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Abstract

We introduce a gauge-invariant measure of the local "abelianicity" of any given lattice configuration in non-abelian lattice gauge theory; it is essentially a comparison of the magnitude of field strength commutators to the magnitude of the field strength itself. This measure, in conjunction with the cooling technique, is used to probe the $SU(2)$ lattice vacuum for a possible large-scale abelian background, underlying the local short-range field fluctuations. We do, in fact, find a substantial rise in abelianicity over 10 cooling steps or so, after which the abelianicity tends to drop again.

It has been suggested on occasion that non-abelian gauge theory is dominated, in the infrared regime, by abelian configurations of some kind. One early example was Saviddy's proposal [1], based on a study of the one-loop effective action, that there is a constant background abelian field strength in the Yang-Mills vacuum. More recently, in the maximal abelian gauge, it has been shown that $SU(2)$ lattice configurations that are "abelian projected" onto $U(1)$ configurations retain information about the asymptotic string tension [2]. This property is known as "abelian dominance," and is widely interpreted as supporting 't Hooft's abelian projection theory of confinement [3] (see, however, ref. [4]). Some other ideas concerning the non-perturbative dynamics have stressed the importance of vortices carrying magnetic flux associated with the center elements of the gauge group [5]. The center subgroup is, of course, abelian by definition.

With this motivation it is interesting to study, via lattice Monte Carlo simulations, the actual degree of "abelianicity" of vacuum fluctuations in non-abelian gauge theories. In order to do this, we must first introduce a quantitative, and preferably gauge-invariant, measure of the abelianicity of a gauge-field configuration. A non-abelian field configuration may be regarded as equivalent to an abelian field if the commutators of its field-strength components vanish everywhere; thus the following gauge-invariant, positive semidefinite quantity

$$B = -\frac{1}{V} \sum_x \frac{1}{n_p(n_p - 1)} \sum_{i>j} \sum_{m>n} \text{Tr}\{[F_{ij}(x), F_{mn}(x)]^2\} \quad (1)$$

vanishes if and only if the configuration is abelian. On the lattice, V is the number of lattice sites, $n_p = D(D - 1)/2$ is the number of plaquettes per site in D dimensions, with lattice field strength taken to be

$$F_{ij} = \frac{1}{2i} [U_i(x)U_j(x+i)U_i^\dagger(x+j)U_j^\dagger(x) - \text{h.c.}] \quad (2)$$

Of course, since B is proportional to the fourth power of field-strengths, it is sensitive not only to abelianicity but also to the magnitude of the field strengths. For the purpose of normalization, we introduce

$$A = \frac{1}{n_p V} \sum_x \sum_{i>j} \text{Tr}\{F_{ij}^4\} \quad (3)$$

and define the average *non*-abelianicity of an ensemble of configurations, which is invariant under a rescaling of the field-strengths, as

$$Q \equiv \frac{\langle B \rangle}{\langle A \rangle} \quad (4)$$

An ensemble of configurations is abelian if the non-abelianicity Q vanishes. This measure of non-abelianicity is also a quantitative measure of abelianicity, in the sense that abelianicity increases as Q decreases.

The observable Q is a local quantity, and will be dominated by high-frequency vacuum fluctuations.¹ However, we are not so much interested in these high-frequency fluctuations, which are perturbative in character, as in the degree of abelianicity of the underlying larger-scale fluctuations. One method that has been suggested for eliminating the higher-frequency fluctuations from a given configuration is the lattice "cooling" technique. We have therefore measured the variation of Q with cooling step, to determine whether the larger scale vacuum fluctuations are more (or less) abelian in character than the high frequency fluctuations.

We follow the constrained cooling technique of ref. [6]. Each cooling step is a sweep through the lattice links, with each link $U_k(x)$ replaced by a new link $U'_k(x)$ which minimizes the lattice action, subject to the constraint

$$\frac{1}{2}\text{Tr}[(U'_k(x) - U_k(x))(U_k^\dagger(x) - U'^\dagger_k(x))] \leq \delta^2 \quad (5)$$

where $\delta = 0.05$. As pointed out by Teper [7], this (or any other) version of cooling can never remove confinement entirely, because cooling can only remove fluctuations of wavelength smaller than a certain scale, which depends on (and increases with) the number of cooling steps. Suppose, then, that we observe an increase in abelianicity with cooling steps up to a certain maximum, followed by a decrease in abelianicity as number of cooling steps continues to increase. A reasonable interpretation of such behavior would be that there are structures in the vacuum, of some intermediate length scale, which are more abelian in character than vacuum fluctuations at larger or smaller scales. This "hill" of abelianicity (or dip in non-abelianicity), is in fact the behavior we find.

Fig. 1 is a plot of the Q observable vs. cooling step, for $\beta = 2.3, 2.4, 2.55$ and 2.7 with Q evaluated on 14, 17, 18 and 25 cooled configurations, respectively. In each case we worked on a 12^4 lattice, thermalized for 3000 iterations before applying the cooling algorithm. What is striking about this data is the evident drop in Q , implying an increase in abelianicity, up to 10–13 cooling steps, which is followed by a rise in Q (drop in abelianicity). The effect seems to become more pronounced as β increases. Again, the simplest interpretation is that vacuum fluctuations at some intermediate length scale are more abelian in character than vacuum fluctuations at smaller and at

¹In zeroth-order lattice perturbation theory, $Q = 1.6$.

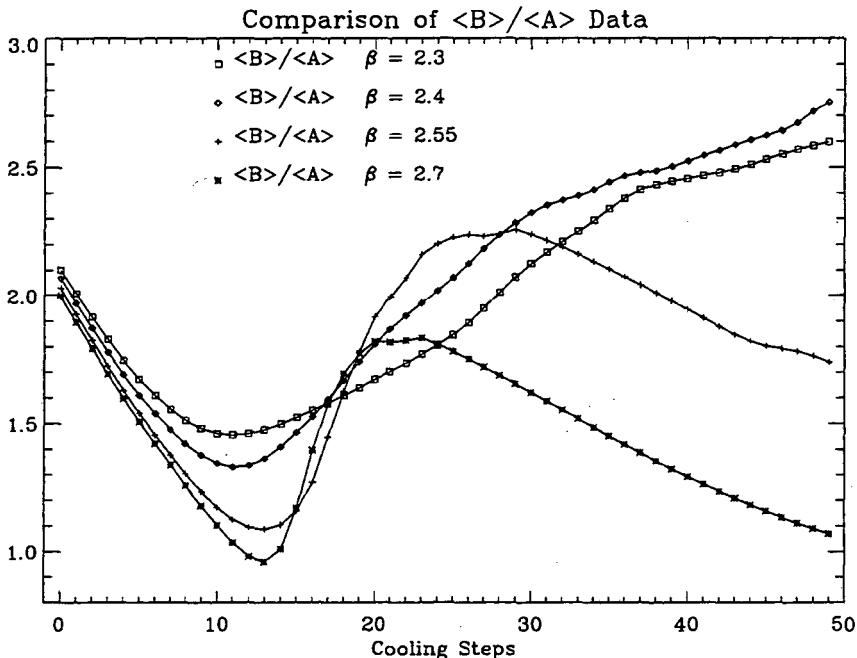


Figure 1: Non-abelianity Q vs. Cooling Step, for several values of β .

somewhat larger length scales. As cooling removes the higher frequency fluctuations, the contribution of these "more abelian" fluctuations becomes relatively larger, and Q decreases. As cooling proceeds, the "more abelian" fluctuations are also removed, and Q increases again.

It is interesting that the abelianity begins to fall off after 10 cooling steps, because this is also where the plateau in Creutz ratios vs. cooling step, noted by Campostrini et al. [6], begins to drop off to zero. However, this simultaneous falloff of abelianity and Creutz ratio could be coincidental, particularly in light of Teper's observation [7] that the string tension should never disappear with cooling, providing one looks at sufficiently large loops.

The asymptotic behavior of Q at large numbers of cooling steps is also of great interest, but here we are rather hesitant in drawing conclusions from our data. In the first place, while the abelianity at $\beta = 2.3, 2.4$ steadily decreases, from 11 cooling steps and beyond, the data for $\beta = 2.55, 2.7$ shows the opposite behavior: the abelianity reaches a minimum at about 23–25 cooling steps, and then increases again. However, at the larger β values a 12^4 lattice is quite small, and it is possible that beyond 25 cooling steps our observable is mainly probing finite-size effects.

The next question is whether the abelianity is correlated with either plaquette

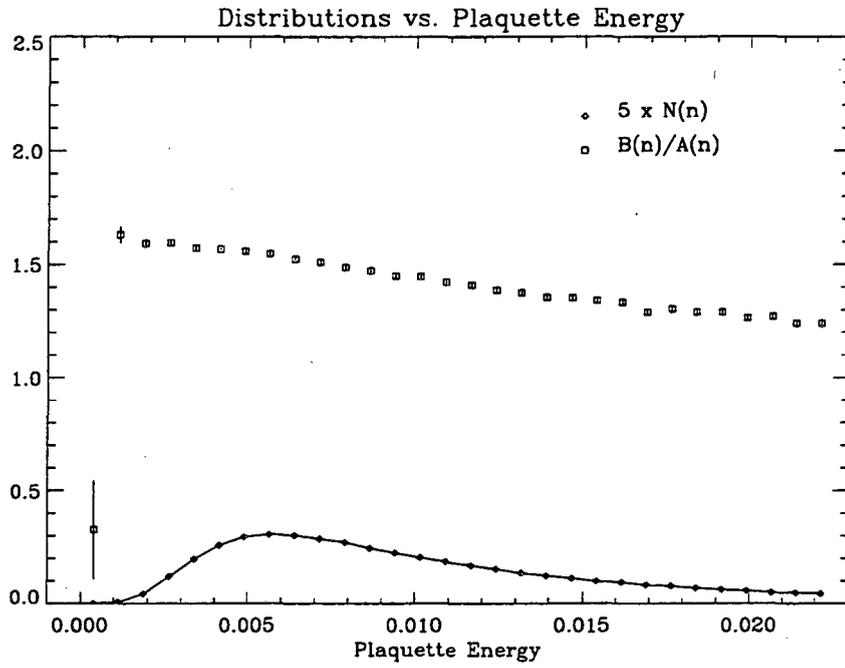


Figure 2: Local non-abelianicity $Q(n)$ vs. Plaquette Energy after 10 cooling steps, at $\beta = 2.55$.

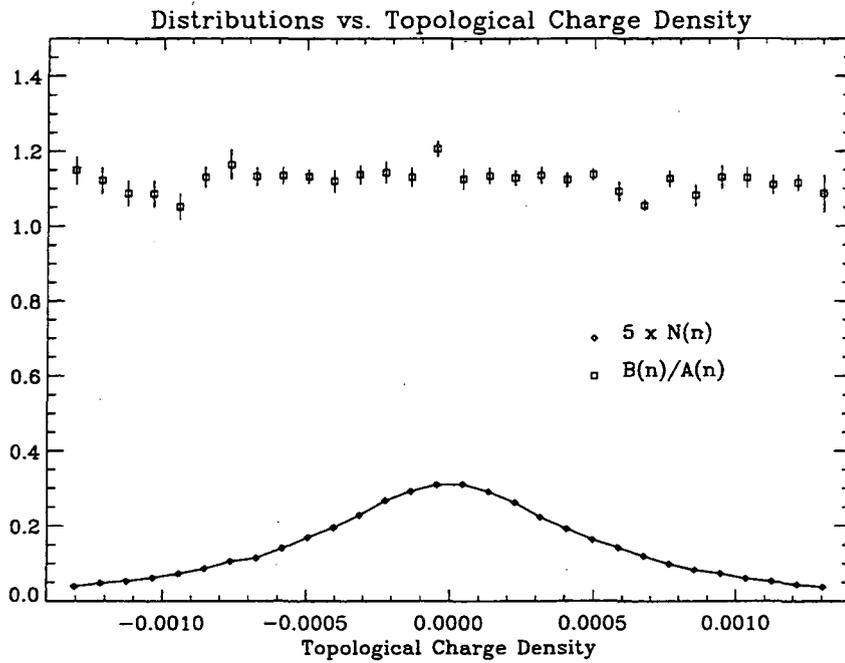


Figure 3: Local non-abelianicity $Q(n)$ vs. Topological Charge Density, after 10 cooling steps, at $\beta = 2.55$.

energy or topological charge. We investigate this issue at $\beta = 2.55$ after 10 cooling steps, which is near the maximum average abelianicity. The relevant data is shown in Figs. 2 and 3, where the quantity $Q(n)$ in Fig. 2 is defined as follows: We first divide the range of plaquette energy into a number of small intervals, indexed by n and define $A(n)$ and $B(n)$ as

$$\begin{aligned} B(n) &= -\frac{1}{N(n)V} \sum_x^{\text{bin } n} \frac{1}{n_p(n_p - 1)} \sum_{i>j} \sum_{m>n} \text{Tr}\{[F_{ij}(x), F_{mn}(x)]^2\} \\ A(n) &= \frac{1}{n_p N(n)V} \sum_x^{\text{bin } n} \sum_{i>j} \text{Tr}\{F_{ij}^4\} \end{aligned} \quad (6)$$

where

$$N(n) = \frac{1}{V} \sum_x^{\text{bin } n} 1 \quad (7)$$

is the fraction of sites x where the averaged plaquette value at the site

$$S_p = \frac{1}{n_p} \sum_{i>j} (1 - \frac{1}{2} \text{Tr}[U_i(x)U_j(x+i)U_i^\dagger(x+j)U_j^\dagger(x)]) \quad (8)$$

is in the n -th interval. Then

$$Q(n) \equiv \frac{\langle B(n) \rangle}{\langle A(n) \rangle} \quad (9)$$

Each data point shown in Fig. 2 represents the data from one interval; the x -component (plaquette energy) of the data point is at the center of the interval, and the width of the interval is the distance, along the x -axis, between neighboring data points. Figure 3 is similar to Figure 2, except that it is the (naive) topological charge density

$$\mathcal{T} = -\frac{1}{32\pi^2} \epsilon_{ijkl} \text{Tr}[U_{ij}(x)U_{kl}(x)] \quad (10)$$

which is subdivided into intervals, where $U_{ij}(x)$ is the product of link variables around a plaquette.

Figures 2 and 3 plot $Q(n)$ vs. plaquette energy S_p and topological charge \mathcal{T} , respectively. We also display the average fraction $N(n)$ of sites with plaquette energy (Fig. 2) or topological charge density (Fig. 3) in the n -th interval (or "bin") associated with each data point. The fraction $N(n)$ has been multiplied by a factor of 5, to make the distribution more visible on the scale of the graphs. Apart from the very first (rather "noisy") data point in the lowest plaquette energy interval, there does not seem to be a very strong correlation, after 10 cooling steps, of abelianicity with either plaquette energy or topological charge in the range shown.

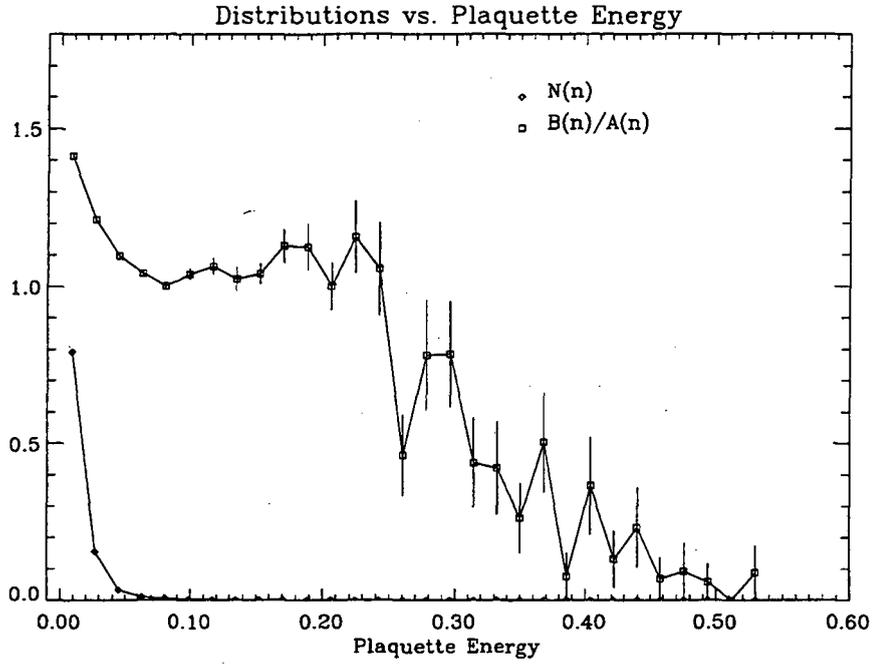


Figure 4: Same as Fig. 2, extended to larger plaquette energies.

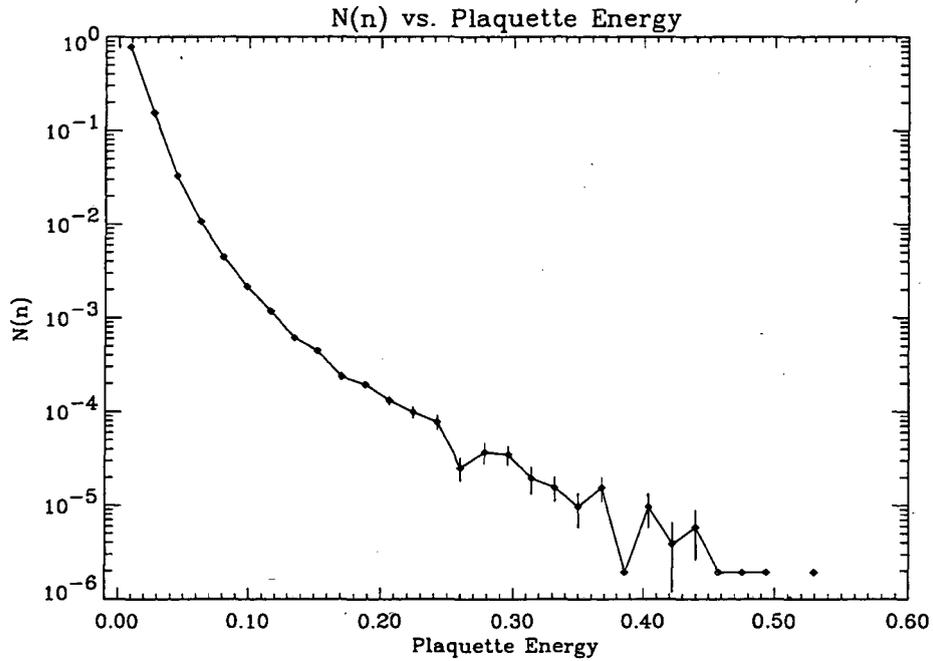


Figure 5: Fraction of sites $N(n)$ with plaquette energies in the n -th bin, corresponding to data points in Figure 4.

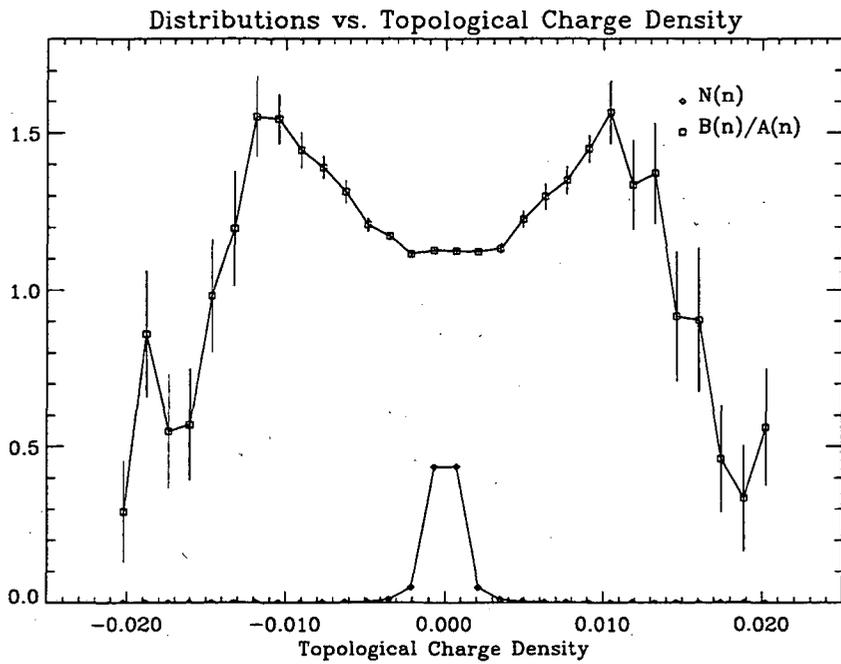


Figure 6: Same as Fig. 3, extended to larger topological charge.

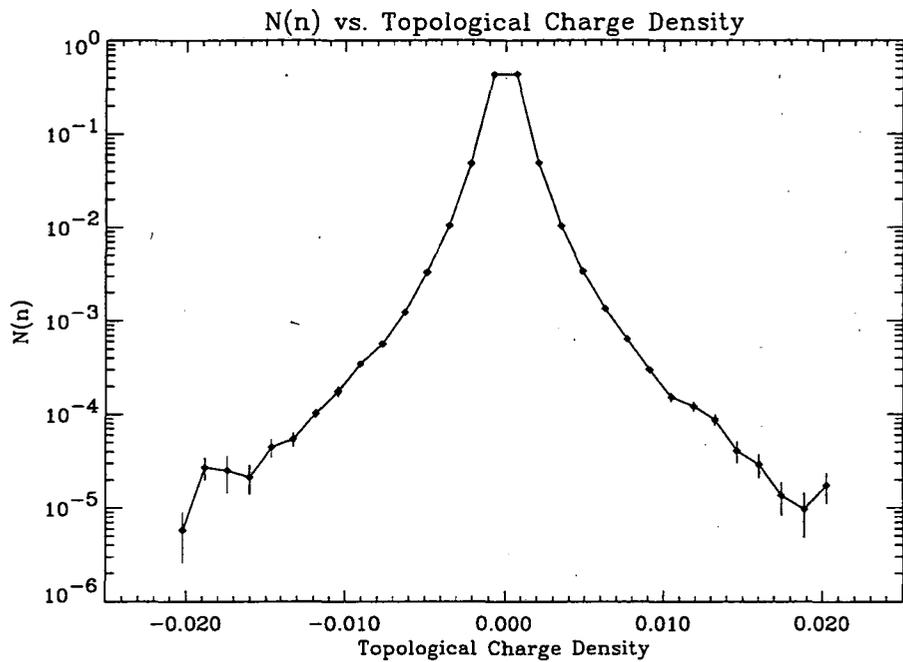


Figure 7: Fraction of sites $N(n)$ with topological charge density in the n -th bin, corresponding to data points in Figure 6.

The situation changes, however, if we include data which is off the scale of Figures 2 and 3. Figure 4 is another plot of $Q(n)$ vs. plaquette energy at $\beta = 2.55$ after 10 cooling steps, with the scale of plaquette energy S_p extended to a maximum of 0.6 (the width of the binning intervals is also increased). It can be seen that as the plaquette energy increases beyond 0.25, there is a steep increase in the abelianicity of the lattice field. Fig. 5 shows the fraction $N(n)$ of sites with averaged plaquette energies S_p in the n -th bin. It is evident that there are very few lattice sites (on the order of 1 in 10,000) with $S_p > 0.25$. Similar behavior is seen for the abelianicity vs. topological charge (Fig. 6), except that the abelianicity decreases at first, up to a minimum at topological charge densities of magnitude 0.01, after which there is again a sharp increase in abelianicity. As seen in Fig. 7, sites where the abelianicity is far different from the average are very rare. It is certainly intriguing that large plaquette energy and large topological charge density are so strongly correlated with large abelianicity. However, the rarity of sites with very large abelianicity means that their physical importance is, as yet, uncertain.

We conclude that there is some modest evidence that vacuum fluctuations at an intermediate scale are more abelian in character than fluctuations at smaller and at somewhat larger scales. Conceivably, this might indicate the presence of abelian domains. The evidence concerning the abelianicity of very long-wavelength fluctuations is ambiguous, perhaps due to our relatively small lattice size. It would be interesting to study the asymptotic abelianicity of cooled configurations on lattice sizes much larger than the 12^4 volume used here.

Acknowledgements

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