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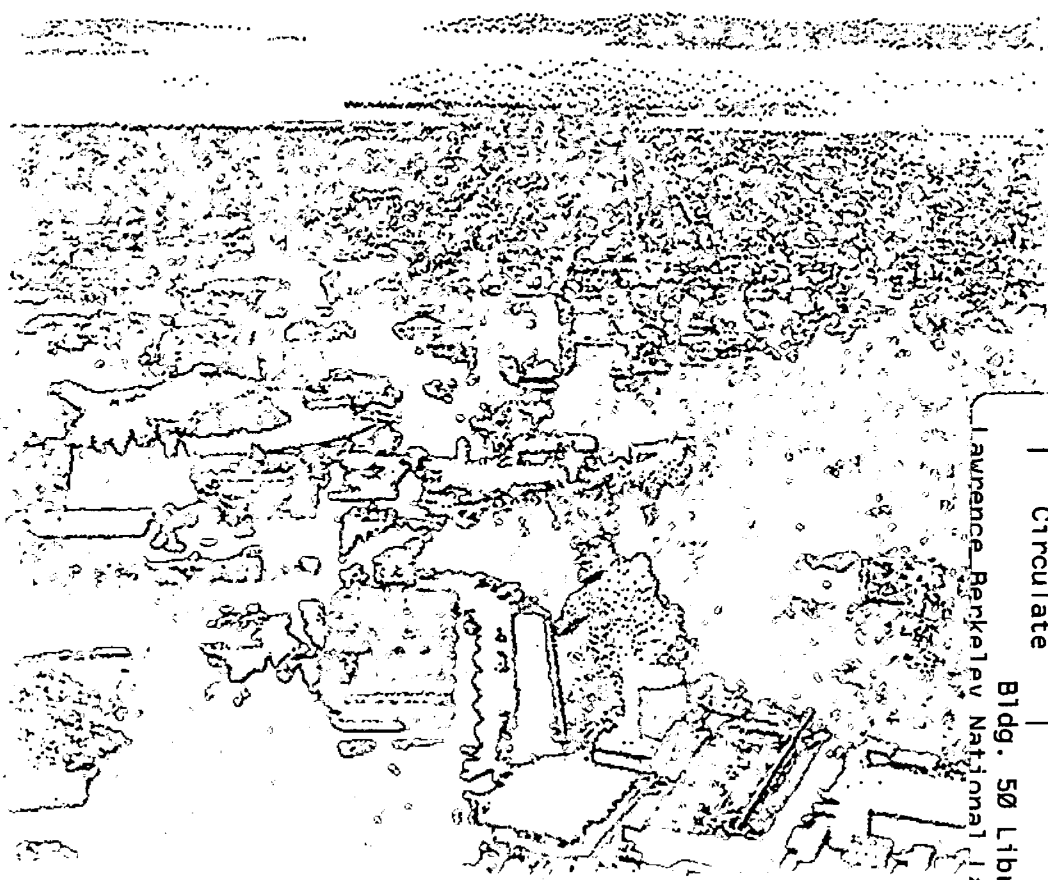

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# Improving the Fine Tuning in Models of Low Energy Gauge Mediated Supersymmetry Breaking [1]

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**Abstract**

The fine tuning in models of low energy gauge mediated supersymmetry breaking required to obtain the correct $Z$ mass is quantified. To alleviate the fine tuning problem, a model with split $(5+\bar{5})$ messenger fields is presented. This model has additional triplets in the low energy theory which get a mass of $O(500)$ GeV from a coupling to a singlet. The improvement in fine tuning is quantified and the spectrum in this model is discussed. The same model with the above singlet coupled to the Higgs doublets to generate the $\mu$ term is also discussed. A Grand Unified version of the model is constructed and a known doublet-triplet splitting mechanism is used to split the messenger $(5+\bar{5})$'s. A complete model is presented and some phenomenological constraints are discussed.

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# 1 Introduction

One of the outstanding problems of particle physics is the origin of electroweak symmetry breaking (EWSB). In the Standard Model (SM), this is achieved by one Higgs doublet which acquires a vacuum expectation value (vev) due to a negative mass squared which is put in by hand. The SM has the well known gauge hierarchy problem [1]. It is known that supersymmetry (SUSY) [2] stabilises the hierachy between the weak scale and some other high scale without any fine tuning if the masses of the superpartners are less than few TeV [3, 4]. The Minimal Supersymmetric Standard Model (MSSM) is considered as a low energy effective theory in which the soft SUSY breaking terms at the correct scale are put in by hand. This raises the question : what is the origin of these soft mass terms, *i.e.*, how is SUSY broken ? If SUSY is broken spontaneously at tree level in the MSSM, then there is a colored scalar lighter than the up or down quarks [5]. So, the superpartners have to acquire mass through radiative corrections. Thus, we need a "hidden" sector where SUSY is broken spontaneously at tree level and then communicated to the MSSM by some "messengers".

There are two problems here: how is SUSY broken in the hidden sector at the right scale and what are the messengers ? There are models in which a dynamical superpotential is generated by non-perturbative effects which breaks SUSY [6]. The SUSY breaking scale is related to the Planck scale by dimensional transmutation. Two possibilities have been discussed in the literature for the messengers. One is gravity which couples to both the sectors [7]. In a supergravity theory, there are non-renormalizable couplings between the two sectors which generate soft SUSY breaking operators in the MSSM once SUSY is broken in the "hidden" sector. In the absence of a flavor symmetry, this theory has to be fine tuned to give almost degenerate squarks and sleptons of the first two generations which is required by Flavor Changing Neutral Current (FCNC) phenomenology [5, 8]. The other messengers are the SM gauge interactions [9]. In these models, the scalars of the first two generations are naturally degenerate since they have the same gauge quantum numbers. This is an attractive feature of these models, since the FCNC constraints are naturally avoided and no fine tuning between the masses of the first two generation scalars is required. If this lack of fine tuning is a compelling argument in favour of these models, then it is important to investigate whether other sectors of these models are fine tuned. In fact, we will argue (and this is also discussed in [10, 11, 12]) that the minimal model (to be defined in section 2) of low energy gauge mediated SUSY breaking requires a minimum 7% fine tuning to generate a correct vacuum ($Z$ mass). Further, if a gauge-singlet is introduced to generate the "$\mu$" and "$B\mu$" terms, then the minimal model of low energy gauge mediated SUSY breaking requires a minimum 1% fine tuning to correctly break the electroweak symmetry. These fine tunings makes it difficult to understand, within the context of these models, how SUSY is to offer some understanding of the origin of electroweak symmetry breaking and the scale of the $Z$ and $W$ gauge boson masses.

Our paper is organized as follows. In section 2, we briefly review both the "messenger sector" in low energy gauge mediated SUSY breaking models that communicates SUSY breaking to the Standard Model and the pattern of the sfermion and gaugino masses that follow. Section 3 quantifies the fine tuning in the minimal model using the Barbieri-Giudice criterion [3]. We show that a fine tuning of $\approx 7\%$ is required in the Higgs sector to obtain $m_Z$. Section 4 describes a toy model with split $(5 + \bar{5})$ messenger representations that improves the fine tuning. To maintain gauge coupling unification, additional triplets are added to the low energy theory. They acquire a mass of $O(500)$ GeV by a coupling to a singlet. The fine tuning in this model is improved to $\sim 40\%$. The sparticle phenomenology of these models is also discussed. In section 5, we discuss a version of the toy model where the above mentioned singlet generates the $\mu$ and $\mu_3^2$ terms. This is identical to the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [13] with a particular pattern for the soft SUSY breaking operators that follows from gauge mediated SUSY breaking and our solution to the fine tuning problem. We show that this model is tuned to $\sim 20\%$, even if LEP does not discover SUSY/light Higgs. We also show that the NMSSM with one complete messenger $(5 + \bar{5})$ is fine tuned to $\sim 1\%$. We discuss, in section 6, how it is possible to make our toy model compatible with a Grand Unified Theory (GUT) [14] based upon the gauge group $SU(5) \times SU(5)$. The doublet-triplet splitting mechanism of Barbieri, Dvali and Strumia [15] is used to split both the messenger representations and the Higgs multiplets. In section 7, we present a model in which all operators consistent with symmetries are present and demonstrate that the low energy theory is the model of section 5. In this model $R$-parity ($R_p$) is the unbroken subgroup of a $Z_4$ global discrete symmetry that is required to solve the doublet-triplet splitting problem. Our model has some metastable particles which might cause a cosmological problem. In the appendix, we give the expressions for the Barbieri-Giudice parameters (for the fine tuning) for the MSSM and the NMSSM.

# 2 Messenger Sector

In the models of low energy gauge mediated SUSY breaking [10, 16] (henceforth called LEGM models), SUSY breaking occurs dynamically in a "hidden" sector of the theory at a scale $\Lambda_{dyn}$ that is generated through dimensional transmutation. SUSY breaking is communicated to the Standard Model fields in two stages. First, a non-anomalous $U(1)$ global symmetry of the hidden sector is weakly gauged. This $U(1)_X$ gauge interaction communicates SUSY breaking from the original SUSY breaking sector to a messenger sector at a scale $\Lambda_{mess} \sim \alpha_X \Lambda_{dyn}/(4\pi)$ as follows. The particle content in the messenger sector consists of fields $\phi_+$, $\phi_-$ charged under this $U(1)_X$, a gauge singlet field $S$, and vector-like fields that carry Standard Model quantum numbers (henceforth called messenger quarks and leptons). In the minimal LEGM model, there is one set of vector-like fields, $\bar{q}$, $l$, and $q$, $\bar{l}$ that together form a $(\bar{5}+5)$ of $SU(5)$. This is a suffucient condition to maintain unification of the SM gauge couplings. The superpotential in the minimal model is

$$W_{mess} = \lambda_\phi \phi_+ \phi_- S + \frac{1}{3}\lambda_S S^3 + \lambda_q S q \bar{q} + \lambda_l S l \bar{l}. \tag{1}$$

The scalar potential is

$$V = \sum_i |F_i|^2 + m_+^2 |\phi_+|^2 + m_-^2 |\phi_-|^2. \tag{2}$$

In the models of [10, 16], the $\phi_+, \phi_-$ fields communicate (at two loops) with the hidden sector fields through the $U(1)$ gauge interactions. Then, SUSY breaking in the original sector generates a negative value $\sim -\left(\alpha_X \Lambda_{dyn}\right)^2/(4\pi)^2$ for the mass parameters $m_+^2$, $m_-^2$ of the $\phi_+$ and $\phi_-$ fields. This drives vevs of $O\left(\Lambda_{mess}\right)$ for the scalar components of both $\phi_+$ and $\phi_-$, and also for the scalar and $F$-component of $S$ if the couplings $\lambda_S$, $g_X$ and $\lambda_\phi$ satisfy the inequalities derived in [11, 17].[4] Generating a vev for both the scalar and $F$-component of $S$ is crucial, since this generates a non-supersymmetric spectrum for the vector-like fields $q$ and $l$. The spectrum of each vector-like messenger field consists of two complex scalars with masses $M^2 \pm B$ and two Weyl fermions with mass $M$ where $M = \lambda S$, $B = \lambda F_S$ and $\lambda$ is the coupling of the vector-like fields to $S$. Since we do not want the SM to be broken at this stage, $M^2 - B \geq 0$. In the second stage, the messenger fields are integrated out. As these messenger fields have SM gauge interactions, SM gauginos acquire masses at one loop and the sfermions and Higgs acquire soft scalar masses at two loops [9]. The gaugino masses at the scale at which the messenger fields are integrated out, $\Lambda_{mess} \approx M$ are [16]

$$M_G = \frac{\alpha_G(\Lambda_{mess})}{4\pi} \Lambda_{SUSY} \sum_m N_R^G(m) f_1\left(\frac{F_S}{\lambda_m S^2}\right). \tag{3}$$

The sum in equation 3 is over messenger fields $(m)$ with normalization $\mathrm{Tr}(T^a T^b) = N_R^G(m)\delta^{ab}$ where the $T$'s are the generators of the gauge group $G$ in the representation $R$, $f_1(x) = 1 + O(x)$, and $\Lambda_{SUSY} \equiv B/M = F_S/S = x\Lambda_{mess}$ with $x = B/M^2$. [5] Henceforth, we will set $\Lambda_{SUSY} \approx \Lambda_{mess}$. The exact one loop calculation [18] of the gaugino mass shows that $f_1(x) \leq 1.3$ for $x \leq 1$. The soft scalar masses at $\Lambda_{mess}$ are [16]

$$m_i^2 = 2\Lambda_{SUSY}^2 \sum_{m,G} N_R^G(m) C_R^G(s_i) \left(\frac{\alpha_G(\Lambda_{mess})}{4\pi}\right)^2 f_2\left(\frac{F_S}{\lambda_m S^2}\right), \tag{4}$$

where $C_R^G(s_i)$ is the Caismir of the representation of the scalar $i$ in the gauge group $G$ and $f_2(x) = 1 + O(x)$. The exact two loop calculation [18] which determines $f_2$ shows that for $x \leq 0.8$ (0.9), $f_2$ differs from one by less than 1%(5%). Henceforth we shall use $f_1(x) = 1$ and $f_2(x) = 1$. In the minimal LEGM model

$$M_G(\Lambda_{mess}) = \frac{\alpha_G(\Lambda_{mess})}{4\pi} \Lambda_{mess}, \tag{5}$$

[4]This point in field space is a local minimum. There is a deeper minimum where SM is broken [11, 17]. To avoid this problem, we can, for example, add another singlet to the messenger sector [11]. This does not change our conclusions about the fine tuning.

[5]If all the dimensionless couplings in the superpotential are of $O(1)$, then $x$ cannot be much smaller than 1.

$$
\begin{aligned}
m^2(\Lambda_{mess}) &= 2\Lambda_{mess}^2 \times \qquad (6)\\
&\left(C_3\left(\frac{\alpha_3(\Lambda_{mess})}{4\pi}\right)^2 + C_2\left(\frac{\alpha_2(\Lambda_{mess})}{4\pi}\right)^2 + \frac{3}{5}\left(\frac{\alpha_1(\Lambda_{mess})Y}{4\pi}\right)^2\right),
\end{aligned}
$$

where $Q = T_{3L} + Y$ and $\alpha_1$ is the $SU(5)$ normalized hypercharge coupling. Further, $C_3 = 4/3$ and $C_2 = 3/4$ for colored triplets and electroweak doublets respectively.

The spectrum in the models is determined by only a few unknown parameters. As equations 3 and 4 indicate, the SUSY breaking mass parameters for the Higgs, sfermions and gauginos are

$$
m_{\tilde{q}}, m_{\tilde{g}} : m_{\tilde{L}}, m_{H_i}, m_{\tilde{W}} : m_{\tilde{e}_R}, m_{\tilde{B}} \sim \alpha_3 : \alpha_2 : \alpha_1. \qquad (7)
$$

The scale of $\Lambda_{mess}$ is chosen to be $\sim 100$ TeV so that the lightest of these particles escapes detection. The phenomenology of the minimal LEGM model is discussed in detail in [19].

## 3 Fine Tuning in the Minimal LEGM

A desirable feature of gauge mediated SUSY breaking is the natural suppression of FCNC processes since the scalars with the same gauge quantum numbers are degenerate [9]. But, the minimal LEGM model introduces a fine tuning in the Higgs sector unless the messenger scale is low. This has been previously discussed in [10, 11] and quantified more recently in [12]. We outline the discussion in order to introduce some notation.

The superpotential for the MSSM is

$$
W = \mu H_u H_d + W_{Yukawa}. \qquad (8)
$$

The scalar potential is

$$
V = \mu_1^2|H_u|^2 + \mu_2^2|H_d|^2 - (\mu_3^2 H_u H_d + h.c.) + \text{D-terms} + V_{1-loop}, \qquad (9)
$$

where $V_{1-loop}$ is the one loop effective potential. The vev of $H_u$ ($H_d$), denoted by $v_u(v_d)$, is responsible for giving mass to the up (down)-type quarks, $\mu_1^2 = m_{H_d}^2 + \mu^2$, $\mu_2^2 = m_{H_u}^2 + \mu^2$ and $\mu_3^2$, [6] $m_{H_u}^2$, $m_{H_d}^2$ are the SUSY breaking mass

[6] $\mu_3^2$ is often written as $B\mu$.

terms for the Higgs fields. [7] Extremizing this potential determines, with $\tan\beta \equiv v_u/v_d$,

$$\frac{1}{2}{m_Z}^2 = \frac{\tilde{\mu}_1^2 - \tilde{\mu}_2^2 \tan^2\beta}{\tan^2\beta - 1}, \tag{10}$$

$$\sin 2\beta = 2\frac{\mu_3^2}{\tilde{\mu}_1^2 + \tilde{\mu}_2^2}, \tag{11}$$

where $\tilde{\mu}_i^2 = \mu_i^2 + 2\partial V_{1-loop}/\partial v_i^2$. For large $\tan\beta$, $m_Z^2/2 \approx -(m_{H_u}^2 + \mu^2)$. This indicates that if $|m_{H_u}^2|$ is large relative to $m_Z^2$, the $\mu^2$ term must cancel this large number to reproduce the correct value for $m_Z^2$. This introduces a fine tuning in the Higgs potential, that is naively of the order $m_Z^2/(2|m_{H_u}^2|)$. We shall show that this occurs in the minimal LEGM model.

In the minimal LEGM model, a specification of the messenger particle content and the messenger scale $\Lambda_{mess}$ fixes the sfermion and gaugino spectrum at that scale. For example, the soft scalar masses for the Higgs fields are $\approx \alpha_2(\Lambda_{mess})\Lambda_{mess}/(4\pi)$. Renormalization Group (RG) evolution from $\Lambda_{mess}$ to the electroweak scale reduces $|m_{H_u}^2|$ due to the large top quark Yukawa coupling, $\lambda_t$, and the squark soft masses. The one loop Renormalization Group Equation (RGE) for $m_{H_u}^2$ is (neglecting gaugino and the trilinear scalar term $(H_u\tilde{Q}\tilde{u}^c)$ contributions )

$$\frac{dm_{H_u}^2(t)}{dt} \approx \frac{3\lambda_t^2}{8\pi^2}(m_{H_u}^2(t) + m_{\tilde{u}^c}^2(t) + m_{\tilde{Q}}^2(t)), \tag{12}$$

which gives

$$m_{H_u}^2(t \approx \ln(\frac{m_{\tilde{t}}}{\Lambda_{mess}})) \approx m_{H_u}^2(0) - \frac{3\lambda_t^2}{8\pi^2}(m_{H_u}^2(0) + m_{\tilde{u}^c}^2(0) + m_{\tilde{Q}}^2(0))\ln(\frac{\Lambda_{mess}}{m_{\tilde{t}}}). \tag{13}$$

On the right-hand side of equation 13 the RG scaling of $m_{\tilde{Q}}^2$ and $m_{\tilde{u}^c}^2$ has been neglected. Since the logarithm $|t| \approx \ln(\Lambda_{mess}/m_{\tilde{t}})$ is small, it is naively expected that $m_{H_u}^2$ will not be driven negative enough and will not trigger electroweak symmetry breaking. However since the squarks are $\approx$ 500 GeV (1 TeV) for a messenger scale $\Lambda_{mess}$ = 50 TeV (100 TeV), the radiative corrections from virtual top squarks are large since the squarks are heavy. A numerical solution of the one loop RGE (including gaugino and the trilinear scalar term $(H_u\tilde{Q}\tilde{u}^c)$ contributions) determines $-m_{H_u}^2$ =(275 GeV)$^2$

[7]The scale dependence of the parameters appearing in the potential is implicit.

$((550\text{ GeV})^2)$ for $\Lambda_{mess}$ =50 TeV (100 TeV) and setting $\lambda_t = 1$. Therefore, $m_Z^2/(2|m_{H_u}^2|)$ ~0.06 (0.01), an indication of the fine tuning required.

To reduce the fine tuning in the Higgs sector, it is necessary to reduce $|m_{H_u}^2|$; ideally so that $m_{H_u}^2 \approx -0.5 m_Z^2$. The large value of $|m_{H_u}^2|$ at the weak scale is a consequence of the large hierarchy in the soft scalar masses at the messenger scale: $m_{\tilde{e}_R}^2 < m_{H_u}^2 \ll m_{\tilde{Q},\tilde{u}^c}^2$. Models of sections 4,5, and 7 attempt to reduce the ratio $m_{\tilde{Q}}^2/m_{H_u}^2$ at the messenger scale and hence improve the fine tuning in the Higgs sector.

The fine tuning may be quantified by applying one of the criteria of [3, 4]. The value $O^*$ of a physical observable $O$ will depend on the fundamental parameters $(\lambda_i)$ of the theory. The fundamental parameters of the theory are to be distinguished from the free parameters of the theory which parameterize the solutions to $O(\lambda_i) = O^*$. If the value $O^*$ is unusually sensitive to the underlying parameters $(\lambda_i)$ of the theory then a small change in $\lambda_i$ produces a large change in the value of $O$. The Barbieri-Giudice function

$$c(O,\lambda_i) = \frac{\lambda_i^*}{O^*}\frac{\partial O}{\partial \lambda_i}|_{O=O^*} \tag{14}$$

quantifies this sensitivity [3]. This particular value of $O$ is fine tuned if the sensitivity to $\lambda_i$ is larger at $O = O^*$ than at other values of $O$ [4]. If there are values of $O$ for which the sensitivity to $\lambda_i$ is small, then it is probably sufficient to use $c(O,\lambda_i)$ as the measure of fine tuning.

To determine $c(m_Z^2, \lambda_i)$, we performed the following. The sparticle spectrum in the minimal LEGM model is determined by the four parameters $\Lambda_{mess}$, $\mu_3^2$, $\mu$, and $\tan\beta$. [8] The scale $\Lambda_{mess}$ fixes the boundary condition for the soft scalar masses, and an implicit dependence on $\tan\beta$ from $\lambda_t$, $\lambda_b$ and $\lambda_\tau$ arises in RG scaling[9] from $\mu_{RG} = \Lambda_{mess}$ to the weak scale, that is chosen to be $\mu_{RG}^2 = m_t^2 + \frac{1}{2}(\tilde{m}_t^2 + \tilde{m}_{t^c}^2)$. The extremization conditions of the scalar potential (equations 10 and 11) together with $m_Z$ and $m_t$ leave two free parameters that we choose to be $\Lambda_{mess}$ and $\tan\beta$ (see appendix for the expressions for these functions).

A numerical analysis yields the value of $c(m_Z^2, \mu^2)$ that is displayed in figure 1 in the $(\tan\beta, \Lambda_{mess})$ plane. We note that $c(m_Z^2, \mu^2)$ is large throughout most of the parameter space, except for the region where $\tan\beta \gtrsim 5$ and

[8]We allow for an arbitrary $\mu_3^2$ at $\Lambda_{mess}$.

[9]The RG scaling of $\lambda_t$ was neglected.

the messenger scale is low. A strong constraint on a lower limit for $\Lambda_{mess}$ is from the right-handed selectron mass. Contours $m_{\tilde{e}_R} = 75$ GeV ($\sim$ the LEP limit from the run at $\sqrt{s} \approx 170$ GeV [20]) and 85 GeV ($\sim$ the ultimate LEP2 limit [21]) are also plotted. The (approximate) limit on the neutralino masses from the LEP run at $\sqrt{s} \approx 170$ GeV, $m_{\chi_1^0} + m_{\chi_2^0} = 160$ GeV and the ultimate LEP2 limit, $m_{\chi_1^0} + m_{\chi_2^0} \sim 180$ GeV are also shown in figures a and c for $sgn(\mu) = -1$ and figures b and d for $sgn(\mu) = +1$. The constraints from the present and the ultimate LEP2 limits on the chargino mass are weaker than or comparable to those from the selectron and the neutralino masses and are therefore not shown. If $m_Z$ were much larger, then $c \sim 1$. For example, with $m_Z = 275$ GeV (550 GeV) and $\Lambda_{mess}$= 50 (100) TeV, $c(m_Z^2; \mu^2)$ varies between 1 and 5 for $1.4 \lesssim \tan\beta \lesssim 2$, and is $\approx 1$ for $\tan\beta > 2$. This suggests that the interpretation that a large value for $c(m_Z^2; \mu^2)$ implies that $m_Z$ is fine tuned is probably correct.

From figure 1 we conclude that in the minimal LEGM model a fine tuning of approximately 7% in the Higgs potential is required to produce the correct value for $m_Z$. Further, for this fine tuning the parameters of the model are restricted to the region $\tan\beta \gtrsim 5$ and $\Lambda_{mess} \approx 45$ TeV, corresponding to $m_{\tilde{e}_R} \approx 85$ GeV. We have also checked that adding more complete $(5 + \bar{5})$'s does not reduce the fine tuning.

# 4 A Toy Model to Reduce Fine Tuning

## 4.1 *Model*

In this section the particle content and couplings in the messenger sector that are suffucient to reduce $|m_{H_u}^2|$ is discussed. The aim is to reduce $m_{\tilde{Q}}^2/m_{H_u}^2$ at the scale $\Lambda_{mess}$.

The idea is to increase the number of messenger leptons ($SU(2)$ doublets) relative to the number of messenger quarks ($SU(3)$ triplets). This reduces both $m_{\tilde{Q}}^2/m_{H_u}^2$ and $m_{\tilde{Q}}^2/m_{\tilde{e}_R}^2$ at the scale $\Lambda_{mess}$ (see equation 4). This leads to a smaller value of $|m_{H_u}^2|$ in the RG scaling (see equation 13) and the scale $\Lambda_{mess}$ can be lowered since $m_{\tilde{e}_R}$ is larger. For example, with three doublets and one triplet at a scale $\Lambda_{mess} = 30$ TeV, so that $m_{\tilde{e}_R} \approx 85$ GeV, we find $|m_{H_u}^2(m_{\tilde{Q}})| \approx (100\text{GeV})^2$ for $\lambda_t = 1$. This may be achieved by the following superpotential in the messenger sector

$$\begin{aligned} W &= \lambda_{q_1} S q_1 \bar{q}_1 + \lambda_{l_1} S l_1 \bar{l}_1 + \lambda_{l_2} S l_2 \bar{l}_2 + \lambda_{l_3} S l_3 \bar{l}_3 + \frac{1}{3}\lambda_S S^3 \\ & \quad + \lambda_\phi S \phi_- \phi_+ + \frac{1}{3}\lambda_N N^3 + \lambda_{q_2} N q_2 \bar{q}_2 + \lambda_{q_3} N q_3 \bar{q}_3, \end{aligned} \tag{15}$$

where $N$ is a gauge singlet. The two pairs of triplets $q_2, \bar{q}_2$ and $q_3, \bar{q}_3$ are required at low energies to maintain gauge coupling unification. In this model the additional leptons $l_2, \bar{l}_2$ and $l_3, \bar{l}_3$ couple to the singlet $S$, whereas the additional quarks couple to a different singlet $N$ that does not couple to the messenger fields $\phi_+$, $\phi_-$. This can be enforced by discrete symmetries (we discuss such a model in section 7). Further, we assume the discrete charges also forbid any couplings between $N$ and $S$ at the renormalizable level (this is true of the model in section 7) so that SUSY breaking is communicated first to $S$ and to $N$ only at a higher loop level.

## 4.2 *Mass Spectrum*

Before quantifying the fine tuning in this model, the mass spectrum of the additional states is briefly discussed. While these fields form complete representations of $SU(5)$, they are not degenerate in mass. The vev and $F$-component of the singlet $S$ gives a mass $\Lambda_{mess}$ to the messenger lepton multiplets if the $F$-term splitting between the scalars is neglected. As the squarks in $q_i + \bar{q}_i$ ($i$=2,3) do not couple to $S$, they acquire a soft scalar mass from the same two loop diagrams that are responsible for the masses of the MSSM squarks, yielding $m_{\tilde{q}} \approx \alpha_3(\Lambda_{mess})\ \Lambda_{SUSY}/(\sqrt{6}\pi)$. The fermions in $q + \bar{q}$ also acquire mass at this scale since, if either $\lambda_{q_2}$ or $\lambda_{q_3} \sim O(1)$, a negative value for $m_N^2$ (the soft scalar mass squared of $N$) is generated from the $\lambda_q N q \bar{q}$ coupling at one loop and thus a vev for $N \sim m_{\tilde{q}}$ is generated. The result is $m_l/m_q \approx \sqrt{6}\pi/\alpha_3(\Lambda_{mess})(\Lambda_{mess}/\Lambda_{SUSY}) \approx 85$.

The mass splitting in the extra fields introduces a threshold correction to $\sin^2\theta_W$ if it is assumed that the gauge couplings unify at some high scale $M_{GUT} \approx 10^{16}$ GeV. We estimate that the splitting shifts the prediction for $\sin^2\theta_W$ by an amount $\approx -7\times\ 10^{-4}\ln(m_l/m_q)n$, where $n$ is the number of split $(5+\bar{5})$.[10] In this case $n$ =2 and $m_l/m_q \sim 85$, so $\delta\sin^2\theta_W \sim -6\ \times$

[10]The complete $(5+\bar{5})$, *i.e.*, $l_1, \bar{l}_1$ and $q_1, \bar{q}_1$, that couples to $S$ is also split because

$10^{-3}$. If $\alpha_3(M_Z)$ and $\alpha_{em}(M_Z)$ are used as input, then using the two loop RG equations $\sin^2\theta_W(\overline{MS}) = 0.233 \pm O(10^{-3})$ is predicted in a minimal SUSY-GUT [22]. The error is a combination of weak scale SUSY and GUT threshold corrections[22]. The central value of the theoretical prediction is a few percent higher than the measured value of $\sin^2\theta_W(\overline{MS}) = 0.231 \pm 0.0003$[23]. The split extra fields shift the prediction of $\sin^2\theta_W$ to $\sim 0.227 \pm O(10^{-3})$ which is a few percent lower than the experimental value. In sections 6,7 we show that this spectrum is derivable from a $SU(5) \times SU(5)$ GUT in which the GUT threshold corrections to $\sin^2\theta_W$ could be $\sim O(10^{-3}) - O(10^{-2})$ [24]. It is possible that the combination of these GUT threshold corrections and the split extra field threshold corrections make the prediction of $\sin^2\theta_W$ more consistent with the observed value.

### 4.3 *Fine Tuning*

To quantify the fine tuning in these class of models the analysis of section 3 is applied. In our RG analysis the RG scaling of $\lambda_t$, the effect of the extra vector-like triplets on the RG scaling of the gauge couplings, and weak scale SUSY threshold corrections were neglected. We have checked *a postiori* that this approximation is consistent. As in section 3, the two free parameters are chosen to be $\Lambda_{mess}$ and $\tan\beta$. Contours of constant $c(m_Z^2, \mu^2)$ are presented in figure 2. We show contours of $m_{\chi_1^0} + m_{\chi_2^0} = 160$ GeV, and $m_{\tilde{e}_R} = 75$ GeV in figure 2 a for $sgn(\mu) = -1$ and 2b for $sgn(\mu) = +1$. These are roughly the present limits from LEP (including the run at $\sqrt{s} \approx 170$ GeV [20]). The (approximate) ultimate LEP2 reaches [21] $m_{\chi_1^0} + m_{\chi_2^0} = 180$ GeV, and $m_{\tilde{e}_R} = 85$ GeV are shown in figure 2c for $sgn(\mu) = -1$ and figure 2d for $sgn(\mu) = +1$. Since $\mu^2 (\approx (100\text{ GeV})^2)$ is much smaller in these models than in the minimal LEGM model, the neutralinos ($\chi_1^0$ and $\chi_2^0$) are lighter so that the neutralino masses provide a stronger constraint on $\Lambda_{mess}$ than does the slepton mass limit. The chargino constraints are comparable to the neutralino constraints and are thus not shown. It is clear that there are areas of parameter space in which the fine tuning is improved to $\sim 40\%$ (see figure 2).

---

$\lambda_l \neq \lambda_q$ at the messenger scale due to RG scaling from $M_{GUT}$ to $\Lambda_{mess}$. This splitting is small and neglected.

While this model improves the fine tuning required of the $\mu$ parameter, it would be unsatisfactory if further fine tunings were required in other sectors of the model, for example, the sensitivity of $m_Z^2$ to $\mu_3^2$, $\Lambda_{mess}$ and $\lambda_t$ and the sensitivity of $m_t$ to $\mu^2$, $\mu_3^2$, $\Lambda_{mess}$ and $\lambda_t$. We have checked that all these are less than or comparable to $c(m_Z^2;\mu^2)$. We now discuss the other fine tunings in detail.

For large $\tan\beta$, the sensitivity of $m_Z^2$ to $\mu_3^2$, $c(m_Z^2;\mu_3^2) \propto 1/\tan^2\beta$, and is therefore smaller than $c(m_Z^2;\mu^2)$. Our numerical analysis shows that $c(m_Z^2;\mu_3^2) \lesssim c(m_Z^2;\mu^2)$ for all $\tan\beta$.

In the one loop approximation $m_{H_u}^2$ and $m_{H_d}^2$ at the weak scale are proportional to $\Lambda_{mess}^2$ since all the soft masses scale with $\Lambda_{mess}$ and there is only a weak logarithmic dependence on $\Lambda_{mess}$ through the gauge couplings. We have checked numerically that $(\Lambda_{mess}^2/m_{H_u}^2)(\partial m_{H_u}^2/\partial\Lambda_{mess}^2) \sim 1$. Then, $c(m_Z^2;\Lambda_{mess}^2) \approx c(m_Z^2;m_{H_d}^2) + c(m_Z^2;m_{H_u}^2)$. We find that $c(m_Z^2;\Lambda_{mess}^2) \approx c(m_Z^2;\mu^2)+1$ over most of the parameter space.

In the one loop approximation, $m_{H_u}^2(t)$ is

$$m_{H_u}^2(t) \approx m_{H_u}^2(0) + (m_{\tilde{Q}_3}^2(0) + m_{\tilde{u}_3^c}^2(0) + m_{H_u}^2(0))(e^{-\frac{3\lambda_t^2}{8\pi^2}t} - 1). \qquad (16)$$

Then, using $t \approx \ln(\Lambda_{mess}/m_{\tilde{Q}_3}) \approx \ln(\sqrt{6}\pi/\alpha_3) \approx 4.5$ and $\lambda_t \approx 1$, $c(m_Z^2;\lambda_t)$ is (see appendix)

$$c(m_Z^2;\lambda_t) \approx \frac{4}{m_Z^2}\frac{\partial m_{H_u}^2(t)}{\partial\lambda_t^2} \approx 50\frac{m_{\tilde{Q}_3}^2}{(600\ \mathrm{GeV})^2}. \qquad (17)$$

This result measures the sensitivity of $m_Z^2$ to the value of $\lambda_t$ at the electroweak scale. While this sensitivity is large, it does not reflect the fact that $\lambda_t(M_{pl})$ is the fundamental parameter of the theory, rather than $\lambda_t(M_{weak})$. We find by both numerical and analytic computations that, for this model with three $(5+\bar{5})$'s in addition to the MSSM particle content, $\delta\lambda_t(M_{weak}) \approx 0.1 \times \delta\lambda_t(M_{pl})$, and therefore

$$c(m_Z^2;\lambda_t(M_{pl})) \approx 5\frac{m_{\tilde{Q}_3}^2}{(600\ \mathrm{GeV})^2}. \qquad (18)$$

For a scale of $\Lambda_{mess} = 50$ TeV ($m_{\tilde{Q}} \approx 600$ GeV), $c(m_Z^2;\lambda_t(M_{pl}))$ is comparable to $c(m_Z^2;\mu^2)$ which is $\approx$ 4 to 5. At a lower messenger scale, $\Lambda_{mess} \approx 35$

TeV, corresponding to squark masses of $\approx 450$ GeV, the sensitivity of $m_Z^2$ to $\lambda_t(M_{pl})$ is $\approx 2.8$. This is comparable to $c(m_Z^2;\mu^2)$ evaluated at the same scale.

We now discuss the sensitivity of $m_t$ to the fundamental parameters. Since, $m_t^2 = \frac{1}{2}v^2\sin^2\beta\lambda_t^2$, we get

$$c(m_t;\lambda_i) = \delta_{\lambda_t\lambda_i} + \frac{1}{2}c(m_Z^2;\lambda_i) + \frac{\cos^3\beta}{\sin\beta}\frac{\partial\tan\beta}{\partial\lambda_i}\lambda_i. \tag{19}$$

Numerically we find that the last term in $c(m_t;\lambda_i)$ is small compared to $c(m_Z^2;\lambda_i)$ and thus over most of parameter space $c(m_t;\lambda_i) \approx \frac{1}{2}c(m_Z^2;\lambda_i)$. As before, the sensitivity of $m_t$ to the value of $\lambda_t$ at the GUT/Planck scale is much smaller than the sensitivity to the value of $\lambda_t$ at the weak scale.

### 4.4 *Sparticle Spectrum*

The sparticle spectrum is now briefly discussed to highlight deviations from the mass relations predicted in the minimal LEGM model. For example, with three doublets and one triplet at a scale of $\Lambda = 50$ TeV, the soft scalar masses (in GeV) at a renormalization scale $\mu_{RG}^2 = m_t^2 + \frac{1}{2}(m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3^c}^2) \approx (630$ GeV$)^2$, for $\lambda_t = 1$, are shown in table 1.

Two observations that are generic to this type of model are: (i) By construction, the spread in the soft scalar masses is less than in the minimal LEGM model. (ii) The gaugino masses do not satisfy the one-loop SUSY-GUT relation $M_i/\alpha_i =$ constant. In this case, for example, $M_3/\alpha_3 : M_2/\alpha_2 \approx$ 1:3 and $M_3/\alpha_3 : M_1/\alpha_1 \approx$ 5:11 to one-loop.

We have also found that for $\tan\beta \gtrsim 3$, the Next Lightest Supersymmetric Particle (NLSP) is one of the neutralinos, whereas for $\tan\beta \lesssim 3$, the NLSP is the right-handed stau. Further, for these small values of $\tan\beta$, the three right-handed sleptons are degenerate within $\approx 200$ MeV.

## 5 NMSSM

In section 3, the $\mu$ term and the SUSY breaking mass $\mu_3^2$ were put in by hand. There it was found that these parameters had to be fine tuned in order to correctly reproduce the observed $Z$ mass. The extent to which this is a "problem" may only be evaluated within a specific model that generates both the $\mu$ and $\mu_3^2$ terms.

For this reason, in this section a possible way to generate both the $\mu$ term and $\mu_3^2$ term in a manner that requires a minimal modification to the model of either section 2 or section 4 is discussed. The easiest way to generate these mass terms is to introduce a singlet $N$ and add the interaction $NH_uH_d$ to the superpotential (the NMSSM)[13]. The vev of the scalar component of $N$ generates $\mu$ and the vev of the $F$-component of $N$ generates $\mu_3^2$.

We note that for the "toy model" solution to the fine tuning problem (section 4), the introduction of the singlet occurs at no additional cost. Recall that in that model it was necessary to introduce a singlet $N$, distinct from $S$, such that the vev of $N$ gives mass to the extra light vector-like triplets, $q_i, \bar{q}_i$ ($i = 2, 3$) (see equation 15). Further, discrete symmetries (see section 7) are imposed to isolate $N$ from SUSY breaking in the messenger sector. This last requirement is necessary to solve the fine tuning problem: if both the scalar and $F$-component of $N$ acquired a vev at the same scale as $S$, then the extra triplets that couple to $N$ would also act as messenger fields. In this case the messenger fields would form complete $(5+\bar{5})$'s and the fine tuning problem would be reintroduced. With $N$ isolated from the messenger sector at tree level, a vev for $N$ at the electroweak scale is naturally generated, as discussed in section 4.

We also comment on the necessity and origin of these extra triplets. Recall that in the toy model of section 4 these triplets were required to maintain the SUSY-GUT prediction for $\sin^2\theta_W$. Further, we shall also see that they are required in order to generate a large enough $-m_N^2$ (the soft scalar mass squared of the singlet $N$). Finally, in the GUT model of section 7, the lightness of these triplets (as compared to the missing doublets) is the consequence of a doublet-triplet splitting mechanism.

The superpotential in the electroweak symmetry breaking sector is

$$W = \frac{\lambda_N}{3}N^3 + \lambda_q N q\bar{q} - \lambda_H N H_u H_d, \tag{20}$$

which is similar to an NMSSM except for the coupling of $N$ to the triplets. The superpotential in the messenger sector is given by equation 15.

The scalar potential is [11]

$$
\begin{aligned}
V &= \sum_i |F_i|^2 + m_N^2|N|^2 + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + \text{D-terms} \\
&\quad -(A_H N H_u H_d + h.c.) + V_{1-loop}. \qquad (21)
\end{aligned}
$$

The extremization conditions for the vevs of the real components of $N$, $H_u$ and $H_d$, denoted by $v_N$, $v_u$ and $v_d$ respectively (with $v = \sqrt{v_u^2 + v_d^2} \approx 250$ GeV), are

$$
v_N(\tilde{m}_N^2 + \lambda_H^2 \frac{v^2}{2} + \lambda_N^2 v_N^2 - \lambda_H \lambda_N v_u v_d) - \frac{1}{\sqrt{2}} A_H v_u v_d = 0, \qquad (22)
$$

$$
\begin{aligned}
\frac{1}{2} m_Z^2 &= \frac{\tilde{\mu}_1^2 - \tilde{\mu}_2^2 \tan^2\beta}{\tan^2\beta - 1}, \qquad (23) \\
\sin 2\beta &= 2\frac{\mu_3^2}{\tilde{\mu}_2^2 + \tilde{\mu}_1^2}, \qquad (24)
\end{aligned}
$$

with

$$
\begin{aligned}
\mu^2 &= \frac{1}{2}\lambda_H^2 v_N^2, \qquad (25) \\
\mu_3^2 &= -\frac{1}{2}\lambda_H^2 v_u v_d + \frac{1}{2}\lambda_H \lambda_N v_N^2 + A_H \frac{1}{\sqrt{2}} v_N, \qquad (26) \\
\tilde{m}_i^2 &= m_i^2 + 2\frac{\partial V_{1-loop}}{\partial v_i^2}; \quad i = (u, d, N). \qquad (27)
\end{aligned}
$$

We now comment on the expected size of the Yukawa couplings $\lambda_q$, $\lambda_N$ and $\lambda_H$. We must use the RGE's to evolve these couplings from their values at $M_{GUT}$ or $M_{pl}$ to the weak scale. The quarks and the Higgs doublets receive wavefunction renormalization from $SU(3)$ and $SU(2)$ gauge interactions respectively, whereas the singlet $N$ does not receive any wavefunction renormalization from gauge interactions at one loop. So, the couplings at the weak scale are in the order: $\lambda_q \sim O(1) > \lambda_H > \lambda_N$ if they all are $O(1)$ at the GUT/Planck scale.

We remark that without the $Nq\bar{q}$ coupling, it is difficult to drive a vev for $N$ as we now show below. The one loop RGE for $m_N^2$ is

$$
\frac{dm_N^2}{dt} \approx \frac{6\lambda_N^2}{8\pi^2} m_N^2(t) + \frac{2\lambda_H^2}{8\pi^2}(m_{H_u}^2(t) + m_{H_d}^2(t) + m_N^2(t)) + \frac{3\lambda_q^2}{8\pi^2}(m_{\tilde{q}}^2(t) + m_{\tilde{\bar{q}}}^2(t)). \qquad (28)
$$

[11]In models of gauge mediated SUSY breaking, $A_H$=0 at tree level and a non-zero value of $A_H$ is generated at one loop. The trilinear scalar term $A_N N^3$ is generated at two loops and is neglected.

Since $N$ is a gauge-singlet, $m_N^2 = 0$ at $\Lambda_{mess}$. Further, if $\lambda_q = 0$, an estimate for $m_N^2$ at the weak scale is then

$$m_N^2 \approx -\frac{2\lambda_H^2}{8\pi^2}(m_{H_u}^2(0) + m_{H_d}^2(0)) \ln\left(\frac{\Lambda_{mess}}{m_{H_d}}\right), \tag{29}$$

*i.e.*, $\lambda_H$ drives $m_N^2$ negative. The extremization condition for $v_N$, equation 22, and using equations 24 and 26 (neglecting $A_H$) shows that

$$m_N^2 + \lambda_H^2 \frac{v^2}{2} \approx \lambda_H^2 \left(\frac{v^2}{2} - \frac{2}{8\pi^2}(m_{H_u}^2(0) + m_{H_d}^2(0)) \ln\left(\frac{\Lambda_{mess}}{m_{H_d}}\right)\right) \tag{30}$$

has to be negative for $N$ to acquire a vev. This implies that $m_{H_u}^2$ and $m_{H_d}^2$ at $\Lambda_{mess}$ have to be greater than $\sim$ (350 GeV)$^2$ which implies that a fine tuning of a few percent is required in the electroweak symmetry breaking sector. With $\lambda_q \sim O(1)$, however, there is an additional negative contribution to $m_N^2$ given approximately by

$$-\frac{3\lambda_q^2}{8\pi^2}(m_q^2(0) + m_{\bar{q}}^2(0)) \ln\left(\frac{\Lambda_{mess}}{m_{\tilde{q}}}\right). \tag{31}$$

This contribution dominates the one in equation 29 since $\lambda_q > \lambda_H$ and the squarks $\tilde{q}$, $\tilde{\bar{q}}$ have soft masses larger than the Higgs. Thus, with $\lambda_q \neq 0$, $m_N^2 + \lambda_H^2 v^2/2$ is naturally negative.

Fixing $m_Z$ and $m_t$, we have the following parameters : $\Lambda_{mess}$, $\lambda_q$, $\lambda_H$, $\lambda_N$, $\tan\beta$, and $v_N$. Three of the parameters are fixed by the three extremization conditions, leaving three free parameters that for convienence are chosen to be $\Lambda_{mess}$, $\tan\beta \geq 0$, and $\lambda_H$. The signs of the vevs are fixed to be positive by requiring a stable vacuum and no spontaneous CP violation. The three extremization equations determine the following relations

$$\begin{aligned}
\lambda_N &= \frac{2}{\lambda_H v_N^2}(\mu_3^2 + \frac{1}{4}\lambda_H^2 \sin 2\beta v^2 - \frac{1}{\sqrt{2}} A_H v_N), && (32)\\
v_N &= \sqrt{2}\frac{\mu}{\lambda_H}, && (33)\\
\tilde{m}_N^2 &= \lambda_N \lambda_H \frac{1}{2}\sin 2\beta v^2 - \lambda_N^2 v_N^2 - \frac{1}{2}\lambda_H^2 v^2 + \frac{1}{2\sqrt{2}} A_H \sin 2\beta \frac{v^2}{v_N}, && (34)
\end{aligned}$$

where

$$\begin{aligned}
\mu^2 &= -\frac{1}{2}m_Z^2 + \frac{\tilde{m}_{H_u}^2 \tan^2\beta - \tilde{m}_{H_d}^2}{1 - \tan^2\beta}, && (35)\\
2\mu_3^2 &= \sin 2\beta(2\mu^2 + \tilde{m}_{H_u}^2 + \tilde{m}_{H_d}^2). && (36)
\end{aligned}$$

The superpotential term $NH_uH_d$ couples the RGE's for $m^2_{H_u}$, $m^2_{H_d}$ and $m^2_N$. Thus the values of these masses at the electroweak scale are, in general, complicated functions of the Yukawa parameters $\lambda_t$, $\lambda_H$, $\lambda_N$ and $\lambda_q$. In our case, two of these Yukawa parameters ($\lambda_q$ and $\lambda_N$) are determined by the extremization equations and a closed form expression for the derived quantities cannot be found. To simplify the analysis, we neglect the dependence of $m^2_{H_u}$ and $m^2_{H_d}$ on $\lambda_H$ induced in RG scaling from $\Lambda_{mess}$ to the weak scale. Then $m^2_{H_u}$ and $m^2_{H_d}$ depend only on $\Lambda_{mess}$ and $\tan\beta$ and thus closed form solutions for $\lambda_N$, $v_N$ and $\tilde{m}^2_N$ can be obtained using the above equations. Once $\tilde{m}^2_N$ at the weak scale is obtained, the value of $\lambda_q$ is obtained by using an approximate analytic solution. An exact numerical solution of the RGE's then shows that the above approximation is consistent.

## 5.1 *Fine Tuning and Phenomenology*

The fine tuning functions we consider below are $c(O;\lambda_H)$, $c(O;\lambda_N)$, $c(O;\lambda_t)$, $c(O;\lambda_q)$ and $c(O;\Lambda_{mess})$ where $O$ is either $m^2_Z$ or $m_t$. The expressions for the fine tuning functions and other details are given in the appendix. In our RG analysis the approximations discussed in subsection 4.3 and above were used and found to be consistent. Fine tuning contours of $c(m^2_Z;\lambda_H)$ are displayed in figures 3 a and 3 b for $\lambda_H = 0.1$ and figures 3 c and 3 d for $\lambda_H = 0.5$. We have found by numerical computations that the other fine tuning functions are either smaller or comparable to $c(m^2_Z;\lambda_H)$. [12]

We now discuss the existing phenomenological constraints on our model and also the ultimate constraints if LEP2 does not discover SUSY/light Higgs($h$). These are shown in figures 3 a,3 c and figures 3 b, 3 d respectively. We consider the processes $e^+e^- \to Zh$, $e^+e^- \to (h + pseudoscalar)$, $e^+e^- \to \chi^+\chi^-$, $e^+e^- \to \chi^0_1\chi^0_2$, and $e^+e^- \to \tilde{e}_R\tilde{e}^*_R$ observable at LEP. Since this model also has a light pseudoscalar, we also consider upsilon decays $\Upsilon \to (\gamma + pseudoscalar)$. We find that the model is phenomenologically viable and requires a $\sim 20\%$ tuning even if no new particles are discovered at

[12] In computing these functions the weak scale value of the couplings $\lambda_N$ and $\lambda_H$ has been used. But since $\lambda_N$ and $\lambda_H$ do not have a fixed point behavior, we have found that $\lambda_H(M_{GUT})/\lambda_H(m_Z)\ \partial\lambda_H(m_Z)/\partial\lambda_H(M_{GUT}) \sim 1$ so that, for example, $c(m^2_Z;\lambda_H(M_{GUT})) \approx c(m^2_Z;\lambda_H(m_Z))$.

LEP2.

We begin with the constraints on the scalar and pseudoscalar spectra of this model. There are three neutral scalars, two neutral pseudoscalars and one complex charged scalar. We first consider the mass spectrum of the pseudoscalars. At the boundary scale $\Lambda_{mess}$, SUSY is softly broken in the visible sector only by the soft scalar masses and the gaugino masses. Further, the superpotential of equation 20 has an $R$-symmetry. Therefore, at the tree level, *i.e.*, with $A_H$ =0, the scalar potential of the visible sector (equation 21) has a global symmetry. This symmetry is spontaneously broken by the vevs of $N^R$, $H_u^R$, and $H_d^R$ (the superscript $R$ denotes the real component of fields), so that one physical pseudoscalar is massless at tree level. It is

$$a = \frac{1}{\sqrt{v_N^2 + v^2 \sin^2 2\beta}} \left( v_N N^I + v \sin 2\beta \cos\beta H_u^I + v \sin 2\beta \sin\beta H_d^I \right), \quad (37)$$

where the superscripts $I$ denote the imaginary components of the fields. The second pseudoscalar,

$$A \sim -\frac{2}{v_N} N^I + \frac{H_u^I}{v \sin\beta} + \frac{H_d^I}{v \cos\beta}, \quad (38)$$

acquires a mass

$$m_A^2 = \frac{1}{2} \lambda_H \lambda_N v_N^2 (\tan\beta + \cot\beta) + \lambda_H \lambda_N v^2 \sin 2\beta \quad (39)$$

through the $|F_N|^2$ term in the scalar potential.

The pseudoscalar $a$ acquires a mass once an $A_H$-term is generated, at one loop, through interactions with the gauginos. Including only the wino contribution in the one loop RGE, $A_H$ is given by

$$\begin{aligned} A_H &\approx 6 \frac{\alpha_2(\Lambda_{mess})}{4\pi} M_2 \lambda_H \ln \left( \frac{\Lambda_{mess}}{M_2} \right), \\ &\approx 20\ \lambda_H \left( \frac{M_2}{280\text{GeV}} \right) \text{GeV}, \end{aligned} \quad (40)$$

where $M_2$ is the wino mass at the weak scale. Neglecting the mass mixing between the two pseudoscalars, the mass of the pseudo-Nambu-Goldstone boson is computed to be

$$\begin{aligned} m_a^2 &= \frac{9}{\sqrt{2}} A v_N v_u v_d / (v_N^2 + v^2 \sin^2 2\beta) \\ &\approx (40)^2 \left( \frac{\lambda_H}{0.1} \right) \frac{M_2}{280\text{GeV}} \sin 2\beta \left( \frac{\frac{v_N}{250\text{GeV}}}{\sin^2 2\beta + \left( \frac{v_N}{250\text{GeV}} \right)^2} \right) (\text{GeV})^2 \end{aligned} \quad (41)$$

If the mass of $a$ is less than 7.2 GeV, it could be detected in the decay $\Upsilon \rightarrow a + \gamma$[23]. Comparing the ratio of decay width for $\Upsilon \rightarrow a + \gamma$ to $\Upsilon \rightarrow \mu^- + \mu^+$ [23, 25], the limit

$$\frac{\sin 2\beta \tan \beta}{\sqrt{(\frac{v_N}{250\text{GeV}})^2 + \sin^2 2\beta}} < 0.43 \tag{42}$$

is found.

Further constraints on the spectra are obtained from collider searches. The non-detection of $Z \rightarrow$ scalar $+$ $a$ at LEP implies that the combined mass of the lightest Higgs scalar and $a$ must exceed $\sim$ 92 GeV. Also, the process $e^+e^- \rightarrow Zh$ may be observable at LEP2. For $\lambda_H = 0.1$, the constraint $m_h + m_a \gtrsim 92$ GeV is stronger than $m_h \gtrsim 70$ GeV which is the limit from LEP at $\sqrt{s} \approx 170$ GeV [20]. The contour of $m_h + m_a = 92$ GeV is shown in figure 3 a. In figure 3 b, we show the contour of $m_h = 92$ GeV ($\sim$ the ultimate LEP2 reach [26]). For $\lambda_H = 0.5$, we find that the constraint $m_h \gtrsim 70$ GeV is stronger than $m_h + m_a \gtrsim 92$ GeV and restricts $\tan\beta \lesssim 5$ independent of $\Lambda_{mess}$. The contour $m_h = 92$ GeV is shown in figure 3 d. We note that the allowed parameter space is not significantly constrained. We find that these limits make the constraint of equation 42 redundant. The left-right mixing between the two top squarks was neglected in computing the top squark radiative corrections to the Higgs masses.

The pseudo-Nambu-Goldstone boson $a$ might be produced along with the lightest scalar $h$ at LEP. The (tree-level) cross section in units of $R = 87/s$ nb is

$$\sigma(e^+e^- \rightarrow h\ a) \approx 0.15 \frac{s^2}{(s - m_Z^2)^2}\ \lambda^2\ v\left(1, \frac{m_h^2}{s}, \frac{m_a^2}{s}\right)^3, \tag{43}$$

where $g\lambda/\cos\theta_W$ is the $Z(a\partial h - h\partial a)$ coupling, and
$v(x,y,z) = \sqrt{(x-y-z)^2 - 4yz}$. If $h = c_N N^R + c_u H_u^R + c_d H_d^R$, then

$$\lambda = \sin 2\beta \frac{\cos\beta c_u - \sin\beta c_d}{\sqrt{(\frac{v_N}{250\text{GeV}})^2 + \sin^2 2\beta}}. \tag{44}$$

We have numerically checked the parameter space allowed by $m_h \gtrsim 70$ GeV and $\lambda_H \leq 0.5$ and have found the production cross section for $ha$ to be less than both the current limit set by DELPHI [27] and a (possible) exclusion limit of 30 fb [26] at $\sqrt{s} \approx 192$ GeV. The production cross-section for $hA$ is larger than for $ha$ and $A$ is therefore in principle easier to detect. However, for the parameter space allowed by $m_h \gtrsim 70$ GeV, numerical calculations show that $m_A \gtrsim 125$ GeV, so that this channel is not kinematically accessible.

The charged Higgs mass is

$$m_{H^\pm}^2 = m_W^2 + m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 \tag{45}$$

which is greater than about 200 GeV in this model since $m_{H_d}^2 \gtrsim (200\mathrm{GeV})^2$ for $\Lambda_{mess} \gtrsim 35$ TeV and as $\mu^2 \sim -m_{H_u}^2$.

The neutralinos and charginos may be observable at LEP2 at $\sqrt{s} \approx 192$ GeV if $m_{\chi^+} \lesssim 95$ GeV and $m_{\chi_1^0} + m_{\chi_2^0} \lesssim 180$ GeV. These two constraints are comparable, and thus only one of these is displayed in figures 3 b and 3 d, for $\lambda_H = 0.1$ and $\lambda_H = 0.5$ repectively. Also, contours of $m_{\chi_1^0} + m_{\chi_2^0} = 160$ GeV ($\sim$ the LEP kinematic limit at $\sqrt{s} \approx 170$ GeV) are shown in figures 3 a and 3 c. Contours of 85 GeV ($\sim$ the ultimate LEP2 limit) and 75 GeV ($\sim$ the LEP limit from $\sqrt{s} \approx 170$ GeV) for the right-handed selectron mass further constrain the parameter space.

The results presented in all the figures are for a central value of $m_t$=175 GeV. We have varied the top quark mass by 10 GeV about the central value of $m_t$= 175 GeV and have found that both the fine tuning measures and the LEP2 constraints (the Higgs mass and the neutralino masses) vary by $\approx 30$ %, but the qualitative features are unchanged.

We see from figure 3 that there is parameter space allowed by the present limits in which the tuning is $\approx 30$ %. Even if no new particles are discovered at LEP2, the tuning required for some region is $\approx 20\%$.

It is also interesting to compare the fine tuning measures with those found in the minimal LEGM model (one messenger $(5+\bar{5})$) with an extra singlet $N$ to generate the $\mu$ and $\mu_3^2$ terms.[13] In figure 4 the fine tuning contours for $c(m_Z^2;\lambda_H)$ are presented for $\lambda_H$=0.1. Contours of $m_{\tilde{e}_R} = 75$ GeV and $m_{\chi_1^0} + m_{\chi_2^0} = 160$ GeV are also shown in figure 4 a. For $\lambda_H = 0.1$, the constraint $m_h + m_a \gtrsim 92$ GeV is stronger than the limit $m_h \gtrsim 70$ GeV and is shown in the figure 4 a. In figure 4 b, we show the (approximate) ultimate LEP2 limits, *i.e.*, $m_h = 92$ GeV, $m_{\chi_1^0}+m_{\chi_2^0} = 180$ GeV and $m_{\tilde{e}_R} = 85$ GeV. Of these constraints, the bound on the lightest Higgs mass (either $m_h + m_a \gtrsim 92$

[13] We assume that the model contains some mechanism to generate $-m_N^2 \sim (100\mathrm{GeV})^2 - (200\mathrm{GeV})^2$; for example, the singlet is coupled to an extra $(5+\bar{5})$.

GeV or $m_h \gtrsim 92$ GeV) provides a strong lower limit on the messenger scale. We see that in the parameter space allowed by present limits the fine tuning is $\lesssim 2\%$ and if LEP2 does not discover new particles, the fine tuning will be $\lesssim 1\%$. The coupling $\lambda_H$ is constrained to be not significantly larger than 0.1 if the constraint $m_h + m_a \gtrsim 92$ GeV (or $m_h \gtrsim 92$ GeV) is imposed and if the fine tuning is required to be no worse than 1%.

# 6 Models Derived from a GUT

In this section, we discuss how the toy model of section 4 could be derived from a GUT model.

In the toy model of section 4, the singlets $N$ and $S$ do not separately couple to complete $SU(5)$ representations (see equation 15). If the extra fields introduced to solve the fine tuning problem were originally part of $(5+\bar{5})$ multiplets, then the missing triplets (missing doublets) necessarily couple to the singlet $S(N)$. The triplets must be heavy in order to suppress their contribution to the soft SUSY breaking mass parameters. If we assume the only other mass scale is $M_{GUT}$, they must acquire a mass at $M_{GUT}$. This is just the usual problem of splitting a $(5+\bar{5})$ [14]. For example, if the superpotential in the messenger sector contains four $(5+\bar{5})$'s,

$$W = \lambda_1 S \bar{5}_{l1} 5_{l1} + \lambda_2 S \bar{5}_{l2} 5_{l2} + \lambda_3 S \bar{5}_{l3} 5_{l3} + \lambda_4 S \bar{5}_q 5_q, \tag{46}$$

then the $SU(3)$ triplets in the $(\bar{5}_l + 5_l)$'s and the $SU(2)$ doublet in $(\bar{5}_q + 5_q)$ must be heavy at $M_{GUT}$ so that in the low energy theory there are three doublets and one triplet coupling to $S$. This problem can be solved using the method of Barbieri, Dvali and Strumia [15] that solves the usual Higgs doublet-triplet splitting problem. The mechanism in this model is attractive since it is possible to make either the doublets or triplets of a quintet heavy at the GUT scale. We next describe their model.

The gauge group is $SU(5) \times SU(5)'$, with the particle content $\Sigma(24,1)$, $\Sigma'(1,24)$, $\Phi(5,\bar{5})$ and $\bar{\Phi}(\bar{5},5)$ and the superpotential can be written as

$$\begin{aligned} W &= \bar{\Phi}^{\beta}_{\alpha'}(M_\Phi \delta^{\alpha'}_{\beta'}\delta^{\alpha}_{\beta} + \lambda \Sigma^{\alpha}_{\beta}\delta^{\alpha'}_{\beta'} + \lambda' \Sigma'^{\alpha'}_{\beta'}\delta^{\alpha}_{\beta})\Phi^{\beta'}_{\alpha} + \\ &\quad + \frac{1}{2} M_\Sigma \mathrm{Tr}(\Sigma^2) + \frac{1}{2} M_{\Sigma'} \mathrm{Tr}(\Sigma'^2) + \end{aligned}$$

$$\frac{1}{3}\lambda_\Sigma \mathrm{Tr}\Sigma^3 + \frac{1}{3}\lambda_{\Sigma'}\mathrm{Tr}\Sigma'^3. \tag{47}$$

A supersymmetric minimum of the scalar potential satisfies the $F$ - flatness conditions

$$\begin{aligned}
0 &= F_{\bar\Phi} = (M_\Phi \delta^{\alpha'}_{\beta'}\delta^{\alpha}_{\beta} + \lambda\Sigma^{\alpha}_{\beta}\delta^{\alpha'}_{\beta'} + \lambda'\Sigma'^{\alpha'}_{\beta'}\delta^{\alpha}_{\beta})\Phi^{\beta'}_{\alpha},\\
0 &= F_\Sigma = \frac{1}{2}M_\Sigma\Sigma^{\beta}_{\alpha} + \frac{1}{2}\left(\lambda\bar\Phi^{\beta}_{\alpha'}\Phi^{\alpha'}_{\alpha} - \lambda\frac{1}{5}\delta^{\beta}_{\alpha}\mathrm{Tr}(\bar\Phi\Phi)\right) + \lambda_\Sigma(\Sigma^2 - \frac{1}{5}\mathrm{Tr}\Sigma^2),\\
0 &= F_{\Sigma'} = \frac{1}{2}M_{\Sigma'}\Sigma'^{\beta'}_{\alpha'} + \frac{1}{2}\left(\lambda'\bar\Phi^{\alpha}_{\alpha'}\Phi^{\beta'}_{\alpha} - \lambda'\frac{1}{5}\delta^{\beta'}_{\alpha'}\mathrm{Tr}(\bar\Phi\Phi)\right) + \lambda_{\Sigma'}(\Sigma'^2 - \frac{1}{5}\mathrm{Tr}\Sigma'^2).
\end{aligned} \tag{48}$$

With the ansatz [14]

$$\Sigma = v_\Sigma\,\mathrm{diag}(2,2,2,-3,-3)\,, \Sigma' = v_{\Sigma'}\,\mathrm{diag}(2,2,2,-3,-3), \tag{49}$$

the $F_{\bar\Phi} = 0$ condition is

$$\mathrm{diag}[M_3, M_3, M_3, M_2, M_2]\cdot\mathrm{diag}[v_3, v_3, v_3, v_2, v_2] = 0, \tag{50}$$

where $M_3 = M_\Phi + 2\lambda v_\Sigma + 2\lambda' v_{\Sigma'}$ and $M_2 = M_\Phi - 3\lambda v_\Sigma - 3\lambda' v_{\Sigma'}$ and the second matrix is the vev of $\Phi$. To satisfy this condition, there is a discrete choice for the pattern of vev of $\Phi$ : i) $v_3 \neq 0$ and $M_3 = 0$ or ii) $v_2 \neq 0$ and $M_2 = 0$. Substituting either i) or ii) in the $F_\Sigma$ and $F_{\Sigma'}$ conditions then determines $v_3$ (or $v_2$). With two sets of fields, $\Phi_1, \bar\Phi_1$ with $v_3 \neq 0$ and $\Phi_2, \bar\Phi_2$ with $v_2 \neq 0$ , we have the following pattern of symmetry breaking

$$\begin{aligned}
SU(5)\times SU(5)' &\stackrel{v_\Sigma, v_{\Sigma'}}{\longrightarrow} (SU(3)\times SU(2)\times U(1))\times(SU(3)\times SU(2)\times U(1))'\\
&\stackrel{v_3, v_2}{\longrightarrow} SM\,\text{(the diagonal subgroup)}.
\end{aligned} \tag{51}$$

If the scales of the two stages of symmetry breaking are about equal, *i.e.* $v_\Sigma$, $v_{\Sigma'}$, $\sim v_3\,, v_2\, \sim M_{GUT}$, then the SM gauge couplings unify at the scale $M_{GUT}$. [15]

The particular structure of the vevs of $\Phi_1$ and $\Phi_2$ can be used to split representations as follows.

[14]The two possible solutions to the $F$-flatness conditions are $\Sigma = v_\Sigma\,\mathrm{diag}(2,2,2,-3,-3)$ and $\Sigma = v_\Sigma\,\mathrm{diag}(1,1,1,1,-4)$.

[15]See [15] and [24] for models which give this structure of vevs for the $\Phi$ fields without using the adjoints.

Consider the Higgs doublet-triplet splitting problem. With the particle content $5_h(5,1)$, $\bar{5}_h(\bar{5},1)$ and $X(1,5)$, $\bar{X}(1,\bar{5})$ and the superpotential

$$W = 5_{h\alpha}\bar{X}^{\alpha'}\bar{\Phi}^{\alpha}_{1\alpha'} + \bar{5}^{\alpha}_{h}X_{\alpha'}\Phi_{1}{}^{\alpha'}_{\alpha}, \tag{52}$$

the $SU(3)$ triplets in $5_h$, $\bar{5}_h$ and $X$, $\bar{X}$ acquire a mass of order $M_{GUT}$ whereas the doublets in $5_h$, $\bar{5}_h$ and $X$, $\bar{X}$ are massless. We want only one pair of doublets in the low energy theory (in addition to the usual matter fields). The doublets in $X$, $\bar{X}$ can be made heavy by a bare mass term $M_{GUT}X\bar{X}$. Then the doublets in $5_h, \bar{5}_h$ are the standard Higgs doublets. But if all terms consistent with symmetries are allowed in the superpotential, then allowing $M_{GUT}\Phi_1\bar{\Phi}_1$, $M_{GUT}X\bar{X}$, $5_h\bar{X}\Phi_1$ and $\bar{5}_hX\bar{\Phi}_1$ implies that a bare mass term for $5_h\bar{5}_h$ is allowed. Of course, we can by hand put in a $\mu$ term $\mu 5_h\bar{5}_h$ of the order of the weak scale as in section 4. However, it is theoretically more desirable to relate all electroweak mass scales to the original SUSY breaking scale. So, we would like to relate the $\mu$ term to the SUSY breaking scale. We showed in section 5 that the NMSSM is phenomenologically viable and "un-fine tuned" in these models.

The vev structure of $\Phi_2$, $\bar{\Phi}_2$ can be used to make the doublets in a $5+\bar{5}$ heavy. Again, we get two pairs of light triplets and one of these pairs can be given a mass at the GUT scale.

We can use this mechanism of making either doublets or triplets in a $(5+\bar{5})$ heavy to show how the model of section 4 is derivable from a GUT. The model with three messenger doublets and one triplet is obtained from a GUT with the following superpotential

$$\begin{aligned} W &= S5\bar{5} + S5_l\bar{5}_l + SX_l\bar{X}_l + \\ &\quad 5_l\bar{X}_l\bar{\Phi}_1 + \bar{5}_lX_l\Phi_1 + \\ &\quad 5_q\bar{X}_q\bar{\Phi}_2 + \bar{5}_qX_q\Phi_2 + \\ &\quad M_{GUT}X_h\bar{X}_h + 5_h\bar{X}_h\bar{\Phi}_1 + \bar{5}_hX_h\Phi_1 + \mu 5_h\bar{5}_h \\ &\quad + N^3 + N5_q\bar{5}_q + NX_q\bar{X}_q. \end{aligned} \tag{53}$$

Here, some of the "extra" triplets and doublets resulting from splitting $(5+\bar{5})$'s are massless at the GUT scale. For example, the "extra" light doublets are used as the additional messenger leptons. After inserting the vevs and integrating out the heavy states, this corresponds to the superpotential in equation 15 with the transcription:

$$\begin{aligned}
5, \bar{5} &\rightarrow q_1, \bar{q}_1 + l_1, \bar{l}_1 \\
5_l, \bar{5}_l &\rightarrow l_2, \bar{l}_2 \\
X_l, \bar{X}_l &\rightarrow l_3, \bar{l}_3 \\
5_q, \bar{5}_q &\rightarrow q_2, \bar{q}_2 \\
X_q, \bar{X}_q &\rightarrow q_3, \bar{q}_3.
\end{aligned} \tag{54}$$

We conclude this section with a remark about light singlets in SUSY-GUT's with low energy gauge mediated SUSY breaking.[16] In a SUSY GUT with a singlet $N$ coupled to the Higgs multiplets, there is a potential problem of destabilising the $m_{weak}/M_{GUT}$ hierarchy, if the singlet is light and if the Higgs triplets have a SUSY invariant mass of $O(M_{GUT})$ [28]. In the LEGM models, a B-type mass for the Higgs triplets and doublets is generated at one loop with gauginos and Higgsinos in the loop, and with SUSY breaking coming from the gaugino mass. Since SUSY breaking (the gaugino mass and the soft scalar masses) becomes soft above the messenger scale, $\Lambda_{mess} \sim 100$ TeV, the B-type mass term generated for the Higgs triplets is suppressed, *i.e.*, it is $O((\alpha/4\pi)M_2\Lambda^2_{mess}/M_{GUT})$. Similarly the soft mass squared for the Higgs triplets are $O(m^2_{weak}\Lambda^2_{mess}/M^2_{GUT})$. Since the triplets couple to the singlet $N$, the soft scalar mass and $B$-term generates at one loop a linear term for the scalar and $F$-component of $N$ respectively. These tadpoles are harmless since the SUSY breaking masses for the triplets are so small. This is to be contrasted with supergravity theories, where the $B$-term$\sim O(m_{weak}M_{GUT})$ and the soft mass $\sim O(m_{weak})$ for the triplet Higgs generate a mass for the Higgs doublet that is at least $\sim O(\sqrt{m_{weak}M_{GUT}}/(4\pi))$.

# 7 One complete Model

The model is based on the gauge group $G_{loc} = SU(5) \times SU(5)'$ and the global symmetry group $G_{glo} = Z_3 \times Z'_3 \times Z_4$. The global symmetry acts universally on the three generations of the SM. The particle content and their $G_{loc} \times G_{glo}$ quantum numbers are given in table 2. The most general

[16]The authors thank H. Murayama for bringing this to their attention.

renormalizable superpotential that is consistent with these symmetries is

$$W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7, \tag{55}$$

where,

$$\begin{aligned}
W_1 &= \frac{1}{2}M_\Sigma \mathrm{Tr}\Sigma^2 + \frac{1}{3}\lambda_\Sigma \mathrm{Tr}\Sigma^3 + \frac{1}{2}M_{\Sigma'}\mathrm{Tr}\Sigma'^2 + \frac{1}{3}\lambda_{\Sigma'}\mathrm{Tr}\Sigma'^3 \\
&\quad +\Phi_2(M_{\Phi_2} + \lambda_{\Phi_2}\Sigma + \lambda'_{\Phi_2}\Sigma')\bar{\Phi}_2 \\
&\quad +\Phi_1(M_{\Phi_1} + \lambda_{\Phi_1}\Sigma + \lambda'_{\Phi_1}\Sigma')\bar{\Phi}_1, && (56)\\
W_2 &= M_1 \bar{X}_l X, && (57)\\
W_3 &= \lambda_1 \bar{5}_h \Phi_1 X_h + \bar{\lambda}_1 5_h \bar{\Phi}_1 \bar{X}_h + \lambda_2 \bar{5}_l \Phi_1 X_l + \bar{\lambda}_2 5_l \bar{\Phi}_1 \bar{X}_l, && (58)\\
W_4 &= \lambda_3 \bar{5}_q \Phi_2 X_q + \bar{\lambda}_3 5_q \bar{\Phi}_2 \bar{X}_q, && (59)\\
W_5 &= \lambda_6 S 5_l \bar{5}_l + \lambda_7 S 5_q \bar{5}_q + \lambda_8 S \bar{X}_h X_l + \lambda_9 S \bar{X} X_h + \frac{1}{3}\lambda_S S^3, && (60)\\
W_6 &= -\lambda_H 5_h \bar{5}_h N + \frac{1}{3}\lambda_N N^3 + \bar{\lambda}_q N X \bar{X} \\
&\quad +\lambda_{10} N' \bar{X} X_q + \lambda_{11} N' \bar{X}_q X + \frac{1}{3}\lambda_{N'} N'^3, && (61)\\
W_7 &= \lambda^D_{ij} \bar{5}_i 10_j \bar{5}_h + \lambda^U_{ij} 10_i 10_j 5_h. && (62)
\end{aligned}$$

The origin of each of the $W_i$'s appearing in the superpotential is easy to understand. In computing the $F$=0 equations at the GUT scale, the only non-trivial contributions come from fields appearing in $W_1$, since all other $W_i$s are bilinear in fields that do not acquire vevs at the GUT scale. The function of $W_1$ is to generate the vevs $\Sigma, \Sigma' \sim$ diag $[2,2,2,-3,-3]$, $\bar{\Phi}_2^T = \Phi_2 \sim$ diag $[0,0,0,1,1]$ and $\bar{\Phi}_1^T = \Phi_1 \sim$ diag $[1,1,1,0,0]$. These vevs are necessary to break $G_{loc} \rightarrow SU(3)_c \times SU(2) \times U(1)_Y$ (this was explained in section 6). The role of $W_3$ and $W_4$ is to generate the necessary splitting within the many $(5+\bar{5})$'s of $G_{loc}$ that is necessary to solve the usual doublet-triplet splitting problem, as well as to solve the fine tuning problem that is discussed in sections 3,4 and 5. The messenger sector is given by $W_5$. It will shortly be demonstrated that at low energies this sector contains three vector-like doublets and one vector-like triplet. The couplings in $W_6$ and $W_7$ at low energies contain the electroweak symmetry breaking sector of the NMSSM, the Yukawa couplings of the SM fields, and the two light vector-like triplets necessary to maintain the few percent prediction for $\sin^2\theta_W$ as well as to generate a vev for $N$.

We now show that the low energy theory of this model is the model that is discussed in section 5.

Inserting the vevs for $\Phi_1$ and $\bar{\Phi}_1$ into $W_3$, the following mass matrix for the colored triplet chiral multiplets is obtained:

$$(\bar{5}_h, \bar{X}_h, \bar{5}_l, \bar{X}_l) \begin{pmatrix} 0 & \lambda_1 v_{\Phi_1} & 0 & 0 & 0 \\ \bar{\lambda}_1 v_{\Phi_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 v_{\Phi_1} & 0 \\ 0 & 0 & \bar{\lambda}_2 v_{\Phi_1} & 0 & M_1 \end{pmatrix} \begin{pmatrix} 5_h \\ X_h \\ 5_l \\ X_l \\ X \end{pmatrix} \tag{63}$$

and all other masses are zero. There are a total of four vector-like colored triplet fields that are massive at $M_{GUT}$. These are the triplet components of $(5_h, \bar{X}_h)$, $(\bar{5}_h, X_h)$, $(\bar{5}_l, X_l)$ and $(\bar{X}_l, T_H)$, where $T_H$ is that linear combination of triplets in $5_l$ and $X$ that marries the triplet component of $\bar{X}_l$. The orthogonal combination to $T_H$, $T_L$, is massless at this scale. The massless triplets at $M_{GUT}$ are $(5_q, \bar{5}_q)$, $(X_q, \bar{X}_q)$ and $(\bar{X}, T_L)$, for a total of three vector-like triplets. By inspection, the only light triplets that couple to $S$ at a renormalizable level are $5_q$ and $\bar{5}_q$, which was desirable in order to solve the fine tuning problem. Further, since $X$ contains a component of $T_L$, the couplings of the other light triplets to the singlets $N$ and $N'$ are

$$W_{eff} = \lambda_{10} N' \bar{X} X_q + \bar{\lambda}_{11} N' \bar{X}_q T_L + \lambda_q N T_L \bar{X}, \tag{64}$$

where $\lambda_q = \bar{\lambda}_q \cos\alpha'$, $\bar{\lambda}_{11} = \lambda_{11} \cos\alpha'$ and $\alpha'$ is the mixing angle between the triplets in $5_l$ and $X$, *i.e.*, $T_L = \cos\alpha' X - \sin\alpha' 5_l$. The $\lambda_q N T_L \bar{X}$ coupling is also desirable to generate acceptable $\mu$ and $\mu_3^2$ terms (see section 5).

In section 4,5 it was also demonstrated that with a total of three messenger doublets the fine tuning required in electroweak symmetry breaking could be alleviated. By inserting the vev for $\Phi_2$ into $W_4$, the doublet mass matrix is given as

$$(\bar{X}_l, \bar{5}_q, \bar{X}_q) \begin{pmatrix} M_1 & 0 & 0 \\ 0 & 0 & \lambda_3 v_{\Phi_2} \\ 0 & \bar{\lambda}_3 v_{\Phi_2} & 0 \end{pmatrix} \begin{pmatrix} X \\ 5_q \\ X_q \end{pmatrix} \tag{65}$$

and all other masses are zero. At $M_{GUT}$ the heavy doublets are $(\bar{X}_l, X)$, $(5_q, \bar{X}_q)$ and $(\bar{5}_q, X_q)$, leaving the four vector-like doublets in $(5_h, \bar{5}_h)$, $(5_l, \bar{5}_l)$,

$(\bar{X}, X_l)$ and $(X_h, \bar{X}_h)$ massless at this scale. Of these four pairs, $(5_h, \bar{5}_h)$ are the usual Higgs doublets and the other three pairs couple to $S$.

The (renormalizable) superpotential at scales below $M_{GUT}$ is then

$$
\begin{aligned}
W \quad = \quad & \lambda_q N \bar{q}_2 q_2 + \frac{1}{3}\lambda_N N^3 + \lambda_{10} N' q_3 \bar{q}_2 \\
& + \lambda_{11} N' q_2 \bar{q}_3 - \lambda_H N H_u H_d + \frac{1}{3}\lambda_{N'} N'^3 \\
& + \lambda_6 S \bar{l}_1 l_1 + \lambda_7 S \bar{q}_1 q_1 + \lambda_8 S \bar{l}_2 l_2 \\
& + \lambda_9 S \bar{l}_3 l_3 + \frac{1}{3}\lambda_S S^3 + W_7,
\end{aligned}
\tag{66}
$$

where the fields have been relabeled to make, in an obvious notation, their $SU(3) \times SU(2) \times U(1)$ quantum numbers apparent.

We conclude this section with comments about both the choice of $Z_4$ as a discrete symmetry and about non-renormalizable operators in our model.

The usual $R$-parity violating operators $10_{SM}\bar{5}_{SM}\bar{5}_{SM}$ are not allowed by the discrete symmetries, even at the non-renormalizable level. In fact, $R$-parity is a good symmetry of the effective theory below $M_{GUT}$. By inspection, the fields that acquire vevs at $M_{GUT}$ are either invariant under $Z_4$ or have a $Z_4$ charge of 2 (for example, $\Phi_1$), so that a $Z_2$ symmetry is left unbroken. In fact, the vevs of the other fields $S$, $N$, $N'$ and the Higgs doublets do not break this $Z_2$ either. By inspecting the $Z_4$ charges of the SM fields, we see that the unbroken $Z_2$ is none other than the usual $R$-parity. So at $M_{GUT}$, the discrete symmetry $Z_4$ is broken to $R_p$. We also note that the $Z_4$ symmetry is suffucient to maintain, to all orders in $1/M_{Pl}$ operators, the vev structure of $\Phi_1$ and $\Phi_2$, *i.e.*, to forbid unwanted couplings between $\Phi_1$ and $\Phi_2$ that might destabilize the vev structure[24]. This pattern of vevs was essential to solve the doublet-triplet splitting problem. It is interesting that both $R$-parity and requiring a viable solution to the doublet-triplet splitting problem can be accommodated by the same $Z_4$ symmetry.

The non-SM matter fields (*i.e.*, the messenger 5's and $X$'s and the light triplets ) have the opposite charge to the SM matter fields under the unbroken $Z_2$. Thus, there is no mass mixing between the SM and the non-SM matter fields.

Dangerous proton decay operators are forbidden in this model by the discrete symmetries. Some higher dimension operators that lead to proton decay are allowed, but are suffuciently suppressed. We discuss these below.

Renormalizable operators such as $10_{SM}10_{SM}5_q$ and $10_{SM}\bar{5}_{SM}\bar{5}_q$ are forbidden by the $Z_3$ symmetries. This is necessary to avoid a large proton decay rate. A dimension-6 proton decay operator is obtained by integrating out the colored triplet scalar components of $5_q$ or $\bar{5}_q$. Since the colored scalars in $5_q$ and $\bar{5}_q$ have a mass $\sim O(50$ TeV), the presence of these operators would have led to an unacceptably large proton decay rate.

The operators $10_{SM}10_{SM}10_{SM}\bar{5}_{SM}/M_{Pl}$ and $10_{SM}10_{SM}10_{SM}\bar{5}_{SM}$ $(\Phi\bar{\Phi}/M_{Pl}^2)^n/M_{Pl}$, which give dimension-5 proton decay operators, are also forbidden by the two $Z_3$ symmetries. The allowed non-renormalizable operators that generate dimension-5 proton decay operators are suffuciently suppressed. The operator $10_{SM}10_{SM}10_{SM}\bar{5}_{SM}N'/(M_{Pl})^2$, for example, is allowed by the discrete symmetries, but the proton decay rate is safe since $v_{N'} \sim 1$ TeV.

The operators $10_i\bar{5}_j\bar{\Phi}_1(\bar{X}$ or $\bar{X}_q)/M_{Pl}$ could, in principle, also lead to a large proton decay rate. Setting $\bar{\Phi}_1$ to its vev, the superpotential couplings, for example, $\lambda_{ij}(U_i^c D_j^c \bar{X}(\bar{3}) + Q_i L_j \bar{X}(\bar{3}))$ are generated with $\lambda_{ij}$ suppressed only by $v_{\Phi_1}/M_{Pl}$. In this model the colored triplet (scalar) components of $\bar{X}$ and $\bar{X}_q$ have a mass $m_{\tilde{q}} \sim 500$ GeV, giving a potentially large proton decay rate. But, in this model these operators are forbidden by the discrete symmetries. The operator $10_i\bar{5}_j\bar{\Phi}_1\bar{X}S/M_{Pl}^2$ is allowed giving a four SM fermion proton decay operator with coefficient $\sim (v_{\Phi_1}\ v_S/M_{Pl}^2)^2/m_{\tilde{q}}^2 \sim 10^{-34}\mathrm{GeV}^{-2}$. This is smaller than the coefficient generated by exchange of the heavy gauge bosons of mass $M_{GUT}$, which is $\sim g_{GUT}^2/M_{GUT}^2 \sim 1/2\ 10^{-32}\mathrm{GeV}^{-2}$ and so this operator leads to proton decay at a tolerable rate.

With our set of discrete symmetries, some of the messenger states and the light color triplets are stable at the renormalizable level. Non-renormalizable operators lead to decay lifetime for some of these particles of more than about 100 seconds. This is a problem from the viewpoint of cosmology, since these particles decay after Big-Bang Nucleosynthesis (BBN). With a non-universal choice of discrete symmetries, it might be possible to make these particles decay before BBN through either small renormalizable couplings to the third generation (so that the constraints from proton decay and FCNC are avoided) or non-renormalizable operators. This is, however, beyond the scope of this paper.

# 8 Conclusions

We have quantified the fine tuning required in models of low energy gauge-mediated SUSY breaking to obtain the correct $Z$ mass. We showed that the minimal model requires a fine tuning of order $\sim$ 7% if LEP2 does not discover a right-handed slepton. We discussed how models with more messenger doublets than triplets can improve the fine tuning. In particular, a model with a messenger field particle content of three $(l+\bar{l})$'s and only one $(q+\bar{q})$ was tuned to $\sim$ 40%. We found that it was necessary to introduce an extra singlet to give mass to some color triplets (close to the weak scale) which are required to maintain gauge coupling unification. We also discussed how the vev and $F$-component of this singlet could be used to generate the $\mu$ and $B\mu$ terms. We found that for some region of the parameter space this model requires $\sim$ 25% tuning and have shown that limits from LEP do not constrain the parameter space. This is in contrast to an NMSSM with one $(5+\bar{5})$ messenger fields, for which we found that a fine tuning of $\sim$ 1% is required and that limits from LEP do significantly constrain the parameter space.

We further discussed how the model with split messenger field representations could be the low energy theory of a $SU(5) \times SU(5)$ GUT. A mechanism similar to the one used to solve the usual Higgs doublet-triplet splitting problem was used to split the messenger field representations. All operators consistent with gauge and discrete symmetries were allowed. In this model $R$-parity is the unbroken subgroup of one of the discrete symmetry groups. Non-renormalizable operators involving non-SM fields lead to proton decay, but at a safe level.

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# 10 Appendix

In this section the Barbieri-Giudice parameters for both the MSSM and NMSSM in a gauge mediated SUSY breaking scenario are presented.

In an MSSM with gauge mediated SUSY breaking, the fundamental parameters of the theory (in the visible sector) are: $\Lambda_{mess}$; $\lambda_t$; $\mu$; and $\mu_3^2$. Once electroweak symmetry breaking occurs, the extremization conditions determine both $m_Z^2$ and $\tan\beta$ as a function of these parameters. To measure the sensitivity of $m_Z^2$ to one of the fundamental parameters $\lambda_i$, we compute the variation in $m_Z^2$ induced by a small change in one of the $\lambda_i$. The quantity

$$\frac{\delta m_Z^2}{m_Z^2} \equiv c(m_Z^2;\lambda_i)\frac{\delta\lambda_i}{\lambda_i}, \tag{67}$$

where

$$c(m_Z^2;\lambda_i) = \frac{\lambda_i}{m_Z^2}\frac{\partial m_Z^2}{\partial\lambda_i}, \tag{68}$$

measures this sensitivity [3]. In the case of gauge mediated SUSY breaking models, there are four functions $c(m_Z^2;\lambda_i)$ to be computed. They are:

$$c(m_Z^2;\mu^2) = \frac{2\mu^2}{m_Z^2}\left(1+\frac{\tan^2\beta+1}{(\tan^2\beta-1)^2}\frac{4\tan^2\beta(\tilde{\mu}_1^2-\tilde{\mu}_2^2)}{(\tilde{\mu}_1^2-\tilde{\mu}_2^2)(\tan^2\beta+1)-m_Z^2(\tan^2\beta-1))}\right), \tag{69}$$

$$\begin{aligned} c(m_Z^2;\mu_3^2) &= 4\tan^2\beta\frac{\tan^2\beta+1}{(\tan^2\beta-1)^3}\frac{\tilde{\mu}_1^2-\tilde{\mu}_2^2}{m_Z^2} \\ &\approx \frac{4}{\tan^2\beta}\frac{\tilde{\mu}_1^2-\tilde{\mu}_2^2}{m_Z^2}, \quad \text{for large } \tan\beta, \end{aligned} \tag{70}$$

$$\begin{aligned} c(m_Z^2;\lambda_t) &= 2\frac{\lambda_t^2}{m_Z^2}\frac{\partial m_Z^2}{\partial m_{H_u}^2}\frac{\partial m_{H_u}^2}{\partial\lambda_t^2} \\ &= \frac{4}{m_Z^2}\lambda_t^2\frac{\tan^2\beta}{\tan^2\beta-1}\frac{\partial m_{H_u}^2}{\partial\lambda_t^2}\left(1+2\frac{\tilde{\mu}_1^2-\tilde{\mu}_2^2}{\tilde{\mu}_1^2+\tilde{\mu}_2^2}\frac{\tan^2\beta+1}{(\tan^2\beta-1)^2}\right) \\ &\approx \frac{4}{m_Z^2}\frac{\partial m_{H_u}^2}{\partial\lambda_t^2}, \quad \text{for large } \tan\beta. \end{aligned} \tag{71}$$

This measures the sensitivity of $m_Z^2$ to the electroweak scale value of $\lambda_t$, $\lambda_t(M_{weak})$. The Yukawa coupling $\lambda_t(M_{weak})$ is not, however, a fundamental parameter of the theory. The fundamental parameter is the value of the coupling at the cutoff $\Lambda^0 = M_{GUT}$ or $M_{pl}$ of the theory. We really should be computing the sensitivity of $m_Z^2$ to this value of $\lambda_t$. The measure of sensitivity is then correctly given by

$$c(m_Z^2; \lambda_t(\Lambda^0)) = \frac{\lambda_t(\Lambda^0)}{\lambda_t(M_{weak})} c(m_Z^2; \lambda_t(M_{weak})) \frac{\partial \lambda_t(M_{weak})}{\partial \lambda_t(\Lambda^0)}. \tag{72}$$

$$\tag{73}$$

We remark that for the model discussed in the text with three $l+\bar{l}$ and one $q+\bar{q}$ messenger fields, the numerical value of $(\lambda_t(\Lambda^0)/\lambda_t(M_{weak}))\partial\lambda_t(M_{weak})/\partial\lambda_t(\Lambda^0)$ is typically $\sim 0.1$ because $\lambda_t(M_{weak})$ is attracted to its infra-red fixed point. This results in a smaller value for $c(m_Z^2; \lambda_t)$ than is obtained in the absence of these considerations.

With the assumption that $m_{H_u}^2$ and $m_{H_d}^2$ scale with $\Lambda_{mess}^2$, we get

$$\begin{aligned} c(m_Z^2; \Lambda_{mess}^2) &= c(m_Z^2; m_{H_u}^2) + c(m_Z^2; m_{H_d}^2) \\ &= 1 + 2\frac{\mu^2}{m_Z^2} - \frac{\tan^2\beta + 1}{(\tan^2\beta - 1)^2} \times \\ &\quad \frac{4\tan^2\beta(m_{H_u}^2 + m_{H_d}^2)(\tilde{\mu}_1^2 - \tilde{\mu}_2^2)/m_Z^2}{(\tilde{\mu}_1^2 - \tilde{\mu}_2^2)(\tan^2\beta + 1) - m_Z^2(\tan^2\beta - 1)}. \end{aligned} \tag{74}$$

The Barbieri-Giudice functions for $m_t$ are similarly computed. They are

$$c(m_t; \mu_3^2) = \frac{1}{2} c(m_Z^2; \mu_3^2) + \frac{1}{1 - \tan^2\beta}, \tag{75}$$

$$c(m_t; \mu^2) = \frac{1}{2} c(m_Z^2; \mu^2) + 2\frac{\mu^2}{\tilde{\mu}_1^2 + \tilde{\mu}_2^2} \frac{1}{\tan^2\beta - 1}, \tag{76}$$

$$c(m_t; \lambda_t) = 1 + \frac{1}{2} c(m_Z^2; \lambda_t) + \frac{\lambda_t}{\tan^2\beta - 1} \frac{1}{\tilde{\mu}_1^2 + \tilde{\mu}_2^2} \frac{\partial m_{H_u}^2}{\partial \lambda_t}, \tag{77}$$

$$c(m_t; \Lambda_{mess}^2) = \frac{1}{2} c(m_Z^2; \Lambda_{mess}^2) - \frac{(\tilde{\mu}_1^2 + \tilde{\mu}_2^2 - 2\mu^2)}{(1 - \tan^2\beta)(\tilde{\mu}_2^2 + \tilde{\mu}_1^2)}. \tag{78}$$

Since $m_Z$ and $m_t$ are measured, two of the four fundamental parameters may be eliminated. This leaves two free parameters, which for conveinence are chosen to be $\Lambda_{mess}$ and $\tan\beta$.

In a NMSSM with gauge mediated SUSY breaking, the scalar potential for $N$, $H_u$ and $H_d$ at the weak scale is specified by the following six parameters: $\lambda_i = m_N^2, m_{H_u}^2, m_{H_d}^2$, the $NH_uH_d$ coupling $\lambda_H$, the scalar $NH_uH_d$ coupling $A_H$, and the $N^3$ coupling, $\lambda_N$. In minimal gauge mediated SUSY breaking, the trilinear soft SUSY breaking term $NH_uH_d$ is zero at tree level and is generated at one loop by wino and bino exchange. In this case, $A_H(\lambda_i) = \lambda_H \tilde{A}(\lambda_i)$. Since the trilinear scalar term $N^3$ is generated at two loops, it is small and is neglected. The extremization conditions which determine $m_Z = g_Z^2 v^2/4$ ($v = \sqrt{v_u^2 + v_d^2}$), $\tan\beta = v_u/v_d$ and $v_N$ as a function of these parameters are given in section 5. Equation 22 can be written, using $\mu = \lambda_H v_N/\sqrt{2}$ as

$$m_N^2 + 2\frac{\lambda_N^2}{\lambda_H^2}\mu^2 - \lambda_H\lambda_N\frac{1}{2}v^2\sin 2\beta + \frac{1}{2}\lambda_H^2 v^2 - \frac{1}{4\mu}A_H v^2 \lambda_H \sin 2\beta = 0. \quad (79)$$

Equation 23 is

$$\frac{1}{8}g_Z^2 v^2 + \mu^2 - m_{H_u}^2\frac{\tan^2\beta}{1-\tan^2\beta} + m_{H_d}^2\frac{1}{1-\tan^2\beta} = 0. \quad (80)$$

Substituting $v_N^2$ from equation 22 in equation 26 and then using this expression for $\mu_3^2$ in equation 24 gives

$$(m_{H_u}^2 + m_{H_d}^2 + 2\mu^2)\sin 2\beta + \frac{\lambda_H}{\lambda_N}(m_N^2 + \frac{1}{2}\lambda_H^2 v^2) + A_H(-\frac{2\mu}{\lambda_H} - \frac{1}{4}\frac{v^2\lambda_H^2\sin 2\beta}{\mu\lambda_N}) = 0. \quad (81)$$

The quantity $c = (\lambda_i/m_Z^2)(\partial m_Z^2/\partial\lambda_i)$ measures the sensitivity of $m_Z$ to these parameters. This can be computed by differentiating equations 79, 80 and 81 with respect to these parameters to obtain, after some algebra, the following set of linear equations:

$$(A + A_{A_H})X^i = B^i + B^i_{A_H}, \quad (82)$$

where

$$A = \begin{pmatrix} \frac{1}{2} & 1 & \frac{\mu_1^2-\mu_2^2}{v^2}\frac{2\tan\beta}{(1-\tan^2\beta)^2} \\ \frac{\lambda_H^3(\lambda_H-\lambda_N\sin 2\beta)}{g_Z^2\lambda_N^2} & 1 & -\frac{1}{2}\frac{\lambda_H^3}{\lambda_N}\frac{1-\tan^2\beta}{(1+\tan^2\beta)^2} \\ \frac{v^2}{g_Z^2(\mu_1^2+\mu_2^2)}\frac{\lambda_H^3}{\lambda_N} & \frac{\sin 2\beta v^2}{\mu_1^2+\mu_2^2} & \frac{1-\tan^2\beta}{(1+\tan^2\beta)^2} \end{pmatrix}, \quad (83)$$

$$A_{A_H} = \frac{A_H}{\mu}\times \quad (84)$$

$$
\begin{pmatrix} 0 & 0 & 0 \\ -\frac{\lambda_H^3 \sin 2\beta}{2g_Z^2 \lambda_N^2} & \frac{\lambda_H^3 \sin 2\beta}{16\lambda_N^2} \frac{v^2}{\mu^2} & \frac{\tan^2\beta - 1}{(1+\tan^2\beta)^2} \frac{\lambda_H^3}{4\lambda_N^2} \\ -\frac{\lambda_H^2}{2g_Z^2 \lambda_N} \frac{v^2 \sin 2\beta}{\mu_1^2+\mu_2^2} & \frac{v^2}{\mu_1^2+\mu_2^2}\left(\frac{\lambda_H^2 \sin 2\beta}{16\lambda_N} \frac{v^2}{\mu^2} - \frac{1}{2\lambda_H}\right) & \frac{\tan^2\beta - 1}{(1+\tan^2\beta)^2} \frac{\lambda_H^2}{4\lambda_N} \frac{v^2}{\mu_1^2+\mu_2^2} \end{pmatrix},
$$

$$
X^{\lambda_H, \lambda_N} = \begin{pmatrix} \frac{1}{v^2} \frac{\partial m_Z^2}{\partial \lambda_i} \\ \frac{1}{v^2} \frac{\partial \mu^2}{\partial \lambda_i} \\ \frac{\partial \tan\beta}{\partial \lambda_i} \end{pmatrix}, \tag{85}
$$

$$
X^{m_i^2} = \begin{pmatrix} \frac{\partial m_Z^2}{\partial m_i^2} \\ \frac{\partial \mu^2}{\partial m_i^2} \\ v^2 \frac{\partial \tan\beta}{\partial m_i^2} \end{pmatrix}, (i = u, d, N), \tag{86}
$$

with $\lambda_i = m_N^2, m_{H_u}^2, m_{H_d}^2, \lambda_H, \lambda_N$, and

$$
B^{m_N^2} + B_{A_H}^{m_N^2} = \begin{pmatrix} 0 \\ -\frac{1}{2} \frac{\lambda_H^2}{\lambda_N^2} \\ -\frac{\lambda_H}{\lambda_N} \frac{v^2}{2(\mu_1^2+\mu_2^2)} \end{pmatrix}, \tag{87}
$$

$$
B^{m_{H_u}^2} + B_{A_H}^{m_{H_u}^2} = \begin{pmatrix} \frac{\tan^2\beta}{1-\tan^2\beta} \\ 0 \\ -v^2 \frac{\sin 2\beta}{2(\mu_1^2+\mu_2^2)} \end{pmatrix}, \tag{88}
$$

$$
B^{m_{H_d}^2} + B_{A_H}^{m_{H_d}^2} = \begin{pmatrix} \frac{1}{\tan^2\beta - 1} \\ 0 \\ -v^2 \frac{\sin 2\beta}{2(\mu_1^2+\mu_2^2)} \end{pmatrix}, \tag{89}
$$

$$
B^{\lambda_H} = \begin{pmatrix} 0 \\ -\frac{\lambda_H^3}{\lambda_N^2} + \frac{3}{4} \frac{\lambda_H^2 \sin 2\beta}{\lambda_N} - \frac{\lambda_H}{\lambda_N^2} \frac{m_N^2}{v^2} \\ -\frac{1}{(\mu_1^2+\mu_2^2)} \left( \frac{1}{2} \frac{m_N^2}{\lambda_N} + \frac{3}{4} v^2 \frac{\lambda_H^2}{\lambda_N} \right) \end{pmatrix}, \tag{90}
$$

$$
B_{A_H}^{\lambda_H} = \frac{A_H}{\mu} \begin{pmatrix} 0 \\ \frac{\lambda_H^2 \sin 2\beta}{2\lambda_N^2} \\ \frac{3}{8} \frac{\lambda_H}{\lambda_N} \frac{v^2 \sin 2\beta}{\mu_1^2+\mu_2^2} \end{pmatrix}, \tag{91}
$$

$$
B^{\lambda_N} = \begin{pmatrix} 0 \\ -\frac{1}{4} \frac{\lambda_H^3 \sin 2\beta}{\lambda_N^2} + \frac{\lambda_H^2}{\lambda_N^3} \frac{m_N^2}{v^2} + \frac{1}{2} \frac{\lambda_H^4}{\lambda_N^3} \\ \frac{1}{2(\mu_1^2+\mu_2^2)} \frac{\lambda_H}{\lambda_N^2} (m_N^2 + \frac{1}{2} v^2 \lambda_H^2) \end{pmatrix}, \tag{92}
$$

$$B_{A_H}^{\lambda_N} = \frac{A_H}{\mu}\begin{pmatrix} 0 \\ -\frac{\lambda_H^3 \sin 2\beta}{4\lambda_N^3} \\ -\frac{\lambda_H^2}{8\lambda_N^2}\frac{v^2 \sin 2\beta}{\mu_1^2+\mu_2^2} \end{pmatrix}. \tag{93}$$

In deriving these equations $A_H(\lambda_i) = \lambda_H \tilde{A}(\lambda_i)$ was assumed and $\partial \tilde{A}/\partial \lambda_H$ was neglected. Inverting these set of equations gives the $c$ functions. We note that these expressions for the various $c$ functions are valid for any NMSSM in which the $N^3$ scalar term is negligible and the $NH_uH_d$ scalar term is proportional to $\lambda_H$. In general, these 6 parameters might, in turn, depend on some fundamental parameters, $\tilde{\lambda}_i$. Then, the sensitivity to these fundamental parameters is:

$$\begin{aligned} \tilde{c}_i &\equiv \frac{\tilde{\lambda}_i}{m_Z^2}\frac{\partial m_Z^2}{\partial \tilde{\lambda}_i} \\ &= \frac{\tilde{\lambda}_i}{m_Z^2}\sum_j \frac{\partial \lambda_j}{\partial \tilde{\lambda}_i}\frac{\partial m_Z^2}{\partial \lambda_j} \\ &= \sum_j \frac{\tilde{\lambda}_i}{\lambda_j} c(m_Z^2; \lambda_j)\frac{\partial \lambda_j}{\partial \tilde{\lambda}_i}. \end{aligned} \tag{94}$$

For example, in the NMSSM of section 5, the fundamental parameters are $\Lambda_{mess}, \lambda_H, \lambda_N, \lambda_t$ and $\lambda_q$ ($A_H$ is a function of $\lambda_H$ and $\Lambda_{mess}$). Fixing $m_Z$ and $m_t$ leaves 3 free parameters, which we choose to be $\Lambda_{mess}, \lambda_H$ and $\tan\beta$. As explained in that section, the effect of $\lambda_H$ in the RG scaling of $m_{H_u}^2$ and $m_{H_d}^2$ was neglected, whereas the sensitivity of $m_N^2$ to $\lambda_H$ could be non-negligible. Thus, we have

$$\tilde{c}(m_Z^2; \lambda_H) = c(m_Z^2; \lambda_H) + c(m_Z^2; m_N^2)\frac{\lambda_H}{m_N^2}\frac{\partial m_N^2}{\partial \lambda_H}. \tag{95}$$

We find, in our model, that $c(m_Z^2; m_N^2)$ is smaller than $c(m_Z^2; \lambda_H)$ by a factor of $\sim 2$. Also, using approximate analytic and also numerical solutions to the RG equation for $m_N^2$, we find that $(\lambda_H/m_N^2)(\partial m_N^2/\partial \lambda_H)$ is $\lesssim 0.1$. Consequently, in the analysis of section 5 the additional contribution to $\tilde{c}(m_Z^2; \lambda_H)$ due to the dependence of $m_N^2$ on $\lambda_H$ was neglected. A similar conclusion is true for $\lambda_N$. Also,

$$\tilde{c}(m_Z^2; \lambda_q) = c(m_Z^2; m_N^2)\frac{\lambda_q}{m_N^2}\frac{\partial m_N^2}{\partial \lambda_q}. \tag{96}$$

We find that $(\lambda_q/m_N^2)(\partial m_N^2/\partial\lambda_q)$ is $\approx 1$ so that $\tilde{c}(m_Z^2; \lambda_q)$ is smaller than $\tilde{c}(m_Z^2; \lambda_H)$ by a factor of 2.

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Captions:

Figure 1: Contours of $c(m_Z^2;\mu^2)$ =(10, 15, 20, 25, 40, 60) for a MSSM with a messenger particle content of one $(5+\bar{5})$. In figures $(a)$ and $(c)$ $sgn(\mu)=-1$ and in figures $(b)$ and $(d)$ $sgn(\mu)=+1$. The constraints considered are: (I) $m_{\tilde{e}_R}$ =75 GeV , (II) $m_{\tilde{\chi}_1^0}+m_{\tilde{\chi}_2^0}$ = 160 GeV, (III) $m_{\tilde{e}_R}$ =85 GeV, and (IV) $m_{\tilde{\chi}_1^0}+m_{\tilde{\chi}_2^0}$ = 180 GeV. A central value of $m_{top}$ =175 GeV is assumed.

Figure 2: Contours of $c(m_Z^2;\mu^2)$ =(1, 2, 3, 5, 7, 10) for a MSSM with a messenger particle content of three $(l+\bar{l})$'s and one $(q+\bar{q})$. In figures $(a)$ and $(c)$ $sgn(\mu)=-1$ and in figures $(b)$ and $(d)$ $sgn(\mu)=+1$. The constraints considered are: (I) $m_{\tilde{e}_R}$ =75 GeV , (II) $m_{\tilde{\chi}_1^0}+m_{\tilde{\chi}_2^0}$ = 160 GeV, (III) $m_{\tilde{e}_R}$ =85 GeV, and (IV) $m_{\tilde{\chi}_1^0}+m_{\tilde{\chi}_2^0}$ = 180 GeV. A central value of $m_{top}$ =175 GeV is assumed.

Figure 3: Contours of $c(m_Z^2;\lambda_H)$ for the NMSSM of Section 5 and a messenger particle content of three $(l+\bar{l})$'s and one $(q+\bar{q})$. In figures $(a)$ and $(b)$, $c(m_Z^2;\lambda_H)$=(4, 5, 6, 10, 15) and $\lambda_H$ =0.1. In figures $(c)$ and $(d)$, $c(m_Z^2;\lambda_H)$ =(3, 4, 5, 10, 15, 20) and $\lambda_H$=0.5. The constraints considered are: (I) $m_h+m_a=m_Z$, (II) $m_{\tilde{e}_R}$ =75 GeV, (III) $m_{\tilde{\chi}_1^0}+m_{\tilde{\chi}_2^0}$ = 160 GeV, (IV) $m_h$ = 92 GeV, (V) $m_{\tilde{e}_R}$ =85 GeV, and (VI) $m_{\tilde{\chi}_1^0}+m_{\tilde{\chi}_2^0}$ = 180 GeV. For $\lambda_H$ =0.5, the limit $m_h \gtrsim$ 70 GeV constrains $\tan\beta \lesssim 5$ (independent of $\Lambda_{mess}$) and is thus not shown. A central value of $m_{top}$ =175 GeV is assumed.

Figure 4: Contours of $c(m_Z^2;\lambda_H)$ =(50, 80, 100, 150, 200) for the NMSSM of Section 5 with $\lambda_H$ =0.1 and a messenger particle content of one $(5+\bar{5})$. The constraints considered are: (I) $m_h+m_a=m_Z$, (II) $m_{\tilde{e}_R}$ =75 GeV, (III) $m_{\tilde{\chi}_1^0}+m_{\tilde{\chi}_2^0}$ = 160 GeV, (IV) $m_h$ =92 GeV, (V) $m_{\tilde{e}_R}$ =85 GeV, and (VI) $m_{\tilde{\chi}_1^0}+m_{\tilde{\chi}_2^0}$ = 180 GeV. A central value of $m_{top}$ =175 GeV is assumed.

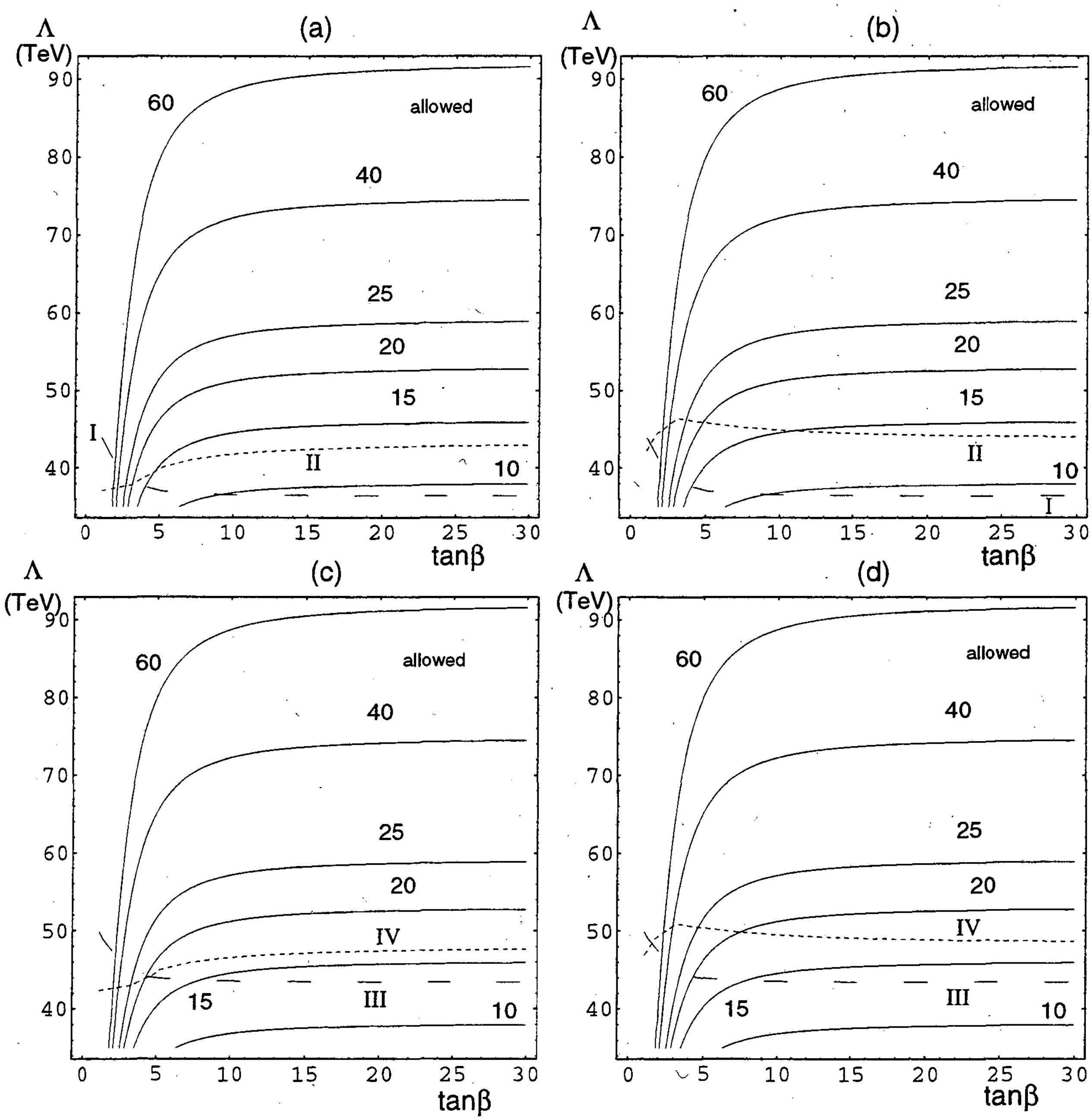
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Figure 1

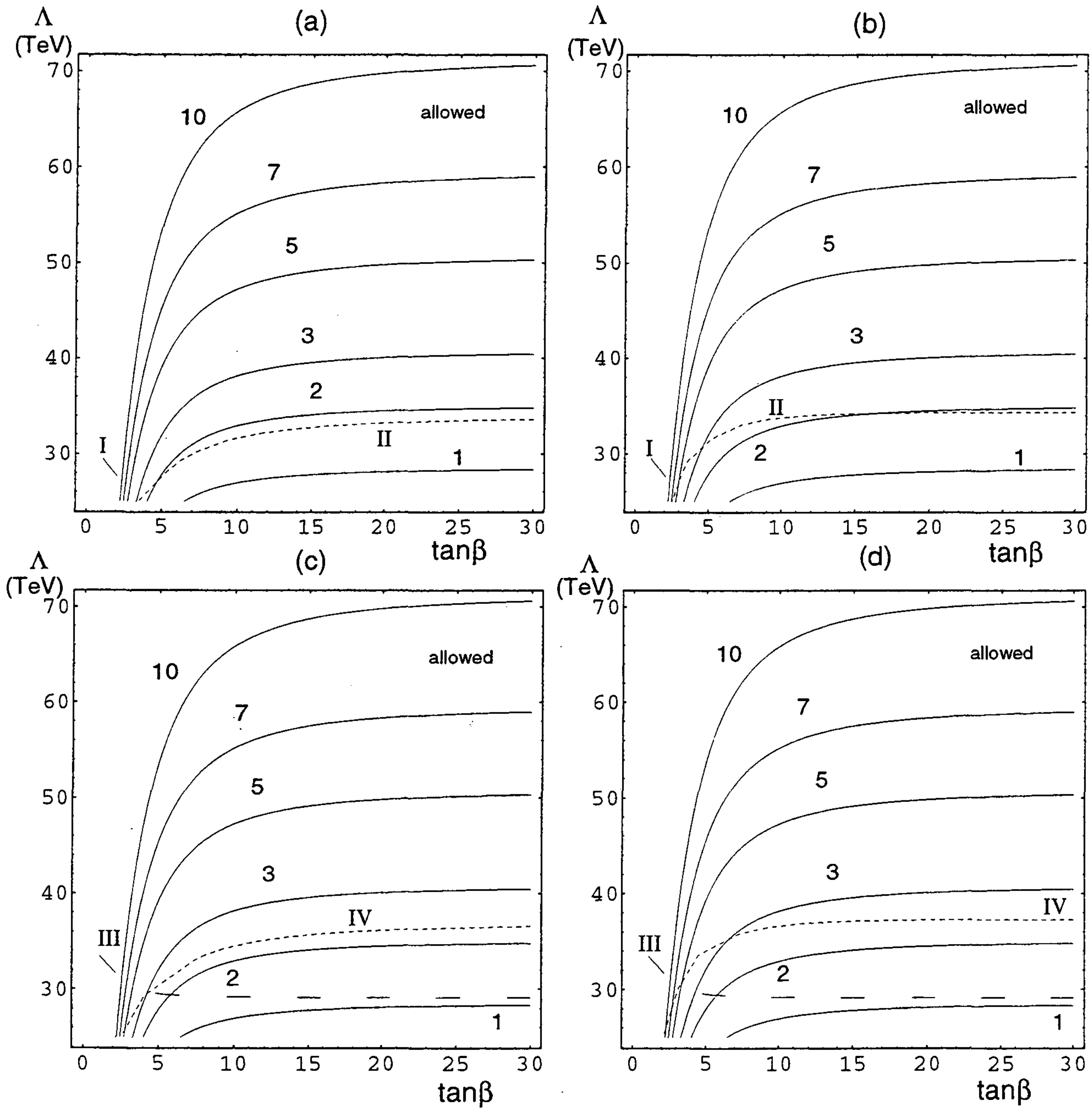
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Figure 2

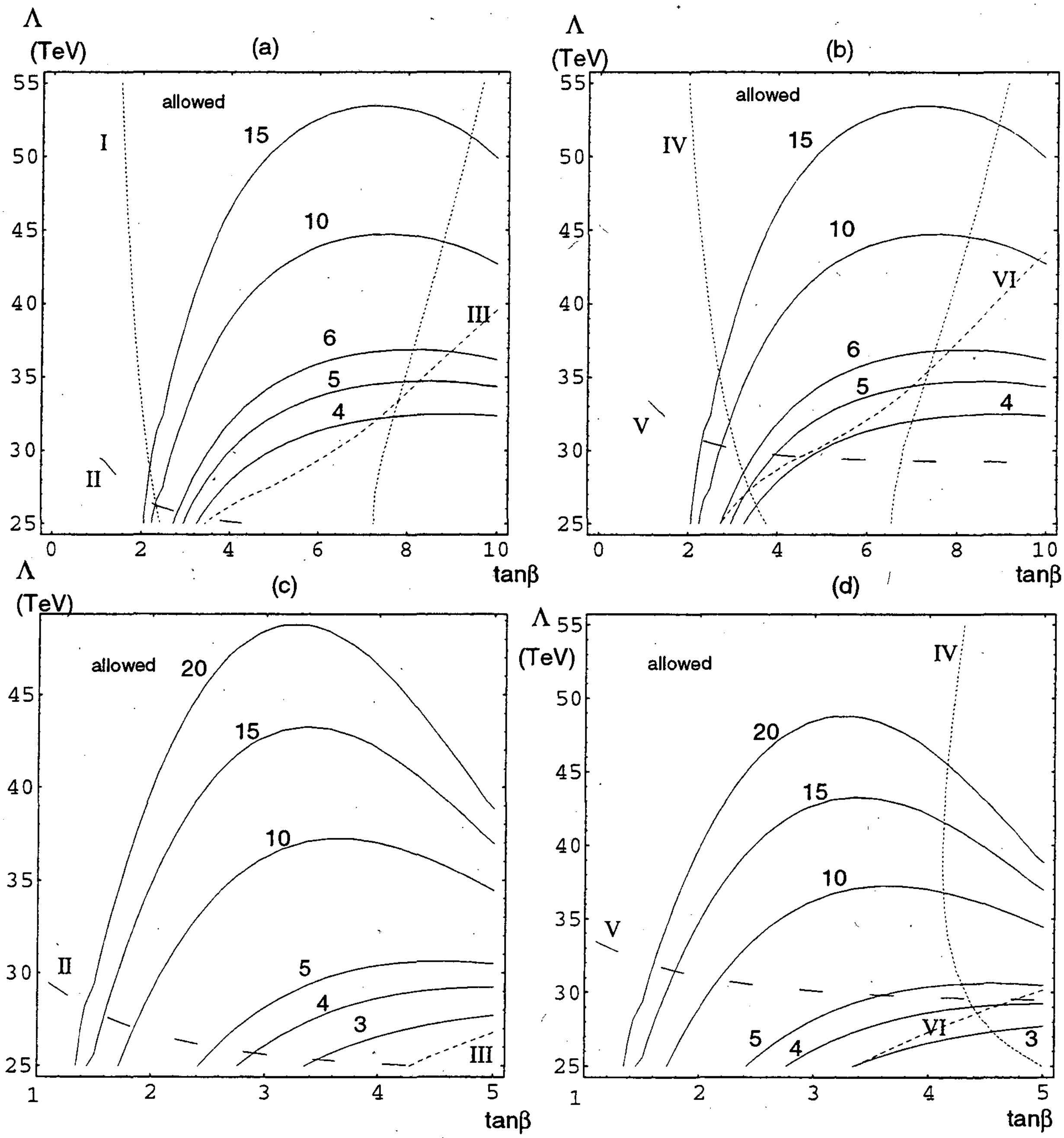
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Figure 3

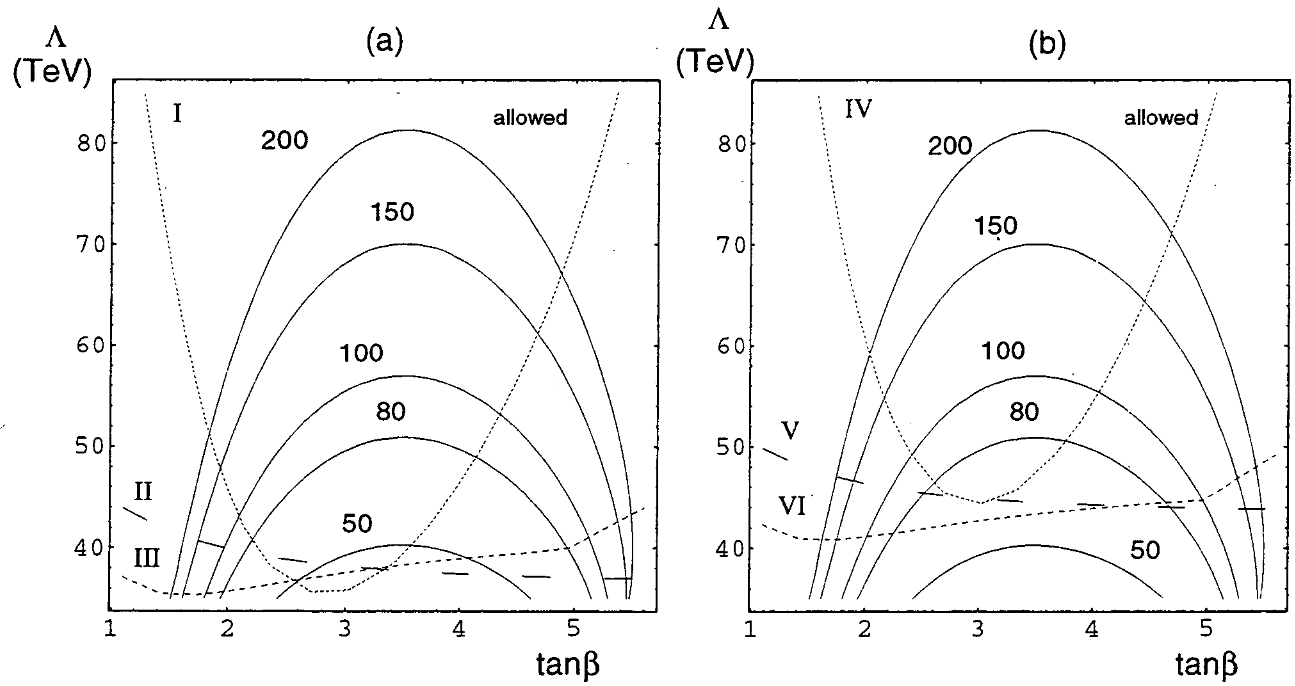

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Figure 4

| $m_{\tilde{Q}_{1,2}}$ | $m_{\tilde{u}^c_{1,2}}$ | $m_{\tilde{d}^c_i}$ | $m_{\tilde{L}_i,H_d}$ | $m_{\tilde{e}^c_i}$ |
|---|---|---|---|---|
| 687 | 616 | 612 | 319 | 125 |

| $m_{\tilde{Q}_3}$ | $m_{\tilde{u}^c_3}$ |
|---|---|
| 656 | 546 |

Table 1: Soft scalar masses in GeV for messenger particle content of three $(l+\bar{l})$'s and one $q+\bar{q}$ and a scale $\Lambda_{mess} = 50$ TeV.

| $\Psi$ | $\bar{5}_i$ | $10_i$ | $5_h$ | $\bar{5}_h$ |
|---|---|---|---|---|
| $G_{loc}$ | $(\bar{5},1)$ | $(10,1)$ | $(5,1)$ | $(\bar{5},1)$ |
| $Z_3$ | $1$ | $a$ | $a$ | $a^2$ |
| $Z_3'$ | $b$ | $1$ | $1$ | $b^2$ |
| $Z_4$ | $c$ | $c$ | $c^2$ | $c^2$ |

| $\Psi$ | $\Sigma$ | $\Sigma'$ | $\bar{\Phi}_2$ | $\Phi_2$ | $\bar{\Phi}_1$ | $\Phi_1$ |
|---|---|---|---|---|---|---|
| $G_{loc}$ | $(24,1)$ | $(1,24)$ | $(\bar{5},5)$ | $(5,\bar{5})$ | $(\bar{5},5)$ | $(5,\bar{5})$ |
| $Z_3$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| $Z_3'$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| $Z_4$ | $1$ | $1$ | $1$ | $1$ | $c^2$ | $c^2$ |

| $\Psi$ | $5_l$ | $\bar{5}_l$ | $X_l$ | $\bar{X}_l$ | $5_q$ | $\bar{5}_q$ |
|---|---|---|---|---|---|---|
| $G_{loc}$ | $(5,1)$ | $(\bar{5},1)$ | $(1,5)$ | $(1,\bar{5})$ | $(5,1)$ | $(\bar{5},1)$ |
| $Z_3$ | $a^2$ | $1$ | $1$ | $a$ | $1$ | $a^2$ |
| $Z_3'$ | $1$ | $1$ | $1$ | $1$ | $b^2$ | $b$ |
| $Z_4$ | $c^2$ | $c^2$ | $1$ | $1$ | $1$ | $1$ |

| $\Psi$ | $X_q$ | $\bar{X}_q$ | $X_h$ | $\bar{X}_h$ | $X$ | $\bar{X}$ |
|---|---|---|---|---|---|---|
| $G_{loc}$ | $(1,5)$ | $(1,\bar{5})$ | $(1,5)$ | $(1,\bar{5})$ | $(1,5)$ | $(1,\bar{5})$ |
| $Z_3$ | $a$ | $1$ | $a$ | $a^2$ | $a^2$ | $a$ |
| $Z_3'$ | $b^2$ | $b$ | $b$ | $1$ | $1$ | $b^2$ |
| $Z_4$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |

| $\Psi$ | $S$ | $N$ | $N'$ | $\phi_+$ | $\phi_-$ |
|---|---|---|---|---|---|
| $Z_3$ | $a$ | $1$ | $a$ | $a$ | $a$ |
| $Z_3'$ | $1$ | $b$ | $b^2$ | $1$ | $1$ |
| $Z_4$ | $1$ | $1$ | $1$ | $1$ | $1$ |

Table 2: $SU(5) \times SU(5)' \times Z_3 \times Z_3' \times Z_4$ quantum numbers for the fields of the model discussed in section 7. $(a,b,c)$ are the generators of $Z_3 \times Z_3' \times Z_4$. The three SM generations are labeled by the index $i$.

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