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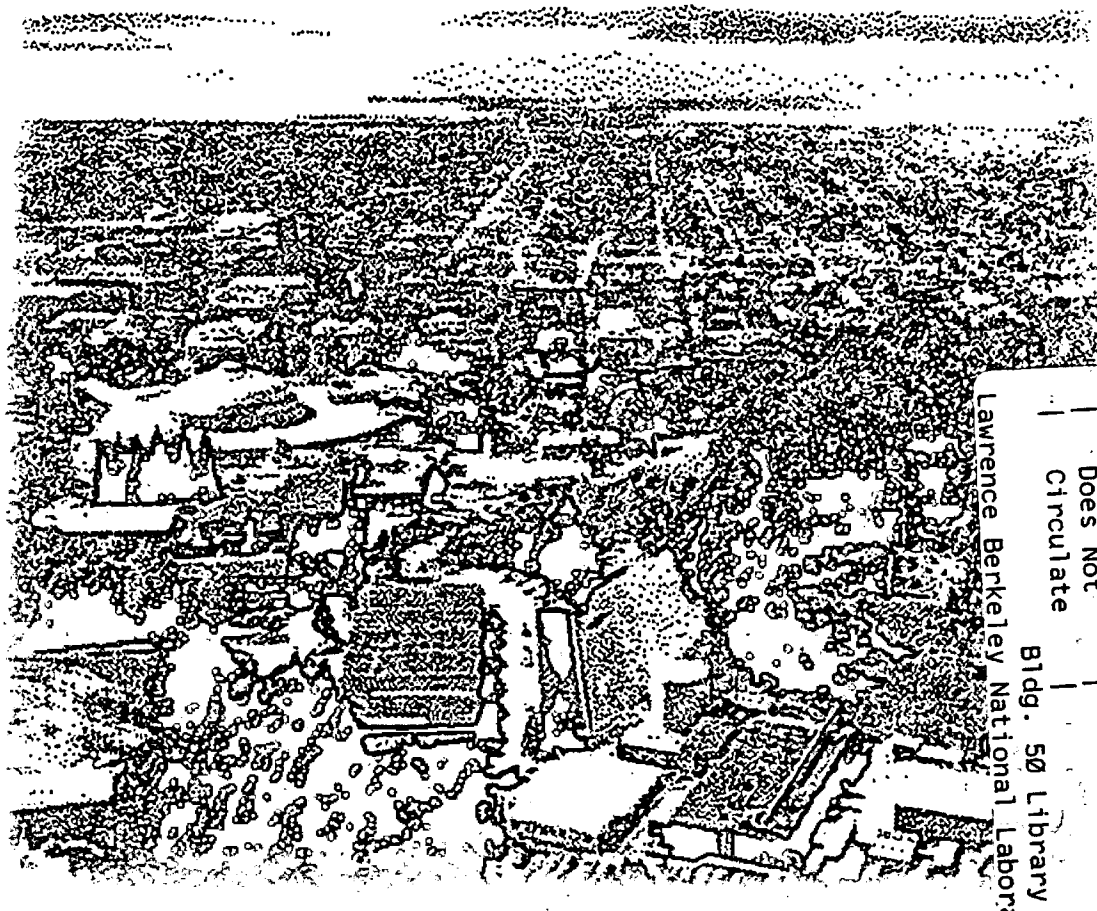

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MULTI-TIME COSMOLOGY

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ABSTRACT

A multi-time cosmological model characterizes by a single parameter(age) the large-scale aspects of a homogeneous universe. Einstein's objective universe within a (single) $3+1$ spacetime manifold is replaced by "aspects of universe perceivable from a standpoint." Using 8-vectors, the model distinguishes a noncompact "cosmological" spacetime housing standpoints from a compact "physical" spacetime housing accelerating matter perceivable from a standpoint. Within regions small on Hubble scale and large on quantum-particle scale, general relativity prevails and provides an objective description of matter. Certain although not all features of Milne's "kinematic cosmology" are manifested. The phenomenological age problem encountered by the Einstein-de Sitter model is alleviated.

I. Introduction

General relativity was developed by Einstein[1] as a local theory of gravity with attention neither to quantum mechanics nor to finiteness of universe age (i.e., "big bang"). Verification of general relativity has been absent not only at quantum-particle scale and below but at Hubble scale and above. The standard (Friedman-Robertson-Walker) cosmological model[2], based on general relativity, has turned out "unesthetic" in the nonuniqueness of its description of homogeneous universe, while the single-parameter Einstein-de Sitter version[3] currently appears to give an observationally-unsatisfactory relation between Hubble time and universe age.[4] Reflecting on the Copenhagen interpretation of quantum mechanics, which through explicit recognition of measurement has undermined classical objective reality, the author has been led to speculate that Einstein's objective universe within a (single) $3+1$ spacetime manifold should be replaced by "aspects of universe perceivable from a standpoint." Objective reality in Einstein's sense may be meaningful only at *intermediate* scales — large compared to quantum particles but small compared to the Hubble-discovered scale. In such spirit this paper presents a multi-time classical standpoint-based model of large scales where homogeneous universe is characterized by a single parameter. The phenomenological age problem encountered by the Einstein-de Sitter model is alleviated.

Although "doubling" of energy-momentum is familiar in the analytic S-matrix representation of particles through the *complex* Poincaré group (quantum particles being associated with S-matrix poles),[5] complexification of energy-momentum has to date not led to enlargement of a $3+1$ spacetime. Nevertheless, *two* different classical

meanings for time are commonplace: (1) "Physical time" based on periodic (reproducible and reversible) motion, associated with stability (permanence) of "objects" such as quantum particles. This is the time of Galilean physics, facilitating the notion of reproducible measurement (and, quantum-mechanically, related inversely to energy difference). It is also the time of S-matrix theory and quantum field theory, where it attaches to localized matter. (2) "Cosmological" and also (locally) "geological-biological" time, based on irreversible aging of systems ("old" vs. "young"). Although system complexity may be presumed in principle to relate physical time with geological or biological age, indefinite expansion of universe undercuts reproducible measurement on Hubble scale. A cosmological model that *explicitly* recognizes two qualitatively-different types of time may prove interesting. Local reproducible physics might be consistently embedded within an expanding universe.

Physical measurement and Poincaré symmetry being intertwined, universe expansion suggests basing an observation-centered cosmological model on the 10-parameter Poincaré group ($3+1$ spacetime displacements plus Lorentz transformations) enlarged by *dilation* to 11-parameters. Complexification to achieve time doubling and representation of gravity then leads to a 22-parameter group, with 8 "inhomogeneous" dimension-carrying generators of complex Poincaré displacements and 14 "homogeneous" dimensionless generators of complex Lorentz transformations plus complex dilations. Elements of the homogeneous group act on "8-vectors". In our model, loosely speaking, an 8-space of dimension length accommodates a $3+1$ "global" non-compact spacetime in which "standpoints" locate, while to each standpoint attaches a $3+1$ "physical" (local) compact spacetime housing accelerating matter representable from that standpoint. (A standpoint is *physically* distinguished from other standpoints of the same age by "surrounding matter".) Although the model allows the local spacetimes belonging to any collection of "neighboring" standpoints — whose separations are small on Hubble scale — to be approximately mapped onto each other via general-relativistic transformations, widely-separated standpoints carry (matter-housing) spacetimes that cannot generally be brought into correspondence. "Slow matter" in the neighborhood of one standpoint may nevertheless be located within the spacetime of a distant standpoint. Also locatable in the spacetimes of two widely-separated standpoints is "fast matter" that transmits a signal between the two standpoint neighborhood. The model displays *approximate* objectivity.

An 8-vector "magnitude" will be defined that is invariant under all homogeneous-group elements *except* real dilation. A special Finsler[6] (non-Riemannian) geometry then results from requiring the magnitude of an infinitesimal 8-vector "matter displacement" to be invariant in such sense. The invariance provides a degree of "objective reality" although not in the full sense of Einstein because the Finsler metric tensor generally exhibits velocity dependence.

The 8-space corresponds to the coset formed by factoring out the 6-parameter complex rotation group from the 14-parameter homogeneous group. The surviving 8 dimensions will be represented by 3 Lorentz boosts, 3 analogous but compact O(4) rotations plus a (real) dilation and an "imaginary dilation" isomorphic to a $U(1)$

transformation. The latter will relate to the local-time displacement of physics, while the (real) dilation relates to growth of standpoint age. Lorentz boosts will be found to control spatial location of one standpoint as perceived from another in a (noncompact) sense similar to that of Milne's "kinematic cosmology."[7]

Striking as a model feature is association of the *compact* coset $U(1) \times O(4)/O(3)$ with local $3+1$ spacetime-location of matter with respect to standpoint. Such association makes sense in a *special* coordinate system belonging to a standpoint. This coordinate system may be arbitrarily rotated but admits neither Lorentz boosts nor displacements. Only when described in these special coordinates does early-universe matter display "homogeneity" (e.g., isotropic cosmic-background radiation). Attachment of special coordinates to standpoint relates to *noncommutation* of Lorentz and $O(4)$ subgroups of our homogeneous group.

Although certain features of Milne's model, such as equality between universe age and Hubble time, are exhibited by ours, predictions from our model do not entirely agree with those of Milne, whose spacetime was flat. Our matter-housing spacetime is "curved", implying a mean energy density that, rather than being negligibly small as in the Milne model, exceeds the standard model's "critical density"; correspondingly our luminosity distance increases less rapidly with redshift than predicted by Milne.

For weak gravity at sub-Hubble scale our Finsler geometry becomes Riemannian and there is concordance with general relativity. Description of strong gravity at sub-Hubble scale ("black hole") requires model development beyond that described in the present paper.

Physical interpretability of multi-time cosmology rests on the Finsler geometry of a standpoint spacetime becoming Riemannian not only in standpoint spatial neighborhood but in the *phase-space* neighborhood of all "physical" geodesics passing through standpoint neighborhood. By "physical" is meant a geodesic along which matter (subject only to gravity) may flow. "Signals" move along physical geodesics.

"Phase-space neighborhood" refers to a 7-dimensional Finsler phase space — 4 spacetime coordinates plus 3 velocities. A general Finsler metric, depending both on coordinates and on velocities, eludes physical interpretation. Objective physics rests on Riemannian geometry where dependence of metric *only* on coordinates leads to local conservation of energy and momentum. The standpoint spacetime becomes Riemannian not only in spatial neighborhood of standpoint but, in a phase-space sense, within the neighborhood of any geodesics that can carry a signal to the selected standpoint.

II. 8-Vectors

An 8-element set of real numbers, transforming into a linear combination of itself under the 14-parameter homogeneous group, will be called an 8-vector. There are two 8×8 real-matrix representations of the homogeneous group acting respectively on two types of 8-vector that behave oppositely in infinitesimal transformations other than complex rotations. The two types are conveniently distinguished by response to

dilation — a distinction associable with "dimensionality". As physically invoked in our model, upper-index 8-vectors

$$\hat{A} = (A^{\mu}, A^{\mu'}) \quad , \quad \mu, \mu' = 0, 1, 2, 3 \tag{1}$$

have the dimensionality of length and time (c = 1), while lower-index 8-vectors

$$\hat{B} = (B_{\mu}, B_{\mu'}) \tag{2}$$

have the dimensionality of energy and momentum. The present paper will be concerned mostly with the former type. As the foregoing notation suggests, any 8-vector may be organized into a pair of Lorentz 4-vectors. Two mixed-type inner products are (separately) invariant under the full 14-parameter group, including complex dilations. Our shorthand for these mixed invariants is

$$\hat{A} \cdot \hat{B} \equiv A^{\mu} B_{\mu} - A^{\mu'} B'_{\mu}, \tag{3}$$

$$\hat{A} * \hat{B} \equiv A^{\mu} B'_{\mu} + A^{\mu'} B_{\mu}. \tag{4}$$

Two analogous quadratic forms built from a *single* 8-vector,

$$\hat{A} \cdot \hat{A} \equiv \eta_{\mu\nu}(A^{\mu} A^{\nu} - A^{\mu'} A^{\nu'}), \tag{5}$$

$$\hat{A} * \hat{A} \equiv 2\eta_{\mu\nu} A^{\mu} A^{\nu'}, \tag{6}$$

with $\eta_{\mu\nu}$ the Minkowski metric tensor, are invariant under complex Lorentz transformations but *not* under either real or imaginary dilations. The *quartic*

$$(\hat{A} \cdot \hat{A})^2 + (\hat{A} * \hat{A})^2 \tag{7}$$

is nevertheless invariant under all homogeneous transformations *except* (real) dilations. Such a quartic form will be the basis for our model's Finsler geometry. The positive fourth root of (7) may be called the "magnitude" of $\hat{A}$.

An arbitrary spacetime-type 8-vector may be constructed through a 4-parameter $U(1) \times O(4)$ "rotation" of the "fiducial" 8 vector $(x^{\mu}, 0)$, where x^{μ} transforms as a 4-vector under real Lorentz transformation. The 4 angular parameters will be designated $-\pi \leq \Theta^{\mu} \leq \pi$, $\mu = 0, 1, 2, 3$. The $U(1)$ transformation is by the angle Θ°; the 3 $O(4)$ angles $\vec{\Theta}$ behave as a 3-vector under rotations, as do the 3 parameters $\vec{x}$. Defining

$$\Theta = \mid \vec{\Theta} \mid, \qquad \vec{u} \equiv \frac{\vec{\Theta}}{\Theta}, \tag{8}$$

and

$$\vec{x}_{\text{long}} \equiv (\vec{x} \cdot \vec{u})\vec{u}, \quad \vec{x}_{tr} \equiv \vec{x} - \vec{x}_{\text{long}}, \tag{9}$$

the general spacetime-type 8-vector may be written

$$\hat{A}(\mathbf{x}, \mathbf{\Theta}) = \begin{pmatrix} x^\circ \cos\Theta^\circ \cos\Theta - \vec{x} \cdot \vec{u} \sin\Theta^\circ \sin\Theta \\ -(\vec{x}_{tr} + \vec{x}_{\text{long}} \cos\Theta) \cos\Theta^\circ + x^\circ \vec{u} \sin\Theta^\circ \sin\Theta \\ -x^\circ \sin\Theta^\circ \cos\Theta - \vec{x} \cdot \vec{u} \cos\Theta^\circ \sin\Theta \\ (\vec{x}_{tr} + \vec{x}_{\text{long}} \cos\Theta) \sin\Theta^\circ + x^\circ \vec{u} \cos\Theta^\circ \sin\Theta \end{pmatrix}, \tag{10}$$

where $\mathbf{x}$ is shorthand for $(x^\circ, \vec{x})$ and $\mathbf{\Theta}$ is shorthand for $(\Theta^\circ, \vec{\Theta})$. The special parameterization (10) exhibits absence of change in x° and $\mid \vec{x} \mid$ when a $U(1) \times O(4)$ transformation is made on $\hat{A}$ and dovetails with a standpoint spacetime isomorphic to the coset $U(1) \times O(4)/O(3)$. Acceleration of matter perceived from a prescribed standpoint will relate to displacements of Θ, while the parameters $\mathbf{x}$ will locate *another* standpoint with respect to the selected standpoint. This *other* standpoint coincides in perceived location with the matter being described from the selected standpoint.

III. Displacement of Matter

Accepting Einstein's association of gravity with geometry (equivalence of gravitational and inertial mass), we propose to identify "gravitational action" with "displacement distance" for arbitrary matter in the spacetime belonging to a selected standpoint. Inspired by Lorentz invariance of the Minkowski metric, we seek a gravitational action (or "distance") that is invariant under all homogeneous transformations except dilation. "Near" the standpoint our metric must reduce in homogeneous-universe approximation to that of Minkowski. More generally, "weak" inhomogeneities in standpoint neighborhood should be representable by a Riemannian metric that accords with general relativity. By "standpoint neighborhood" we mean a standpoint – containing region that is small on Hubble scale.

Let us associate to a "particle of matter" m, described from standpoint i, the spacetime-type 8-vector $\hat{\mathbf{x}}_i^m = \hat{A}(\mathbf{x}_i^m, \mathbf{\Theta}_i^m)$, where each of the 8 parameters $(\mathbf{x}_i^m, \mathbf{\Theta}_i^m)$ is a prescribed differentiable function over a compact 4-manifold that is spanned by the coset $U(1) \times O(4)/O(3)$ and parameterized by 4 angles ϕ. By $\mathbf{x}_i^m$ and $\mathbf{\Theta}_i^m$ we mean $\mathbf{x}_i(\phi^m)$, $\mathbf{\Theta}_i\ (\phi^m)$.

The 4 parameters $\mathbf{x}_i^m$ locate, in the i coordinate system, that *other* standpoint which coincides in location with our particle. In other words, without the superscript m, $\mathbf{x}_i$ represents the *matter-based* perception from i standpoint of a spacetime that houses other standpoints. To the extent that *all* matter representable from one standpoint need not be representable from another, the spacetime of standpoint i fails to house *all* standpoints. There will nevertheless be i-spacetime housing of any standpoint in i neighborhood.

More difficult to describe is physical significance of the angles $\mathbf{\Theta}_i^m$; they relate to particle displacement — to velocity and acceleration. It is the *change* of $\mathbf{\Theta}_i^m$ as particle location changes that has physical significance. (Shift of $\mathbf{\Theta}_i^m$ at fixed $\mathbf{x}_i^m$ might be described as a physically-empty "gauge" transformation.) Change in $\mathbf{\Theta}_i^m$

correlates with particle-location through the functions $\mathbf{x}_i(\boldsymbol{\phi}^m)$ and $\boldsymbol{\Theta}_i\ (\boldsymbol{\phi}^m)$. This correlation will be seen below to control the metric of i-standpoint spacetime.

When only gravitational forces are to be considered, the functions $\mathbf{x}_i(\boldsymbol{\phi})$ and $\boldsymbol{\Theta}_i\ (\boldsymbol{\phi})$ are the same for all particles, although these functions generally depend on i–providing physical distinction between different standpoints. We shall see that the functions $\mathbf{x}_i(\boldsymbol{\phi})$ depend only on standpoint age (meaning of "standpoint age" will be made precise below), while the functions $\boldsymbol{\Theta}_i(\boldsymbol{\phi})$ depend also on standpoint spatial location. Henceforth in the present paper because we ignore nongravitational forces, the particle superscript m will be omitted. In *homogeneous-universe* approximation, furthermore, the functions $\boldsymbol{\Theta}_i(\boldsymbol{\phi})$ will not even depend on i. (All standpoints of the same age are equivalent in a homogeneous universe.)

Meaning here for the term "particle" is classical, merely implying spatial localization of (positive) energy within some small region. "Small" means by comparison to distance from standpoint, so that particle "trajectory" in standpoint spacetime is meaningful. Otherwise we put no upper limit on "particle diameter" and no requirement as to "structure". Neglect of non-gravitational ("short-range") forces implies particle size sufficiently large that all short-range forces are "internal" to individual particles. With respect to light, "particle size" is large compared to wavelength.

It is convenient to choose the origin of the compact $\boldsymbol{\phi}$ space so that matter with $\boldsymbol{\phi} = 0$ locates at standpoint. With no loss of generality the functions $\boldsymbol{\Theta}_i(\boldsymbol{\phi})$ and $\mathbf{x}_i(\boldsymbol{\phi})$ may be chosen so that $\boldsymbol{\Theta}_i(0) = 0$ and $\vec{x}_i(0) = 0$, while $x_i^\circ(0) = R_i$. The real positive parameter R_i will completely control the functions $\mathbf{x}_i(\boldsymbol{\phi})$ and will emerge as central to our model. "Neighborhood" of standpoint will mean $\mid \phi^\circ \mid << 1$, $\mid \vec{\phi} \mid << 1$ and, correspondingly, $\mid x_i^\circ - R_i \mid << R_i$, $\quad \mid \vec{x}_i \mid << R_i$.

Subject to later verification, we presume the functions $\boldsymbol{\Theta}_i(\boldsymbol{\phi})$ to be such that, so long as black holes do not occur in standpoint proximity, "neighborhood of standpoint" will mean $\mid \vec{\Theta}_i \mid << 1, \mid \Theta_i^\circ \mid << 1$. Particle displacement in standpoint neighborhood is then seen from (10) to associate with the 8-vector

$$d\hat{x}_i = \begin{pmatrix} dx_i^\circ \\ -d\vec{x}_i \\ -R_i d\Theta_i^\circ \\ R_i d\vec{\Theta}_i \end{pmatrix}, \tag{11}$$

whose magnitude according to (7) is the positive fourth root of

$$\begin{aligned} &[dx_i^{\circ 2} - d\vec{x}_i^{\,2} - R_i^2(d\Theta_i^{\circ 2} - d\vec{\Theta}_i^{\,2})]^2 + 4R_i^2[dx_i^\circ d\Theta_i^\circ - d\vec{x}_i \cdot d\vec{\Theta}_i]^2 \\ &\qquad = [\eta_{\mu\nu}(dx_i^\mu dx_i^\nu - R_i^2 d\Theta_i^\mu d\Theta_i^\nu)]^2 + 4R_i^2[\eta_{\mu\nu} dx_i^\mu d\Theta_i^\nu]^2. \end{aligned} \tag{12}$$

Because each of the 8 variables x_i^μ and Θ_i^ν is a specified differentiable function of a set of 4 parameters $\boldsymbol{\phi}$, we may define functions

$$g_i^{\mu}{}_{\nu} \equiv R_i \frac{\partial \Theta_i^\mu}{\partial x_i^\nu} \tag{13}$$

over the compact 4-manifold. It is then possible to rewrite (12) as

$$[(\eta_{\mu\nu} - g_{i\gamma\mu}g_{i\nu}^{\gamma})dx_i^{\mu}dx_i^{\nu}]^2 + 4[g_{i\mu\nu}dx_i^{\mu}dx_i^{\nu}]^2, \tag{12'}$$

the raising and lowering of indices here being controlled by the Minkowski tensor $\eta_{\mu\nu}$.

A standpoint-neighborhood metric proportional to the positive fourth root of (12′) is not Riemannian; we are dealing here with a special case of Finsler geometry.[6]. Nevertheless, to the extent that

$$h_i^{\mu}{}_{\nu} \equiv g_i^{\mu}{}_{\nu} - \eta^{\mu}{}_{\nu} \tag{14}$$

is *small* compared to $\eta^{\mu}{}_{\nu} = \delta_{\mu\nu}$, the first term of (12′) is negligible and our metric becomes *approximately* Riemannian in standpoint neighborhood — with a metric tensor proportional to $g_{i\mu\nu}$. In homogeneous-universe approximation our choice below of the functions $\mathbf{x}_i$ and $\mathbf{\Theta}_i$ gives $h_i^{\mu}{}_{\nu} = 0$ at standpoint, and more generally (absent black holes) our model will conform to general relativity in standpoint neighborhood by virtue of $h_i^{\mu}{}_{\nu}$ smallness.

To achieve coincidence with the Minkowski metric near standpoint in homogeneous-universe approximation we generally define "increment of distance" in the spacetime of standpoint i as the positive fourth root of

$$ds_i^4 \equiv [\eta_{\mu\nu}dA_i^{\mu}dA_i^{\nu'}]^2 + \frac{1}{4}\,[\eta_{\mu\nu}(dA_i^{\mu}dA_i^{\nu} - dA_i^{\mu'}dA_i^{\nu'})]^2. \tag{15}$$

It is necessary now to specify the functions $\mathbf{x}_i(\boldsymbol{\phi})$ and $\mathbf{\Theta}_i(\boldsymbol{\phi})$ that determine $\hat{A}_i$ over the spacetime of standpoint i.

IV. Choice of the Functions $x_i^{\mu}(\boldsymbol{\phi})$

Requirement that $\mathbf{x}_i$ locates standpoints *other* than i in the i coordinate system, and that "objective reality" be provided by approximate mappings onto spacetimes belonging to those other standpoints, places a severe restriction on the mapping between $\boldsymbol{\phi}$ and $\mathbf{x}_i$. Only one satisfactory mapping has been found. Defining $\phi \equiv |\vec{\phi}|$ and $\vec{v} \equiv \vec{\phi}/\phi$, the following single-parameter mapping will be shown to gravitationally confine matter within standpoint spacetime while accommodating an objective representation of matter.

$$\begin{aligned} t_i &= R_i(1 + \sin\phi^{\circ}\cos\phi), \\ \vec{x} &= R_i\vec{v}\sin\phi\cos\phi^{\circ}, \end{aligned} \tag{16}$$

with $t_i \equiv x_i^{\circ}$ and $R_i \geq 0$. Equivalent to (16) with $r_i \equiv |\vec{x}_i|$ are the relations

$$t_i \pm r_i = R_i(1 + \sin\phi_{\pm}), \tag{17}$$

where

$$\phi_{\pm} \equiv \phi_o \pm \phi. \tag{18}$$

Notice that, as forecast above, $t_i(0) = R_i$ and $\vec{x}_i(0) = 0$. Notice also that $t_i \geq 0$ and $r_i \leq t_i$ and that, at standpoint,

$$\frac{1}{R_i} \frac{\partial x_i^\mu}{\partial \phi^\nu} = \delta_{\mu\ \nu} = \eta^\mu{}_\nu. \tag{19}$$

This latter relation will in homogeneous-universe approximation yield Minkowski metric near standpoint.

The $\mathbf{x}_i$ domain decreed by (17),

$$0 \leq t_i \pm r_i \leq 2R_i, \tag{20}$$

is the intersection of interiors of certain forward and backward light cones — a "double cone" with standpoint at center. The vertices of both cones share the standpoint's spatial location at $\vec{x}_i = 0$, while vertex times are equally displaced from standpoint by the interval R_i, one vertex in the standpoint's past and one in its future. We shall associate the forward cone (vertex in past of standpoint) with "big bang." The backward cone with vertex in future will receive an interpretation in black-hole spirit — the universe as perceived from the standpoint resembling the interior of a classical black hole (from which matter cannot escape) with standpoint at center and Schwartzschild radius of order R_i. The parameter R_i also relates to "age of standpoint i". We later will give the precise relation between R_i and both Hubble constant and standpoint age.

The double cone in $t_i, \vec{x}_i$ is multiply covered by the coset space. Efforts to interpret the classical redundancy in terms of antiparticles and CPT have been encouraging but will not be described here. In what follows we confine attention to the coset sector corresponding to

$$0 \leq \phi \leq \pi/2, -\pi/2 \leq \phi^\circ \leq \pi/2, \tag{21}$$

which maps one to one onto the double-cone interior.

V. Homogeneous Universe

From (13) and (19) follows the standpoint-proximity relation

$$g_i^\mu{}_\nu = \frac{\partial \theta_i^\mu}{\partial \phi^\nu}, \tag{22}$$

while Section III has shown that in standpoint neighborhood (so long as $h_i^\mu{}_\nu$ is small) our model has Riemannian metric equal to $g_{i\mu\nu}$. This metric takes Minkowski form, according to (22), for the trivial functions

$$\Theta_i^\mu = \phi^\mu. \tag{23}$$

Extending these trivial standpoint-independent functions to the entire (compact) standpoint spacetime provides our representation of "homogeneous universe." Representation of *inhomogeneity* will be through nontrivial i-dependent functions $\Theta_i^\mu(\phi)$ addressed below in Section VII.

It is straightforward to calculate the Finsler metric that follows when the functions (16) and (23) are applied to (15). One finds

$$ds_i = \frac{1}{\sqrt{2}}\{[\mathcal{G}(dt_i^2 - dr_i^2) - gr_i^2 d\vec{n}_i^{\,2}]^2 + [\mathcal{G}'(dt_i^2 - dr_i^2) - g'r_i^2 d\vec{n}_i^{\,2}]^2\}^{1/4}, \qquad (24)$$

where the coefficients $\mathcal{G}, g, \mathcal{G}', g'$ depend symmetrically on the two dimensionless variables

$$x_\pm \equiv \sin\ \phi_\pm = \frac{t_i \pm r_i}{R_i} - 1, \quad (-1 \leq x_\pm \leq +1), \qquad (25)$$

and where $\vec{n}_i \equiv \vec{x}_i/r_i$ is a 3-vector of unit length on which the coefficients in (24) do *not* depend.

Defining the 6 auxiliary functions of t_i, r_i,

$$\begin{aligned} h_\pm &\equiv 1 + 2x_\pm, \qquad f_\pm \equiv (1 - 2x_\pm)(\frac{1 + x_\pm}{1 - x_\pm})^{1/2}, \\ u &\equiv 1 + x_+ + x_-, \quad v \equiv \frac{(1 + x_+)(1 - x_+^2)^{1/2} - (1 + x_-)(1 - x_-^2)^{1/2}}{x_+ - x_-}, \end{aligned} \qquad (26)$$

the 4 coefficients in (24) determining our homogeneous-universe Finsler geometry are found to be

$$\begin{aligned} \mathcal{G} &= h_+ f_- + f_+ h_- \quad , \mathcal{G}' = h_+ h_- - f_+ f_- \ , \\ g &= 2uv \quad , \qquad\qquad g' = u^2 - v^2 \ . \end{aligned} \qquad (27)$$

The metric (24) exhibits a type of velocity dependence absent from any Riemannian metric. Except in certain approximations to (24) there is no velocity-independent metric tensor. The expanded role for velocity will make it useful to speak of "slow" and "fast" matter displacements. A "slow" displacement has $(d\vec{x}_i/dt_i)^2 << 1$ in standpoint spatial neighborhood while a "fast" displacement has $1-(d\vec{x}_i/dt_i)^2 << 1$. Although definition of "slow" and "fast" displacement rests on coordinate systems belonging to standpoints spatially located near the matter, meaning of these terms will prove to be coordinate independent. We shall find slow matter always objectively describable in multi-time cosmology whereas fast matter presents nonobjective aspects. The present paper focuses on slow matter; Reference (9) deals with light propagation — an important example of fast geodesic.

At standpoint i, $x_\pm = 0$ and here (27) gives $\mathcal{G} = g = 2$, $\mathcal{G}' = g' = 0$, so the metric (24) is indeed Minkowskian in the standpoint coordinates t_i, $\vec{x}_i$. More generally, at the *spatial* location of standpoint i ($\vec{x}_i = 0$) where $x_+ = x_-$ so $h_+ = h_- = u$, $f_+ = f_- = v$, we find $\mathcal{G} = g$ and $\mathcal{G}' = g'$ so the metric takes Riemannian form. Reference (9) shows

the slow-matter deviation of (24) from Riemannian form to be of order $(r_i/R_i)^4$. Deviation of order $(r_i/R_i)^2$ from Minkowski-equivalent metric is there shown to be representable through general relativity and associable with Riemannian spacetime curvature that implies mean energy density near standpoint i of order $(GR_i^2)^{-1}$, where G is the gravitational constant.

Although in Riemannian approximation "invariant curvature" at standpoint is of order R_i^{-2}, the Ricci curvature tensor vanishes for all components that carry a time index.[9] This feature reflects the Finsler metric (24) becoming "flat" for purely radial displacements. For displacements with $d\vec{n}_i = 0$, (24) reduces to the Riemannian form

$$ds_i = [\frac{1}{(1-x_+)(1-x_-)}]^{1/4}(dt_i^2 - dr_i^2)^{1/2}. \tag{28}$$

Although there might seem here to be curvature, the factorized dependence on x_+ and x_- in (28) allows transformation to coordinates in which the radial metric is Minkowskian. Introducing coordinates τ_i, ρ_i that relate to t_i, r_i by

$$1 - \frac{\tau_i \pm \rho_i}{4R_i} = (1 - \frac{t_i \pm r_i}{2R_i})^{1/2} = (\frac{1-x_\pm}{2})^{1/2}, \tag{29}$$

the radial metric becomes

$$ds_i = \frac{1}{\sqrt{2}}(d\tau_i^2 - d\rho_i^2)^{1/2}. \tag{30}$$

Thus, invariant "radial curvature" vanishes. This feature of multi-time cosmology will provide a link to Milne cosmology.[7] Lorentz boosts that preserve (30) will provide a partial mapping between spacetimes belonging to *different* standpoints.

Implied by (30) is that any radial geodesic is characterized by a constant value of

$$\beta_i \equiv d\rho_i/d\tau_i, \tag{31}$$

physical radial trajectories being limited to the interval $-1 \leq \beta_i \leq +1$ where $ds_i^2 \geq 0$. Along such trajectories ds_i is interpretable as elapse of "proper time." Although the radial velocity

$$\dot{r}_i \equiv dr_i/dt_i \tag{32}$$

is not generally constant along a geodesic, from (29) it can be shown that the physical constraint on the range of β_i implies $-1 \leq \dot{r}_i \leq +1$. Lightlike radial geodesics — with $\beta_i = \pm 1$ — have $\dot{r}_i = \pm 1$; in the lightlike case $\dot{r}_i$ *is* constant.

An important special slow category of radial geodesics comprises those "stationary" geodesics that "start" from the big-bang vertex at $r_i = t_i = \rho_i = \tau_i = 0$ and carry a (constant) β_i lying between 0 and 1 ($\dot{r}_i$ is here constant only for β_i equal to 0 or 1). Stationary geodesics associate with "Hubble flow." Each β_i is equivalent to a "radial rapidity"

$$\Delta_i = ln(\frac{1+\beta_i}{1-\beta_i})^{1/2} \tag{33}$$

lying between zero and $+\infty$. Although rapidity is not constant in the i standpoint coordinate system, (29) reveals that Hubble-flow *initial* rapidity equals Δ_i. Stationary geodesics completely and non-redundantly span the i-standpoint's double cone. It is correspondingly possible to coordinate this compact manifold, instead of by $\mathbf{x}_i$ or by $(\tau_i, \rho_i, \vec{n}_i)$, instead through an initial rapidity

$$\Delta_i = ln(\frac{\tau_i + \rho_i}{\tau_i - \rho_i})^{1/2} \tag{34}$$

and direction $\vec{n}_i$ ($\vec{\Delta}_i \equiv \Delta_i \vec{n}_i$) together with "distance" — or "proper time" from big bang. The invariant "distance from big bang" is

$$s = \frac{1}{\sqrt{2}}(\tau_i^2 - \rho_i^2)^{1/2}. \tag{35}$$

Choosing to coordinate i-standpoint spacetime by $(s, \vec{\Delta}_i)$, it is necessary to recognize a bound on the invariant s that depends both on Δ_i and on R_i:

$$0 \leq s \leq 2\sqrt{2}R_i e^{-\Delta_i}. \tag{36}$$

Any *other* standpoint j perceivable from the i standpoint is conveniently labeled by $(s_j, \vec{\Delta}_{ij})$, where we shall see that s_j may appropriately be called the "age" of standpoint j. Absence of index i from s_j reflects *independence* of standpoint age on the spacetime in which the standpoint locates. This important feature of multi-time cosmology is essential to slow-matter objectivity. Note that standpoints of age exceeding the upper bound in (36) are not representable in i spacetime. Note also that the age of standpoint i — which within its own spacetime lies on the zero-rapidity Hubble-flow geodesic at $t_i = R_i$ or $\tau_i = 4R_i(1 - \frac{1}{\sqrt{2}})$, relates to R_i by

$$s_i = 2(\sqrt{2} - 1)R_i. \tag{37}$$

It is stationary matter, moving along the i-system Hubble-flow geodesic that starts from big bang at $r_i = 0$ with rapidity $\vec{\Delta}_{ij}$ and is perceived from standpoint i to coincide in location with j standpoint when matter proper age is s_j, which allows physical identification of s_j with "age of standpoint j". Such matter is at rest in the j coordinate system. By this stage of our exposition, readers familiar with Milne cosmology will have recognized a connection. Linkage is reinforced by deduction from (29) that, at the point along *any* radial geodesic in i coordinates where $r_i = 0$, the radial velocity $\dot{r}_i$ is equal to the (constant) value of β_i labeling this geodesic. Thus, although velocity varies along a geodesic, we may physically interpret the geodesic label $\vec{\beta}_i \equiv \beta_i \vec{n}_i$ as particle velocity at the spatial location of standpoint i, measured in the i coordinate system. Invariance of the Minkowski radial metric (30) permits between i and j coordinate systems a mapping (detailed in the appendix of Reference (9)) of any geodesic passing through both i and j standpoint spatial locations. According

to this mapping, Lorentz transformation of the i-standpoint particle-velocity $\vec{\beta}_i$ by the boost Δ_{ij} leads to a $\vec{\beta}_j$ equal to velocity along the *same* particle trajectory at a point coincident with standpoint j and measured in j coordinates.

We encounter here in the notion of a particle trajectory representable *either* in i *or* in j coordinates, the limited degree of "objectivity" accommodated by multi-time cosmology: Particle trajectories ("signals") connecting spatial neighborhood of i standpoint with that of j are representable *either* in i *or* in j coordinates. In particular, the velocity at i in i coordinates relates to the velocity at j in j coordinates by a Lorentz boost of magnitude Δ_{ij} along the direction connecting the two standpoints. The boost depends only on the two standpoints (not on the particle trajectory).

As in Milne cosmology, sign of boost means a universe perceived from any standpoint to be expanding. Specifically, the redshift z_{ij} of a source located at standpoint j and at rest in j coordinates, as observed through a light signal at (an older) standpoint i, relates to the boost Δ_{ij} by

$$1 + z_{ij} = e^{\Delta_{ij}}, \quad 0 \leq z_{ij} \leq +\infty. \tag{38}$$

Milne cosmology and, more generally, any "standard" cosmology accommodates a broader degree of "objectivity" wherein a particle trajectory represented in the coordinates used by some observer is unambiguously representable by *any* observer through coordinate transformation whether or not the trajectory passes close to the spatial location of the observer. All coordinate representations may consistently be mapped on each other. In multi-time cosmology, on the other hand, comparison of trajectory representations depends on the trajectory's intersecting the spatial neighborhoods of the standpoints being compared. Reference (9) extends the "exactly-radial" geodesics considered here to geodesics whose impact parameter with respect to standpoint is small on Hubble scale. Notice that slow matter is always objectively describable in multi-time cosmology because any slow geodesic enters the common spatial neighborhood of *all* standpoints as matter age approaches zero. It is the representation of fast matter at great distance that may fail to be objective.

Only for exactly-radial displacements can the Finsler standpoint-spacetime metric (24) be transformed to Minkowski form. When all 3 spatial dimensions are considered[9], there is curvature of order R_i^{-2} in Riemannian approximation. Two phenomenological features of Milne's flat-space model nevertheless survive:

1. Defining Hubble time (or length) H_i^{-1} of standpoint i through

$$H_i \equiv \lim_{r_{ij} \to 0} \frac{z_{ij}}{r_{ij}}, \tag{39}$$

   where r_{ij} is distance to j observed by i, one finds $H_i s_i = 1$.

2. The age ratio between observer and source is

$$\frac{s_i}{s_j} = e^{\Delta_{ij}} = 1 + z_{ij}, \tag{40}$$

when source lies on observer's backward light cone.

Thus, although luminosity distance, as shown in Reference (9), is affected by spacetime curvature (as also, of course, is energy density), multi-time cosmology maintains Milne's "kinematic" relation between age and redshift. For a given Hubble constant, a multi-time universe is older than an Einstein-de Sitter universe by a factor 3/2.

We conclude the present section by calling attention to points of analogy between the compact double-cone spacetime with standpoint i at center and the interior of a spherically–symmetrical "standard" black hole whose Schwartzschild radius is of order R_i.

(a) Because black-hole mass would be of order R_i/G, black-hole energy density would be of order $\frac{1}{R_i^3}\frac{R_i}{G} \sim \frac{1}{GR_i^2}$.

(b) Matter inside double cone is gravitationally confined thereto, as shown via (29) by examination of radial geodesics near the boundary at $r_i^2 = (2R_i - t_i)^2$ in the standpoint's future.

(c) Geodesics originating along the big-bang light-cone in the standpoint's past, but *not* near the vertex thereof and thus not "stuck to a standpoint from the beginning", may associate with matter "falling into" the double cone.

VI. Poincaré Symmetry in Standpoint Neighborhood

This brief and perhaps unnecessary section makes explicit the sense in which standpoint cosmology is compatible with "usual" classical physics inside any homogeneous-universe neighborhood that is small on Hubble scale. Consider two different standpoints i and j whose "boosts" with respect to a (third) reference standpoint, labeled "o", are $\vec{\Delta}_{oi}$ and $\vec{\Delta}_{oj}$, respectively. Let us suppose $\mid R_i - R_o \mid << R_o$ and $\mid R_j - R_o \mid << R_o$ and further that $\mid \vec{\Delta}_{oi} \mid << 1$, $\mid \vec{\Delta}_{oj} \mid << 1$. The two standpoints being compared, that is to say, are close to each other on Hubble scale. "Neighborhoods" of these standpoints, defined by

$$\mid t_i - R_i \mid << R_o, \mid \vec{r}_i \mid << R_o, \mid t_j - R_j \mid << R_o, \mid \vec{r}_j \mid << R_o, \tag{41}$$

(a region where angles $\phi_\pm$ are small for all 3 standpoints) then approximately map onto each other via Minkowskian geometry even through the full i, j spacetimes do not.

To leading order the mapping is

$$\begin{aligned} (t_i - R_i) - (t_j - R_j) &= s_j - s_i \\ \vec{r}_i - \vec{r}_j &= (\vec{\Delta}_{oj} - \vec{\Delta}_{oi})s_o, \\ &\approx \vec{\Delta}_{ij} s_o. \end{aligned} \tag{42}$$

Change of standpoint thus amounts to a familiar Poincaré displacement. Adding the consideration that, to the foregoing order, metric is Minkowskian for all 3 coordinate systems within the neighborhood (41), one recognizes usual Poincaré covariance of a unique noncompact spacetime. For physics within this neighborhood all Poincaré frames are equivalent. Reference (9) provides extension to general relativity.

VII. Weak Inhomogeneity

Standpoint cosmology needs extension, through energy-momentum-type 8 vectors attached to particles, in order to connect derivatives of nontrivial functions $\mathbf{\Theta}_i(\boldsymbol{\phi}^m)$ for particle m with energy-momentum of *other* particles. It is natural to associate with particle l, as observed from standpoint i, an 8-vector of energy-momentum type obtained by a 4-parameter $U(1)\times O(4)$ "rotation" of a fiducial 8-vector specified by a 4-vector p^l_μ. This 8-vector would represent not only particle energy-momentum but particle location in the compact 4-dimensional coset space parameterized by $\boldsymbol{\phi}^l$. Dynamics will involve invariants of the (mixed) type (3) and (4) as well as (7).

Although such a model extension remains for the future, we here note a "weak-gravity" formula for $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ from general relativity that illustrates the relation to be sought in standpoint neighborhood. In a coordinate system where $h_{\mu\nu}$ is small, the retarded Lienard-Wiechert "gravitational potential" at $\mathbf{x}$ generated by a particle of energy-momentum $\mathbf{p}^l$ is given by the Poincaré-covariant formula

$$h^l_{\mu\nu}(\mathbf{x}) = G\frac{p^l_\mu p^l_\nu}{\mathbf{p}^l\cdot(\mathbf{x}^l-\mathbf{x})}, \tag{43}$$

where $\mathbf{x}^l$ locates the intersection of "source-particle" trajectory with the backward light cone of the point $\mathbf{x}$. The advanced potential is given by a similar formula. In the sense of Wheeler and Feynman[10] a superposition of advanced and retarded potentials is required. That is, the acceleration experienced by a *test particle* at $\mathbf{x}$ combines its gravitational interaction with source particles in past *and* in future. (If a source particle intersects both forward and backward light cones of the point $\mathbf{x}$ with no intervening change of energy-momentum, its contributions to advanced and retarded potentials at $\mathbf{x}$ are equal; but generally the retarded contributions are independent of advanced.)

Average energy density of order $(GR_i^2)^{-1}$ means that (absent black holes) the Formula (43) contribution to $h_{\mu\nu}$ near standpoint from *all* particles within a distance r_i is of order $(r_i/R_i)^2$. There is no reason to believe (43) for source distances on Hubble scale ($r_i \sim R_i$); a natural "cut off" rule follows from the homogeneous-universe feature that $h_{\mu\nu} = 0$ at standpoint. From the contribution according to (43) of sources between r_i and $r_i + dr_i$ one subtracts the contribution that a *homogeneous* particle distribution would give. Because homogeneity is observed to be a good approximation for $r_i/R_i > 10^{-2}$,[8] an effective cutoff will then develop at such distances. Only proximity of (local) black holes can disturb smallness of $h_{\mu\nu}$ near standpoint. Such proximity is exceptional in "dilute" standpoint neighborhoods — characterized by R_i

sufficiently large that quantum-particle Compton wavelength is much smaller than average distance between particles. The presently-proposed classical model makes sense only for dilute standpoint neighborhoods.

VIII. Conclusion

A companion paper [9] deals in homogeneous-universe approximation with small deviations from radial motion—i.e., with standpoint-impact parameters nonzero but small on Hubble scale. (*All* motion within standpoint neighborhood is thereby encompassed.) From the acceleration implied by the Finsler metric (24), Ref. (9) infers mean energy density near standpoint and luminosity distance as a function of redshift. An important contribution is found from "vacuum energy density" which, in our model, parallels matter energy density at all standpoint ages.

Because matter energy density varies inversely with square of standpoint age, quantum-particle mean free paths near sufficiently young standpoints become small (in time as well as in 3-space) on Hubble scale. A thermodynamic approximation is thereby suggested (even for standpoint neighborhoods that are "dilute" in the sense of Section VII). Would a "hot universe" calculation of primordial nucleosyntheseis make sense within our model? Although the model's emphasis on "perception" suggests importance for "transparency" as well as "diluteness" in standpoint neighborhood, one might nevertheless ask what differences from standard-model nucleosynthesis would arise if our model were to foray unquestioningly into this domain of phenomenology. Two differences are easy to identify: (1) In the standard-model hot universe, age is $\frac{1}{2}H^{-1}$ whereas in our model age is always H^{-1}. To the extent that H correlates with temperature through energy density, a specified lowering of temperature then requires a longer time in the standpoint-based model. (2) For the same H, hot-matter energy density as deduced in Reference (9) is smaller than in the standard model[10] by a factor 0.77 (although *total* energy density, including vacuum, is *larger* by a factor 1.54). The foregoing two differences have opposing impacts, with difference #1 prevailing in the time-temperature relation. Because our model allows more neutrons to decay during helium synthesis, achievement of a specified helium abundance presumably would require a larger ratio of baryons to photons than in the standard model.

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