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Gaugino Condensation, Loop Corrections and S-Duality Constraint[1]

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Abstract

This talk is a brief review of gaugino condensation in superstring effective field theories and some related issues (such as renormalization of the gauge coupling in the effective supergravity theories and modular anomaly cancellation). As a specific example, we discuss a model containing perturbative (1-loop) corrections to the Kähler potential and approximate S-duality symmetry.

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Introduction

Amongst the candidates for fundamental unified theories, heterotic superstring theory with gauge group $E_8 \times E_8$ seems to be the most promising one. This is because the spectrum of the theory easily accomodates the Standard Model spectrum and gauge structure. In addition, the underlying gauge group contains an extra factor of E_8 which provides an 'hidden sector', which couples to the observable sector only through gravity, and, as will be discussed below, plays a crucial role in the mechanism of supersymmetry breaking. Furthermore, the effective theories that describe the heterotic string in 4 dimensions below the Planck scale are (nonrenormalizable) locally supersymmetric effective field theories. Indeed, requiring supersymmetry at energies well above M_W in order to stabilize the gauge hierarchy, in some sense forces one to consider locally supersymmetric theories: A unified field theory must include gravity. Within the framework of General Relativity, a supersymmetric theory has to be *locally* supersymmetric. This follows from the fact that the supersymmetry transformation on the metric, or on the vielbein must include general coordinate transformations. Supergravity theories are nonrenormalizable, but can be consistently viewed as low-energy effective field theories (LEEFT) for the massless modes of superstring.

A basic feature of superstring constructions in four dimensions is the presence of massless moduli in the effective field theory. These fields whose vevs parameterize the continuously degenerate string vacua, are gauge-singlet chiral fields; furthermore, they are *exact* flat directions of the low energy effective field theory (LEEFT) scalar potential. Generically, the moduli appear in the couplings of the LEEFT. For example, the tree level gauge couplings at the string scale depend on the dilaton, S, and the Yukawa couplings as well as the kinetic terms depend on the T-moduli (and S through the Kähler potential) . There is mixing of the moduli beyond tree level, due to both string threshold corrections [1] and field-theoretical loop effects, as we shall dicuss.

Since the supersymmetric vacua of heterotic strings consist of continuously degenerate families (to all orders of perturbation theory), parameterized by the moduli vevs, the latter remain perturbatively undetermined. This degeneracy can only be lifted by a nonperturbative mechanism which would induce a nontrivial superpotential for moduli, and at the same time break supersymmetry. We shall assume that this nonperturbative mechanism takes place in the LEEFT and is not intrinsically stringy. This certainly appears to be the most "tractible" possibility. A popular candidate for such a mechanism has been gaugino condensation which is the focus of this talk.

As a specific model, we later consider gaugino condensation in a superstring-inspired effective field theory, with approximate S-duality invariance [2, 3] and exact T-modular invariance (generalization of the work in ref. [3]) and incorporate an intermediate scale M_I ($M_{\rm cond} \ll M_I \ll M_{\rm string}$), [4] in order to see how the intermediate-scale threshold corrections will affect gaugino condensation and supersymmetry breaking. This part of the talk is based on the work in ref. [4]. Incorporating the intermediate-scale threshold corrections into gaugino condensation is non-trivial in the sense that the field-theoretical threshold corrections at M_I are dilaton-dependent. Hence, these modifications can have non-trivial implications for supersymmetry breaking by gaugino condensation. Furthermore, *a priori*, nothing prohibits intermediate scales in the hidden sector.

The outline of this talk is as follows. In the next section, we review gaugino condensation, and of duality symmetries (modular and S-duality). We shall discuss our model in section 3.1, and give the renormalized Kähler potential including 1-loop threshold

corrections at an intermediate mass, and constrained by duality symmetries. The issues related to the scalar potential, dilaton run-away, and supersymmetry breaking , as well as the role of the intermediate mass are discussed in section 3.2. Concluding remarks are given in section 4. Due to the significance of renormalization of the field-dependent gauge coupling in such models and its connection with modular anomaly cancellation in the effective theory, we give a review of these ideas in the Appendix.

Generalities

Gaugino Condensation

A possible mechanism for breaking supersymmetry within the framework of ($N = 1$, $D = 4$) LEEFT of superstring is gaugino condensation in the hidden sector. In this scenario, the nonperturbative effects arise from the strong coupling of the asymptotically free gauge interactions at energies well below M_{Pl}. Corresponding to this strong coupling is the condensation of gaugino bilinear $\langle\bar{\lambda}\lambda\rangle_{h.s.}$. Let us briefly remind the reader the overview of the development of gaugino condensation. It was recognized many years ago that gaugino condensation in globally supersymmetric Yang-Mills theories without matter does not break supersymmetry [6]. In fact, that dynamical supersymmetry breaking cannot be achieved in pure SYM theories was shown by topological arguments of Witten [7]. In the locally supersymmetric case the picture is rather different, namely, gaugino condensation can break supersymmetry [8], and the gauge coupling is itself generally field-dependent. When the gauge coupling becomes strong, it gives rise to gaugino condensation at the scale[1]

$$M_{cond} \sim M_{string}\langle \mathrm{Re}T\rangle^{-1/2} e^{-\mathrm{Re}S/2b_0} = M_{string}\langle \mathrm{Re}T\rangle^{-1/2} e^{-1/b_0 g_{st}^2},$$

which breaks local supersymmetry spontaneously ($M_{cond}^3 \sim \langle\bar{\lambda}\lambda\rangle_{h.s.}$), and S is the dilaton/axion chiral field. Supersymmetry breaking in the obesrvable sector is induced by gravitational interactions which act as 'messenger' between the two otherwise decoupled sectors.

However, there are generally two problems associated with the above scenario. First, the destabilization of S — the only stable minimum of the potential in the S-direction being at $S \to \infty$; *i.e.,* in the direction where exact supersymmetry is recovered and the coupling vanishes! This is contrary to the expectation that the vacuum is in the strongly coupled, confining regime. This problem, the so-called dilaton runaway problem, is present in most formulations of gaugino condensation, in particular the so-called 'truncated superpotential' approach [10] , where the condensate field is assumed to be much heavier than the dilaton and therefore is integrated out below M_{cond}. In fact, the dilaton runaway problem is perhaps a more generic problem in string phenomenology where the underlying string theory is assumed to be weakly coupled. We shall return to the dilaton runaway later.

The second difficulty is the large cosmological constant that arises from the vacuum energy associated with gaugino condensation. An early attempt to remedy these difficulties was proposed by Dine *et al.* [10], in the context of no-scale supergravity whereby a constant term, c, is introduced in the superpotential which independently

[1]These arguments are modified by, for instance, the requirement of modular invariance [9].

breaks supersymmetry and cancels the cosmological constant. The origin of c is traced to the vev of the 3 form in 10D supergravity, and is quantized in units of order M_{pl}. Therefore, this approach has the unsatisfactory feature of breaking supersymmetry at the scale of the fundamental theory.

Duality Symmetries (Modular Invariance and S-Duality)

Modular symmetry, with the group $SL(2,\mathcal{Z})$ subgroup of $SL(2,\mathcal{R})$ duality transformations, written in its simplest form:

$$T \to \frac{\alpha T - i\beta}{i\gamma T + \delta}, \tag{1}$$

where $\alpha\delta - \beta\gamma = 1$ and $\alpha, \beta, \gamma, \delta$ are integers,[2] is an exact invariance of the underlying string theory. However, this symmetry is anomalous in the LEEFT. Cancellation, or partial cancellation, of this anomaly in the effective theory can be achieved by the Green-Schwarz (GS) mechanism, which is especially clear in the linear-multiplet formulation of the LEEFT [11, 12, 13]. In the corresponding chiral formulation, the adding of GS counter-terms amounts to modifying the dilaton Kähler potential:

$$\ln(S + \bar{S}) \to \ln(S + \bar{S} - bG),$$

where $b = -\frac{2}{3}b_0$, and b_0 is the E_8 one-loop β-function coefficient. $G = \Sigma_i \ln(T^i + \bar{T}^i - \Sigma|\Phi|^2)$, and Φ is any untwisted sector (non-modulus) chiral field in the theory. For simplicity, here we only consider models where modular anomalies are completely cancelled by GS mechanism, for example, the (2,2) symmetric abelian orbifolds with no $N = 2$ fixed planes, like Z_3 or Z_7 [11, 12, 13]. The role of the gauge coupling and its renomalization in superstring effective theories, and the connection with modular anomaly cancellation are reviewed in Appendix.

Recently, another type of duality symmetry has been receiving much attention in string theories. In this case the group of duality transformations is $SL(2,\mathcal{Z})$, but acting on the field S instead of T^i, and is referred to as S-duality. Like its T-analogue, this group has a generator which is the transformation $S \to 1/S$, and since S is related to the gauge coupling, this duality transformation is also referred to as 'strong-weak' duality. Font *et al.* [14] have conjectured that S-duality, like T-duality is an exact symmetry of string theory. More recently, it has been mounting evidence that S-duality is a symmetry of certain string theories [15]. However, these theories all have $N = 4$ or $N = 2$ supersymmetries. At the level of string theory, there are two different types of S-duality, namely (i) those that map different theories into one another, and (ii) those that map strongly and weakly coupled regimes of the same theory into each another. Indeed, presently there is no evidence of an S-dual $N = 1$ theory, and it is therefore difficult to justify the use of S-duality as a true symmetry in the corresponding LEEFT. However, it has been shown that in the effective theory, the full $SL(2,\mathcal{R})$ duality transformation is a symmetry of the equations of motion of the gravity, gauge, and dilaton sector in the limit of weak gauge coupling [2, 3]. As in [3], we shall take S-duality as a guiding principle in constructing the Kähler potential for the gaugino condensate, which is, so far, the least understood element in the description of the effective theory for gaugino condensation. That is, we assume that S-duality invariance is recovered in limit of vanishing gauge coupling, $S + \bar{S} \to \infty$.

[2]There is, generally, one copy of the group per modulus field T^i.

A Specfic Model

This model basically generalizes the momel of gaugino condensation with S-duality of ref. [3] to the case in which an intermediate scale is present. For details of the calculation and a more complete discussion, the reader is referred to ref [4].

The scheme of generating the intermediate scale considered here does not involve the spontaneous breaking of the hidden-sector gauge group. Here, we couple the hidden-sector gauge non-singlet fields Φ_i to a gauge singlet A. When A dynamically gets a vev, Φ_i become massive and the intermediate scale is thus generated. Since A is a singlet, the hidden-sector gauge group does not break. Such a scheme has interesting implications for gauge coupling unification [5]. For consistency, the pattern $M_{cond} \ll M_I \ll M_{string}$ is always assumed. Therefore, we shall integrate out the hidden-matter fields below M_I and the effective lagrangian at M_{cond} will consist of the moduli and the gauge composites only.

The superpotential for the hidden sector matter fields in our toy model is:

$$W_{HM} = \frac{1}{2}\lambda^{ij} A \Phi_i \Phi_j + \frac{1}{3}\lambda' A^3. \tag{2}$$

When constructing our model, two symmetry principles have been used to constrain the Lagrangian: First, the LEEFT must be T-modular invariant to all orders, according to all-loop string calculations. Second, S-duality is a symmetry in the weak-coupling limit $\langle S + \bar{S} \rangle \to \infty$. We will include the renormalization and intermediate-scale threshold corrections only in the dilatonic part of the Kähler potential. We simply write down the Kähler potential and for the full discussion we again refer the reader to ref. [4].

$$K = -\ln m - 3\ln(1 - m^{1/3}Q) + G \tag{3}$$

where[3],

$$m = 2/g^2_{eff}(M_{cond}) = S + \bar{S} - bG + 3\left[b^{>}\ln\left(\frac{M_I^2}{M^2_{string}}\right) + b^{<}\ln\left(\frac{Q/(S+\bar{S}-bG)}{M_I^2}\right)\right], \tag{4}$$

is the renormalized coupling including the 1-loop threshold corrections at the canonically normalized, modular invariant intermediate mass M_I which can be computed to be:

$$M_I^2 = e^K (K^{\varphi\bar{\varphi}})^2 |\lambda A|^2 = \frac{|\lambda A|^2 e^{G/3}}{9(s+\bar{s}-bG)}\left(1 + \frac{b}{s+\bar{s}-bG}\right)^{-2}. \tag{5}$$

In these relations, G is the Green-Schwarz term,

$$G = -3\ln(T + \bar{T} - |A|^2), \tag{6}$$

and $Q = |H|^2 e^{G/3}$ (where H is the condensate superfield) is the modular invariant condensation scale. Various group theoretical factors are as foolws:

$$-3b = 2b_0 = \frac{1}{8\pi^2}C(E_8); \qquad b^{>} = (3C_G - C_M)/24\pi^2, \qquad b^{<} = C_G/8\pi^2, \tag{7}$$

[3]Here, $M_{pl} = 1$; and notice that the UV cut-off is taken to be $M_{string} = (S + \bar{S} - bG)^{-1/2}$ meaning that the condensation scale is really in these units, $Q/(S + \bar{S} - bG)$.

where C_G and C_M are the quadratic Casimirs:

$$C_G = T(adj); \quad C_M = \sum_r n_r T(r); \quad T(r) = \mathrm{Tr}_r(T^2), \tag{8}$$

with r labelling the representations of the gauge group, and n_r being the number of fields in the r representation.

To summarize, our Kähler potential given in eq. (3) includes the one loop renormalization of the dilaton with the intermediate threshold corrections, as well GS counter terms that ensure modular anomaly cancellation. It is also constrained by approximate S-duality symmetry as discussed in references [3, 4].

The dynamical fields at the condensation scale in our model are S, H, and T. The scalar potential is given by:

$$V = e^K \left[K^{i\bar{j}}(K_i W + W_i)(K_{\bar{j}}\bar{W} + \bar{W}_{\bar{j}}) - 3|W|^2\right] \tag{9}$$

and the Kähler metric written in terms of $m = 2/g^2_{eff}(M_{cond})$ (eq. (5)), $Q = |H|^2 e^{G/3}$, and their derivatives with respect to the scalar fields is given by:

$$\begin{aligned} K_{i\bar{j}} = m^{-2}\{m_i m_{\bar{j}}\tilde{x} \;&+\; m(\xi - 1)m_{i\bar{j}} + (\xi + \xi^2)(m_i q_{\bar{j}} + m_{\bar{j}} q_i) \\ &+\; 3m^2[\xi q_{i\bar{j}} + (\xi + \xi^2) q_i q_{\bar{j}}] + m^2 G_{i\bar{j}}\}, \end{aligned} \tag{10}$$

where

$$x = m^{1/3}Q, \quad \xi = \frac{x}{1-x}, \quad \tilde{x} = 1 - 2\xi/3 + \xi^2/3,$$

and

$$m_i = \partial_i m, \quad q = \ln Q, \quad q_i = \partial_i \ln Q, \quad etc.$$

Notice that $G_{i\bar{j}} = 0$ unless $i = j = t$, $m_{h\bar{j}} = 0$, and $q_s = 0$. The nonperturbative part of the superpotential is of the form

$$W_{NP} = \alpha e^{-S/b} Y^n \left(\ln \frac{Y}{\mu}\right)^k, \quad Y = H e^{S/3b^<}, \tag{11}$$

with $n < 3$ (the Veneziano-Yankielowicz superpotential is the special case of $n = 3$ and $k = 1$). The reason the exponents n and k are introduced is because it is the Kähler potential (3) that already includes the gaugino condensate wave function renormalization, and so the superpotential should not.

We summarize the results of the numerical computation and analytic expansions of this model as follows. The scalar potential, V, is positive semi-definite and has a nontrivial minimum at finite values of the dilaton and the condensate field, at which the following relations are satisfied:

$$\langle W \rangle \simeq 0, \quad \text{and} \quad m = 2/g^2_{eff}(M_{cond}) \to 0. \tag{12}$$

This is in addition to the usual 'runaway' solution at $S \to \infty$ and $H = 0$. Notice that the second relation tells us that the coupling $g_{eff}(M_{cond})$ blows up (at a finite value of $S + \bar{S}$). However the nontrivial minimum occurs at the boundary of the kinematically forbidden region of the (S, H) plane. In other words, the potential runs in this direction as well! But this the correct direction, as the value that it runs to corresponds to strong coupling at the condensation scale, with a nonzero value of the condensate. The

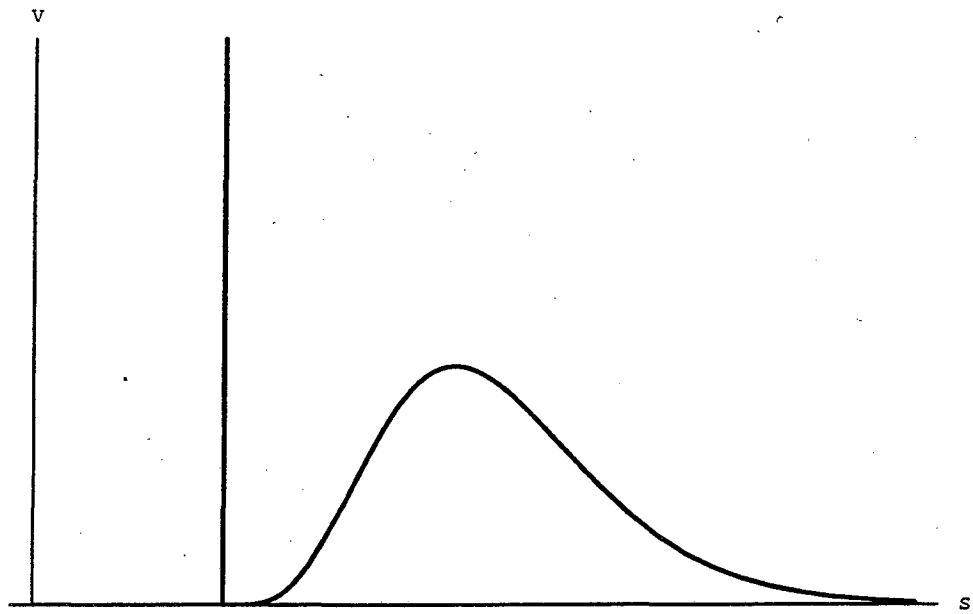

Figure 1: The running behaviour of the dilaton. The minimum on left is at the boundary of the kinematically forbidden region.

similar running behaviours (see Fig. 1) in both strong and weak coupling directions is attributed to S-duality. Notice however that the relations $\langle V \rangle = \langle W \rangle = 0$ imply that supersymmetry remains unbroken.

As for the effect of the intermediate mass, the two independent conditions $m = \langle W \rangle = 0$ imply that [4] the parameter μ of the nonperturbative superpotential in eq. (11) is 'locked' to M_I; and that the parameters of the superpotential which generates M_I allows for a phenomenologically sensible hierarchy between the condensation and the string scales.

Conclusion

We have discussed hidden sector gaugino condensation as a possible mechanism for supersymmetry breaking. In the model which was presented in the last section, which included some perturbbative corrections to the Kähler potential, as well as a nonperturbative constraint (S-dulaity), we saw that suprsymmetry remains unbroken. Perhaps the most peculiar feature of our model is the running behaviour of the dilaton, which is schematically shown in Fig. 1. Because the 'minimum' on the left hand side is at the boundary of the kinematically forbidden region, we hesitate to call this stabilization of the dilaton. The perturbative breakdown of supersymmetry and stabilization of moduli of string theory may require the full 1-loop corrections to the effective supergravity theory which have been recently calculated [16]. On the nonperturbative side, perhaps other stringy nonperturbative effects are more crucial as pointed ot in ref. [17]. A realization of this proposal in the context of linear multiplet formulation of gaugino condensate appears in ref [18]. Of course, the exact form of these nonperturbative

corrections are not yet understood. But one can perhaps expect that the recent developments in string dualities can shed some light on the latter, and on the stabilization of string moduli and supersymmetry breaking.

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Appendix - The Role of the Gauge Coupling

In this appendix, we recall a few facts about the perturbative corrections of the gauge coupling function in the superstring effective field theories, as well as the connection with modular invariance of the effective theory.

As mentioned earlier, in our approach, the one-loop renormalization of the gauge coupling is completely included in the Kähler potential K, *i.e.,* the renormalization effects are completely absorbed into K by replacing the tree-level gauge coupling $S+\bar{S}$ in K by the one-loop renormalized gauge coupling. Therefore, it is worthwhile to discuss the renormalization of gauge couplings in supestring LEEFTs.

Let us first recall the Lagrangian for supergravity plus super-YM. In the Kähler covariant formalism [20] the classical superfield Lagrangian is given by:

$$\mathcal{L} = \int d^4\theta\{-3E + \frac{E}{8R} f_{ab}(Z) W^{\alpha a} W^b_\alpha + \frac{E}{2R} e^{K/2} W(Z)\} + h.c., \qquad \text{(A.1)}$$

where $E = \mathrm{Sdet} E_M{}^A$, R is the curvature scalar of the superspace, and Z stands for the chiral fields in the theory. The first term in eq. (A.1) corresponds to the kinetic energy for the gravity sector as well as the chiral fields. The chiral fields enter through the dependence of the spinorial derivatives of E on the Kähler potential, $K(Z,\bar{Z})$. The second term describes the super-YM coupling to the theory, with the (holomorphic) gauge coupling function $f_{ab}(Z)$ and the YM 'field-strength' superfield

$$W_\alpha = W_{\alpha a} T^a = (\frac{1}{8}\bar{\mathcal{D}}^2 - R) e^{-2V} \mathcal{D}_\alpha e^{2V},$$

where V is the vector multiplet containing the YM gauge potential. We shall take $f_{ab} = f\delta_{ab} = S\delta_{ab}$ corresponding to the bare coupling of the effective superstring theories where S is the dilaton/axion chiral superfield. The component form of the second term contains:

$$\int d^4x \sqrt{g} \left(-\frac{1}{4} \mathrm{Re} f \, \mathrm{Tr}(F^2) - \frac{1}{4} \mathrm{Im} f \, \mathrm{Tr}(F\tilde{F}) \right),$$

and thus Ref is the YM gauge coupling, while Imf gives the axionic coupling.

Finally in the last term of eq. (A.1), $W(Z)$ is the superpotential which is a holomorphic function of the chiral matter fields (independent of S and other internal moduli, until supersymmetry is broken nonperturbatively).

In discussing the gauge couplings in effective theories, it is important to to distinguish between the *Wilsonian* couplings, and the physical, or *effective* couplings. In particular in the effective supersymmetric theories that we are considering, there are powerful statements that can be made about the two types of gauge coupling. The (holomorphic) Wilsonian gauge couplings in supersymmetric YM theories, which appear in the Wilson effective action, do not renormalize beyond one loop. These are funcions that appear in the Wilson effective action, $S_W(\mu)$, the local functional of quantum operators. In $S_W(\mu)$, only momenta between the scale μ and the UV cut-off contibute to loops. The physically measurable 'effective' (or running) couplings appear in the c-number valued generating functional of 1PI graphs, Γ; this is in general a nonlocal functional of background fields that contain the IR momenta $p < \mu$ running through loops, as well. Right at the UV cut off, the Wilsonian couplings, *i.e.,* the coefficients appearing in front of the operator terms in S_W are the bare couplings of the theory. The relation between the two effective actions may formally be written as [21]

$$e^{i\Gamma[\Phi_{cl},\mu]} = \langle e^{iS_W[\Phi,\mu]}\rangle,$$

where the expectation value on the right hand side is taken in the the presence of background fields. In the supersymmetric YM theories, it is known that, unlike the Wilsonian gauge coupling, the effective coupling renormalizes perturbatively at all orders, and that, indeed, higher order corrections introduce nonholomorphicities [21]. The generalizations of these results to supergravity effective theories of superstrings have been carried out more recently [1, 13, 19, 22].

The gauge coupling in all $N = 1$ effective heterotic string constructions is given at tree level by:

$$g_\alpha^{-2} = k_\alpha \mathrm{Re}S = k_\alpha g_{string}^{-2}. \tag{A.2}$$

ReS is the 'universal' gauge coupling at string scale, and k_α is the level of the affine Lie algebra associated with the factor G_α of the product gauge group. Subsequently, we shall set $k_\alpha = 1$, and throughout the analysis G_α refers to the IR strong group with gaugino condensation. The exact Wilsonian coupling is given by the holomorphic function: $f_W = S + f^{(1)}$, and the moduli dependent one-loop (*i.e.,* all-loop) correction $f^{(1)}(T^i)$ has been determined [22] (see below). The effective gauge coupling, with LEEFT-loop corrections to all orders is given by [19, 21]:

$$\begin{aligned} g_{eff}^{-2}(p^2) = \mathrm{Re}S \quad &+ \quad b_0 \ln\frac{\Lambda^2}{p^2} + cK + \frac{T(adj)}{8\pi^2}\ln g_{eff}^{-2}(p^2) \\ &- \quad \frac{1}{8\pi^2}\sum_r T(r)\ln\det Z_{eff}^{(r)}(p^2), \end{aligned} \tag{A.3}$$

where, $b_0 \equiv (-3T(adj) + \sum_r n_r T(r))/16\pi^2$ (the YM β-function coefficient), and $c \equiv (-T(adj) + \sum_r n_r T(r))/16\pi^2$, and Z is the kinetic normalization matrix. To one-loop order, one has to evaluate the r.h.s. of the above equation at tree level, at 2-loop the r.h.s. is evaluated to one loop, etc. The one-loop result has also been obtaind in [13]. Threshold corrections due to integrating out the heavy string modes have been calculated in reference [1]. These corrections are only dependent on the moduli

T^i, and not on the dilaton. All the perturbative dilaton dependences in the effective gauge coupling arise from *field-theoretical* loop effects. We have seen in section (3) that threshold corrections in the effective field theory also introduce dilaton-dependent terms in the running coupling.

Let us now turn to the question of modular invariance. As inputs from string theory, for general fields Φ^I (ignoring for the moment the GS counter terms), we have the normalisation matrix for the kinetic term, and the Kähler function. The former is given by:

$$Z_{I\bar{J}} = \delta_{I\bar{J}} \prod_i (T^i + \bar{T}^i)^{-q_I^i} + \mathcal{O}(\Phi^2), \tag{A.4}$$

where the rational numbers q_I^i are the modular weights of the field Φ^I. They depend on the twist sector of the orbifold which gives rise to the matter fields Φ^I, and the modulus field T^i. The Kähler function at the tree level is given by $K = -\ln(S+\bar{S}) - \sum_i \ln(T^i + \bar{T}^i) + \mathcal{O}(\Phi^2)$. For the modular transformation given in eq. (1) of the text, K transforms by the usual transformation law:

$$K \to K + F + \bar{F}, \qquad F = \sum_i \ln(i\gamma_i T^i + \delta_i). \tag{A.5}$$

Under a modular transformation, the non-modulus chiral fields, transforms as:

$$\Phi^I \to C^I_J(T^i)\Phi^J. \tag{A.6}$$

Hence, the kinetic matrix $Z_{I\bar{J}}$ transforms according to:

$$Z \to (C^\dagger)^{-1} Z C^{-1}. \tag{A.7}$$

It follows from eq. (A.5 – A.7) that the reparametrization induced on the matter fields by modular tranformations is given by:

$$C^I_J = C^I_{0J} \prod_i (i\gamma_i T^i + \delta_i)^{q_I^i}, \tag{A.8}$$

where C^I_{0J} is moduli independent.

For a generic supergravity theory with super-YM, under the combined transformations: $K \to K + F + \bar{F}$ and $\Phi^I \to C^I_J \Phi^J$ with C^I_J holomorphic function of the moduli Φ^I, the Kähler invariance of the (exact) integral of the RGE's, *i.e.,* eq. (A.3) imply that:

$$f_W \to f_W + cF - \frac{1}{2\pi^2} \sum_r T(r) \mathrm{tr} \ln C^{(r)}, \tag{A.9}$$

where c is the group theoretical factor given after eq. (A.3) above. For C^I_J and F corresponding to modular transformations, eq's (A.5) and (A.8), this gives:

$$\mathrm{Re} f_W \to \mathrm{Re} f_W - \frac{1}{16\pi^2} \sum_i 2\alpha^i \ln |(i\gamma_i T^i + \delta_i)|^2, \tag{A.10}$$

with

$$\alpha^i = \sum_I T(\Phi^I)(1 - 2q_I^i) - T(adj); \quad T(\Phi^I) = \sum_r \mathrm{tr}(T^2(r)), \tag{A.11}$$

and $T_a(r)$ are the generators of the representations of the fields Φ^I.

Furthermore, the transformation law (A.10) corresponds, up to a modular invariant function, to the transformation of the logarithm of Dedekind function. In fact it will

give the complete modular dependent perturbative correction, $f^{(1)}$ to the Wilsonian coupling [1, 12, 13] :

$$\mathrm{Re} f^{(1)} = \delta \mathrm{Re} f_W = -\frac{1}{4\pi^2} \sum_i \alpha^i \ln |\eta(iT^i)|^2, \tag{A.12}$$

modulo a moduli independent part which has been argued to be a constant in most orbifold models [22]. These equations are interpreted as a parametrization of the string threshold corrections to the gauge couplings [1].

Modular invariance is restored by including factors of $\eta(iT^i)$ in the superpotential (see eq (2)), and in the definition of the fields, so as to cancel the above modular dependent correction of the gauge coupling, as well as by introducing GS counter term as discussed in the text. However, the inclusion of the η factors tends to spoil the boundedness from below of the scalar potential. To avoid this, we may restrict ourselves to the orbifold models which do not receive string threshold corrections. These models have been classified [1, 12, 22]. For such models, the modular anomaly is solely cancelled by the GS counter term.

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