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Next-to-Minimal Supersymmetric Standard Model with the Gauge Mediation of Supersymmetry Breaking*

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Abstract

We study the Next-to-Minimal Supersymmetric Standard Model (NMSSM) as the simplest candidate solution to the μ-problem in the context of the gauge mediation of supersymmetry breaking (GMSB). We first review various proposals to solve the μ-problem in models with the GMSB. We find none of them entirely satisfactory and point out that many of the scenarios still lack quantitative studies, and motivate the NMSSM as the simplest possible solution. We then study the situation in the Minimal Supersymmetric Standard Model (MSSM) with the GMSB and find that an order 10% cancellation is necessary between the μ-parameter and the soft SUSY-breaking parameters to correctly reproduce M_Z. Unfortunately, the NMSSM does not to give a phenomenologically viable solution to the μ-problem. We present quantitative arguments which apply both for the low-energy and high-energy GMSB and prove that the NMSSM does not work for either case. Possible modifications to the NMSSM are then discussed. The NMSSM with additional vector-like quarks works phenomenologically, but requires an order a few percent cancellation among parameters. We point out that this cancellation has the same origin as the cancellation required in the MSSM.

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1 Introduction

The primary motivation for supersymmetry (SUSY) is to stabilize the smallness of the electroweak scale against radiative corrections [1, 2, 3], which can be as large as the Planck scale if the Higgs bosons are truly elementary. Once the electroweak scale is set in the tree-level Lagrangian, it only receives logarithmic radiative corrections, and hence its order of magnitude is not changed. Moreover, the electroweak symmetry remains unbroken in the Minimal Supersymmetric Standard Model (MSSM) in the absence of explicit SUSY-breaking parameters. Therefore, one can view the electroweak symmetry breaking as being triggered by the soft SUSY breaking. Indeed, the soft SUSY-breaking mass-squared of the Higgs boson can be driven negative due to the top quark loop [4] while all the other scalar bosons still have positive mass-squared. In this sense, there is nothing special about the Higgs boson. It is just one of many scalar bosons, which happens to acquire a negative mass-squared due to the top quark loop. This idea eliminates one of the least appealing features of the Standard Model. However, there are at least two open questions. First, SUSY by itself does not explain why the electroweak scale is small to begin with. Therefore, SUSY makes the smallness of the electroweak scale "technically natural," but not truly natural. Second, the MSSM contains one dimensionful parameter (the μ-parameter), allowed by SUSY, in the superpotetial. The natural values of μ are either the Planck mass (the only natural dimensionful parameter available) or zero, but recent experimental constraints imposed by LEP2 imply that a nonzero $\mu \lesssim 50$ GeV is required [5].

SUSY, fortunately, can potentially explain the smallness of the electroweak scale if it is broken dynamically [2]. The perturbative non-renormalization theorem forbids the generation of a mass scale in the superpotential if it is absent at the tree-level. However, non-perturbative effects can violate the non-renormalization theorem, and a mass scale can be generated by a dimensional transmutation: $\Lambda_{\not{S}USY} \sim M_{\rm Planck} e^{-8\pi^2/g^2|b_0|}$, if an asymptotically free gauge theory is responsible for SUSY breaking. There has been major progress in building models of dynamical SUSY breaking [6, 7, 8, 9, 10, 11, 12], which became possible with the detailed understanding of the non-perturbative dynamics of SUSY gauge theories [13]. Furthermore, the so-called gauge mediation of SUSY breaking (GMSB) [3, 14] can generate soft SUSY-breaking parameters in the SUSY Standard Model in a phenomenologically desired form. Therefore, there is hope of understanding the smallness of the electroweak scale in a truly natural manner.

However, the other question remains largely unanswered: how can the dimensionful parameters in the superpotential naturally be of the order of the SUSY-breaking parameters? There have been extensive discussions on this subject in the literature which we briefly summarize in Section 2. Unfortunately, many of the proposed mechanisms rely on either small parameters, accidental cancellations, or the absence of interactions allowed by symmetries. We find the current situation to be rather unsatisfactory.

A natural direction to follow is to start with a superpotential which does not contain a dimensionful parameter and hope that the electroweak scale is generated solely due to the soft SUSY-breaking parameters. The simplest model which can potentially work along this line is the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [15], which replaces the μ-parameter by the vacuum expectation value of an electroweak singlet superfield. We revisit this possibility with detailed quantitative studies in this paper. Unfortunately, our conclusion is negative. The NMSSM by itself does not produce a phenomenologically viable electroweak symmetry breaking even if we vary the messenger scale. The major experimental constraints include Higgs boson and slepton searches. Certain simple modifications can evade phenomeno-

logical constraints, but require a cancellation among parameters accurate to a few percent. We present all of these points quantitatively in this paper, and hope that our results prompt further investigations in understanding the origin of the μ-parameter in models with the GMSB.

The paper is organized as follows. In the next section, we review the situation of the μ-problem in models with the GMSB, and discuss various proposals to explain the origin of the μ-parameter. We, however, find none of them entirely satisfactory. Even if we accept one of the proposed models, it is still necessary to check whether the generated μ-parameter is phenomenologically allowed. We address this question in Section 3, and find that the currently available experimental lower bounds on superparticle masses already require a cancellation of order 10% between the μ-parameter and soft SUSY-breaking parameters to reproduce the observed M_Z. Then, as the major part of our study, we present the quantitative results of the electroweak symmetry breaking in the NMSSM with the GMSB in Section 4 and find that there is no phenomenologically viable parameter set even if we vary the messenger scale from 10^5 to 10^{16} GeV. We study various simple modifications of the NMSSM in Section 5, and find that they either do not break electroweak symmetry in a phenomenologically viable manner or require a cancellation among parameters of order 1%. We finally conclude in Section 6.

2 The μ-problem in the GMSB

In this section, we review the μ-problem in the supersymmetric Standard Model in general, and also various attempts to solve it in the context of the GMSB.

The parameter μ is the only dimensionful quantity present in the superpotential of the MSSM

$$W = \mu H_u H_d + \lambda^l_{ij} L_i e_j H_d + \lambda^d_{ij} Q_i d_j H_d + \lambda^u_{ij} Q_i u_j H_u. \tag{2.1}$$

Here, Q_i, L_i, u_i, d_i, e_i are the matter chiral superfields with the obvious notation, and H_u, H_d the Higgs doublets. Note that μ is part of the supersymmetric Lagrangian, and hence its origin is, naively, unrelated to the origin of the soft SUSY-breaking terms

$$\begin{aligned} V_{soft} &= m^2_{H_d}|H_d|^2 + m^2_{H_u}|H_u|^2 \\ &\quad + m^{2ij}_{\tilde{Q}} \tilde{Q}^\dagger_i \tilde{Q}_j + m^{2ij}_{\tilde{L}} \tilde{L}^\dagger_i \tilde{L}_j + m^{2ij}_{\tilde{u}} \tilde{u}^\dagger_i \tilde{u}_j + m^{2ij}_{\tilde{d}} \tilde{d}^\dagger_i \tilde{d}_j + m^{2ij}_{\tilde{e}} \tilde{e}^\dagger_i \tilde{e}_j \\ &\quad - m^2_3 H_u H_d + \mathcal{A}^{ij}_d \tilde{Q}_i \tilde{d}_j H_d + \mathcal{A}^{ij}_u \tilde{Q}_i \tilde{u}_j H_u + \mathcal{A}^{ij}_l \tilde{L}_i \tilde{e}_j H_d. \end{aligned} \tag{2.2}$$

Phenomenology, on the other hand, dictates that the values of both μ and the soft SUSY-breaking masses should be around the weak scale (100 GeV), if SUSY is to be responsible for stabilizing the Higgs mass. Therefore, the important question is how the mechanism of SUSY breaking can induce a μ-parameter naturally, at the same order of magnitude as the other soft SUSY-breaking parameters in the Lagrangian.

One popular scenario of SUSY breaking is the so-called "hidden sector" SUSY breaking in supergravity (SUGRA) [16]. In hidden sector models, SUSY is broken in the hidden sector by some mechanism, such as the Polonyi model [17], gaugino condensation [18], or the O'Rafeartaigh model [19], and the effects of SUSY breaking are mediated to the fields in the supersymmetric Standard Model only by interactions suppressed by the Planck scale. It therefore requires SUSY breaking at a scale $\Lambda \sim 10^{10}$ GeV if the soft SUSY-breaking masses are generated as $\Lambda^2/M_{\rm Planck}$. This class of models is able to generate the appropriate soft SUSY-breaking masses and μ-parameter given that the μ-term is forbidden in the supersymmetric limit by appropriate

symmetries, and arises due to SUSY breaking (see, for example, the Giudice–Masiero mechanism [20]). Hidden sector models have, on the other hand, to face serious bounds imposed by flavor-changing neutral currents (FCNC) [21]. Low-energy constraints such as the smallness of K^0–$\overline{K^0}$ mixing require the matrices $m^{2ij}_{\tilde{Q}}$, $m^{2ij}_{\tilde{d}}$ to have eigenvalues degenerate to a few percent, or their eigenvectors to be strongly "aligned" with the eigenvectors of the Yukawa matrices λ^d_{ij} (the same is true for $\mathcal{A}^{ij}_d$). Within the SUGRA framework alone, there is no natural mechanism to guarantee the degeneracy or the alignment [22]. In this case, flavor symmetries are probably necessary to ensure either degeneracy [23] or alignment [22] and suppress FCNC, and some of the models presented are also capable of generating the μ-term through flavor symmetry breaking [24, 25]. There is also the possibility that string theory generates degenerate squark masses if, for instance, the dilaton field provides the dominant contribution to the soft SUSY-breaking masses [26].

The gauge mediation of supersymmetry breaking is an alternative mechanism which can naturally ensure the degeneracy of squarks masses and therefore suppress the dangerous FCNC effects. SUSY is somehow broken (hopefully dynamically via dimensional transmutation to generate a large hierarchy), and SUSY-breaking effects are mediated to the fields in the supersymmetric Standard Model by the Standard Model gauge interactions. Mediating SUSY breaking via gauge interactions is not a novel idea [3, 14]. It allows for SUSY breaking at a lower scale (when compared to SUGRA inspired models) and, because all SUSY-breaking effects are transmitted by flavor blind interactions (the Standard Model gauge interactions), squarks of different families have the same mass. This scheme has attracted a lot of interest after the pioneering works by the authors of references [6, 7, 8], which showed that one can successfully mediate the SUSY-breaking effects via gauge interactions with the help of a so-called "messenger sector." Their scheme can easily incorporate dynamical SUSY breaking and can explain the origin of the large hierarchy between the Planck (string, grand unified (GUT)) scale and the weak scale.

The GMSB itself, however, has nothing to say about the μ-parameter unless one introduces extra fields which couple to the particle content of the MSSM. The μ-problem in the GMSB is the primary interest of this paper. Many solutions to the μ-problem have been suggested by different authors and all of them require the introduction of new fields and/or interactions. Some of these solutions will be reviewed shortly.

In the original models [6, 7, 8], SUSY is broken dynamically in a so-called SUSY-breaking sector and the breaking effects are transmitted to the supersymmetric Standard Model via a messenger sector. The energy scale of the messenger sector is given by $\Lambda \simeq 10^4$–10^5 GeV. There are, however, models which do not have a separate messenger sector so that the sector which breaks SUSY dynamically is directly coupled to the Standard Model gauge group [10, 11, 12]. In this case, the effective messenger scale tends to be much higher. For our purposes it is enough to employ a simple version of the messenger sector, as in the original models, and take the messenger scale Λ as a free parameter.

The messenger sector can be described by the superpotential

$$W_S = \frac{1}{3}\lambda S^3 + \kappa S\Phi^+\Phi^- + \kappa_q S q\bar{q} + \kappa_l S l\bar{l}, \tag{2.3}$$

where S is a singlet superfield, $\Phi^\pm$ are charged under a $U(1)$ associated with the SUSY-breaking sector and are singlets under the Standard Model $SU(3) \times SU(2) \times U(1)$ gauge group. The superfield q ($\bar{q}$) transforms as a $(\mathbf{3}(\bar{\mathbf{3}}), \mathbf{1}, \pm 1/3)$ under the Standard Model, while l ($\bar{l}$) transforms as $(\mathbf{1}, \mathbf{2}, \mp 1/2)$.

We assume that the scalar components of $\Phi^{\pm}$ acquire negative SUSY-breaking masses-squared due to its interaction with the SUSY-breaking sector (usually accomplished by the so-called "messenger $U(1)$" gauge interaction [7, 8]), and the potential associated with the scalar component of S reads

$$V_S = -|m_\Phi|^2(|\Phi^+|^2 + |\Phi^-|^2) + |\kappa S\Phi^+|^2 + |\kappa S\Phi^-|^2 + |\kappa\Phi^+\Phi^- + \lambda S^2|^2, \tag{2.4}$$

neglecting terms containing l or q. It is easy to see that the scalar and the F components of S acquire vacuum expectation values (VEVs) $\langle S\rangle$ and $\langle F_S\rangle$ and therefore q and l acquire supersymmetric masses proportional to $\langle S\rangle$ and SUSY-breaking masses-squared proportional to $\langle F_S\rangle$.[†] This effect feeds down to the MSSM through loop corrections. Gauginos acquire Majorana masses at one loop, while sfermions acquire SUSY-breaking masses-squared at two loops. The calculation of these soft SUSY-breaking parameters was done long ago (see [3, 14]) and its result is well known. At the messenger scale:

$$M_i = \frac{\alpha_i}{4\pi} nB, \tag{2.5}$$

$$m_{\tilde{f}_i}^2 = 2nB^2\left(\frac{3}{5}Y_i^2\left(\frac{\alpha_1}{4\pi}\right)^2 + C_{2i}\left(\frac{\alpha_2}{4\pi}\right)^2 + C_{3i}\left(\frac{\alpha_3}{4\pi}\right)^2\right). \tag{2.6}$$

Here and below, all $\alpha_i = g_i^2/4\pi$ are in the $SU(5)$ normalization, $B = \langle F_S\rangle/\langle S\rangle$ in the messenger sector discussed above, and n determines the number of messenger sector superfields responsible for mediating SUSY breaking. In the example we described above, which will be referred to as the model with the minimal GMSB, $n = 1$. Y is the hypercharge of the particle, $C_2 = 3/4$ for weak $SU(2)$ doublets (zero for singlets) and $C_3 = 4/3$ for color triplets (zero for color singlets). Eq. (2.6) guarantees that squarks of different families are degenerate at the messenger scale and therefore FCNC effects are safely suppressed. It is interesting to note that, for small n, gaugino masses and sfermion masses are comparable. For very large n, on the other hand, sfermion masses can be significantly smaller than gaugino masses (by a factor $\sqrt{n}$).

In the mechanism described above, trilinear couplings are not generated at the same order (in loop expansion) at the messenger scale. This is not the case in general, and some models can generate trilinear couplings with values comparable to the other soft SUSY-breaking parameters even at the messenger scale [11]. We will, for most of our discussions, consider

$$\mathcal{A}_{l,u,d}^{ij}(\Lambda) = 0, \tag{2.7}$$

unless otherwise noted.

The GMSB does not generate a μ-term because of the non-renormalization theorem. Therefore μ is an input of the model, and, because it has dimensions of mass, its only nonzero natural value is M_{Planck} (M_{string}, M_{GUT}). This is clearly not allowed phenomenologically. The μ-term must, therefore, be forbidden at the Planck scale (by, say, a Z_3 symmetry) and generated dynamically. Below, we review various attempts to generate the μ-term in the context of the GMSB. The following list is not meant to be exhaustive and our descriptions of the various attempts are by no means complete. The review below only intends to show that many attempts have been made while none of them appears to be entirely satisfactory.

[†]There is a run-away direction $q = \bar{q}$, $l = \bar{l}$ in this potential [27]. This problem can be avoided by introducing more S fields to the messenger sector. Such details are, however, irrelevant for the rest of our discussion.

The simplest solution would be to introduce a term in the superpotential [7]

$$W \supset kSH_dH_u, \tag{2.8}$$

where S is the singlet superfield in Eq. (2.3). In such a scenario $\mu = k\langle S\rangle$ and $m_3^2 = k\langle F_S\rangle$. m_3^2 is the SUSY-breaking Higgs mixing mass-squared in Eq. (2.2).

Phenomenology imposes that both μ and $\sqrt{m_3^2}$ are of the order of the weak scale, unless one is willing to accept a drastic cancellation among parameters to reproduce the observed M_Z. Therefore,

$$(k\langle S\rangle)^2 \sim k\langle F_S\rangle \sim (100 \text{ GeV})^2, \tag{2.9}$$

$$\frac{\langle F_S\rangle}{\langle S\rangle} \sim k\langle S\rangle \sim 100 \text{ GeV} \tag{2.10}$$

and

$$m_3^2 = \mu\frac{\langle F_S\rangle}{\langle S\rangle}. \tag{2.11}$$

This situation is already excluded experimentally. Eq. (2.5) states that the gluino mass is given by $(\alpha_3/4\pi)\langle F_S\rangle/\langle S\rangle$, and if Eq. (2.10) is satisfied one would arrive at $M_{\tilde{g}} \simeq 1$ GeV, which is unacceptable. The same is true for all the other soft SUSY-breaking masses. This is a general consequence of Eq. (2.11). It implies that $\sqrt{m_3^2} \gg \mu$ if all experimental bounds on the SUSY spectrum are to be satisfied, while $SU(2)\times U(1)$ breaking requires Eq. (2.9). Some authors refer to this puzzle as the μ-problem in the GMSB [28].

Another simple solution that does not require the introduction of any extra superfields into the theory couples the Higgs superfields to the q superfields present in Eq. (2.3) [28]. In the minimal messenger sector [7, 8], one may have, instead of q and l, a complete $\mathbf{5}+\bar{\mathbf{5}}$ multiplet of $SU(5)$ to preserve the gauge coupling unification. One can also use a $\mathbf{10}+\overline{\mathbf{10}}$ for this purpose, and generate gaugino masses and scalar masses-squared with $n = 3$. In this case, one can couple the components Q in $\mathbf{10}$ that have the same quantum numbers as left-handed quark doublets and the components u that have the same quantum numbers as right-handed up quarks (or their corresponding components in $\overline{\mathbf{10}}$) to the Higgs doublets. Explicitly, $W \supset \lambda_1 H_d\bar{Q}\bar{u} + \lambda_2 H_u Q u$. This will induce, in the Lagrangian, a one-loop term proportional to

$$\frac{\lambda_1\lambda_2}{16\pi^2}\int d^4\theta \frac{H_dH_uS^\dagger S^\dagger}{S^\dagger S}. \tag{2.12}$$

The F vacuum expectation value of S will generate $\mu \simeq \frac{\lambda_1\lambda_2}{16\pi^2}\frac{\langle F_S\rangle}{\langle S\rangle}$ and $m_3^2 \simeq \frac{\lambda_1\lambda_2}{16\pi^2}\left(\frac{\langle F_S\rangle}{\langle S\rangle}\right)^2$. Again one runs into Eq. (2.11) and must hunt for other solutions.

All of the models described above couple the MSSM Higgs superfields to those in the messenger sector. Not only did we encounter the problem of Eq. (2.11), but some of the coupling constants introduced had to be made fairly small because of the magnitude of $\langle S\rangle$ and $\langle F_S\rangle$. Another class of solutions tries to get around this issue by introducing another singlet superfield, whose vacuum expectation value would generate the μ-term.

One motivation for such models is to utilize the extra singlet to solve the doublet-triplet Higgs splitting in $SU(5)$ grand unified theories via a sliding singlet mechanism [29]. This mechanism is known to be unstable against radiative corrections if the soft SUSY-breaking parameters are generated at a scale higher than the GUT scale, but can be stable for the low-energy GMSB [30]. Ciafaloni and Pomarol [31] claim that such a solution would generate a viable μ-term. We

believe, however, that the conditions that they impose on the soft SUSY-breaking parameters can never be satisfied in the context of the GMSB, where all soft SUSY-breaking masses are tightly related. We will comment on this in Section 5.

The simplest model with the addition of an extra singlet one can imagine, referred to as the NMSSM [15], involves substituting the μ-term in the MSSM superpotential by

$$\lambda H_d H_u N - \frac{k}{3} N^3. \tag{2.13}$$

The minimization of the scalar potential for H_d, H_u and N at the weak scale should produce VEVs v_d and v_u for both Higgs bosons, thus breaking $SU(2) \times U(1)$, and x for the singlet. μ would be equal to λx. The m_3^2 term would arise due to renormalization group (RG) running of the A-term $\lambda A_\lambda H_d H_u N$ from the messenger scale to the weak scale. m_3^2 would be equal to $\lambda A_\lambda x$.

Dine and Nelson [6] claim that this model does not work for the low-energy GMSB. A detailed analysis was not presented in their paper, and we will explain the problem in Section 4. They suggest the introduction of an extra light pair of $q' + \bar{q}'$ and $l' + \bar{l}'$ as a means to produce a viable spectrum. They did not, however, publish a quantitative analysis of the model, and say nothing about its naturalness. Agashe and Graesser [32] study this scenario and show that there is indeed a solution, but it is fine-tuned. They present a possibility to ease the fine-tuning by employing many lepton-like messengers while keeping the number of quark-like messengers small. In Sections 4 and 5, we analyze in great detail the case for both the high- and low-energy GMSB.

There are ways of giving N a VEV which are not related to electroweak symmetry breaking. In Ref. [7] two mechanisms are introduced, neither of them very appealing, where the N VEV is generated at the messenger scale. Namely,

$$W \supset -\frac{1}{2} k_S N^2 S \tag{2.14}$$

or

$$W \supset -k_q N q \bar{q} - k_l N l \bar{l} \tag{2.15}$$

in addition to the NMSSM. S, q and l are the messenger sector superfields present in Eq. (2.3).

In the case of Eq. (2.14), a potential

$$V_N = |kN^2 + k_S \langle S \rangle N|^2 - k_S N^2 \langle F_S \rangle \tag{2.16}$$

is generated for N in the presence of $\langle S \rangle$ and $\langle F_S \rangle$ VEVs. If one assumes k_S to be small, N develops a VEV $x = \sqrt{\frac{k_S \langle F_S \rangle}{k^2}}$, and $\mu = \frac{\lambda}{k}\sqrt{k_S \langle F_S \rangle}$ assuming all other couplings to be of order one. It is easy to see, a posteriori, that k_S must indeed be small if one is to generate a phenomenologically viable μ. Unfortunately this case requires that the soft SUSY-breaking masses-squared $\sim (\alpha_i/4\pi)^2(\langle F_S \rangle / \langle S \rangle)^2$ and $\mu^2 \sim k_S \langle F_S \rangle$ are accidentally of the same order of magnitude.[‡]

The superpotential coupling Eq. (2.15) would lead to a potential

$$V_N \supset |kN^2|^2 - k_q N \frac{1}{32\pi^2} \frac{(\kappa_q \langle F_S \rangle)^2}{\kappa_q \langle S \rangle} - (l \leftrightarrow q). \tag{2.17}$$

[‡]Although k_S has to be small, its smallness is natural in the sense of 't Hooft. It can be interpreted as being generated due to the breaking of some global symmetry, such as $N \to e^{2\pi i/3} N$ and $H_{1,2} \to e^{2\pi i/3} H_{1,2}$, while S is invariant. This type of symmetry would also explain the suppression of a term NS^2 in the superpotential, which would be of order $(k_S)^2$.

The linear terms in N arise via tadpole one-loop diagrams involving q's and l's. This would lead to $x^3 = \frac{k_q}{k^2}\frac{1}{32\pi^2}\frac{\kappa_q\langle F_S\rangle^2}{\langle S\rangle} + (l \leftrightarrow q)$. Again k_q and k_l would have to be small. This solution still faces the problem of explaining why a term NS^2 is not present in the superpotential. Note that the presence of such a term would lead to an unacceptably large VEV for N. One may argue, however, that this is "technically natural" because the absence of a term in the superpotential is preserved by radiative corrections. An even more serious problem is the need to suppress the kinetic mixing $\int d^4\theta S^\dagger N + h.c.$ to ensure $F_N \ll F_S$; an unacceptably large $m_3^2 = \lambda F_N$ would be generated otherwise. An order unity kinetic mixing can be induced via radiative corrections between the ultraviolet cutoff, say the Planck scale, and the messenger scale, and the bare parameter has to be chosen very carefully so that the unwanted mixing term can be canceled at the messenger scale. This kinetic mixing can be forbidden if there are two sets of messenger fields and if the field N couples off-diagonally, *e.g.*, $W = Nq_1\bar{q}_2$ etc [33]. Then the tadpole term mentioned above is also forbidden, but a negative mass squared for the N field can be generated instead. This would lead to the NMSSM in a successful manner; again the parameters must be carefully chosen as in the NMSSM with extra light quark pairs (see Section 5).

Another solution with extra singlets, which points an interesting way around Eq. (2.11), was suggested by Dvali, Giudice and Pomarol [28]. Their idea is to generate the μ-term via the following one-loop effective term in the Lagrangian:

$$\int d^4\theta \frac{H_d H_u D^\alpha D_\alpha (S^\dagger S)}{\langle S\rangle^3}, \tag{2.18}$$

where D_α is the supersymmetric covariant derivative. This works because D^2 cancels θ^2 in S, while leaving $\bar{\theta}^2$ in $S^\dagger$. Then the integral over $d\bar{\theta}^2$ can be done and the μ-term is generated, while m_3^2 is not. The m_3^2 term would arise at higher loops, or via some other mechanism.

An explicit realization of this mechanism [28] is the following. Suppose a singlet field N acquires a linear term M^2N in the superpotential due to its coupling to the messenger sector. Then the superpotential $W = N(Y^2 + H_uH_d - M^2)$ leads to a minimum with $\langle Y\rangle = M$, $N = 0$. However, by further coupling N to the messenger superfields, *i.e.* $Nq\bar{q}$ etc, a one-loop diagram of messenger fields generates the operator $\frac{1}{16\pi^2}\int d^4\theta NS^\dagger(S^\dagger S)/\langle S^\dagger S\rangle$, which contains $V \sim \frac{1}{16\pi^2}N\langle F_S\rangle^2/\langle S\rangle$. Note that this is the same linear potential generated in the case of Eq. (2.15). This tadpole term induces a VEV for N of order $\langle N\rangle \sim \frac{1}{16\pi^2}\langle F_S\rangle^2/\langle S\rangle/\langle Y\rangle^2$ which is of the order of the weak scale if $\langle Y\rangle^2 = M^2 \sim \langle F_S\rangle$. The Y field plays a crucial role: it *slides* to cancel the F-component VEV of N before the tadpole is added and, after SUSY is broken, its VEV is shifted and leads to $\langle F_N\rangle = m_3^2 \sim \mu^2$, as required by phenomenology. Note that the μ-parameter obtained here can be understood as a consequence of the effective Lagrangian Eq. (2.18), which is generated upon integrating out N and Y before substituting the effect of the VEVs of S.

The necessary linear term (M^2N) in the superpotential for N can be easily generated by the kinetic mixing between N and S or also by other mechanisms, as pointed out in reference [34]. One apparent drawback of this realization is that one needs a set of new fields whose interactions are arranged in a rather special way. Furthermore one would expect the presence of a term proportional to SH_uH_d in the superpotential. This happens because both S and N couple to the messengers, that is, $W \supset Sq\bar{q} + Nq\bar{q}$, and have, therefore, the same quantum numbers. We have already argued that a coupling SH_uH_d has to vanish (see Eq. (2.8)). Finally we point out that this model also suffers from the cancellation problem present in the MSSM (see Section 3).

Dine, Nelson, Nir and Shirman [8] suggest yet another way of generating a μ-term with the introduction of an extra singlet. It was inspired by flavor symmetry models in [24], and resembles a modified version of the NMSSM + Eq. (2.14):

$$W \supset \lambda_n \frac{N^{n+1}}{M_{Pl}^n} H_d H_u + \lambda_m \frac{N^{m+3}}{M_{Pl}^m} + \lambda_p \frac{N^{2+p}}{M_{Pl}^p} S \tag{2.19}$$

where M_{Pl} is the Planck mass. When $m = 2$, $n = 1$ and $p = 2$, it is easy to check that $\mu \sim \lambda_n \sqrt{\langle F_S \rangle}$. We assume the other couplings to be of order 1. It is also easy to see that one would require a very small, carefully chosen coupling λ_n in order to guarantee $\mu \sim 100$ GeV. It is worth noting that this mechanism does not generate an m_3^2 term.

At last we would like to mention another interesting possibility, pointed out by Yanagida [35] and Nilles and Polonsky [36]. Their models utilize the accidental equality $(\Lambda_{\rm DSB}/M_*)^{1/3} \sim (\alpha/4\pi)^2$, where $\Lambda_{\rm DSB} \sim 10^7$ GeV is the scale of dynamical SUSY breaking (DSB) in models with the low-energy GMSB and $M_* = M_{Pl}/\sqrt{8\pi}$ the reduced Planck mass. By introducing a new SUSY-preserving sector with strong gauge dynamics, Yanagida's model generates a VEV for the superpotential which cancels the cosmological constant from the DSB sector. The constant superpotential in turn generates a μ-term of order $\Lambda_{\rm DSB}(\Lambda_{\rm DSB}/M_*)^{1/3} \sim (\alpha/4\pi)^2 \Lambda_{\rm DSB} \sim 1$ TeV. The phenomenology of this model is the same as the previous one (see Eq. (2.19)). The model by Nilles and Polonsky makes use of the Planck-scale suppressed Kähler potential, $\int d^4\theta N(z^* z)/M_*$, where z is a chiral superfield in the DSB sector with an F-component VEV. This operator may be present at the tree-level, but may also be generated by gravitational effects. It generates a tadpole for the singlet N: $V = (\Lambda_{\rm DSB}^4/M_*)N$. Together with the $\frac{k}{3}N^3$ superpotential of the NMSSM, it generates a VEV for N of order $\langle N \rangle \sim (\Lambda_{\rm DSB}^4/M_*))^{1/3}$. Even though these models generate the correct μ-term of order the weak scale in the models with the low-energy GMSB, this would not work for the high-energy GMSB.

We consider that none of the mechanisms outlined above are entirely satisfactory. Most of them require a very specific choice of parameters and the introduction of extra matter at or slightly above the weak scale. Furthermore, most of them have not been studied quantitatively (see, however, Ref. [37]), and there is no guarantee that they indeed generate the correct electroweak symmetry breaking pattern and an experimentally viable spectrum. And last, but not least, there is no study of how natural such a solution is, given that a viable pattern of electroweak symmetry breaking can be generated.

It is, therefore, part of our goal to study the simplest of the models mentioned above in detail. Before that, we would like to review the status of electroweak symmetry breaking in the MSSM, where the μ-term is introduced "by hand." We will point out that, in the case of the GMSB, the current lower bounds on superparticle masses already require an order 10% cancellation between the μ-parameter and the soft SUSY-breaking parameters.

3 The μ-parameter in the MSSM

We reviewed various proposals to generate the μ-parameter in models with the GMSB. In this section, we review how electroweak symmetry breaking occurs in the MSSM, assuming that the μ-parameter and m_3^2 are somehow generated. In particular, we point out a need for an order 10% cancellation between μ-parameter and soft SUSY-breaking parameters in models with the GMSB given the current experimental lower bounds on superparticle masses. Note that the case of the

NMSSM is different because the μ-parameter is generated together with electroweak symmetry breaking and hence the two problems cannot be clearly separated. This will be discussed in the next two sections.

The tree-level Higgs potential in the MSSM is given by

$$V = m_1^2|H_d|^2 + m_2^2|H_u|^2 - m_3^2(H_d H_u + c.c.) + \frac{g_2^2}{8}(H_d^\dagger \vec{\sigma} H_d + H_u^\dagger \vec{\sigma} H_u)^2 + \frac{g'^2}{8}(|H_d|^2 - |H_u|^2)^2, \quad (3.1)$$

where the mass parameters involve both the supersymmetric μ-term and the soft SUSY-breaking terms,

$$m_1^2 = \mu^2 + m_{H_d}^2, \quad (3.2)$$
$$m_2^2 = \mu^2 + m_{H_u}^2. \quad (3.3)$$

In the MSSM, one can show that the vacuum can always be gauge rotated to the following configuration

$$H_d = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad H_u = \begin{pmatrix} 0 \\ v_u \end{pmatrix}. \quad (3.4)$$

The two expectation values need to satisfy $v_d^2 + v_u^2 = v^2 = (174 \text{ GeV})^2$ in order to reproduce the observed M_Z, and it is conventional to parametrize them by $v_d = v\cos\beta$, $v_u = v\sin\beta$. The minimization condition of the potential can be rewritten in the following form:

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}, \quad (3.5)$$
$$2m_3^2 = (2\mu^2 + m_{H_d}^2 + m_{H_u}^2)\sin 2\beta. \quad (3.6)$$

It has been claimed that electroweak symmetry breaking is natural in the MSSM because $m_{H_u}^2$ is easily driven negative due to the presence the top Yukawa coupling in its RG evolution. In models with the minimal GMSB such as the original ones in [7, 8], the boundary condition for the supersymmetry breaking parameters are given by Eqs. (2.5, 2.6, 2.7). A simple one-loop approximation is valid in the case of the low-energy GMSB because of the small logarithm between the messenger scale Λ and the electroweak scale, and one finds

$$m_{H_u}^2(M_Z) \simeq m_{H_d}^2(M_Z) - \frac{6}{16\pi^2} h_t^2 2m_{\tilde{t}}^2 \log\frac{\Lambda}{M_Z}, \quad (3.7)$$

which is always negative.

The need for a cancellation between the μ-parameter and soft SUSY-breaking masses can be seen as follows. Experimental constraints bound the superparticle masses from below, which hence set a lower limit for the ratio $B = \langle F_S\rangle/\langle S\rangle$. Therefore one finds that $|m_{H_u}^2|$ is bounded from below. On the other hand, in order for the observed M_Z^2 to be reproduced, the μ-parameter is constrained by Eq. (3.5). For a moderately large $\tan\beta \gtrsim 2$, $m_{H_d}^2$ can be completely neglected and one finds

$$\frac{M_Z^2}{2} \sim -\mu^2 - m_{H_u}^2. \quad (3.8)$$

This equation requires a cancellation between μ^2 and (negative) $m_{H_u}^2$ to reproduce $M_Z^2/2 \sim (70 \text{ GeV})^2$ correctly. The degree of cancellation is given by $(M_Z^2/2)/\mu^2$.[§]

[§]The degree of cancellation is defined as follows: it is a percentage quantity that measures how much a given input parameter (in this case μ^2) is free to vary before a given output parameter (in this case M_Z^2) changes significantly. Explicitly, the degree of cancellation is $(\mathrm{d}(\log M_Z^2)/\mathrm{d}(\log\mu^2))^{-1}$. This definition corresponds to the inverse of the Barbieri–Giudice function [38].

To determine the lower limit on $|m_{H_u}^2|$, we consider a number of experimental constraints [39]. One is that the gluino must be heavier than 190 GeV, which becomes stronger if the squarks have comparable masses. The second is that the right-handed selectron must be heavier than 80 GeV.¶ For large $\tan\beta$, the right-handed stau may become rather light; we then require $m_{\tilde{\tau}} > 55$ GeV if it decays into tau and a neutralino or gravitino, and $m_{\tilde{\tau}} > 73$ GeV if it does not decay inside the detector. We also considered the lightest chargino to be heavier than 63 GeV. The most recent lower bound on the chargino mass [5] is $m_{\chi^+} \gtrsim 67$ GeV, which leaves our analysis virtually unchanged.

Let us first discuss the case of the minimal low-energy GMSB with small $\tan\beta$ to make the argument clear. Here we consider $\Lambda \sim 10^5$ GeV. The gluino mass constraint requires

$$B > 23 \text{ TeV}. \tag{3.9}$$

This bound itself is independent from the messenger scale. However, the gluino mass bound depends on the mass of the squarks, and it strengthens if the squark masses are comparable to the gluino mass. For the minimal low-energy GMSB, squarks are significantly heavier than the gluino and we can use the bound above. A more stringent constraint is derived from the requirement that the right-handed sleptons are heavier than 80 GeV. Including the one-loop renormalization group evolution and the D-term, we find

$$\begin{aligned} m_{\tilde{e}}^2 &= 2\frac{3}{5}\left(\frac{\alpha_1}{4\pi}\right)^2 B^2 + \frac{2}{11}\left(\left(\frac{\alpha_1(M_{mess})}{\alpha_1(M_Z)}\right)^2 - 1\right) M_1^2(M_Z) + M_Z^2 \sin^2\theta_W \cos 2\beta \\ &= 2.89 \times 10^{-6} B^2 - 0.232 M_Z^2 \cos 2\beta. \end{aligned} \tag{3.10}$$

Therefore we find

$$B > 39 \text{ TeV} \tag{3.11}$$

for the most conservative case $\cos 2\beta = -1$. With this lower bound we find

$$m_{\tilde{t}}^2 > 2\frac{4}{3}\left(\frac{\alpha_3(M_{mess})}{4\pi}\right)^2 B^2 > (430 \text{ GeV})^2. \tag{3.12}$$

Using the one-loop running of $m_{H_u}^2$, we obtain

$$m_{H_u}^2(M_Z) < -(260 \text{ GeV})^2 \tag{3.13}$$

and as a result of the minimization condition,

$$\mu > 250 \text{ GeV}. \tag{3.14}$$

This requires a cancellation of 7% in order to obtain the correct M_Z^2. Even though this level of cancellation is not of immediate concern, this analysis shows the need for a certain amount of cancellation which will become worse as experimental lower bounds on superparticle masses improve.

¶This bound depends on the mass of the neutralino into which the selectron decays. However, since μ turns out to be large, it is *a posteriori* justified to assume that the lightest neutralino is almost pure bino. Then the GMSB predicts the relation between selectron and the bino masses, and hence we have a fairly reliable lower bound.

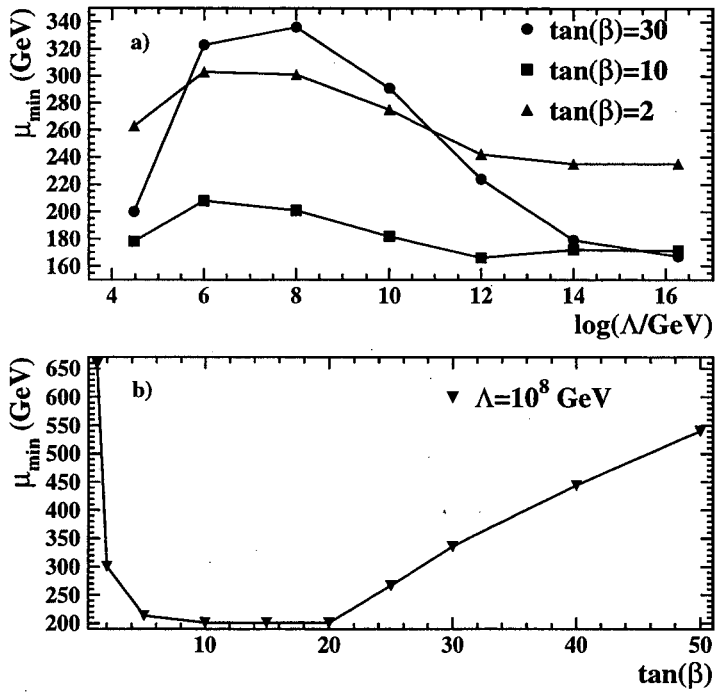

Figure 1: Lower bounds on μ in models with the GMSB subject to the constraint $M_Z = 91$ GeV and to the lower bounds on superparticle masses (see text), (a) as a function of the messenger scale, for $\tan\beta = 2$, 10, and 30, and (b) as a function of $\tan\beta$ for a fixed messenger scale of 10^8 GeV.

As it is clear from the argument above, the actual lower bound on μ depends on the messenger scale and $\tan\beta$. We have studied this issue numerically using the experimental bounds quoted above and found the lowest possible value of μ as a function of the messenger scale. In Fig. 1(a) we present bounds for three values of $\tan\beta$. The lower bound on μ comes from one of the various experimental constraints. For instance, the $\tan\beta = 2$ case is dominated by the lower bound on $m_{\tilde{e}_R}$ up to a messenger scale of 10^{12} GeV, after which the gluino mass bound is more important.[‖] The case of $\tan\beta = 10$ has a similar behavior. The situation is more complex and interesting for $\tan\beta = 30$. For a messenger scale of up to 10^{10} GeV, the stau is the lightest supersymmetric particle (except for the gravitino). It decays inside the detector to tau and gravitino for the lowest messenger scale, but leaves the detector without decaying for higher messenger scales. This stable stau provides the strongest constraint. From messenger scales above $\sim 10^{12}$ GeV the stau decays inside the detector to tau and neutralino. This bound dominates up to $\sim 10^{16}$ GeV, when the gluino bound dominates. The chargino bound is comparable to that of the gluino for the GUT scale ($M_{GUT} = 1.86 \times 10^{16}$ GeV).

In Fig. 1(b) we show the minimum value of μ as a function of $\tan\beta$ for a fixed messenger scale ($\Lambda = 10^8$ GeV). The $\tan\beta$ dependence can be easily understood as follows. Starting from

[‖]We only analyzed the case for one messenger (n=1). For larger n the gluino bound becomes less important and the slepton bounds dominate up to the GUT scale.

low $\tan\beta$, increasing $\tan\beta$ decreases the top Yukawa coupling, and hence $m_{H_u}^2$ receives a less negative contribution from the top-stop loop. Therefore a lower value of μ is allowed. This part is dominated by the $\tilde{e}_R$ bound. However beyond $\tan\beta \sim 20$, the bottom and tau Yukawa coupling become important. In fact, the scalar tau mass is pushed down both because of the loop effect and left-right mixing, and the experimental lower bound on B becomes stronger. Beyond $\tan\beta \sim 30$, the stau does not decay inside the detector for this choice of the messenger scale and the constraint is even more stringent. This in turn leads to a more negative $m_{H_u}^2$ and hence a larger μ.

Combining both the messenger scale dependence and $\tan\beta$ dependence, we conclude that the most conservative current limit is

$$\mu > 160 \text{ GeV}. \tag{3.15}$$

The required cancellation between μ^2 and soft SUSY-breaking parameters in order to reproduce the observed M_Z is $M_Z^2/2\mu^2 = 16\%$. Note that this level of cancellation is the absolute minimum, and a more accurate cancellation is required for most of the parameter space.

In the case of minimal supergravity models, where all scalars have the universal SUSY-breaking mass-squared m_0^2, all gauginos have mass $M_{1/2}$ and all A-terms are given by $\mathcal{A}_f^{ij} = A_0\lambda_f^{ij}$ for $f = u, d, l$, at the GUT scale, the situation appears to be much better. The renormalization group equations can be solved numerically for each choice of $\tan\beta$. As an example we take $\tan\beta = 2$ and find

$$m_{H_1}^2 = m_0^2 + 0.50M_{1/2}^2, \tag{3.16}$$

$$m_{H_2}^2 = -0.32m_0^2 - 2.49M_{1/2}^2 - 0.05A_0^2 - 0.20M_{1/2}A_0, \tag{3.17}$$

$$m_{\tilde{e}}^2 = m_0^2 + 0.15M_{1/2}^2. \tag{3.18}$$

By requiring $m_{\tilde{e}} > 80$ GeV and $M_{1/2} > 60$ GeV (this is a rough bound inferred from the gluino bound $M_{\tilde{g}} \gtrsim 190$ GeV), we find $\mu > 82$ GeV. This basically does not require any cancellation, since $M_Z^2/2\mu^2 = 65\%$.

The situation can be somewhat ameliorated in the MSSM if there is a Fayet–Illioupoulos D-term for the $U(1)_Y$ gauge group. Such a D-term is known to arise in many ways, such as kinetic mixing of the $U(1)_Y$ and $U(1)_{mess}$ gauge fields [40]. The running of all the parameters remains the same except that one adds another contribution from the D_Y at the weak scale. If the sign is appropriate, it increases $m_{\tilde{e}}^2 \to m_{\tilde{e}}^2 + D_Y$ and $m_{H_u}^2 \to m_{H_u}^2 + \frac{1}{2}D_Y$ (less negative) while decreasing $m_{H_d}^2 \to m_{H_d}^2 - \frac{1}{2}D_Y$. All of these help push the parameters relevant for electroweak symmetry breaking in the right direction. Larger $m_{\tilde{e}}^2$ reduces the lower bound on $\langle F_S\rangle/\langle S\rangle$, and a less negative $m_{H_u}^2$ is also welcome. Therefore the sensitivity to μ (required cancellation between μ and soft SUSY-breaking parameters) in the MSSM can be improved in the presence of a D_Y with the appropriate sign.

We will see in the next two sections that the situation in the NMSSM is much worse. There is no phenomenologically viable solution to electroweak symmetry breaking. One can modify the model to generate a large negative mass-squared for the singlet field and then find a viable solution. This solution also requires a cancellation among parameters which has the same origin as the cancellation present in the MSSM. We will also see that the addition of the Fayet–Illiopoulos D-term does not improve the situation within the NMSSM.

4 The NMSSM with the GMSB

In this section we study the feasibility of implementing the GMSB in the framework of the Next-to-Minimal Supersymmetric Standard Model (NMSSM). We begin our presentation by introducing the NMSSM: its particle content, superpotential, and soft SUSY-breaking terms. We briefly review the major steps in our analysis: the boundary conditions for the breaking terms, the RG evolution, and the minimization of the weak-scale one-loop effective potential. We then describe the results of a numerical scan of a large portion of the model's parameter space. We find that it is impossible to evade the present-day experimental constraints. We further strengthen this argument by providing a semi-analytical explanation for the inevitability of this conclusion.

4.1 The NMSSM

The NMSSM represents an attempt to solve the μ-problem of the MSSM in the simplest and most direct way: the spectrum of the MSSM is augmented by a gauge singlet superfield N, which couples to $H_d H_u$ and plays the role of the μ-term once it develops a nonzero vacuum expectation value [15]. The original μ-term is banned from the theory so that there are no dimensionful parameters left in the superpotential.

The VEV of the scalar component of N is determined by minimizing the scalar potential with respect to H_d, H_u, and N simultaneously. It is natural to expect the VEVs to be of the same order of magnitude for all three fields, thus generating an effective μ-parameter of order the weak scale, as required by phenomenology.

The complex scalar N introduces two additional degrees of freedom to the Higgs sector. Therefore, the particle spectrum of the NMSSM contains three CP–even Higgs scalars, two CP–odd Higgs scalars, and one charged Higgs scalar. Immediately, there is a problem: one of the pseudoscalar Higgs bosons is massless. This happens because the superpotential $W = \lambda N H_d H_u$ has a Peccei-Quinn symmetry $N \to N e^{i\alpha}, H_d H_u \to H_d H_u e^{-i\alpha}$. This symmetry is spontaneously broken by the VEVs of the fields, making one of the pseudoscalars massless.

The standard solution to this problem is to introduce a term cubic in N, which explicitly breaks the symmetry mentioned above. This term is allowed by the gauge symmetries of the model and does not contain a dimensionful coupling constant, so it is generically expected to be present in the superpotential. One, however, still has to worry about a light pseudoscalar Higgs boson. As we will show shortly, its mass can also be small because of the presence of a different (approximate) $U(1)$ symmetry.

Overall, the only change made to the MSSM superpotential is the following:

$$\mu H_d H_u \longrightarrow \lambda N H_d H_u - \frac{k}{3} N^3, \tag{4.1}$$

while the corresponding change to the soft SUSY-breaking part of the potential is:

$$-m_3^2 (H_d H_u + c.c.) \longrightarrow -(\lambda A_\lambda N H_d H_u + \frac{1}{3} k A_k N^3 + h.c.) + m_N^2 |N|^2. \tag{4.2}$$

One can determine the VEVs of the Higgs fields H_d, H_u, and N by minimizing the scalar potential, which at the tree-level consists of the F-terms, D-terms, and soft SUSY-breaking

terms:

$$
\begin{aligned}
V_{Higgs}^{tree} &= V_F + V_D + V_{soft}, \\
V_F &= |\lambda H_d H_u - kN^2|^2 + \lambda^2 |N|^2 (|H_d|^2 + |H_u|^2), \\
V_D &= \frac{{g_2}^2}{8}(H_d^\dagger \vec{\sigma} H_d + H_u^\dagger \vec{\sigma} H_u)^2 + \frac{g'^2}{8}(|H_d|^2 - |H_u|^2)^2, \\
V_{soft} &= m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 + m_N^2 |N|^2 - (\lambda A_\lambda H_d H_u N + h.c.) - \left(\frac{k}{3} A_k N^3 + h.c.\right).
\end{aligned} \tag{4.3}
$$

An important fact to notice is that both V_F and V_D remain unchanged when H_d, H_u, and N are all rotated by the same phase. In fact, only the soft SUSY-breaking A-terms are not invariant under this transformation. This can be potentially dangerous, because we, in general, consider the A-terms to be zero at the messenger scale, and their sizes at the weak scale are determined by the RG evolution. If the generated values of A_λ and A_k are not large enough, our scalar potential has an approximate $U(1)$ symmetry. This symmetry is spontaneously broken by the vacuum expectation values of the Higgs fields, and, as before, we have to worry about a light pseudoscalar Higgs boson.

We denote the VEVs of the neutral components of the Higgs fields by v_d and v_u, as in Section 3, and the VEV of the singlet field by x:

$$
\langle N \rangle = x. \tag{4.4}
$$

As a function of these VEVs, the potential has the form

$$
\begin{aligned}
V_{neutral}^{tree} &= |\lambda v_d v_u - kx^2|^2 + \lambda^2 |x|^2 (|v_d|^2 + |v_u|^2) + m_{H_d}^2 |v_d|^2 + m_{H_u}^2 |v_u|^2 + m_N^2 |x|^2 - \\
&\quad -(\lambda A_\lambda v_d v_u x + h.c.) - (\frac{k}{3} A_k x^3 + h.c.) + \frac{g_2^2 + g'^2}{8}(|v_d|^2 - |v_u|^2)^2.
\end{aligned} \tag{4.5}
$$

It is well known that some of the Higgs boson masses receive significant contributions from radiative corrections. In our numerical analysis we account for that by employing the one-loop effective potential

$$
V_{neutral}^{1-loop}(v_i) = V_{neutral}^{tree}(v_i, \mu) + \frac{1}{64\pi^2} \mathrm{STr} \mathcal{M}^4(v_i) \left(\log \frac{\mathcal{M}^2(v_i)}{\mu^2} - \frac{3}{2} \right). \tag{4.6}
$$

In this expression $\mathcal{M}^2(v_i)$ is a field-dependent scalar mass-squared matrix, and μ is the $\overline{\mathrm{MS}}$ renormalization scale. As we have indicated explicitly, the values of the various parameters entering V^{tree} depend on the choice of this scale. To the leading order this dependence is canceled when the second term on the right hand side of Eq. (4.6) is included, and the result of minimizing V^{1-loop} is less sensitive to the choice of the scale where one stops running the RG equations. (Canceling out this dependence completely would require calculating radiative corrections to all orders.)

The matrix $\mathcal{M}^2$ depends on the field VEVs v_i through the Yukawa couplings of the Higgs fields to various other particles. What plays a crucial role here is not the absolute values of the masses, but rather the rate of their change as one changes v_i. Therefore, the most important contribution comes from the field-dependent masses of the top quark and squarks, which have

the largest Yukawa coupling. Denoting their mass eigenvalues by m_t, $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ respectively, the contribution to V^{1-loop} from radiative corrections due to these states is

$$\Delta V = \frac{3}{32\pi^2}\left[m_{\tilde{t}_1}^4(v_i)\left(\log\frac{m_{\tilde{t}_1}^2(v_i)}{\mu^2}-\frac{3}{2}\right)+m_{\tilde{t}_2}^4(v_i)\left(\log\frac{m_{\tilde{t}_2}^2(v_i)}{\mu^2}-\frac{3}{2}\right)\right.$$
$$\left.-2m_t^4(v_i)\left(\log\frac{m_t^2(v_i)}{\mu^2}-\frac{3}{2}\right)\right]. \tag{4.7}$$

4.2 Numerical Analysis

In models with the GMSB the values of the soft SUSY-breaking terms are specified at the messenger scale by Eqs. (2.5), (2.6) and (2.7). Their values at the weak scale can be determined by solving the RG equations given in Appendix A.

The model has five input parameters: h_t, λ, k, B, and n. (Note that the only dimensionful input parameter is B, and its magnitude will determine the overall scale of the VEVs and the soft SUSY-breaking masses.) There are, however, two constrains which must be satisfied at the weak scale: $v \equiv \sqrt{v_d^2+v_u^2} = 174$ GeV and $h_t v_u = 165 \pm 5$ GeV.** A common approach is to use the minimization conditions and RG equations to solve for the inputs, given a phenomenologically allowed set of weak–scale outputs. In the case of a high messenger scale, however, no easily invertible solution for the RG equations is available. Instead, we simply choose to tackle the problem numerically. After running down the RG equations and minimizing the Higgs potential once, we iterate this procedure, each time adjusting the value of the parameter B to fix the overall scale of the VEVs and masses, while simultaneously changing the dimensionless couplings to correctly reproduce the top quark mass. This iteration process, in fact, converges fairly quickly.

Using the procedure above, we perform a numerical scan of a large portion of the parameter space. We study the low-energy particle spectrum for various messenger scales Λ, numbers of messengers n, and values of the couplings λ and k. It is interesting to note that it is very easy to generate non-zero VEVs for H_d, H_u, and N, even when m_N^2 is a small *positive* number. This is because the terms $|\lambda v_d v_u - kx^2|^2$ and $A_\lambda \lambda v_d v_u x$, when $\lambda v_d v_u$ and k are of the same sign, both "push" the VEV of the real component of the singlet away from the origin. Unfortunately, we find that, for any choice of values of the input parameters, there are always particles with unacceptably small masses. To illustrate the situation, we present in Table 1 our numerical results for several representative points in different "corners" of the parameter space. The first two points represent the typical situation for the case of the low-energy GMSB, the next two are representative of the case of the high-energy GMSB, and the last one explores the extreme case of $\Lambda = 10^{15}$ GeV. Points 1 and 3 have relatively large values of k, while points 2 and 4 have $k \ll 1$. Notice that in the table we did not consider a similar limiting case for λ. This is not a coincidence. It turns out that, for $\lambda \lesssim 0.2$, the dominant term in the potential is V_D, and $\tan\beta$ is forced to values very close to one. In this case, in order to correctly reproduce the top quark mass, one is forced to choose h_t at the weak scale such that h_t hits the Landau pole below the GUT scale. We have chosen to list only the cases where the couplings in the superpotential remain perturbative up to the GUT scale. We make, however, no such assumption in our analysis in the next subsection.

**Notice that this number is not equal to the top quark pole mass, the experimentally measured quantity, because of QCD corrections. The relationship between the two is given, at 1-loop, by $m_{pole} = \bar{m}(1+\frac{4}{3}\frac{\alpha_s}{\pi})$.

Table 1: The numerically determined NMSSM parameters for five sample points in the parameter space. Here m_{h_i} and m_{A_i} refer to the eigenvalues of the scalar and pseudoscalar Higgs mass matrices respectively, and $m_{\tilde{e}}$ denotes the mass of the right-handed selectron. The values of λ, k, and h_t are given at the weak scale. All the other quantities have been defined earlier in the text.

	Input Parameters					
point	Λ (GeV)	λ	k	B (TeV)	n	h_t
1	5×10^4	0.25	0.1	6.4	1	1.12
2	10^5	0.28	3×10^{-4}	3.6	3	1.08
3	10^{12}	0.32	0.3	0.99	10	1.07
4	10^{12}	0.25	3×10^{-4}	6.0	1	1.11
5	10^{15}	0.28	0.3	6.9	1	1.07

	Soft SUSY-breaking Parameters at the Weak Scale				
point	$m^2_{H_u}$ (GeV^2)	$m^2_{H_d}$ (GeV^2)	m^2_N (GeV^2)	A_λ (GeV)	A_k (GeV)
1	-2.4×10^3	5.3×10^2	4.6	-1.5	-4.0×10^{-3}
2	-2.8×10^3	5.7×10^2	6.8	-2.6	-6.2×10^{-3}
3	-3.1×10^3	4.8×10^2	29	-11.4	-0.15
4	-2.5×10^3	6.8×10^2	12	-8.0	-0.11
5	-2.9×10^3	1.0×10^3	-8.1	-9.4	-6.0×10^{-3}

	Field VEVs		Particle Masses			
point	$\tan\beta$	x (GeV)	M_3 (GeV)	$m_{\tilde{e}}$ (GeV)	m_{h_i} (GeV)	m_{A_i} (GeV)
1	1.59	-3.7	61	32	85, 39, 35	51, 1.8
2	1.84	-3.7	103	35	87, 48, 38	48, 0.2
3	1.97	-40	94	36	87, 53, 28	76, 25
4	1.63	-14	57	34	85, 43, 37	44, 0.5
5	1.88	-49	66	40	88, 50, 27	71, 24

It can be seen that, in all the cases presented in Table 1, there are particles with unacceptably small masses. The result for the low-energy GMSB is not new and has been known for several years [6]. On the other hand, the situation with a high messenger scale had not been quantitatively studied in the literature to date. One expected feature that we indeed see in points 3 and 5 is the increase of the pseudoscalar Higgs boson mass with Λ. This happens because the magnitude of A_λ, generated by running the RG equations, increases with the messenger scale, and it is A_λ that breaks the $U(1)$ symmetry of the potential, as discussed before. Another result that could have been anticipated is the smallness of the mass of the light pseudoscalar Higgs when $k \ll 1$ (points 2 and 4). This is due to the Peccei-Quinn symmetry, which is restored in this limit. What is surprising is that raising the messenger scale by 10 orders of magnitude does not bring any other significant changes to the particle spectrum. The masses of the gluino, right-handed selectron, and scalar Higgs boson still remain small.

4.3 Analytical Considerations

In this subsection we present a rather simple semi-analytical argument which explains why there can be no phenomenologically acceptable solution to the NMSSM with the GMSB. We show that if one assumes that such a solution exists, one arrives at a contradiction. We also explain some of the features of the numerical solutions presented in the previous subsection.

Suppose that for some point in the parameter space an acceptable solution exists. The problem to be addressed is the smallness of the selectron, gluino, and Higgs masses. We choose to base our analysis on the right-handed selectron mass constraint. The magnitude of $m_{\tilde{e}}$ is directly proportional to the size of the B-parameter. In our numerical procedure the value of B is chosen in such a way that $v = 174$ GeV. A typical value of B obtained in this way yields a very small selectron mass ($m_{\tilde{e}} \sim 35$ GeV), gluino mass ($M_3 \lesssim 100$ GeV), and soft SUSY-breaking masses for the Higgs bosons ($m^2_{H_u} \sim -3000$ GeV^2, $m^2_{H_d} \sim 500$ GeV^2).

It is, therefore, obvious that the only chance of obtaining an acceptable value for the selectron mass is to raise B, by a factor of three or more, and try to arrange the other parameters in such a way that $\sqrt{v_d^2 + v_u^2}$ remains 174 GeV. Since B feeds into all soft SUSY-breaking masses, their absolute values will also increase. For example, imposing $m_{\tilde{e}} > 80$ GeV forces $m^2_{H_u} < -(215 \text{ GeV})^2$ for a messenger scale of 10^{16} GeV. For different messenger scales the bound becomes even more stringent, as shown in Fig 2(a).

To determine the consequences of raising the soft SUSY-breaking masses, we analyze the Higgs potential (Eq. (4.5)). The extremization conditions at tree level are

$$\frac{\partial V^{tree}_{neutral}}{\partial v_d} = 2(\lambda v_d v_u - kx^2)\lambda v_u + 2\lambda^2 x^2 v_d + 2m^2_{H_d} v_d - 2A_\lambda \lambda v_u x + \frac{g'^2 + g_2^2}{4} 2v_d(v_d^2 - v_u^2), \quad (4.8)$$

$$\frac{\partial V^{tree}_{neutral}}{\partial v_u} = 2(\lambda v_d v_u - kx^2)\lambda v_d + 2\lambda^2 x^2 v_u + 2m^2_{H_u} v_u - 2A_\lambda \lambda v_d x + \frac{g'^2 + g_2^2}{4} 2v_u(v_u^2 - v_d^2), \quad (4.9)$$

$$\frac{\partial V^{tree}_{neutral}}{\partial x} = 2(\lambda v_d v_u - kx^2)(-2kx) + 2x\lambda^2(v_d^2 + v_u^2) + 2m_N^2 x - 2A_\lambda \lambda v_d v_u - 2kA_k x^2. \quad (4.10)$$

The first two equations (Eqs. (4.8) and (4.9)) closely resemble the corresponding ones in the MSSM case. In fact, the only difference in the NMSSM is the presence of the first term on the right-hand side of Eq. (4.8) and Eq. (4.9). This term originates from $|\frac{\partial W}{\partial x}|^2 = |\lambda v_d v_u - kx^2|^2$ and is, therefore, absent in the MSSM. Dividing Eq. (4.8) by v_u, Eq. (4.9) by v_d, and subtracting the

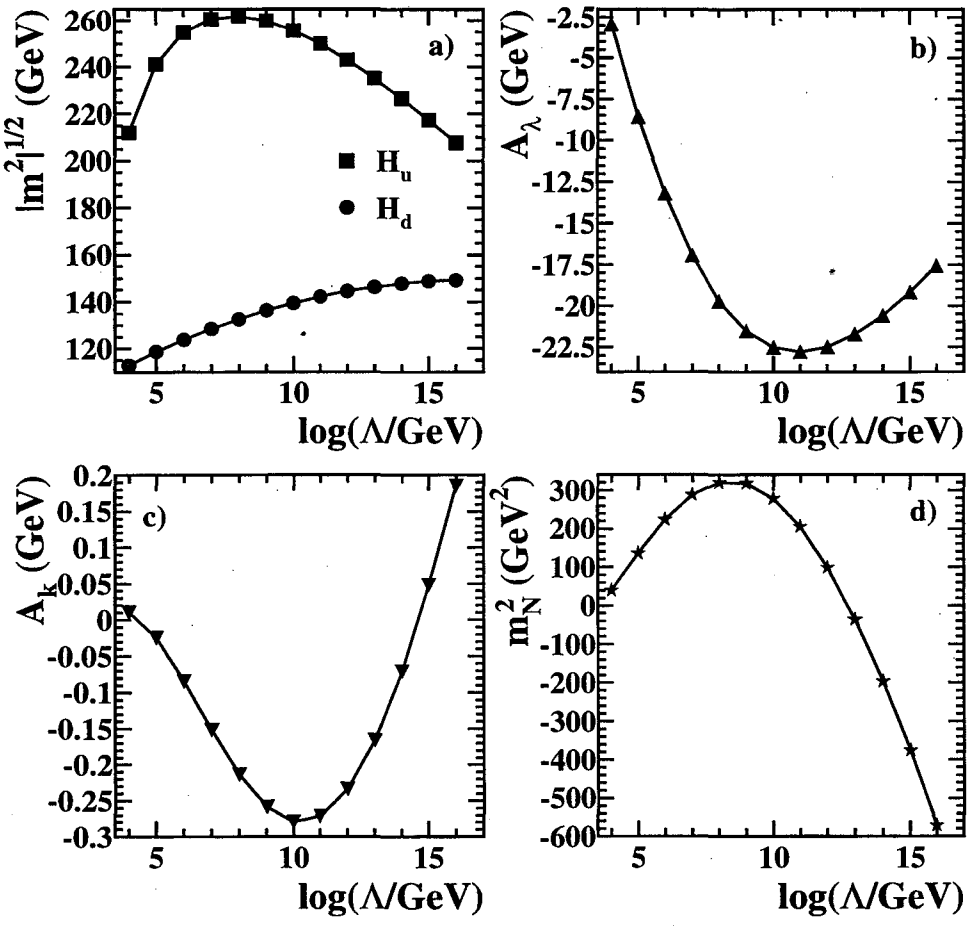

Figure 2: (a) Lower bounds on $|m^2_{H_u}|^{1/2}$ and $|m^2_{H_d}|^{1/2}$ as a function of the messenger scale Λ from the selectron mass constraint $m_{\tilde{e}} > 80$ GeV. Here $n = 1$, $h_t = 1.07$, $k = 0.3$ and $\lambda = 0.29$ at the weak scale. These bounds do not change for different values of k or λ. The other plots show typical values of (b) A_λ, (c) A_k, and (d) m^2_N, for the same choice of parameters that yielded (a). The values of these parameters do not change significantly for different values of k or λ.

two expressions, we can cancel out this term. As a result, we obtain

$$\lambda^2 x^2 = -\frac{M_Z^2}{2} - \frac{1}{2}(m_{H_d}^2 + m_{H_u}^2) - \frac{1}{2}\frac{m_{H_d}^2 - m_{H_u}^2}{\cos 2\beta} = -\frac{M_Z^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}. \tag{4.11}$$

Note that this equation is identical to Eq. (3.5) with $\mu \equiv \lambda x$. To obtain the NMSSM analog of Eq. (3.6) we divide Eq. (4.8) by v_d, Eq. (4.9) by v_u and *add* the two. Solving for $A_\lambda \lambda x$ ($A_\lambda \lambda x \equiv m_3^2$) we find:

$$A_\lambda \lambda x = (m_{H_d}^2 + m_{H_u}^2 + 2\lambda^2 x^2)\frac{\sin 2\beta}{2} + \lambda(\lambda v_d v_u - k x^2). \tag{4.12}$$

Eqs. (3.6) and (4.12) differ only by the contribution from $|\frac{\partial W}{\partial x}|^2$.

Eq. (4.11) states that the value of the effective μ-parameter generated in this model is subject to a rather stringent bound: $\lambda^2 x^2 > -m_{H_u}^2 - M_Z^2/2$, which, if one imposes $m_{H_u}^2 < -(212\ \text{GeV})^2$, translates into $\lambda x > 200$ GeV. Notice that the origin of this bound is the same as of the bound on the size of the μ-parameter derived in Section 3, since the condition given by Eq. (4.11) is the same in both cases. In the present case, however, the bound is stronger because $\tan\beta$ is no longer a free parameter but is determined by minimizing the Higgs potential.

So far we have only looked at the first two extremization conditions. We now turn our attention to Eq. (4.10). Solving for x^2 in Eq. (4.11), one can rewrite Eq. (4.10) as

$$2\frac{k^2}{\lambda^2}\left(\frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2}\right) = \lambda v^2(k \sin 2\beta - \lambda) - m_N^2 + A_\lambda \lambda v^2 \frac{\sin 2\beta}{2x} + k A_k x. \tag{4.13}$$

While we have shown that phenomenology requires the expression in parenthesis on the left-hand side to be larger than $(200\ \text{GeV})^2$, the terms on the right-hand side are all much smaller, because m_N^2, A_λ, and A_k are zero at the messenger scale and the effects of the RG running are relatively small (see Fig. 2). This means that the above equation can never be satisfied unless $k \ll \lambda$.

An immediate consequence of the $k \to 0$ limit is that the mass of the lightest pseudoscalar Higgs goes to zero, as it becomes a Nambu-Goldstone boson. (It is for this reason that k was introduced in the first place.) Furthermore in the limit of large μ and small k the determinant of the scalar Higgs mass-squared matrix becomes negative, which means that the extremum point given by Eqs. (4.8-4.10) ceases to be a minimum. To show this we first derive a relationship between k and $\sin 2\beta$. That relationship can be derived from Eq. (4.11) and Eq. (4.12). Neglecting M_Z^2, $A_\lambda \lambda x$, and $\lambda^2 v^2$ in comparison to $m_{H_d}^2$ and $m_{H_u}^2$, we find that

$$\sin 2\beta = \frac{k}{\lambda}\frac{1}{1 + r^2\left(\frac{k}{\lambda}\right)^2}\left(1 + r\sqrt{1 + (r^2 - 1)\left(\frac{k}{\lambda}\right)^2}\right). \tag{4.14}$$

Here $r \equiv -(m_{H_d}^2 + m_{H_u}^2)/(m_{H_d}^2 - m_{H_u}^2)$.[††] For $k \ll \lambda$, Eq. (4.14) reduces to

$$\sin 2\beta \approx 2\frac{k}{\lambda}\left[\frac{-m_{H_u}^2}{m_{H_d}^2 - m_{H_u}^2}\right]. \tag{4.15}$$

[††]In deriving Eq. (4.14) it was necessary to assume that $\sin 2\beta > k/\lambda$. This translates into two requirements: $r > 0$ and $k/\lambda < 1$. We conclude that for large soft SUSY-breaking Higgs masses-squared it is necessary to have $\lambda > k$.

Equipped with the last result, we consider the determinant of the scalar Higgs mass-squared matrix. The full expression for it is given in Appendix B; here we only need to identify the leading terms. We are interested in the case $\mu > \lambda v$, and, as we have argued before, the soft trilinear couplings A_λ and A_k can be neglected. For this reason, the dominant terms will be the ones containing the highest power of μ:

$$\det \mathcal{M}^2_{scalar} \simeq \frac{2v^2\mu^4}{\lambda^3 \sin(2\beta)} \left(-4k\lambda^4 + k^3\bar{g}^2 + k^3\cos(4\beta)\bar{g}^2 + 8k^2\lambda^3\sin(2\beta)\right) . \tag{4.16}$$

That these are indeed the largest terms was checked numerically.

Taking into account the fact that k and $\sin 2\beta$ are proportional to each other for small k, one can easily see that, in the limit $k \to 0$, the first term dominates and the determinant is negative.[‡‡]

This completes our argument, and we are now able to state that there can be no phenomenologically viable solution in the context of the NMSSM. We could have also based our argument on the gluino mass bound. The experimental constraint $M_3 > 190$ GeV translates into the requirement $m^2_{H_u} < -(212 \text{ GeV})^2$ (assuming $n = 1$), and the rest of the argument follows unchanged. Notice, however, that the bound on $m^2_{H_u}$ weakens if the number of messenger fields is taken to be very large.

We now turn to the issue of interpreting the numerical results of the previous subsection. We would like to understand, for instance, why the values of the singlet VEV x in Table 1 are always smaller than the VEVs of the Higgs doublets and, furthermore, why x is only several GeV for a low messenger scale.

The answer comes from considering the extremization condition for x:

$$2k^2x^3 + \lambda\left(\lambda - k\sin(2\beta)\right)v^2x - \frac{v^2\lambda\sin(2\beta)A_\lambda}{2} \simeq 0 , \tag{4.17}$$

where we omitted the terms m^2_N and kA_kx ($|m^2_N| \ll \lambda^2v^2$ for all the points in the table). For most of the parameter space the cubic term in x can also be neglected, giving

$$x \simeq \frac{A_\lambda}{\lambda}\frac{\sin(2\beta)}{2\left(1 - \frac{k}{\lambda}\sin(2\beta)\right)} . \tag{4.18}$$

Thus the smallness of x is related to the fact that A_λ is small. The above approximation holds as long as

$$A^2_\lambda < \lambda^2v^2\frac{\lambda^2}{k^2}\frac{2\left(1 - \frac{k}{\lambda}\sin 2\beta\right)^3}{\sin^2 2\beta} , \tag{4.19}$$

which is not satisfied only for point 5 in Table 1. For point 5 the value of x can be approximated by

$$x \simeq \left(A_\lambda v^2\frac{\lambda\sin 2\beta}{4k^2}\right)^{1/3} . \tag{4.20}$$

Again $x < v$ and therefore $\lambda x \ll 175$ GeV.

[‡‡]Because $\sin 2\beta \propto k/\lambda$ there is no ambiguity with sign redefinitions of λ or k in Eq. (4.16).

Knowing that x is small in this model we can derive another interesting relation. Neglecting all the terms containing x in Eq. (4.12), we obtain:

$$\lambda^2 v^2 \simeq -(m_{H_d}^2 + m_{H_u}^2)\,. \tag{4.21}$$

This explains why the values of the soft SUSY-breaking masses for the Higgs bosons are so similar for very different values of the messenger scale.

Finally, we can say a few words about the scalar Higgs boson masses. In the limit of small x (and hence small μ), the dominant term in the determinant of the scalar Higgs mass-squared matrix (see Appendix B) is

$$\det \mathcal{M}_{scalar}^2 \simeq \frac{3A_\lambda v^6 \lambda^4 \bar{g}^2}{32\mu \sin(2\beta)}\,. \tag{4.22}$$

Taking into account the fact that, for small x, $\mu \equiv \lambda x \sim A_\lambda$ (see Eq. (4.18)), the equation above gives:

$$m_{h_1}^2 m_{h_2}^2 m_{h_3}^2 \sim \frac{3v^6 \lambda^4 \bar{g}^2}{32 \sin(2\beta)}\,. \tag{4.23}$$

This explains why changes in the messenger scale have almost no effect on the product of the scalar Higgs boson masses (see Table 1), as long as λ is unchanged. For $\sin(2\beta) \sim 0.8 - 0.9$, which is what we typically find in this case, Eq. (4.23) gives a "geometrical average" value of the scalar Higgs boson mass of only about 50 GeV. This means that, as long as x is small, the model necessarily yields phenomenologically unacceptable Higgs boson masses.

5 Possible Modifications to the NMSSM

In this section we reexamine the expressions derived in Section 4 and attempt to modify the NMSSM to make it phenomenologically viable. We study several possibilities and comment on the problems that arise. Overall, we find none of these possibilities entirely satisfactory.

5.1 Extra Vector-like Quarks

We want to modify the NMSSM in a way that allows one to avoid the conclusions of Section 4. Recall that the crucial step in our analysis there was the observation that Eq. (4.13) could not be satisfied: the left-hand side was always greater than the right-hand side. To obtain a consistent solution one has to somehow make both sides equal. One possibility is to make m_N^2 of the same order of magnitude (and sign) as $m_{H_u}^2$. That could be accomplished by coupling the singlet to some new fields and arranging the parameters in such a way that the SUSY-breaking mass-squared of the singlet is driven sufficiently negative. This idea was first proposed by Dine and Nelson in Ref. [6], who introduced new color-triplet fields q' and $\bar{q}'$ and coupled them to N. The corresponding superpotential is

$$W = h_u Q H_u u^c + h_d Q H_d d^c + h_e L H_d e^c + \lambda N H_d H_u - \frac{1}{3} k N^3 + \lambda_q N q' \bar{q}'\,. \tag{5.1}$$

According to Eq. (2.6), the scalar components of q' and $\bar{q}'$ acquire large SUSY-breaking masses, which can drive m_N^2 sufficiently negative.

Agashe and Graesser in Ref. [32] did a quantitative study of this scenario for the case of the low-energy GMSB. They showed that it is indeed possible to generate a large negative m_N^2, in the range $-(150\text{ GeV})^2$ to $-(200\text{ GeV})^2$, and further demonstrated that, with m_N^2 of this magnitude, one can choose the input parameters in such a way that $v = 174$ GeV and all experimental constraints are evaded. They also pointed out that in this scenario the input parameters need to be fine-tuned in order to reproduce the above value of v. In what follows we give a set of input parameters that yields an acceptable particle spectrum, and then proceed to analyze the sensitivity of the Higgs boson VEVs to the NMSSM coupling constants. We clarify the origin of this sensitivity and also extend the analysis to the case of the high-energy GMSB.

As an example of an allowed solution, we consider the case of the low-energy GMSB with $B = 50$ TeV, $n = 1$, and $\Lambda = 100$ TeV. For $m_N^2 = -(190\text{ GeV})^2$, to correctly reproduce M_Z and m_t we take h_t=0.99, $k = -0.045$ and λ=0.11 at the weak scale. We find that $\tan\beta$ equals -2.9 for this point. Because the magnitude of the product Bn is now quite large, the masses of the gluino and right–handed selectron are safe: $M_3 = 477$ GeV, $m_{\tilde{e}}$ =93 GeV. The vacuum expectation value of the singlet is also large, x=2.97 TeV, which, as was argued earlier, is required by Eq. (4.11). The eigenvalues of the scalar Higgs mass matrix are 404, 270, and 90 GeV, and those of the pseudoscalar Higgs mass matrix are 400 and 6.7 GeV. The last number appears alarmingly small at first sight but, as shown in Ref. [32], has not been excluded. The reason is that the corresponding eigenstate a is almost a pure singlet:

$$|a\rangle = 0.031|H_d\rangle - 0.011|H_u\rangle - 0.999|N\rangle \tag{5.2}$$

The quantitative criterion given in Ref. [32], based on the constraint from the $\Upsilon \to a\gamma$ decay, is

$$\frac{\sin 2\beta \tan\beta}{\sqrt{\left(\frac{x}{250\text{ GeV}}\right)^2 + \sin^2 2\beta}} < 0.43, \tag{5.3}$$

and for the parameter set above the left-hand side equals 0.15.

In this scheme it is, therefore, possible to find a point in the parameter space which leads to a phenomenologically viable solution. Unfortunately, as we already mentioned, this solution is very sensitive to the choice of the superpotential coupling constants λ and k. In the remainder of this subsection we discuss this issue in detail.

The values of the parameters for the set that we have just described had to be chosen in such a way that the top quark and Z–boson masses were fixed at their known experimental values. It is interesting to investigate what values of M_Z would be predicted for a generic choice of the parameters. In Fig. 3 we plot the magnitude of the quantity $v \equiv \sqrt{v_d^2 + v_u^2}$ as a function of λ and k. The figure shows that small changes in both λ and k lead to large changes in v. This is very similar to the situation in the MSSM which was considered in Section 3. There we showed that the value of the μ-parameter had to be chosen very carefully in order to yield the correct value of v. In the present case, the points in the parameter space that correspond to values of v around 174 GeV lie in a very thin band on the $\lambda - k$ plane. Also notice that, for this range of λ and k, the slope is the steepest. (See Appendix C for comments on this point.)

It is possible to perform the same type of analysis for a higher messenger scale. The same problem is found in that case as well. In Fig. 4 we plot the dependence of v on λ for fixed values of k. For comparison, the curve for $\Lambda = 10^{12}$ GeV is plotted next to the curve for $\Lambda = 100$ TeV. From the slopes of these curves one can determine the degree of sensitivity with respect to λ,

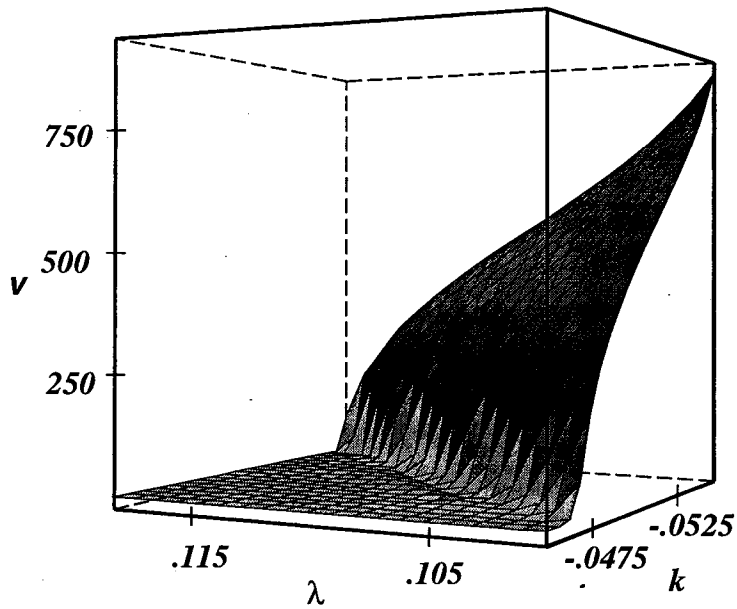

Figure 3: The value of $v \equiv \sqrt{v_d^2 + v_u^2}$ as a function of λ and k. The inputs are $n = 1$, $m_N^2 = -(190\ \text{GeV})^2$, $B = 50$ TeV, $\Lambda = 100$ TeV, $h_t = 0.99$.

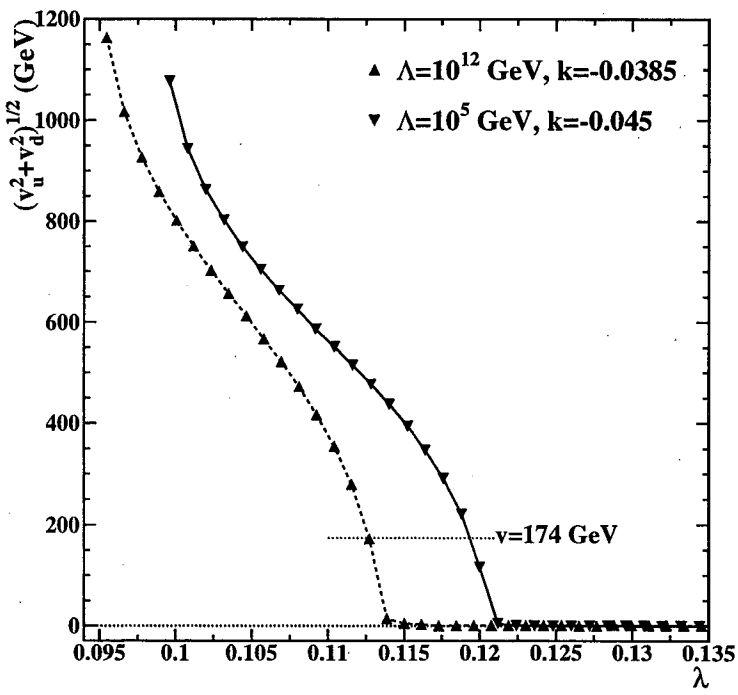

Figure 4: The dependence of v on the value of λ for the high- and low-energy GMSB. The other input parameters are the same as in Fig. 3.

using the definition in Section 3. The degree of sensitivity, given by $\mathrm{d}(\log v)/\mathrm{d}(\log \lambda)$, is 2% for the low-energy curve and 1% for the high-energy curve. Our numerical results agree with those in Ref. [32] for the low-energy GMSB if the same inputs parameters are used.

In order to understand this behavior, we once again turn to the extremization conditions Eqs. (4.8-4.10). First, we present some qualitative observations. Recall that phenomenology requires $|x|$ to be rather large (of the order $\sqrt{|m_{H_u}^2|}/\lambda \gtrsim 1$ TeV), while v has to remain "small" ($v = 174$ GeV) to correctly reproduce M_Z. As a result, the terms containing high powers of x and the terms containing m_i^2 ($i = H_d, H_u, N$) dominate, while the terms with v_u and v_d are not fixed, and have to absorb the residual difference between the dominant terms. Therefore, small percentile changes in the dominant terms can result in large percentile changes in the Higgs boson VEVs. This is to be contrasted with the situation in the previous section, where $\lambda^2 v^2$ was tied to the value of the sum $m_{H_d}^2 + m_{H_u}^2$ (see Eq. (4.21)).

Next, we try to identify the main source of this sensitivity. We first consider the dependence of v on λ for fixed B, k, and h_t. One can use Eq. (4.11) to solve for v^2 and then isolate the largest contribution to $\partial v/\partial\lambda$.

$$\frac{\partial v^2}{\partial \lambda} = \frac{4}{\bar{g}^2}\left[-2\lambda x^2 - 2\lambda^2 x\frac{\partial x}{\partial \lambda} + \frac{\partial}{\partial \tan\beta}\left(\frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}\right)\frac{\partial \tan\beta}{\partial \lambda} + \frac{1}{\tan^2\beta - 1}\frac{\partial m_{H_d}^2}{\partial \lambda} - \frac{\tan^2\beta}{\tan^2\beta - 1}\frac{\partial m_{H_d}^2}{\partial \lambda}\right]. \tag{5.4}$$

Using the data that led to Fig. 3, we numerically evaluate the derivative around the point $\lambda = 0.11$, $k = -0.045$. The following are the results of evaluating each of the terms on the right-hand side, respectively: -1.4×10^7, -2.4×10^6, -3.9×10^6, -1.2×10^3, 1.9×10^1 (GeV^2). The largest term is the first one, the next two terms combined provide a 45% correction, and the derivatives of the soft SUSY-breaking masses can be completely neglected. In Appendix D we show how these numbers can be understood by studying the minimization conditions.

The fact that the dominant contribution to $\partial v/\partial\lambda$ comes from the first term in Eq. (5.4) has a very important implication. It means that the problems of cancellation in the NMSSM and the MSSM are not merely similar, but have *exactly* the same origin. Indeed, Eq. (4.11) is the same as Eq. (3.5), and, because in the NMSSM v depends on λ mainly through the combination λx, which plays the role of the μ–term, the two models require roughly the same degree of cancellation. The degree of cancellation quoted in Section 3 for the MSSM is most conservatively 16%, but this is so because one can choose $\tan\beta$ freely in the MSSM. On the other hand, $\tan\beta$ is determined by minimizing the potential for the NMSSM and cannot be chosen arbitrarily to ease the cancellation. For the value of $\tan\beta$ which we obtained in the NMSSM, the degree of cancellation is actually comparable (order a few percent) in the MSSM. The small difference between the two models is due to the dependence of x and $\tan\beta$ on λ.

We have discussed the λ dependence of the Higgs boson VEV, and now turn to the k dependence. Fig. 3 shows that the points that yield $v = 174$ GeV form an almost straight line on the $\lambda - k$ plane. It can be shown (see Appendix D) that in order to keep v constant one has to change k and λ according to $\Delta k/k = \Delta\lambda/\lambda$. The sensitivity of v to k is, thus, related to the sensitivity of v to λ, which, in turn, originates from the need to carefully choose the μ-parameter in the MSSM as discussed in Section 3.

To summarize, we have shown that this model requires a very particular choice of parameters to yield the correct Z-boson mass. Furthermore, we explained that the sensitivity of the Z-boson

mass to the NMSSM couplings has the same origin as the sensitivity of the Z-boson mass to the value of the μ-parameter in the MSSM. We emphasize that the problem is present for both high and low messenger scales, simply because the bound on the μ-parameter does not weaken as one raises the messenger scale.

5.2 Hypercharge D-term

Next, we investigate what happens if the D-term contributions described at the end of Section 3 are included. First, we consider the case of the NMSSM with no extra particles added. We try to determine if, by introducing the D-terms, it is possible to make v smaller. If that happened, v could be rescaled back by increasing B, and that would raise all masses in the model, as desired. We find that this is not the case. Upon adding the D–terms both $\tan\beta$ and $\langle x\rangle$ change, but $v_d^2 + v_u^2$, curiously enough, remains virtually constant. This happens because, in the limit $x^2 \ll v^2$, v^2 is constrained by Eq. (4.21), and the change $m_{H_d}^2 \to m_{H_d}^2 - \frac{1}{2}D_Y$, $m_{H_u}^2 \to m_{H_u}^2 + \frac{1}{2}D_Y$ preserves the quantity $m_{H_d}^2 + m_{H_u}^2$.

The next question to ask is whether the D-terms can decrease the degree of cancellation for the case with q' and $\bar{q}'$ added. The answer is again negative and the reason can be seen from Eq. (4.12). Recall that the degree of cancellation is controlled by the magnitude of x^2. As long as $A_\lambda \lambda x$ and $\lambda^2 v_d v_u$ can be neglected compared to $m_{H_d}^2 + m_{H_u}^2$, Eq. (4.12) yields

$$x^2 \simeq -\frac{(m_{H_d}^2 + m_{H_u}^2)}{2\lambda(\lambda - \frac{k}{\sin 2\beta})}, \tag{5.5}$$

and the relevant quantity is again $m_{H_d}^2 + m_{H_u}^2$.

5.3 Large Trilinear Couplings

At last, we consider the scenario proposed by Ciafaloni and Pomarol [31]. They consider a modified version of the NMSSM, where $k = 0$, $\lambda \ll 1$ and the value of A_λ is large at the messenger scale. Their model also contains, in the potential at the weak-scale, a linear term in N which is generated by tadpole diagrams and solves the problem of a light pseudoscalar. They find that the requirement of the positivity of the determinant of the scalar Higgs boson mass-squared matrix is very restrictive. We repeat part of their analysis to determine if their choice of parameters could indeed lead to a phenomenologically viable electroweak symmetry breaking spectrum. Note that, as far as the following is concerned, their model is identical to the NMSSM.

The full expression for the determinant can be found in Appendix B. In the limit of $k \to 0$ and $\lambda \to 0$

$$\det \mathcal{M}^2_{scalar} \simeq \frac{A_\lambda^2 v^2 M_Z^2 \lambda^2 \sin^2(4\beta)}{4(1+y)^3}\left[1 + y - \frac{y^2}{\cos^2(2\beta)}\left(\frac{A_\lambda^2}{M_Z^2} + 1\right) - \frac{y^3}{\cos^2(2\beta)}\right], \tag{5.6}$$

where we introduced a variable $y \equiv \bar{g}^2 m_N^2/(2M_W^2)$ to conform to the notation used in Ref. [31]. From the extremization conditions for the potential, Eqs. (4.8-4.10), one can show that $\mu = A_\lambda \sin(2\beta)/(2(1+y))$. There are two intervals of y over which the determinant is greater than zero. One interval is where both the expression in the brackets and the denominator are

positive. It is given approximately by the following bound on $|y|$:

$$|y| < \left| \cos 2\beta \left(1 + \frac{A_\lambda^2}{M_Z^2}\right)^{-1/2} \right| . \tag{5.7}$$

The other interval, not mentioned in [31], is approximately $\left(-(\frac{A_\lambda^2}{M_Z^2}+1), -1\right)$, where both the denominator and the bracketed expression are negative.

The first interval, for $A_\lambda > M_Z$, corresponds to rather small values of m_N^2 and

$$\mu \simeq \frac{1}{2} A_\lambda \sin(2\beta) . \tag{5.8}$$

Using this equation together with Eq. (4.12), one can derive the following result:

$$A_\lambda^2 \left(1 - \frac{\sin^2(2\beta)}{2}\right) \simeq (m_{H_u}^2 + m_{H_d}^2) . \tag{5.9}$$

The above equation is impossible to satisfy in models with the GMSB, because the combination $(m_{H_u}^2 + m_{H_d}^2)$ is always negative at the weak scale for the messenger-scale boundary conditions given by Eqs. (2.5) and (2.6). To satisfy Eq. (5.9), a drastic modification of the boundary conditions would be required.

We now turn our attention to the second possibility. It requires a relatively large negative value of the singlet soft SUSY-breaking mass-squared: $m_N^2 < -2/(\bar{g}^2) \times M_W^2 = -(132 \text{ GeV})^2$. This value is impossible to generate unless, as before, one introduces fields q' and $\bar{q}'$ and couples them to N. Even with the introduction of these fields, if $k = 0$, $\lambda \ll 1$, the extremization conditions cannot be simultaneously satisfied. This can be seen in the following way. For $k = 0$ Eq. (4.13) takes on the form

$$x = \frac{A_\lambda \lambda v^2 \sin(2\beta)}{m_N^2 + \lambda^2 v^2} , \tag{5.10}$$

which implies $x \to 0$ as $\lambda \to 0$. This is incompatible with Eq. (4.11), which requires that $x \to \infty$ as $\lambda \to 0$.

6 Conclusion

We studied the issue of electroweak symmetry breaking in models with the gauge mediation of supersymmetry breaking (GMSB). We first reviewed various proposals in the literature to generate the μ-parameter of the MSSM with the same order of magnitude as the soft SUSY-breaking parameters such as squark, slepton, and gaugino masses. We find that most of them require small parameters which are accidentally of the same magnitude as the loop factors, cancellation of the kinetic mixing terms at the level of 10^{-4}, omission of interactions allowed by symmetries, or many new degrees of freedom not motivated otherwise.

Even if one could generate the μ-parameter with the same order of magnitude as the soft SUSY-breaking parameters, it has to have particular values to reproduce $M_Z = 91$ GeV. We studied this question numerically and found the following. The current experimental lower bounds on superparticle masses limit the overall scale of SUSY breaking from below, which in turn limits $m_{H_u}^2 < 0$ from above (*i.e.*, $|m_{H_u}^2|$ from below). To reproduce M_Z, μ^2 needs to cancel

(too-negative) $m_{H_u}^2$ and is hence bounded from below. Therefore, there is some cancellation required between μ^2 and $m_{H_u}^2$. Even with the most conservative set of parameters, we found that a cancellation of 16% is necessary. The situation is worse for most of the parameter space. This situation was contrasted to the supergravity scenario where the current experimental lower bounds on superparticle masses do not require a significant cancellation among parameters.

The simplest mechanism to generate the μ-parameter would be the NMSSM, the minimal extension of the MSSM without dimensionful parameters in the superpotential. The NMSSM is known not to work with the low-energy GMSB, but there was hope that it may work with higher messenger scales. We have shown that this is unfortunately not the case. The current bounds on the superparticles masses are already strong enough to exclude the model completely. We presented a semi-analytic discussion to clarify why the NMSSM fails.

We also discussed various possible modifications to the NMSSM and whether they could lead to a viable electroweak symmetry breaking. The introduction of extra vector-like quarks coupled to the NMSSM singlet produces a large negative mass-squared for the singlet, and leads to a viable electroweak symmetry breaking. One needs to adjust the parameters to a few percent, which is comparable to the MSSM case for the same $\tan\beta$ range. A Fayet–Illiopoulos D-term for $U(1)_Y$ does not improve the situation.

The overall prospect of electroweak symmetry breaking with the GMSB remains unclear. We hope our detailed investigation prompts further studies on this issue.

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Appendix A The Renormalization Group Equations of the NMSSM

In this appendix we list all of the RG equations for the NMSSM, at 1-loop [41]. These are the equations used, in section 4, to determine the coupling constants and SUSY-breaking parameters of the NMSSM at the weak scale, given their values at the messenger scale.

$$16\pi^2 \frac{d}{dt} g' = 11 g'^3, \tag{A.11}$$

$$16\pi^2 \frac{d}{dt} g_2 = g_2^3, \tag{A.12}$$

$$16\pi^2 \frac{d}{dt} g_3 = (-3) g_3^3, \tag{A.13}$$

$$16\pi^2 \frac{d}{dt} h_t = (6h_t^2 + h_b^2 + \lambda^2 - \frac{13}{9} g'^2 - 3g_2^2 - \frac{16}{3} g_3^2) h_t, \tag{A.14}$$

$$16\pi^2\frac{d}{dt}h_b = (6h_b^2 + h_t^2 + h_\tau^2 + \lambda^2 - \frac{7}{9}g'^2 - 3g_2^2 - \frac{16}{3}g_3^2)h_b, \tag{A.15}$$

$$16\pi^2\frac{d}{dt}h_\tau = (4h_\tau^2 + 3h_b^2 + \lambda^2 - 3g'^2 - 3g_2^2)h_\tau, \tag{A.16}$$

$$16\pi^2\frac{d}{dt}\lambda = (4\lambda^2 + 2k^2 + 3h_t^2 + 3h_b^2 + h_\tau^2 - g'^2 - 3g_2^2)\lambda, \tag{A.17}$$

$$16\pi^2\frac{d}{dt}k = 6(\lambda^2 + k^2)k. \tag{A.18}$$

In the above equations g' is the $U(1)_Y$ gauge coupling; explicitly $g' = e/\cos\theta_W$. g_2 and g_3 are, respectively, the weak and strong coupling constants. One defines g_1 to be the hypercharge coupling constant in the GUT normalization, *i.e.* $g_1 \equiv \sqrt{\frac{5}{3}}g'$ and $\alpha_1 \equiv \frac{5}{3}\alpha'$. Gauge couplings at the messenger scale are defined in such a way that they match their experimental values at the Z-mass. We only consider the effect of third generation Yukawa couplings, namely, h_t, h_b and h_τ.

$$\begin{aligned} 16\pi^2\frac{d}{dt}A_{u_a} &= 6h_t^2(1+\delta_{a3})A_t + 2h_b^2\delta_{a3}A_b + 2\lambda^2 A_\lambda \\ &- 4(\frac{13}{18}g'^2M_1 + \frac{3}{2}g_2^2M_2 + \frac{8}{3}g_3^2M_3), \end{aligned} \tag{A.19}$$

$$\begin{aligned} 16\pi^2\frac{d}{dt}A_{d_a} &= 6h_b^2(1+\delta_{a3})A_b + 2h_t^2\delta_{a3}A_t + 2h_\tau^2\delta_{a3}A_\tau + 2\lambda^2 A_\lambda \\ &- 4(\frac{7}{18}g'^2M_1 + \frac{3}{2}g_2^2M_2 + \frac{8}{3}g_3^2M_3), \end{aligned} \tag{A.20}$$

$$\begin{aligned} 16\pi^2\frac{d}{dt}A_{e_a} &= 2h_\tau^2(1+3\delta_{a3})A_\tau + 6h_b^2A_b + 2\lambda^2A_\lambda \\ &- 6(g'^2M_1 + g_2^2M_2), \end{aligned} \tag{A.21}$$

$$\begin{aligned} 16\pi^2\frac{d}{dt}A_\lambda &= 8\lambda^2A_\lambda - 4k^2A_k + 6h_t^2A_t + 6h_b^2A_b + 2h_\tau^2A_\tau \\ &- 2(g'^2M_1 + 3g_2^2M_2), \end{aligned} \tag{A.22}$$

$$16\pi^2\frac{d}{dt}A_k = 12(k^2A_k - \lambda^2A_\lambda). \tag{A.23}$$

A_i are the soft SUSY-breaking trilinear couplings, given in Sections 2 and 4. Note that we only consider third generation trilinear couplings, namely $A_th_t = \mathcal{A}_u^{33}$, $A_bh_b = \mathcal{A}_d^{33}$, $A_\tau h_\tau = \mathcal{A}_l^{33}$. M_i (i=1,2,3) are the soft SUSY-breaking gaugino masses and they evolve, at one loop, identically to α_i. Explicitly

$$\frac{M_i(Q)}{M_{\frac{1}{2}}} = \frac{g_i^2(Q)}{g_X^2}, \tag{A.24}$$

where g_X is the value of all g_i at the GUT scale, while $M_{\frac{1}{2}}$ is the common gaugino mass at the GUT scale.

$$\begin{aligned} 16\pi^2\frac{d}{dt}m_{\tilde{Q}_a}^2 &= 2\delta_{a3}h_t^2(m_{\tilde{Q}_3}^2 + m_{H_u}^2 + m_{\tilde{t}}^2 + A_t^2) + 2\delta_{a3}h_b^2(m_{\tilde{Q}_3}^2 + m_{H_d}^2 + m_{\tilde{b}}^2 + A_b^2) \\ &- 8(\frac{1}{36}g'^2M_1^2 + \frac{3}{4}g_2^2M_2^2 + \frac{4}{3}g_3^2M_3^2) + \frac{1}{3}g'^2\xi, \end{aligned} \tag{A.25}$$

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m^2_{\tilde{u}_a} &= 4\delta_{a3} h_t^2 (m^2_{\tilde{Q}_3} + m^2_{H_u} + m^2_{\tilde{t}} + A_t^2) \\
&- 8(\frac{4}{9} g'^2 M_1^2 + \frac{4}{3} g_3^2 M_3^2) - \frac{4}{3} g'^2 \xi, \qquad \text{(A.26)} \\
16\pi^2 \frac{d}{dt} m^2_{\tilde{d}_a} &= 4\delta_{a3} h_b^2 (m^2_{\tilde{Q}_3} + m^2_{H_d} + m^2_{\tilde{b}} + A_b^2) \\
&- 8(\frac{1}{9} g'^2 M_1^2 + \frac{4}{3} g_3^2 M_3^2) + \frac{2}{3} g'^2 \xi, \qquad \text{(A.27)} \\
16\pi^2 \frac{d}{dt} m^2_{\tilde{L}_a} &= 2\delta_{a3} h_\tau^2 (m^2_{\tilde{L}_3} + m^2_{H_d} + m^2_{\tilde{\tau}} + A_\tau^2) \\
&- 8(\frac{1}{4} g'^2 M_1^2 + \frac{3}{4} g_2^2 M_2^2) - g'^2 \xi, \qquad \text{(A.28)} \\
16\pi^2 \frac{d}{dt} m^2_{\tilde{e}_a} &= 4\delta_{a3} h_\tau^2 (m^2_{\tilde{L}_3} + m^2_{H_d} + m^2_{\tilde{\tau}} + A_\tau^2) \\
&- 8 g'^2 M_1^2 + 2 g'^2 \xi, \qquad \text{(A.29)} \\
16\pi^2 \frac{d}{dt} m^2_{H_d} &= 6 h_b^2 (m^2_{\tilde{Q}_3} + m^2_{H_d} + m^2_{\tilde{b}} + A_b^2) + 2 h_\tau^2 (m^2_{\tilde{L}_3} + m^2_{H_d} + m^2_{\tilde{\tau}} + A_\tau^2) \\
&+ 2\lambda^2 (m^2_{H_d} + m^2_{H_u} + m^2_N + A_\lambda^2) - 8(\frac{1}{4} g'^2 M_1^2 + \frac{3}{4} g_2^2 M_2^2) - g'^2 \xi, \qquad \text{(A.30)} \\
16\pi^2 \frac{d}{dt} m^2_{H_u} &= 6 h_t^2 (m^2_{\tilde{Q}_3} + m^2_{H_u} + m^2_{\tilde{t}} + A_t^2) + 2\lambda^2 (m^2_{H_d} + m^2_{H_u} + m^2_N + A_\lambda^2) \\
&- 8(\frac{1}{4} g'^2 M_1^2 + \frac{3}{4} g_2^2 M_2^2) + g'^2 \xi, \qquad \text{(A.31)} \\
16\pi^2 \frac{d}{dt} m^2_N &= 4\lambda^2 (m^2_{H_d} + m^2_{H_u} + m^2_N + A_\lambda^2) + 4k^2 (3 m^2_N + A_k^2). \qquad \text{(A.32)}
\end{aligned}
$$

ξ is the hypercharge-weighted sum of all soft SUSY-breaking masses-squared

$$
\xi = \sum_i Y_i m_i^2, \qquad \text{(A.33)}
$$

where i runs over all scalar particles. With the boundary conditions in Eqs. (2.5,2.6), $\xi = 0$ and remains zero throughout the RG evolution. All soft SUSY-breaking mass-squared terms were taken to be diagonal. Again, we only consider the running of third generation soft SUSY-breaking masses-squared. m_N^2 is defined in Section 4.

Appendix B Scalar Higgs Mass-Squared Matrix

In this appendix we explicitly show the 3×3 scalar Higgs mass-squared matrix of the NMSSM.

$$
\mathcal{M}^2_{scalar} = \frac{1}{2} \frac{\partial^2 V_{tree}}{\partial v_i \partial v_j} = \frac{1}{2} \times
$$

$$
\begin{pmatrix}
\bar{g}^2 v_1^2 + \left(A_\lambda + \frac{k\mu}{\lambda}\right) \frac{2\mu v_2}{v_1} & (4\lambda^2 - \bar{g}^2) v_1 v_2 - 2\mu \left(A_\lambda + \frac{k\mu}{\lambda}\right) & 4\lambda\mu v_1 - 2A_\lambda \lambda v_2 - 4k\mu v_2 \\
(4\lambda^2 - \bar{g}^2) v_1 v_2 - 2\mu \left(A_\lambda + \frac{k\mu}{\lambda}\right) & \bar{g}^2 v_2^2 + \left(A_\lambda + \frac{k\mu}{\lambda}\right) \frac{2\mu v_1}{v_2} & -2A_\lambda \lambda v_1 - 4k\mu v_1 + 4\lambda\mu v_2 \\
4\lambda\mu v_1 - 2A_\lambda \lambda v_2 - 4k\mu v_2 & -2A_\lambda \lambda v_1 - 4k\mu v_1 + 4\lambda\mu v_2 & \frac{-2A_k k\mu}{\lambda} + \frac{8k^2\mu^2}{\lambda^2} + \frac{2A_\lambda \lambda^2 v_1 v_2}{\mu}
\end{pmatrix},
\qquad \text{(B.1)}
$$

where v_i for $i = 1, 2, 3$ corresponds, respectively, to v_d, v_u and x. All other parameters were defined in previous sections.

The determinant of the matrix above can be evaluated explicitly, and its full expression is given bellow. Various limits of this determinant are considered in the body of the paper.

$$
\begin{aligned}
&\det \mathcal{M}^2_{scalar} = \\
&\frac{v^2}{32\lambda^3\mu\sin(2\beta)}\Big(-6A_\lambda v^4\lambda^9 - 32A_\lambda^3\lambda^5\mu^2 - 64A_\lambda v^2\lambda^7\mu^2 - 32A_kA_\lambda k\lambda^4\mu^3 - 160A_\lambda^2 k\lambda^4\mu^3 - \\
&128kv^2\lambda^6\mu^3 - 32A_k k^2\lambda^3\mu^4 - 128A_\lambda k^2\lambda^3\mu^4 - 256A_\lambda\lambda^5\mu^4 - 256k\lambda^4\mu^5 + 8A_\lambda v^4\lambda^9\cos(4\beta) + \\
&32A_\lambda^3\lambda^5\mu^2\cos(4\beta) + 64A_\lambda v^2\lambda^7\mu^2\cos(4\beta) + 32A_kA_\lambda k\lambda^4\mu^3\cos(4\beta) + 160A_\lambda^2 k\lambda^4\mu^3\cos(4\beta) + \\
&128kv^2\lambda^6\mu^3\cos(4\beta) + 32A_k k^2\lambda^3\mu^4\cos(4\beta) + 128A_\lambda\, k^2\lambda^3\mu^4\cos(4\beta) - 2A_\lambda v^4\lambda^9\cos(8\beta) + \\
&3A_\lambda v^4\lambda^7\bar{g}^2 + 32A_\lambda v^2\lambda^5\mu^2\bar{g}^2 - 16A_kA_\lambda k\lambda^2\mu^3\bar{g}^2 + 64kv^2\lambda^4\mu^3\bar{g}^2 - 16A_k k^2\lambda\mu^4\bar{g}^2 + \\
&64A_\lambda k^2\lambda\mu^4\bar{g}^2 + 64k^3\mu^5\bar{g}^2 - 4A_\lambda v^4\lambda^7\cos(4\beta)\bar{g}^2 - 32A_\lambda v^2\lambda^5\mu^2\cos(4\beta)\bar{g}^2 - \\
&16A_kA_\lambda k\lambda^2\mu^3\cos(4\beta)\bar{g}^2 - 64kv^2\lambda^4\mu^3\cos(4\beta)\bar{g}^2 - 16A_k k^2\lambda\mu^4\cos(4\beta)\bar{g}^2 + \\
&64A_\lambda k^2\lambda\mu^4\cos(4\beta)\bar{g}^2 + 64k^3\mu^5\cos(4\beta)\bar{g}^2 + A_\lambda v^4\lambda^7\cos(8\beta)\bar{g}^2 + 48A_\lambda^2 v^2\lambda^7\mu\sin(2\beta) + \\
&24A_k kv^2\lambda^6\mu^2\sin(2\beta) + 120A_\lambda kv^2\lambda^6\mu^2\sin(2\beta) + 256A_\lambda^2\lambda^5\mu^3\sin(2\beta) + 96v^2\lambda^7\mu^3\sin(2\beta) + \\
&768A_\lambda k\lambda^4\mu^4\sin(2\beta) + 512k^2\lambda^3\mu^5\sin(2\beta) - 12A_\lambda^2 v^2\lambda^5\mu\bar{g}^2\sin(2\beta) - 12A_k kv^2\lambda^4\mu^2\bar{g}^2\sin(2\beta) - \\
&60A_\lambda kv^2\lambda^4\mu^2\bar{g}^2\sin(2\beta) - 16k^2v^2\lambda^3\mu^3\bar{g}^2\sin(2\beta) - 48v^2\lambda^5\mu^3\bar{g}^2\sin(2\beta) - 16A_\lambda^2 v^2\lambda^7\mu\sin(6\beta) - \\
&8A_k kv^2\lambda^6\mu^2\sin(6\beta) - 40A_\lambda kv^2\lambda^6\mu^2\sin(6\beta) - 32v^2\lambda^7\mu^3\sin(6\beta) + 4A_\lambda^2 v^2\lambda^5\mu\bar{g}^2\sin(6\beta) + \\
&4A_k kv^2\lambda^4\mu^2\bar{g}^2\sin(6\beta) + 4A_\lambda kv^2\lambda^4\mu^2\bar{g}^2\sin(6\beta) - 16k^2v^2\lambda^3\mu^3\bar{g}^2\sin(6\beta) + 16v^2\lambda^5\mu^3\bar{g}^2\sin(6\beta)\Big).
\end{aligned}
\tag{B.2}
$$

All parameters were defined previously. Recall that $\mu = \lambda x$.

Appendix C Comments on Naturalness

We have studied the NMSSM with extra vector-like quarks in Section 5.1 and discussed that the model requires a delicate cancellation among independent parameters. In this appendix, we make further comments on the naturalness of this model.

From Fig. 4, one can easily note that not only does the experimentally allowed value of v lie on a steep region of the parameter space, which requires a degree of cancellation of order 1%, but it lies on the *steepest* region of the parameter space.

One may, therefore, try to address the following question: if all parameters are kept fixed (and this choice of parameters yields an experimentally allowed spectrum) except one (*e.g.* λ), what is the likelihood of obtaining a certain value of v upon a random choice of the free parameter? In other words, what is the probability $P(v)\,\mathrm{d}v$ of finding the value of $\sqrt{v_u^2 + v_d^2}$ between v and $v + \mathrm{d}v$ given a random choice of λ? This line of reasoning is related to the definition of fine-tuning introduced by Anderson and Castaño [42]. It is easy to note that

$$
P(v) \propto \left(\frac{\mathrm{d}v}{\mathrm{d}\lambda}\right)^{-1}. \tag{C.1}
$$

This "probability density" is plotted in Fig. 5. Note that we restrict λ to lie on a range where the

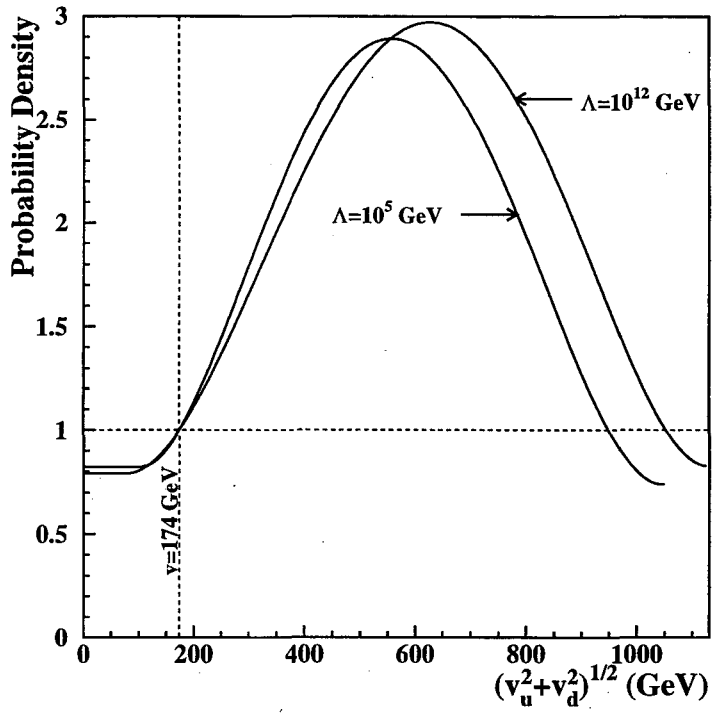

Figure 5: The probability densities of finding specific values of v in the NMSSM with extra vector-like quarks upon random choices of λ. All other parameters are the same as in Fig. 4. The probability densities are normalized so that $P(v = 174 \text{ GeV}) = 1$.

same "qualitative" physics is obtained, that is, electroweak symmetry is broken and $\tan(\beta) > 1$. The plot has been normalized in such a way that $P(v = 174 \text{ GeV}) = 1$.

We note that, in some sense, the probability of living in our universe is smaller, if this model is to be taken seriously, than the probability of living in a universe where $v \simeq 600$ GeV by a factor of three. One can turn this picture around and say that the NMSSM, with the above choice of parameters, "prefers" (or predicts) $v \simeq 600$ GeV.

This does not happen in the MSSM. The analog of Fig. 4 would be Eq. (3.5), which is a straight line ($M_Z^2 = M_Z^2(\mu^2)$) if all parameters except μ^2 are kept fixed. In the language introduced above, the MSSM does not "prefer" (or predict) any particular value of M_Z^2, that is, the "probability density" of M_Z^2 upon random choices of μ^2 is flat.

Appendix D The Dependence of the Higgs VEVs on the couplings of the modified NMSSM

In Subsection 5.1 we showed that the values of the Higgs boson VEVs were extremely sensitive to small variations of the superpotential couplings λ and k. These variations were evaluated numerically after Eq. (5.4) for one particular set of λ and k. In this appendix we study this issue analytically and show how one can estimate the effects of small variations $\Delta\lambda$ and Δk on

v, $\tan\beta$, and x.

We will use the following three equations, derived in Section 4:

$$\begin{aligned} \frac{\bar{g}^2 v^2}{4} &= -\lambda^2 x^2 + f\,, && \text{(D.1)} \\ 2k^2 x^2 &\simeq -m_N^2 + \lambda(k\sin 2\beta - \lambda)v^2\,, && \text{(D.2)} \\ \sin 2\beta &\simeq 2\frac{k}{\lambda}\left[\frac{-m_{H_u}^2}{m_{H_d}^2 - m_{H_u}^2}\right]\,, && \text{(D.3)} \end{aligned}$$

where

$$f \equiv -\frac{1}{2}(m_{H_d}^2 + m_{H_u}^2) - \frac{1}{2}\frac{m_{H_d}^2 - m_{H_u}^2}{\cos 2\beta} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}\,, \tag{D.4}$$

and we dropped the A-terms in Eq. (D.2). Notice that, because the term on the left-hand side of Eq. (D.1) is much smaller then each of the terms on the right-hand side, $f \simeq \lambda^2 x^2$.

For the purpose of the following estimates we will keep only the largest terms in the variations. According to the numbers presented after Eq. (5.4), for a small variation of λ the largest variation on the right-hand side of Eq. (D.1) is $2\lambda x^2 \Delta\lambda$. We can therefore write

$$\Delta(v^2) \approx -4\frac{2\lambda x^2 \Delta\lambda}{\bar{g}^2}\,. \tag{D.5}$$

We will justify this approximation *a posteriori*. Also, in our analysis we will completely neglect the dependence of the soft-breaking masses-squared on λ and k. This dependence is very weak, as seen in the numbers presented after Eq. (5.4).

A small change in λ results in a large change in v. Hence, to determine the corresponding change in x, one can use Eq. (D.2) and only consider the variation of v^2, which is approximately given by Eq. (D.5). We find

$$4k^2 x\Delta x \approx \lambda(k\sin 2\beta - \lambda)\Delta(v^2) \approx \lambda(k\sin 2\beta - \lambda)(-4\frac{2\lambda x^2\Delta\lambda}{\bar{g}^2})\,, \tag{D.6}$$

so that

$$\frac{\Delta x}{x} \approx -\frac{2\lambda^3(k\sin 2\beta - \lambda)}{k^2\bar{g}^2}\frac{\Delta\lambda}{\lambda}\,. \tag{D.7}$$

For the point considered in the text ($\lambda = 0.11$, $k = -.045$, $\tan\beta = -2.9$) one finds $(\Delta x)/x \approx 0.2(\Delta\lambda)/\lambda$.

Under a small change Δk, again using Eqs. (D.2,D.5),

$$4(k\Delta k x^2 + k^2 x\Delta x) \approx \lambda(k\sin 2\beta - \lambda)(-4\frac{2\lambda^2 x\Delta x}{\bar{g}^2}). \tag{D.8}$$

Solving for $\Delta x/x$.

$$\frac{\Delta x}{x} \approx \frac{\Delta k}{k}\left(-1 - \frac{2\lambda^3(k\sin 2\beta - \lambda)}{k^2\bar{g}^2}\right)^{-1}\,. \tag{D.9}$$

Numerically, $(\Delta x)/x \approx -1.2(\Delta k)/k$.

Next, we consider the effect of $\Delta\lambda$ on f. The problem comes down to estimating $\Delta(\cos 2\beta)^{-1}$, which can be done with the aid of Eq. (D.3):

$$\Delta(\cos 2\beta)^{-1} = -\frac{\sin 2\beta}{\cos^3 2\beta}\Delta(\sin 2\beta) \approx -\frac{\sin 2\beta}{\cos^3 2\beta}\left(-2\frac{k}{\lambda^2}\frac{-m_{H_u}^2}{m_{H_d}^2 - m_{H_u}^2}\right)\Delta\lambda\,. \tag{D.10}$$

Thus,

$$\frac{\Delta f}{f} \approx -\frac{k}{\lambda}\frac{(-m_{H_u}^2)}{\lambda^2 x^2}\frac{\sin 2\beta}{\cos^3 2\beta}\frac{\Delta\lambda}{\lambda}\,. \tag{D.11}$$

Plugging in the numerical values of the parameters, we find that the right-hand side of Eq. (D.11) equals $-0.5\Delta\lambda/\lambda$. Thus, a 1% change in λ results in a 0.5% change in the value of f. Since $\lambda^2 x^2$ changes by 2% in this case, the contribution of f to the variation of v is approximately one fourth of that of $\lambda^2 x^2$, consistent with the numbers given in Subsection 5.1.

The above argument can be repeated to find the effect of Δk on f. Notice that $\sin 2\beta$ depends on the ratio k/λ (Eq. (D.3)), and hence changing k by +1% has the same effect on f as changing λ by −1%.

Finally, we show that the condition for v to remain constant is $\Delta k/k = \Delta\lambda/\lambda$. We have already argued that $\sin 2\beta$, and therefore f, stays unchanged in this case and now show that the same is true for $\lambda^2 x^2$. Under $\lambda \to \lambda + \Delta\lambda$ the term $\lambda^2 x^2$ changes by $2\lambda^2 x^2((\Delta\lambda/\lambda) + (\Delta x/x)) = 2\lambda^2 x^2(1+0.2)(\Delta\lambda/\lambda)$, while under $k \to k+\Delta k$ it changes by $2\lambda^2 x^2(\Delta x/x) = 2\lambda^2 x^2(-1.2)(\Delta k/k)$. These variations can be made to cancel by imposing $\Delta k/k = \Delta\lambda/\lambda$.

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