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Summing up D-Instantons

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We investigate quantum corrections to the moduli space for hypermultiplets for type IIA near a conifold singularity. We find a unique quantum deformation based on symmetry arguments which is consistent with a recent conjecture. The correction can be interpreted as an infinite sum coming from multiple wrappings of the Euclidean D-branes around the vanishing cycle.

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1. Introduction

Non-perturbative aspects of string theory have been studied vigorously recently. These have been both in the form of solitonic objects as well as instanton corrections to various physical quantities. An important class of such objects is in the form of D-branes or D-instantons [1], [2]. Most applications to date involve D-branes as solitons. However one can consider Euclidean p-branes wrapped around non-trivial cycles of the compactification manifold to obtain instanton corrections to various physical quantities. This aspect has been far less studied, however, except for the considerations of Euclidean membranes [3] and Euclidean fivebranes [4] in the context of M-theory compactifications.

In this paper, we study D-brane instanton corrections to the hypermultiplet moduli space of type II string compactification on a Calabi-Yau (CY) 3-fold[1]. In particular, we examine the moduli space near the conifold singularity where the non-perturbative aspects are expected to be crucial. In the type IIA string, the complex moduli of CY 3-fold belong to the hypermultiplet, and the conifold singularity is realized when there is a non-trivial 3-cycle in CY whose period

$$z = \int \Omega \tag{1.1}$$

is small. In the limit $z \to 0$, the classical hypermultiplet moduli space develops a singularity, as we will explain in the next section.

Before describing the resolution of the conifold singularity in the hypermultiplet moduli space, let us remind ourselves how a similar singularity was resolved in the vector multiplet moduli space. In type IIB string, the complex moduli belong to the vector multiplet and the same limit $z \to 0$ generates a singularity in the classical vector multiplet moduli space. In this case, however, we know that the moduli space will not be corrected by quantum string effect, perturbative or non-perturbative. In particular the exact leading singular part of the metric for the vector multiplet moduli is given by

$$ds^2 = -\log(z\bar{z})dzd\bar{z}.$$

It was pointed out by Strominger [5] that a D3-brane wrapping on the vanishing 3-cycle has a mass of order $|z|/\lambda$, where λ is the string coupling, and that the conifold singularity is a reflection of the fact that we ignore the light solitonic particle arising from the D3-brane

[1] Our results also have a natural interpretation in the context of M-theory compactifications near a conifold singularity.

in string perturbation theory. If we include it, the low energy effective theory is regular even at the conifold point.

Now let us come back to the hypermultiplet moduli space. Since the type IIA string does not have a D3-brane, there is no solitonic state which can become massless at the conifold. On the other hand, we may consider the Euclidean D2-brane which is wrapped around the vanishing 3-cycle. The hypermultiplet moduli space is not protected against quantum corrections, and the D2-brane instanton would have an effect of order $\exp(-|z|/\lambda)$. It was conjectured in [3] that the instanton effect should resolve the conifold singularity. Recently a precise form for this resolution was conjectured in [6]. In this paper we show that, if we take into account various symmetries, there is a unique quantum deformation to the conifold singularity in the classical moduli space and that this result agrees with the conjecture of [6]. Moreover the modification to the classical metric is exactly of the form expected for the multiply wrapped Euclidean D2-branes (or Euclidean membranes of M-theory) around the vanishing S^3. This explicit result, which in effect sums up the contribution of infinitely many D-instantons, may shed light on how to sum up D-instantons in other cases as well.

2. Classical moduli space

Let us consider the type IIA string on a CY 3-fold M. If $n = \dim H^{2,1}(M)$, the hypermultiplet moduli space is complex $(2n+2)$ dimensional; n of which come from the complex moduli of M, $(n+1)$ from the RR 3-form gauge potential and 1 from the dilaton and axion S. Since we are interested in the universal behavior of the moduli space near the conifold limit $z \to 0$, we will send the string coupling constant $\lambda \to 0$ while keeping $|z|/\lambda$ finite. In this limit we may hope to extract a universal deformation of moduli space, including the important instanton effects of the order $\exp(-|z|/\lambda)$, which would be independent of how the vanishing cycle is embedded in the rest of CY. Even though the hypermultiplet moduli space is quaternionic [7][8], in the limit we are considering the relevant piece of the singularity is a hyperkähler manifold of real dimension 4. The complex moduli z is paired with 2 real coordinates x and t which are expectation values of the RR 3-form corresponding to the vanishing cycle and its dual respectively. In the following, we will concentrate on the subspace of moduli space spanned by z and (t, x).

Since the RR charges carried by D-branes are quantized, the moduli space must be periodic in the RR 3-forms x and t. We normalize them so that each has period 1. Moreover there is a monodromy action on $H_3(M)$ as z goes around the conifold point, and this mixes

$(t,x) \to (t+x,x)$. Thus the moduli space geometry near the conifold is described by the elliptic fibration

$$\tau(z) = \frac{1}{2\pi i}\log z. \tag{2.1}$$

This is similar to the situation of the stringy cosmic string [9]. In fact at weak coupling, the leading singularity in the classical moduli space metric computed using the result of [8] agrees with that of [9]. The Kähler form for the classical metric is given by

$$k = \partial\bar{\partial}\left(\frac{(\zeta-\bar{\zeta})^2}{2(S+\bar{S})\tau_2}\right) + \tau_2 dz d\bar{z} \tag{2.2}$$

where $\zeta = t+\tau x$ and $\tau_2 = \mathrm{Im}\tau(z)$. The metric has a $U(1)_t \times U(1)_x$ translational invariance in t and x. This is to be expected since there is no perturbative string state which carries the RR charges. Since $(S+\bar{S})$ is the dilaton from the NS-NS sector, $(S+\bar{S}) \sim 1/\lambda^2$ where λ is the string coupling constant (for a precise definition of the string coupling constant in the present context see [10]). In the following, we will use $1/\lambda^2$ in place of $(S+\bar{S})$.

The metric of the classical moduli space discussed in the above is singular at the conifold point $z=0$ as shown in [9]. In the neighborhood of $z=0$, however, we expect large instanton effects due to Euclidean D2-branes wrapping the vanishing 3-cycle. It has been conjectured in [3] that such effects would resolve the singularity at $z=0$. In [6] based on some field theory considerations, it was conjectured more precisely that the exact corrected metric is the unique hyperkähler metric where the Kähler class of the elliptic fiber is λ^2. In the following, we will *derive* the form of the corrected metric based on symmetry considerations for Euclidean membranes wrapped around vanishing cycle and the assumption of resolution of the singularity [3] and find agreement with the conjecture in [6]. This also leads us to an explicit realization of the metric for which the classical part and quantum corrections can be identified. Moreover the quantum corrections can be naturally reinterpreted as D-instanton contributions to the metric.

3. Quantum moduli space

In order to exhibit the symmetries of the metric (2.2), it is convenient to rewrite it as

$$ds^2 = \frac{\lambda^2}{\tau_2}(dt+\tau(z)dx)(dt+\bar{\tau}(\bar{z})dx) + \tau_2 dz d\bar{z}.$$

Note that the metric has $U(1)_t \times U(1)_x$ symmetries corresponding to the translations in t and x, as we explained in section 2. One then recognizes that it takes the form of the ansatz [11] [12] for a self-dual metric:

$$ds^2 = \lambda^2 \left[V^{-1}(dt - \mathbf{A}\cdot\mathbf{dy})^2 + V d\mathbf{y}^2\right], \tag{3.1}$$

with $\mathbf{y} = (x, z/\lambda, \bar{z}/\lambda)$ and

$$V = \tau_2 = \frac{1}{4\pi}\log\left(\frac{1}{z\bar{z}}\right)$$
$$A_x = -\tau_1 = \frac{i}{4\pi}\log\left(\frac{z}{\bar{z}}\right), \quad A_z = 0, \quad A_{\bar{z}} = 0$$

This metric is singular at $z = 0$. Moreover we are taking t and x to be periodic with period 1.

Now let us discuss how the quantum corrections could modify the metric, paying attention to the fate of the $U(1)_t \times U(1)_x$ translational invariance. Since the variable x corresponds to the expectation value of RR 3-form on the vanishing cycle, the D2-instanton wrapping on it would break the translational invariance in the x-direction. In particular if we consider a Euclidean 2-brane wrapped m times around S^3 it couples to RR expectation value on it and gives us a factor of $\exp(2\pi i m x)$. Note that this is still consistent with periodicity of x, i.e. the $U(1)_x$ has been broken to $\mathbf{Z}$. On the other hand t couples to a cycle dual to the vanishing S^3, and its translational invariance will not be broken, as we are considering a limit where the dual period is not vanishing[2] and thus is irrelevant in the leading order as $\lambda \to 0$. Thus it is appropriate to work in the ansatz (3.1).

There are various requirements that the potential V has to satisfy:

(1) The metric is hyperkähler if and only if V and $\mathbf{A}$ obey

$$V^{-1}\Delta V = 0, \quad \nabla V = \nabla \times \mathbf{A}.,$$

where

$$\Delta = \partial_x^2 + 4\lambda^2 \partial_z \bar{\partial}_{\bar{z}}.$$

The factor V^{-1} in the first equation means that we allow delta-function singularities in ΔV. Thus we can think of V as the electro-magnetic scalar potential for a collection of charges in 3-dimensions.

[2] Note that if we had been considering a case where the dual cycle also has vanishing period the translation in t would also be broken. This should be interesting to study.

(2) For large z when the instanton effects are suppressed, the metric should reduce to the classical one.

$$V \to \frac{1}{4\pi}\log\left(\frac{1}{z\bar{z}}\right) \qquad (|z| \to \infty).$$

(3) The metric should be periodic, but not translationally invariant, in x with the period 1.

(4) Since the Calabi-Yau geometry near the conifold is invariant under the phase rotation of z and that Euclidean membranes only probe the overall size $|z|$ of S^3, the $dzd\bar{z}$ part of the moduli space metric should be independent of the phase. This means that the potential V is a function of x and $|z|$ only.

(5) For a single conifold we assume the quantum metric has no singularity. This means in particular that the singularities of V must be such that they can be removed by appropriate coordinate transformation.

The conditions (1), (3) and (4) means that we are to find the electro-magnetic potential V which is periodic in x and axial symmetric in the z-plane. The condition (2) says that the electric charges are distributed near the axis $z = 0$ and its density per unit length in x is 1. The condition (5) requires that these charges to be quantized in the unit of 1 and that no two charges are at the same point. In particular if we have N charges at the same point the space will develop $\mathbf{C}^2/\mathbf{Z}_N$ singularity. There is a unique solutions satisfying these conditions, and it is given by

$$V = \frac{1}{4\pi}\sum_{n=-\infty}^{\infty}\left(\frac{1}{\sqrt{(x-n)^2 + z\bar{z}/\lambda^2}} - \frac{1}{|n|}\right) + \text{const.} \tag{3.2}$$

To exhibit the D-instanton effects, it is convenient to take the Poisson resummation of this potential. We then find

$$V = \frac{1}{4\pi}\log\left(\frac{\mu^2}{z\bar{z}}\right) + \sum_{m\neq 0}\frac{1}{2\pi}e^{2\pi i m x}K_0\left(2\pi\frac{|mz|}{\lambda}\right), \tag{3.3}$$

where μ is some constant. K_0 is the modified Bessel function, whose appearance is natural in the axially symmetric potential problem. By construction, the metric is regular at $z = 0$ and reduces to the classical one $V \sim \frac{1}{4\pi}\log(1/z\bar{z})$ for $|z| \to \infty$.

4. Interpretation

When z is large, we can use the asymptotic formula of the Bessel function to expand (3.3) as

$$V = \frac{1}{4\pi}\log\left(\frac{\mu^2}{z\bar{z}}\right) + \sum_{m\neq 0}\exp\left[-2\pi\left(\frac{|mz|}{\lambda} - imx\right)\right] \times \\ \times \sum_{n=0}^{\infty}\frac{\Gamma(\frac{1}{2}+n)}{2\sqrt{\pi}n!\Gamma(\frac{1}{2}-n)}\left(\frac{\lambda}{4\pi|mz|}\right)^{n+\frac{1}{2}} . \tag{4.1}$$

Notice that the correction to the classical term $\frac{1}{4\pi}\log(1/z\bar{z})$ is exponentially suppressed by the factor $\exp[-2\pi(|mz|/\lambda - imx)]$. This is exactly what we expect for the instanton effect due to D2-branes wrapping the vanishing S^3. The D2-instanton configuration should preserve 1/2 of the spacetime supersymmetry, which means in particular that the volume form on the membrane worldvolume is proportional to the holomorphic 3-form Ω [3]. Thus $2\pi|mz|/\lambda$ in the exponent is nothing but the Born-Infeld action for the m-instanton (m times wrapping of S^3). Since x is the integral of the RR 3-form on S^3, the second term $2\pi imx$ in the exponent describes the coupling of the D2-brane to the RR field.

Note that this result implies a number of things: First of all there is no perturbative correction to the leading singularity of hypermultiplet moduli near the conifold singularity. Secondly, perhaps surprisingly, *all* instanton numbers are present for the correction to the metric. This is in contract with the count of stable solitons in the type IIB near the conifold where the multiply wrapped state is not expected to be stable [5][13]. Thirdly for each D-instanton we have an infinite 'perturbative' sum. It would be interesting to connect this to perturbative string computations around the D-instanton background. In this connection the open topological string theory on T^*S^3 [14] may be relevant[3] (See in particular [15]). It is also surprising that the power of the coupling λ is shifted from an integer by 1/2. This may be related to a precise definition of the coupling constant [10].

Note that if we consider the case of N vanishing 3-cycles instead of 1, then our considerations naturally lead to $V \to NV$. This space will have $\mathbf{C}^2/\mathbf{Z}_N$ singularity. This is in agreement with the conjecture in [6] and the fact that the Euclidean membranes treat each vanishing S^3 independently of each other.

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