U-gauge but replace the Higgs boson exchange diagram(s) by s-channel exchange of P_X with the WW"H" vertex given by $gm_W\ g^{\mu\nu}$ as in equation (5). Here we make the usual, unavoidable extrapolation, required in any approach to strong WW scattering, from massless (in Landau gauge) Goldstone boson to m_W in the on-shell gauge boson amplitude to space-like $-q^2 \simeq O(m_W^2)$ for the initial state virtual W's in the WW fusion amplitude. As always, the extrapolation contributes to the inevitable $O(m_W/E)$ correction.

BRS and gauge invariance

In [4] the U-gauge transcription was verified by explicit calculation for the K-matrix model and the heavy Higgs boson standard model in the W^+W^+ channel. We now demonstrate its validity in general for s-wave scattering by constructing a BRS invariant[6] set of amplitudes that relate the input model $\mathcal{M}_R^X(ww \to ww)$ to the corresponding gauge boson amplitude $\mathcal{M}^X(W_LW_L \to W_LW_L)$. Beginning from the Goldstone boson amplitude $\mathcal{M}_R^X(ww \to ww) = < wwww >$ we use BRS invariance to construct the family of amplitudes $< wwwW >$, $< wwWW >$, $< wWWW >$ and $< WWWW >$ in the generalized R_ξ gauge. The gauge boson amplitude $\mathcal{M}^X(WW \to WW)$ is then explicitly gauge invariant (ξ independent) and for the longitudinal modes is precisely the previously formulated U-gauge transcription. BRS invariance is verified explicitly for the *amplitudes*, even though there may ($W^+W^- \to ZZ$) or may not ($W^+W^+ \to W^+W^+$) be an underlying effective Lagrangian.

The construction of the BRS invariant set of amplitudes is accomplished by following a Feynman diagram algorithm, using as a template the set of diagrams for $W^+W^- \to ZZ$ in the standard model. That is, we write each amplitude ($< wwww >$, $< wwwW >$, $< wwWW >$, $< wWWW >$ and $< WWWW >$) as a sum of terms corresponding to the tree Feynman diagrams for the $W^+W^- \to ZZ$ channel in the standard model. By maintaining the diagrammatic form of the amplitude and the essential relationships between vertices and propagators as they are in the standard model, we automatically preserve BRS invariance. This $W^+W^- \to ZZ$ standard model template is applied to strong scattering in the $W^+W^- \to ZZ$ channel and, less obviously, also to the $W^+W^+ \to W^+W^+$ channel.

The diagrammatic algorithm is specified by the Feynman rules for vertices and propagators. The WWH vertex[5] and the propagator $P_X(s)$ were defined already in the U-gauge transcription, equations 5-6. The three and four gauge boson vertices and the propagators of the gauge and unphysical Goldstone bosons are determined by gauge sector dynamics and keep their standard model values.

The quartic Higgs sector coupling λ_X and the Goldstone-Higgs wwH vertex are related as in the standard model, $\lambda_{wwH} = -2\lambda_X v$. The quartic coupling λ_X is then determined by expressing the input model $\mathcal{M}_R^X(ww \to ww)$ as the sum of the four-point contact interaction and s-channel Higgs exchange amplitude,

$$\mathcal{M}_R^X(ww \to ww) = -2\lambda_X \eta - (2\lambda_X v)^2 P_X \tag{8}$$

where η is defined in equation 7. Using equations (6) and (7) we solve equation (8) to obtain

$$\lambda_X = \frac{s}{2v^2} \frac{\mathcal{M}_R^X}{\mathcal{M}_R^X - \mathcal{M}_{\rm LET}} \tag{9}$$

and

$$P_X = \frac{\eta}{s - 2\lambda_X v^2}. \tag{10}$$

While the effective coupling "constant" λ_X is not in fact constant but is in general a function of s, equation 10 shows that we have preserved (up to the factor η) the standard model relationship between the effective Higgs propagator and the Higgs sector vertices which is crucial for maintaining BRS invariance.

The remaining interaction vertices are fixed by requiring that the $WWWW$, $WWWw$, $WWww$, $Wwww$, and $wwww$ amplitudes satisfy the BRS identities

$$(\partial W + \xi m_W w)^n = 0 \tag{11}$$

for n = 1,2,3.4. For the $W^+W^- \to ZZ$ channel all vertices not specified above are given precisely by their standard model values. The amplitudes obtained from our algorithm then trivially satisfy BRS invariance and $\mathcal{M}(W^+W^- \to ZZ)$ is trivially gauge invariant (ξ independent in R_ξ gauge), because equation 10 assures that the necessary cancellations occur just as in the standard model.

It is less trivial but no less straightforward to verify that the prescription can be made to work for $W^+W^+ \to W^+W^+$. In this case it is necessary to define

[5] WWH denotes generically W^+W^-H and ZZH with reference to the $W^+W^- \to ZZ$ channel and $W^+W^+H^{++}$ for $W^+W^+ \to W^+W^+$.

some nonstandard interaction vertices, since the U-gauge transcription defined above already mutilates the standard model structure for $W^+W^+ \to W^+W^+$ by substituting an s-channel effective H^{++} exchange for the t and u-channel H^0 exchanges of the standard model. Clearly all interactions of the effective H^{++} boson are nonstandard and require definition. The U-gauge transcription already specifies the $H^{++}W^-W^-$ vertex as $gm_W g^{\mu\nu}$ (see equation 5), equal[6] to the standard model HW^+W^- vertex. The remaining nonstandard vertices are fixed by insisting on the validity of the BRS identities, equation 11, applied to $< W^+W^+W^-W^- >$ for n = 1,2,3,4. With the vertices chosen to satisfy BRS invariance we find that ξ independence of $\mathcal{M}(W^+W^+ \to W^+W^+)$ in R_ξ gauge is also automatically assured.

In addition to defining the interactions of the effective H^{++} boson we must adopt nonstandard quartic couplings for the $w^+w^+w^-w^-$ and $W^+W^+w^-w^-$ vertices: the former is -1/2 times its standard model value while the latter does not exist at all in the standard model. Vertices that do not exist in the standard model or that differ from their standard model values are given in table 1.

To illustrate how the diagrammatic algorithm satisfies BRS invariance, consider the identity equation 11 with $n = 2$, applied to the two initial state bosons in WW scattering. We can consider $W^+W^- \to ZZ$ and $W^+W^+ \to W^+W^+$ concurrently, since in our approach they are given by the same set of Feynman diagrams. The BRS identity for the scattering amplitude $W_1W_2 \to W_3W_4$ is

$$\epsilon_{3\alpha}\epsilon_{4\beta}\left(k_{1\mu}k_{2\nu}\mathcal{M}^{\mu\nu\alpha\beta} + im_W(k_{1\mu}\mathcal{M}^{\mu\alpha\beta}_{w_2}k_{2\nu}\mathcal{M}^{\nu\alpha\beta}_{w_1}) - m_W^2\mathcal{M}^{\alpha\beta}_{w_1w_2}\right) = 0. \qquad (12)$$

The subscript w_i indicates the amplitude in which gauge boson W_i is replaced by Goldstone boson w_i.

Using the Feynman rules defined above to evaluate the amplitudes in equation 12 in R_ξ gauge, we find after trivial cancellations that the remaining terms are

$$\delta^2_{\rm BRS} = \frac{1}{2}g^2m_W^2\epsilon_3\cdot\epsilon_4\left((s - 2\lambda_X v^2)P_X - \eta\right). \qquad (13)$$

Using equation 10 the right side vanishes, confirming the BRS identity equation 12. All other BRS identities can be similarly verified.

[6] I follow the phase conventions of the CORE compendium.[7]

Discussion

The U-gauge transcription has been verified for s-wave strong WW scattering models by demonstrating its consistency with BRS invariance for the complete set of gauge and Goldstone boson scattering amplitudes. A Feynman diagram algorithm was defined, including an effective "Higgs" propagator that carries the strong scattering dynamics and a related energy dependent "effective" ϕ^4 Higgs sector coupling constant. The transcription is useful both in high energy applications, to strong WW scattering at pp and e^+e^- colliders, and in low energy applications, to estimate the effect of strong WW scattering on electroweak radiative corrections.

As discussed in [4] the U-gauge transcription is more accurate and more complete than the effective W approximation for computing strong WW scattering. It is more accurate because it retains the interference between the strong WW scattering amplitude and the gauge sector background amplitude, and also because it provides the transverse momentum of the WW diboson which is neglected in the EWA. It is more complete because it provides the full three-momentum distribution for the final state quark jets in the $qq \to qqWW$ process, while the EWA neglects the jet transverse momenta (also introducing an error in the determination of the jet rapidities).

The final state jet distributions are needed to compute the efficiency of detection strategies such as the central jet veto[8] and the forward jet tag[9]. The former is very effective against the gluon exchange and electroweak gauge sector backgrounds, and the latter may be very useful against the surprisingly large $\overline{q}q \to WZ$ background[10] to the $W^+W^+ \to W^+W^+$ strong scattering signal. In previous studies the necessary jet distributions have been estimated assuming the same shape for strong scattering as for the standard model with a heavy (typically 1 TeV) Higgs boson. This assumption can now be tested using the U-gauge transcription. I find that it works well at low enough energy colliders, for which the strong scattering and heavy Higgs cross sections are "squashed" into roughly the same region in s_{WW}, but not at higher energy colliders with enough phase space to allow the differences in the WW energy spectrums to emerge. From this perspective the LHC is a "low" energy collider, while at SSC energies (R.I.P.) the differences begin to be important.

The effective Higgs sector propagator defined in the U-gauge transcription may also be used to estimate the direct effect of strong WW scattering on low energy radiative corrections. Unlike the typically large corrections predicted by technicolor models, the correction due just to strong nonresonant dynamics in WW scattering is not very much bigger than the effect of the 1 TeV standard model Higgs boson. These results and other applications of the U-gauge transcription will be presented elsewhere.

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Table 1. Nonstandard interaction vertices for $W^+W^+ \to W^+W^+$ scattering. Also shown are analogous vertices for $W^+W^- \to ZZ$, which agree precisely with the standard model. All momenta are inflowing, phase conventions are as in the CORE compendium[7], and η is defined below equation 7.

$W^+W^+ \to W^+W^+$	$W^+W^- \to ZZ$	Interaction
$H^{++}(k)W^-_\mu(p)W^-_\nu(q)$	$H^0(k)W^+_\mu(p)W^-_\nu(q)$	$gm_W g^{\mu\nu}$
$H^{++}(k)W^-_\mu(p)w^-(q)$	$H^0(k)W^+_\mu(p)w^-(q)$	$ig(q^\mu - k^\mu)/2$
$H^{++}(k)w^-(p)w^-(q)$	$H^0(k)w^+(p)w^-(q)$	$-2\lambda_X v$
$w^+(p_1)w^+(p_2)W^-_\mu(p_3)W^-_\nu(p_4)$	$w^+(p_1)w^-(p_2)Z_\mu(p_3)Z_\nu(p_4)$	$\eta g^2 g^{\mu\nu}/2$
$w^+(p_1)w^+(p_2)w^-(p_3)w^-(p_4)$	$w^+(p_1)w^-(p_2)z(p_3)z(p_4)$	$-2\eta\lambda_X$

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