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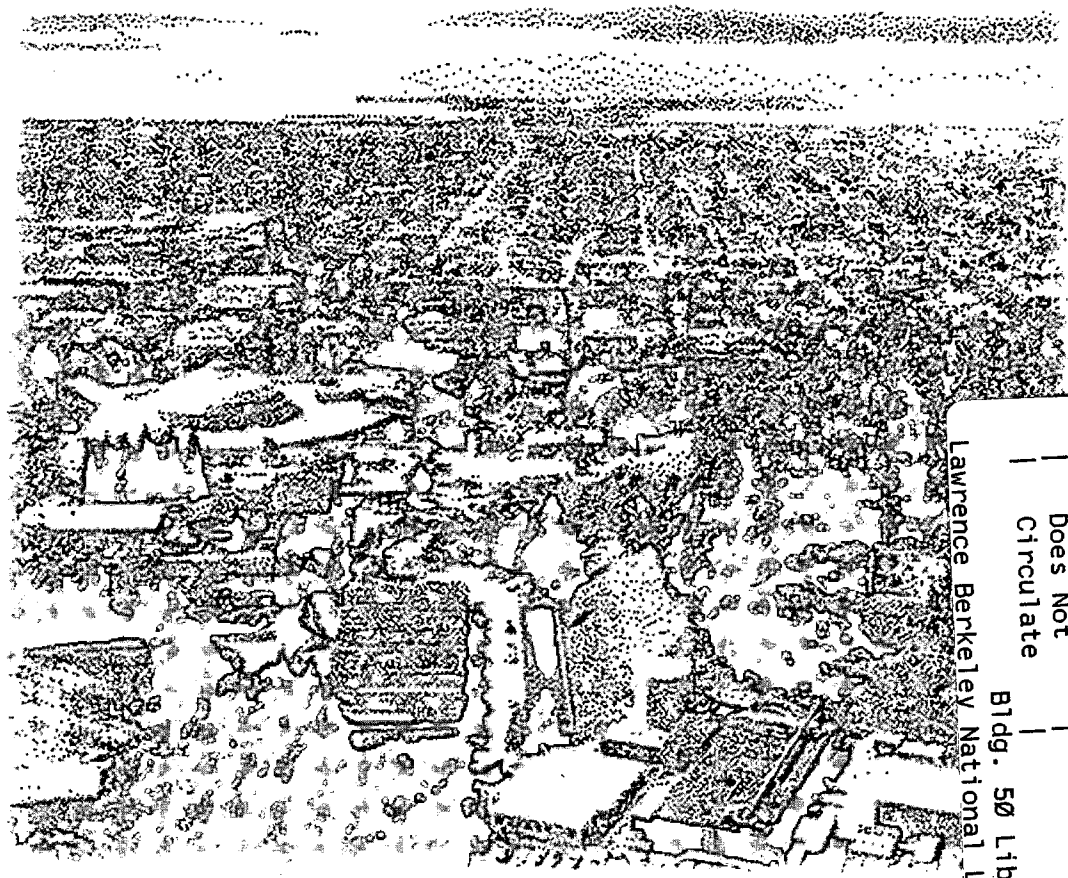

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Neutrino physics from a U(2) flavor symmetry*

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Abstract

We consider the neutrino physics of models with a sequentially broken U(2) flavor symmetry. Such theories yield the observed pattern of quark and lepton masses, while maintaining sufficient degeneracies between superparticles of the first two generations to solve the supersymmetric flavor problem. Neutrino mass ratios and mixing angles in these models may differ significantly from those of the charged leptons, even though the neutrinos and charged leptons transform identically under the flavor group. A wide class of well-motivated U(2) theories yield order one ν_μ-ν_τ mixing, without a fine-tuning of parameters. These models provide a natural solution to the atmospheric neutrino deficit, and also have distinctive signatures at long-baseline neutrino oscillation experiments.

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1 Introduction

The question of neutrino masses is one of the most interesting in particle physics – especially in view of persistent observations suggesting non-zero neutrino masses and mixing angles. It is straightforward to construct models for neutrino masses, and it is certainly very easy to understand why neutrino masses are much lighter than the charged leptons and quarks. However, the predictions for neutrino masses and mixing angles depend strongly on theoretical assumptions, since the neutrino mass matrix has a symmetry structure which is very different from that of the charged fermions[†]. This leads to a decoupling of the unknown neutrino masses from the known charged fermion masses. For example, one factor which determines the overall neutrino mass scale is the breaking of lepton number, about which we have no experimental information. We will not attempt to predict this overall scale of neutrino masses. In this paper we are able to make predictions for neutrino mass ratios and mixing angles by assuming a symmetry which re-couples, to a large degree, the neutrino and charged fermion masses.

Neutrino masses are part of the larger question of flavor physics – how are the flavor symmetries of the standard model gauge interactions broken to yield fermion masses and mixing angles? In supersymmetric theories, flavor physics becomes much richer as the squarks and sleptons must also have mass matrices. Furthermore, flavor physics becomes constrained in new ways, because some form of "super-GIM" mechanism is necessary to suppress the flavor changing neutral current effects induced by supersymmetric gauge interactions.

An approximate flavor $U(2)$ symmetry has recently been proposed as a simple and economical framework for understanding flavor in supersymmetric theories [1, 2]. The idea is that $U(2)$, and its breaking pattern, provide a basic order to the spectrum of quarks, leptons, and their superpartners, in the same way that the $SU(2)$ isospin symmetry provided for nuclear states, and the flavor $SU(3)$ provided for hadronic physics. The small values of the light quark and lepton masses are governed by two small $U(2)$ symmetry breaking

[†]The flavor symmetry group of the leptons of the standard model is $U(3)_L \times U(3)_R$, with the left(right)-handed leptons transforming as $[3,1]$ ($[1,3]$). While the charged lepton masses transform as $[3,3]$, the neutrino masses transform as $[6,1]$.

parameters, as are the small CKM mixing angles. The same two symmetry breaking parameters are also responsible for the small non-degeneracies among the squarks and sleptons – leading to a "super-GIM" mechanism. The choice of $U(2)$ transformations for the symmetry breaking parameters leads to relations between the CKM mixing angles and ratios of quark masses – these are the analogue of the Gell-Mann-Okubo mass relation. These relations are in agreement with current measurements, and will be more precisely tested by future measurements [3]. The $U(2)$ theory also provides significant motivation for pursuing searches for $\mu \to e$ conversion and for electric dipole moments of the electron and neutron. The virtual effects of superpartners are also expected to contribute to $B\bar{B}$ mixing, changing the pattern of the CP asymmetries in B meson decays. Grand unified theories with a flavor U(2) symmetry give more complete and predictive theories of fermion masses.

An important consequence of the U(2) symmetry is that the symmetry structure of neutrino masses becomes similar, but not identical, to that of the charged fermion masses. The similarities ensure that predictions can be made, while the differences lead to an unusual result from the seesaw mechanism.

The $U(2)$ theories of flavor are based on three assumptions:

i.) Flavor physics is governed ultimately by a flavor symmetry $U(3)$, under which the three generations transform as a $\mathbf{3}$.

This assumption identifies the flavor space as the horizontal space of the three generations. The flavor group acts identically on all charge components of a generation. This simple assumption follows directly from theories having a unified vertical gauge symmetry, but can also occur in non-unified theories. This assumption greatly constrains the flavor structure of theories; for example, charged fermion masses transform as $\bar{\mathbf{3}} + \mathbf{6}$, while Majorana neutrino masses transform as $\mathbf{6}$ – there is a crucial connection between charged and neutral fermion masses. In this paper we study theories containing right-handed neutrinos. They are assumed to be part of the generations so that they have Majorana masses which transform as $\mathbf{6}$, and Dirac masses which transform as the charged fermion masses.

ii.) $U(3)$ is broken strongly to $U(2)$ in all charged sectors.

The large mass of the top quark is a signal that the $U(3)$ symmetry is strongly broken, by couplings of order unity, to $U(2)$. We assume that

this large breaking is also manifest in the other charged sectors[‡]. The three generations transform as $\mathbf{2}+\mathbf{1}$: $\psi_a + \psi_3$. The entries in the Dirac fermion mass matrices therefore transform as: $\psi_3\psi_3, \psi_3\psi_a, \psi_a\psi_b$, and so are generated by the vevs of fields which transform as ϕ^a, S^{ab} and A^{ab}, where S and A are symmetric and antisymmetric tensors respectively.

Since $U(2)$ has rank 2, it can be broken in two stages. The only breaking pattern which leads to a hierarchy of masses for the three generations is

$$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} 0$$

where ϵ and ϵ' are two small symmetry breaking parameters.

iii.) The vevs of all components of ϕ^a, S^{ab} and A^{ab} are restricted to be of order ϵ, ϵ' or 0.

This ensures that the magnitude of every entry in both the charged fermion and neutrino mass matrices has a magnitude which is controlled by the $U(2)$ symmetry and its breaking.

The assumptions stated above, together with the phenomenological considerations discussed in Section 3, lead to definite predictions in the U(2) model for neutrino mass ratios and mixing angles. The neutrino physics of other viable supersymmetric flavor models can be found in the recent literature, for both Abelian [4] and non-Abelian [5] flavor groups.

2 Canonical Models

If we assume that a flavor symmetry G_F acts identically on all members of a 16-plet generation, and that the symmetry is broken sequentially by a set of flavon fields $\{\Phi_i\}$ that are symmetric under interchange of the matter fields, then we will obtain mass matrices for the quarks, charged leptons, and neutrinos that have identical textures, up to factors of order unity. It will be instructive to consider the implications for neutrino physics in this simple class of models before we move on to more complicated possibilities later. In theories of this type, the differences between the up, down and

[‡]In view of the lightness of the b quark and τ lepton, relative to the t quark, this assumption could be questioned. However, we take the view that there is an overall suppression of the down quark and charged lepton masses due to effects in the Higgs sector.

charged lepton mass matrices must originate from fluctuations in the order one coefficients. Such theories, however, are far from satisfactory. While the down quark Yukawa couplings fall in the approximate ratio

$$h_d :: h_s :: h_b \approx \lambda^4 :: \lambda^2 :: 1 \ , \tag{2.1}$$

the up quark Yukawa couplings are even more hierarchical

$$h_u :: h_c :: h_t \approx \lambda^8 :: \lambda^4 :: 1 \ , \tag{2.2}$$

where $\lambda \approx 0.22$ is the Cabibbo angle. In order to explain the difference between Eqs. (2.1) and (2.2), some 'order one' coefficients must differ from unity by more than one power of λ. Hence, the fermion masses are not completely determined by the flavor symmetry breaking pattern and naive dimensional analysis. In any realistic model where G_F acts identically on the matter fields in a full generation, and where the 'order one' coefficients really are near unity, flavor symmetry breaking in the up sector must occur at higher order. As we will see later, this can happen if the flavons transform nontrivially under the grand unified group.

With this in mind, there are now a number of possibilities for the flavor symmetry breaking pattern in the neutrino sector. First, it is possible that each entry of the Dirac and Majorana neutrino mass matrices, M_{LR} and M_{RR}, will experience G_F breaking at the same order as the corresponding entry of the down quark mass matrix M_D. In this case, all three mass matrices will have the same texture, while the up quark mass matrix will differ due to some additional mechanism. In a model of this type, the eigenvalues of $M_{LR} \approx M_{RR}$ will be in the approximate ratio $\lambda^4 :: \lambda^2 :: 1$ and all mixing angles will be CKM-like. For example, we might have

$$M_{LR} \sim M_{RR} \sim M_D \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \ , \tag{2.3}$$

where $\sim$ indicates that we are interested only in the relevant hierarchies, and not in overall mass scales or in coefficients of order unity. The left-handed Majorana mass matrix M_{LL} is then given by the seesaw mechanism [6]

$$M_{LL} \approx M_{LR} \, M_{RR}^{-1} \, M_{LR}^T \ , \tag{2.4}$$

and we find

$$M_{LL} \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} . \tag{2.5}$$

In this example, the left-handed neutrino spectrum has mass ratios and mixing angles that are comparable to those of the down quarks or charged leptons. We will refer to models of this type as "canonical". In such models, the neutrino mass matrices M_{RR}, M_{LR}, and M_{LL} each have eigenvalues in the ratio $\lambda^4 :: \lambda^2 :: 1$ and mixing angles bounded by the corresponding CKM angles, $\theta_{12} \lesssim \lambda$, $\theta_{13} \lesssim \lambda^3$ and $\theta_{23} \lesssim \lambda^2$. Since canonical models have no order one mixing angles, they cannot explain the atmospheric neutrino deficit, nor can they account for the large angle MSW or vacuum oscillation solutions to the solar neutrino problem [§]. The small angle MSW solution is possible in a canonical model only if the Cabibbo angle originates in the up quark sector, and the 12 mixing in M_D is of order λ^2.

More interesting results are obtained when M_{LR}, M_{RR} and M_D have differing textures. In theories where a full generation of the matter fields transforms identically under G_F, this may happen for two reasons:

i.) There are flavons in the theory that are purely antisymmetric under interchange of the matter fields. These may contribute to all the mass matrices except M_{RR}, which is purely symmetric.

ii.) Other symmetries restrict the form of the mass matrices. For example, some of the flavons may transform nontrivially under the grand unified group, G_{GUT}, so that the corresponding mass matrix elements are generated only after both G_F and G_{GUT} are broken. This may produce the desired suppression of the up and charm quark masses, but may also lead to a suppression of entries in the neutrino mass matrices as well.

In our previous example, it is simple to show that even a modest variation in the form of M_{RR} away from Eq. (2.3) can lead to bizarre results. For example, we could imagine a theory where M_{RR} has CKM-like mixing angles,

[§]See Ref. [7], and references therein.

but eigenvalues in the ratio $\lambda^6 :: \lambda^4 :: 1$, as follows from

$$M_{RR} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} . \tag{2.6}$$

This form is obtained when the light two-by-two block is suppressed by λ^2 compared to the canonical example of Eq. (2.3). The seesaw mechanism now gives

$$M_{LL} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} . \tag{2.7}$$

Notice that the neutrino mass eigenvalues are in the ratio $\lambda^2 :: 1 :: 1$, and the mixing angles are not all CKM-like. This is a result that we would not have anticipated based on our knowledge of flavor symmetry breaking in the down quark or charged lepton sector, and our intuition alone.

In the remainder of this paper, we will consider the phenomenology of models with a U(2) flavor symmetry [1, 2]. In these models, complete generations transform identically under the flavor group, and either *i*, *ii*, or both are true, so that the neutrino mass ratios and mixing angles are often noncanonical. In particular, we will see that a wide class of U(2) models predict order one ν_μ-ν_τ mixing, even though they involve no special assumptions that would allow us to anticipate such a result.

3 The Standard U(2) Model and a Simple Modification

Models with flavons in antisymmetric representations of the flavor group may yield noncanonical neutrino mass ratios and mixing angles, even when this is the only factor that distinguishes the Dirac and Majorana neutrino mass matrices. This fact is particularly significant in models with a U(2) flavor symmetry [1, 2]. In these models, the full 16-plet of matter fields ψ transforms as a $\mathbf{2}+\mathbf{1}$ under U(2). Flavor symmetry breaking is achieved via three flavons,

$$S^{ab}, \ A^{ab}, \ \text{and} \ \phi^a, \tag{3.1}$$

where S and A are symmetric and antisymmetric tensors, respectively, and ϕ is a doublet. To obtain a viable texture for the down quark Yukawa matrix, we require U(2) to be sequentially broken:

$$U(2) \longrightarrow U(1) \longrightarrow \text{nothing} \ , \tag{3.2}$$

where the first stage of symmetry breaking is achieved via the vevs

$$\langle \phi^2 \rangle / M \approx \langle S^{22} \rangle / M = \epsilon \ , \tag{3.3}$$

where M is the flavor scale. In the "standard" U(2) model, the remaining U(1) symmetry is broken at a lower scale via the antisymmetric tensor, so that

$$\langle A^{12} \rangle / M = \epsilon' \ . \tag{3.4}$$

With this pattern of symmetry breaking, the down quark Yukawa matrix has the texture

$$h_D \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} \ , \tag{3.5}$$

where we have omitted the order one coefficients. Eq. (3.5) yields acceptable mass ratios and mixing angles with $\epsilon \sim \lambda^2$ and ϵ' between λ^3 and λ^4. Precise values for these parameters and the order one coefficients, obtained from a global fit, can be found in Ref. [2].

The crucial issue that must be addressed in any realistic U(2) model is the origin of the differing mass hierarchies in the down and up quark sectors, Eqs. (2.1) and (2.2). The difference between the top and bottom quark masses may be explained by a large value for the ratio of Higgs vevs (i.e. $\tan\beta = \langle H_u \rangle / \langle H_d \rangle \sim 40$) or by an overall small parameter in h_D originating from mixing in the Higgs sector of the theory. With the choice of ϵ and ϵ' given above, however, all the Yukawa matrices will have eigenvalue ratios that are characteristic of the down quarks. Clearly, the sequential breaking of the flavor symmetry cannot account for the differing up and down quark mass hierarchies alone. Therefore, the transformation properties of the flavons under the grand unified group G_{GUT}, and perhaps also the orientation of the flavon vevs in GUT space must explain why the up and charm masses are generated at higher order in the flavor symmetry breaking.

The precise mechanism that is responsible for suppressing m_u and m_c in U(2) theories is in fact a model-dependent question. The relevant issue is whether this mechanism also affects the entries of the neutrino mass matrices, so that their sizes are not what we would expect naively from a sequential breaking of the U(2) symmetry. In the next section, we will address this issue explicitly in the context of a well-motivated effective theory, the SU(5)×U(2) model. We will find that a suppression of some entries of the neutrino mass matrices does occur and has interesting consequences. In the remainder of this section, however, we will consider the class of model in which the neutrino mass matrix elements are determined only by the scales of sequential U(2) breaking. We will first comment briefly on U(2) models without right-handed neutrinos, and then focus on the models of interest, which have complete 16-plet generations.

In U(2) models without right-handed neutrinos, the left-handed Majorana mass matrix originates from a higher-dimension operator of the form $LHLH/\Lambda$, where Λ is some high scale, perhaps the ratio of the Planck scale squared to the scale where lepton number is violated. The form of M_{LL} is determined by the pattern of symmetry breaking in Eqs. (3.2), (3.3), and (3.4), and we find

$$M_{LL} \approx \frac{H^2}{\Lambda} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} . \tag{3.6}$$

Notice that the 12 and 21 entries have vanished due to the antisymmetry of A^{ab}. The muon and tau neutrinos have masses in the ratio $\epsilon :: 1$, while the electron neutrino is massless. The 23 mixing angle is of order $\epsilon \approx 0.02$, while the 13 mixing is negligible. The 12 mixing angle originates from diagonalization of the charged lepton mass matrix, and is given by $\theta_{12} = \sqrt{m_e/m_\mu} \approx \epsilon'/(3\epsilon) \approx 0.07$. This angle is too large by about a factor of 2 to yield the small angle MSW solution to the solar neutrino problem, but may explain the LSND neutrino oscillation signal [8] if the muon neutrino mass squared is in the range 0.3-0.6 eV^2.

In considering U(2) models with right-handed neutrinos, we will also assume here that both the Dirac and Majorana neutrino mass matrices have entries determined by the pattern of U(2) breaking, without any additional suppression. An example of a theory of this type is the second SO(10)×U(2)

model of Ref. [2], with all flavons transforming as adjoints of SO(10), and a flavor-singlet **126** added to generate the right-handed neutrino scale. In this model, the orientation of the flavon vevs in GUT space assures a suppression of the lowest order contributions to m_u and m_c, but does not alter the form of the remaining Yukawa matrices, when all operators are taken into account.

In models of this type, the neutrino Dirac mass matrix M_{LR} has the same form as h_D, while M_{RR} is given by

$$M_{RR} \approx \Lambda_R \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} , \tag{3.7}$$

where Λ_R is the right-handed neutrino mass scale. The absence of a contribution from the antisymmetric flavon not only has given us a different texture from h_D, but also has created a serious problem: Eq. (3.7) has a zero eigenvalue. If the seesaw mechanism is to be effective for all three generations, we must decide how to modify the theory (or our assumptions) so that all the eigenvalues of M_{RR} are nonvanishing.

A simple solution that does not require us to modify the field content of the theory, is to relax the assumption made in Refs. [1, 2] that each flavon is involved only in a single stage of the symmetry breakdown. Thus, we will consider the possibility that S^{11}, S^{12}, and ϕ^1 have nonvanishing vacuum expectation values of order ϵ'. Notice that these tensor components cannot acquire a vacuum expectation value until the U(1) symmetry is spontaneously broken, so in general

$$S^{11} \lesssim \epsilon', \quad S^{12} \lesssim \epsilon', \quad \text{and} \quad \phi^1 \lesssim \epsilon' . \tag{3.8}$$

We will assume that these relations are equalities, so that the size of every nonvanishing Yukawa matrix element is set by one of the possible scales of sequential U(2) breaking. The alternative, that S^{11}, S^{12} and ϕ^1 develop vevs at scales far below the U(1) breaking scale, yields Yukawa textures that cannot be understood solely in terms of a symmetry breaking pattern. Since this possibility leads to less predictivity in the neutrino sector, we will consider it separately in the Appendix.

If we allow S^{11}, S^{12} and ϕ^1 to be either 0 or $\mathcal{O}(\epsilon')$, we must consider the effects of our choice on the phenomenology of the quark and charged lepton sectors:

• $S^{11} \approx \epsilon'$. This leads to a texture for the down-strange Yukawa matrix

$$\begin{pmatrix} \epsilon' & \epsilon' \\ -\epsilon' & \epsilon \end{pmatrix} , \tag{3.9}$$

which implies that the Cabibbo angle θ_c is given approximately by m_d/m_s. The measured value of the θ_c is described quite accurately by $\sqrt{m_d/m_s}$, so our result is not phenomenologically acceptable. Thus we must choose $S^{11} = 0$.

• $S^{12} \approx \epsilon'$. Since the only antisymmetric flavon in the theory contributes to the 12 entries of the Yukawa matrices, the choice $S^{12} \approx \epsilon'$ guarantees that no Yukawa entry is dominated by the contribution of an antisymmetric flavon. Thus, we obtain a canonical model, with mass ratios and mixing angles similar to those in the charged lepton sector. With both symmetric and antisymmetric flavons present in the theory, the 12 and 21 entries of h_U and h_D no longer have a definite symmetry under interchange, and the successful prediction of the original theory $\theta_c = \sqrt{m_d/m_s}$ is violated at the 100% level. Since this relation is known to be valid within 20%, taking into account the allowed range $m_s/m_d = 17$ to 25 [9], we conclude that theories with both A^{12} and S^{12} nonvanishing are not favored. Thus, we are led to consider $S^{11} = S^{12} = 0$ and $\phi^1 \approx \epsilon'$ as the most promising U(2) breaking pattern for both the neutral and charged fermion masses.

• $\phi^1 \approx \epsilon'$. Notice that adding 13 and 31 entries to h_D of order ϵ' corrects the down quark mass at the percent level, which is negligible. However, there are now new contributions to m_u and V_{ub} that are of the same order as the ones in the original theory. The only predicted relation involving these observables that is known accurately enough to be affected significantly by these new contributions is $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$. Since $V_{ub}/V_{cb} = 0.08 \pm 0.02$, a 50% correction to this relation would be within the range allowed at the 95% confidence level. Thus, if the ϕ^1 vev is slightly smaller than ϵ', say $\epsilon'/3$, then the only effect on the phenomenology of the quark sector would be to alter some of the detailed predictions of the $\phi^1 = 0$ theory, obtained via a global fit in Ref. [3]. The phenomenological viability of the model, however, would not be affected.

In light of these arguments, we will adopt the choice $\phi^1 \sim \epsilon'$, $S^{11} = S^{12} = 0$, and proceed with the analysis of the neutrino sector. A somewhat smaller choice for ϕ^1 will not affect the form of our results, which are only valid up

to order 1 factors. The right-handed neutrino mass matrix is now given by

$$M_{RR} = \Lambda_R \begin{pmatrix} 0 & 0 & \epsilon' \\ 0 & \epsilon & \epsilon \\ \epsilon' & \epsilon & 1 \end{pmatrix} , \tag{3.10}$$

and the seesaw mechanism gives

$$M_{LL} = \frac{H^2}{\Lambda_R} \begin{pmatrix} \epsilon'^2/\epsilon & \epsilon' & \epsilon' \\ \epsilon' & 1 & 1 \\ \epsilon' & 1 & 1 \end{pmatrix} . \tag{3.11}$$

Note that we have not included operators involving the flavon product $\phi^a \phi^b$ for simplicity. If these operators are present, it is straightforward to check that they have no effect on the form of our result[¶]. The interesting feature of Eq. (3.11) is the order 1 mixing in the 2-3 block, which allows for a possible solution to the atmospheric neutrino problem, via ν_μ-ν_τ oscillation. The preferred parameter range for this solution, $\delta m^2_{23} \approx 10^{-2\pm0.5}$ eV^2 and $\sin^2 2\theta_{23} \approx 0.4 - 0.6$ [7], may be obtained by appropriate choices for Λ_R and the order 1 coefficients[‖]. Neutrino oscillations in this parameter range would be observable at proposed long-baseline experiments, such as the KEK-SuperKamiokande, MINOS, or CERN-ICARUS experiments [10]. The 13 mixing angle in Eq. (3.11) is of order $\epsilon' \approx \lambda^3$–$\lambda^4$, and is unlikely to have measurable consequences if the overall neutrino mass scale is determined by the atmospheric neutrino deficit.

[¶]Note that the results presented here and in the next section remain unchanged in form by the field redefinitions required to place the neutrino kinetic terms in canonical form after the small U(2)-breaking corrections to the Kahler potential are taken into account.

[‖]One might worry that a ϕ^1 vev somewhat smaller than ϵ' might alter our conclusion that the 2-3 mixing angle is of order one. Let us assume that the ϕ^1 vev is $a\epsilon'$ and that the operator involving the antisymmetric flavon that contributes to M_{LR} has a coefficient b. Then the 2-3 block of Eq. (3.11) scales as

$$\begin{pmatrix} (b/a)^2 & (b/a) \\ (b/a) & 1 \end{pmatrix} \tag{3.12}$$

Any systematic deviation away from the order 1 entries in Eq. (3.11) due to a slightly smaller choice for the ϕ^1 vev can be compensated by a slightly smaller choice for the coefficient b. Thus, we obtain the large 2-3 mixing angle without a significant fine-tuning.

The 12 mixing angle, however, is actually larger than ϵ' since it originates at leading order from the diagonalization of the charged lepton mass matrix. Thus, we know θ_{12} quite accurately,

$$\theta_{12} = \sqrt{m_e/m_\mu} \ , \tag{3.13}$$

or $\sin^2 2\theta_{12} \approx 0.02$. This may be large enough to allow ν_μ-ν_τ and ν_μ-ν_e oscillations to be observed simultaneously at least at some of the long-baseline experiments mentioned above. The neutrino mixing matrix U, defined by $\nu_{mass} = U\nu_{flavor}$, is given approximately by the product of successive two dimensional rotations in the 23 and 12 subspaces. Thus, neglecting CP violation, we obtain the simple form

$$U = \begin{pmatrix} 1 & -s_{12} & 0 \\ s_{12}c_{23} & c_{23} & s_{23} \\ -s_{12}s_{23} & -s_{23} & c_{23} \end{pmatrix} \ , \tag{3.14}$$

where c_{ij} (s_{ij}) is the cosine (sine) of the ij mixing angle. The ν_μ-ν_e oscillation probability is then given by

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta_{12} \left(c_{23}^2 \sin^2 \delta_{12}t + s_{23}^2 \sin^2 \delta_{13}t - s_{23}^2 c_{23}^2 \sin^2 \delta_{23}t \right) \ , \tag{3.15}$$

where $\delta_{ij} = (m_i^2 - m_j^2)/4E$, and E is the beam energy. If we set θ_{23} to the central value suggested by the atmospheric neutrino anomaly, then Eq. (3.15) may be written

$$P(\nu_\mu \to \nu_e) = 0.0171 \sin^2 \delta_{12}t + 0.0029 \sin^2 \delta_{13}t - 0.0025 \sin^2 \delta_{23}t \ . \tag{3.16}$$

The MINOS experiment is expected to measure the ν_μ-ν_e oscillation probability to an accuracy of 0.0044 [11], and the ICARUS experiment may achieve a comparable sensitivity [10]. Thus, we have hope of measuring the first term in Eq. (3.16), which might provide a 2 sigma signal if the $\sin^2 \delta_{12}t$ factor is approximately 1/2. While this factor depends effectively on one free parameter, the muon neutrino mass, the amplitude of this term, $A = \sin^2 2\theta_{12} \cos^2 \theta_{23}$, is a fixed prediction of the theory. For $\sin^2 2\theta_{23}$ in the range 0.4-0.6, A must fall in the range

$$0.016 \leq A \leq 0.018 \tag{3.17}$$

if the models presented in this section are correct. Since the muon neutrino oscillates primarily to ν_τ in our model, one might worry that this small ν_e oscillation signal would be swamped by the background electrons coming from τ decays. Fortunately, these electrons have a softer energy spectrum than those produced directly via ν_e charged current scattering. Thus, the ν_μ-ν_e oscillation signal may be isolated by placing an appropriate cut on the electron energy spectrum [11].

Finally, on a more speculative note, it is possible that our model can also account for the $\overline{\nu}_\mu$-$\overline{\nu}_e$ oscillation signal reported by the LSND experiment [8]. Given our prediction that $\sin^2\theta_{12} \approx 0.02$, the LSND results favor a Δm^2_{12} in the range 0.3-0.6 eV^2 [8]. At face value, this mass scale seems too large to account for the atmospheric neutrino deficit, in the absence of a 10% fine-tuning. However, the only obstacle to solving the atmospheric neutrino problem via ν_μ-ν_τ oscillation with Δm^2_{23} in a similar range is a bound coming from the observed flux of upward-going muons at the IMB experiment [12]. The observed flux is roughly comparable to theoretical expectations, and can be used to exclude a region of the $\sin^2 2\theta$-Δm^2 plane that overlaps with the region preferred by the atmospheric neutrino data for Δm^2_{23} larger than 0.03. A possible loophole is that this bound depends sensitively on the absolute neutrino flux, which has a large theoretical uncertainty. The most optimistic estimates for this flux (from our point of view) yield no constraint on the region of parameter space favored by the atmospheric neutrino deficit, beyond those already available from other experiments [12], and allow a solution with Δm^2_{23} as large as 0.4 eV^2 [13]. This would be sufficient to explain both the atmospheric and LSND phenomena, without any fine-tuning. If this interpretation is correct, it would also imply that the ν_μ-ν_e mixing angle would lie only a factor of 2 below the current bounds from reactor experiments [8].

4 The SU(5)×U(2) Model

We have seen in the previous section that U(2) models with flavons in antisymmetric representations of the flavor group may yield textures for M_{LL} that have noncanonical mass ratios and mixing angles. Another factor that may contribute to deviations from the canonical result is an additional suppression

of some of the flavor-symmetry-breaking operators due to the transformation properties of the flavon fields under the grand unified group. This is a possibility we will take into account in this section. We will work in the context of SU(5), which is contained in all other grand unified groups. In principle, the number of possible effective U(2) theories for neutrino masses grows considerably if we also allow the flavons to have nontrivial transformation properties under SU(5). However, we will argue (as in Ref. [2]) that one particular set of quantum number assignments for the flavons seems favored by the known phenomenological differences between the up, down, and charged lepton Yukawa matrices. This will enable us to make specific predictions in the neutrino sector as well.

In the SU(5)×U(2) model of Ref. [2], fermion masses originate from the operators

$$T_3 H T_3 + T_3 \overline{H}\,\overline{F}_3 \tag{4.1}$$

$$+ \frac{1}{M}\left(T_3 \phi^a H T_a + T_3 \phi^a \overline{H}\,\overline{F}_a + \overline{F}_3 \phi^a \overline{H} T_a\right) \tag{4.2}$$

$$+ \frac{1}{M}\left(T_a (S^{ab} H + A^{ab} H) T_b + T_a (S^{ab} \overline{H} + A^{ab} \overline{H}) \overline{F}_b\right) \quad , \tag{4.3}$$

where T and $\overline{F}$ are the **10** and $\overline{\mathbf{5}}$ matter multiplets, while H and $\overline{H}$ are the **5** and $\overline{\mathbf{5}}$ Higgs fields. If all the flavons were SU(5) singlets, then the Yukawa matrices h^U, h^D, and h^E would have the same U(2) breaking texture, and we would have no explanation for the differing mass hierarchies in the down- and up-quark sectors. Thus, the SU(5) transformation properties of the flavons must account for the known differences between h^U, h^D, and h^E.

The Yukawa matrices for the first two generations of up and down quarks originate from the first pair and last pair of terms in Equation (4.3), respectively. The simplest way of obtaining the differing mass hierarchies in Eqs. (2.1) and (2.2) is to choose SU(5) transformation properties for S^{ab} and A^{ab} such that they contribute at leading order to h^D, but not to h^U. The crucial observation is that

$$10 \times \overline{5} = 5 + 45$$

while

$$10 \times 10 = \overline{5}_s + \overline{45}_a + \overline{50}_s \ .$$

The representations that contain a Higgs doublet (the **5** and **45**) are distinguished in the up sector by their definite symmetry under interchange of the two **10**'s. Thus, if we choose SH$\sim$**45** and AH$\sim$**5**, the up quark mass will vanish at leading order, while a charm mass may originate via the nonvanishing (2,3) and (3,2) entries of h^U, as we describe below. To realize this scenario, the flavons A and S must transform as a **1** and **75**, respectively. Any other choice for the transformation properties of A and S that allows AH and SH to contain the desired SU(5) representations, also yields undesired representations as well**. Thus, the quantum number assignments for the symmetric and antisymmetric flavons are significantly restricted. In fact, there is additional evidence that the choice $A \sim \mathbf{1}$ and $S \sim \mathbf{75}$ is a compelling one. The products $S\overline{H}$ and $A\overline{H}$ then transform as a $\overline{\mathbf{45}}$ and $\overline{\mathbf{5}}$, respectively, leading to a factor of 3 enhancement in the (2,2) entry of h^E. We then automatically obtain the Georgi-Jarlskog mass relations at the GUT scale:

$$m_e = \frac{1}{3} m_d \qquad m_\mu = 3 m_s \,. \tag{4.4}$$

Therefore, we will assume $S \sim \mathbf{75}$ and $A \sim \mathbf{1}$ in our subsequent analysis. The remaining doublet flavons ϕ^a are needed to generate the mixing between the second and third generations, and therefore must contribute to either (or both) the up and down sectors at lowest order in the flavor symmetry breaking. Since the components of the Yukawa matrices generated by ϕ have no definite symmetry under interchange of the matter fields, we expect ϕ to contribute to both the up and down quark sectors, regardless of their SU(5) transformation properties. A viable model is obtained with the minimal choice $\phi \sim \mathbf{1}$, which we will assume henceforth. If there are additional doublets in the theory that transform nontrivially under SU(5), their effects will be no larger than the SU(5) singlet contribution, and will not alter our results. Notice that ϕ contributes to the (2,3) and (3,2) entries of h^U at lowest order, so we generate a charm mass $m_c \sim \epsilon^2 \sim \lambda^4$, as desired.

Given these quantum number assignments, all the masses and mixing angles of the standard model are obtained, with the exception that the up quark is massless, $m_u = 0$, as a consequence of the combined grand unified and flavor symmetries. An up quark mass can be generated at higher order,

For example, if A where to transform as a **24 then AH would indeed contain a **5**, but would also have a component transforming as a **45**.

however, if we introduce additional fields [2]. Let us suppose that we also have a flavor-singlet, SU(5) adjoint field, whose vev points in the hypercharge direction, Σ_Y. This is the smallest representation whose vev can break SU(5) down to the standard model gauge group. Then at order $1/M^2$, we have the operators

$$\frac{1}{M^2}\left(T_a\phi^a\phi^b HT_b + T_a S^{ab}\Sigma_Y HT_b + T_a A^{ab}\Sigma_Y HT_b\right) \quad , \tag{4.5}$$

which always generate an up quark mass via the second and third terms. To obtain an up Yukawa coupling of the appropriate magnitude, we find that $\Sigma_Y/M \approx \epsilon$, which is exactly the vev that we would have expected based on dimensional analysis: Since $S^{22} \approx \epsilon$, and S transforms nontrivially under SU(5), we know that the flavor scale is approximately $1/\epsilon$ times higher than the unification scale. For any purely SU(5) breaking vev v, we estimate that $v/M \approx \epsilon$, which is exactly what we need to generate m_u via the operators in Eq. (4.5). Note in addition that the second operator gives us another contribution to m_c that is of order λ^4.

Since we have found that the possible variations on the basic SU(5)×U(2) effective theory are significantly restricted, we have some hope for predictivity in the neutrino sector [††]. The Majorana mass matrix for the right-handed neutrinos M_{RR} is generated at leading order by the operators

$$\Lambda_R\left(\nu_3\nu_3 + \frac{1}{M}\phi^a\nu_a\nu_3 + \frac{1}{M^2}\phi^a\phi^b\nu_a\nu_b + \frac{1}{M^3}S^{ab}\Sigma_Y\Sigma_Y\nu_a\nu_b\right) \quad , \tag{4.6}$$

where the two factors of Σ_Y in the fourth term are necessary to form an SU(5) singlet. As in the model presented in Section 3, we assume that $\phi^1 \approx \epsilon'$ so that we lift the zero eigenvalue in M_{RR} without spoiling the most successful phenomenological predictions in the quark sector. We then obtain

$$M_{RR} = \Lambda_R \begin{pmatrix} \epsilon'^2 & \epsilon\epsilon' & \epsilon' \\ \epsilon\epsilon' & \epsilon^2 & \epsilon \\ \epsilon' & \epsilon & 1 \end{pmatrix} \quad . \tag{4.7}$$

Similarly, the neutrino Dirac mass matrix is generated by the operators

$$\overline{F}_3 H\nu_3 + \frac{1}{M}(\phi^a\overline{F}_3 H\nu_a + \phi^a\overline{F}_a H\nu_3 + A^{ab}\overline{F}_a H\nu_b)$$

[††]The SU(5) theory without right-handed neutrinos has a phenomenology identical to the corresponding theory in Section 3, except that the the muon and tau neutrino masses fall in the ratio $\epsilon^2 :: 1$. Therefore, we do not discuss this case in the text.

$$+\frac{1}{M^2}(\phi^a\phi^b\overline{F}_aH\nu_b+S^{ab}\Sigma_Y\overline{F}_aH\nu_b) \tag{4.8}$$

leading to the texture

$$M_{LR}=H\begin{pmatrix}\epsilon'^2 & \epsilon' & \epsilon' \\ -\epsilon' & \epsilon^2 & \epsilon \\ \epsilon' & \epsilon & 1\end{pmatrix}\ . \tag{4.9}$$

The seesaw mechanism then give us solutions with two possible textures:

$$M_{LL}=\frac{H^2}{\epsilon\Lambda_R}\begin{pmatrix}(\epsilon'/\epsilon)^2 & \epsilon'/\epsilon & \epsilon\epsilon' \\ \epsilon'/\epsilon & 1 & \epsilon \\ \epsilon\epsilon' & \epsilon & \epsilon\end{pmatrix} \tag{4.10}$$

if there is a single doublet flavon in the theory, or

$$M_{LL}=\frac{H^2}{\epsilon\Lambda_R}\begin{pmatrix}(\epsilon'/\epsilon)^2 & \epsilon'/\epsilon & \epsilon'/\epsilon \\ \epsilon'/\epsilon & 1 & 1 \\ \epsilon'/\epsilon & 1 & 1\end{pmatrix} \tag{4.11}$$

if there are two or more doublet flavons. The smaller entries in Eq. (4.10) result from a cancellation in leading terms due to the proportionality between the entries generated by a single ϕ^a in M_{RR} and M_{LR}. In the second case, we obtain $\mathcal{O}(1)$ mixing between the second and third generation neutrinos, as in the model of Section 3. The significant difference in this case is that the additional suppression of the operators involving the symmetric flavon S^{ab} has yielded an enhancement in the 12 and 13 mixing angles, which are now both of order $\epsilon'/\epsilon \approx \lambda$. Unlike our earlier models, which had negligible 13 mixing, the neutrino mixing matrix U in the present case does not assume a particularly simple form. Moreover, the 12 mixing angle comes primarily from the diagonalization of M_{LL}, and therefore is known only up to an order one factor. While these results prevent us from achieving the (rather surprising) level of predictivity that we obtained in the models of Section 3, it is significant consolation that the ν_μ-ν_e mixing probability is nearly an order of magnitude larger in the present model. If the neutrino mass scale is the proper one to solve the atmospheric neutrino problem, then it seems very likely in this model that ν_μ-ν_τ and ν_μ-ν_e oscillations would be observed together at long-baseline neutrino oscillation experiments, assuming the anticipated sensitivity of the MINOS experiment.

5 Conclusions

We have considered the implications of a non-Abelian flavor symmetry on neutrino masses and mixing angles. In models where complete generations transform identically under the flavor symmetry, we argued that neutrino mass matrix textures can differ dramatically from those of the charged leptons. This may happen if there are flavons in the theory that transform nontrivially under G_{GUT}, or that are antisymmetric under G_F. In theories with a U(2) flavor symmetry, we found that some noncanonical models predict a large 2-3 mixing angle, and therefore may provide a natural solution to the atmospheric neutrino problem. Assuming that this consideration sets the mass scale for the muon neutrino, the ν_μ-ν_e mixing angle in these models is large enough to be measured at proposed long-baseline neutrino oscillation experiments. We would then expect ν_μ-ν_τ and ν_μ-ν_e oscillations to be observed simultaneously, with events falling in the approximate ratio 1 :: 0.02 in the models of Section 3, or 1 :: 0.1 in the model of Section 4.

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A Other Models

In the text, it was assumed that each nonvanishing entry of the Yukawa matrices was associated with one of the scales of sequential U(2) breaking. The possible values for these matrix elements were then given by ϵ, ϵ', or 0. This assumption was particularly important in determining the flavon vevs needed to lift the zero eigenvalue in Eq. (3.7). In this appendix, we point out that smaller vevs for S^{11}, S^{12}, and ϕ^1 lead to neutrino mass spectra with a distinct qualitative feature – a heavy, nearly decoupled muon neutrino mass eigenstate. The generalization of our previous analysis is straightforward, and we work with the model of Section 3 for the purposes of illustration.

If we assume that the S^{11}, S^{12}, and ϕ^1 vevs are of order δ_1, δ_2, and δ_3, respectively, then the Dirac and Majorana neutrino mass matrices become

$$M_{LR} = \begin{pmatrix} \delta_1 & \epsilon' + \delta_2 & \delta_3 \\ -\epsilon' + \delta_2 & \epsilon & \epsilon \\ \delta_3 & \epsilon & 1 \end{pmatrix} , \tag{A.1}$$

$$M_{RR} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ \delta_2 & \epsilon & \epsilon \\ \delta_3 & \epsilon & 1 \end{pmatrix} , \tag{A.2}$$

and again $M_{LL} = M_{LR} M_{RR}^{-1} M_{LR}^T$. The general form for M_{LL} can be easily computed, but is somewhat cumbersome, so we will not display it explicitly. However, the important qualitative result is easy to appreciate by considering some simplifying limits.

$\delta_1 \geq \delta_2, \delta_3$:

$$M_{LL} = \frac{H^2}{\Lambda_R} \begin{pmatrix} \epsilon'^2/\epsilon & \epsilon' & \epsilon' \\ \epsilon' & \epsilon'^2/\delta_1 & \epsilon \\ \epsilon' & \epsilon & 1 \end{pmatrix} \tag{A.3}$$

$\delta_2 \gg \delta_1, \delta_3$:

$$M_{LL} = \frac{H^2}{\delta_2 \Lambda_R} \begin{pmatrix} 0 & \epsilon'^2 & 0 \\ \epsilon'^2 & \epsilon\epsilon'^2/\delta_2 & \epsilon\epsilon' \\ 0 & \epsilon\epsilon' & \delta_2 \end{pmatrix} \tag{A.4}$$

$\delta_2 \sim \delta_3 \gg \delta_1$:

$$M_{LL} = \frac{H^2}{\delta_2 \Lambda_R} \begin{pmatrix} \epsilon'^2\delta_2 & \epsilon'^2 & \epsilon'\delta_2 \\ \epsilon'^2 & \epsilon\epsilon'^2/\delta_2 & \epsilon\epsilon' \\ \epsilon'\delta_2 & \epsilon\epsilon' & \delta_2 \end{pmatrix} \tag{A.5}$$

$\delta_3 \gg \delta_1, \delta_2$:

$$M_{LL} = \frac{H^2}{\delta_3 \Lambda_R} \begin{pmatrix} \epsilon'^2\delta_3/\epsilon & \epsilon'^2 & \epsilon'\delta_3 \\ \epsilon'^2 & \epsilon'^2/\delta_3 & \epsilon' \\ \epsilon'\delta_3 & \epsilon' & \delta_3 \end{pmatrix} \tag{A.6}$$

In each case it was assumed that the δ_i were the smallest scales in the problem. Notice also in the last case that we recover Eq. (3.11) when $\delta_3 \approx \epsilon'$. While the general form for M_{LL} implied by Eqs. (A.1) and (A.2) does not allow us to make very definite statements about the phenomenology of the

neutrino sector, we do see from these limiting cases that a widely split neutrino spectrum, with a heavy muon neutrino, is another possibility in U(2) models with an antisymmetric flavon.

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