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Physics Division

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Classification of the $N = 1$ Seiberg-Witten Theories

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Abstract

We present a systematic study of $N = 1$ supersymmetric gauge theories which are in the Coulomb phase. We show how to find all such theories based on a simple gauge group and no tree-level superpotential. We find the low-energy solution for the new theories in terms of a hyperelliptic Seiberg-Witten curve. This work completes the study of all $N = 1$ supersymmetric gauge theories where the Dynkin index of the matter fields equals the index of the adjoint ($\mu = G$), and consequently all theories for which $\mu < G$.

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1 Introduction

The past four years have witnessed a tremendous progress in our understanding of strongly coupled supersymmetric gauge theories. The number of theories for which exact results have been established is ever growing since the initial breakthrough by Seiberg and Witten [1], who gave a complete solution of the $N = 2$ $SU(2)$ theory, and by Seiberg, who described the low-energy dynamics of $N = 1$ supersymmetric QCD with varying number of flavors [2].

From these solutions the following basic picture emerges: there are six known phases of supersymmetric gauge theories: Higgs, confining, abelian Coulomb, non-abelian Coulomb, free magnetic and infrared free phases. The actual phase of a given theory usually depends on the size of its matter content. This size can be characterized by the relative value of the Dynkin index μ of the chiral superfields compared to the value of the Dynkin index G of the vector superfields.

In this paper we will focus on the $N = 1$ theories which are in the abelian Coulomb phase everywhere on their moduli spaces. We examine theories based on simple gauge groups and no tree-level superpotential. We will argue that in order for such a theory to be in the Coulomb phase, the theory has to satisfy the index condition $\mu = G$. The essence of the argument can be summarized in the following: one expects the low-energy solutions of such a theory to be given in terms of an auxiliary Riemann surface, defined by a curve (which in most cases is hyperelliptic). The classical curve is smoothed out by quantum corrections, which are proportional to the dynamical scale Λ of the theory. We will show, that in the absence of a tree-level superpotential the condition for Λ to appear in the curve is $\mu = G$. Theories with an adjoint chiral superfield (the pure $N = 2$ theories) satisfy this condition, and we will give a complete list of other theories which do so as well. After restricting ourselves to these theories it is easy to actually find all of those which are in the Coulomb phase by checking the unbroken gauge group on a generic point of the moduli space. This way we obtain a complete list of $N = 1$ Seiberg-Witten theories based on simple gauge groups and no tree-level superpotential. For these theories we determine the Seiberg-Witten curves providing the solution for the low-energy effective gauge kinetic couplings by flowing to theories for which the curve is already known.

This work completes the study of theories satisfying the index condition $\mu = G$. It has been known for a while, that confining theories with a quantum deformed moduli space have to satisfy this index condition [3, 4]. All such confining theories have been systematically analyzed in Refs. [3, 4, 5, 6]. We find, that if the matter content is in a faithful representation of the gauge group, then a $\mu = G$ theory is confining with a quantum deformed moduli space. However, if the matter content is not in a faithful representation of the gauge group, then the theory is in the Coulomb phase, and the low-energy solution can be given in terms of a Seiberg-Witten curve. Therefore the low-energy dynamics of all theories with $\mu = G$ has now been determined. Since all theories with $\mu < G$ can be obtained from the $\mu = G$ theories by adding mass terms for fields in vector-like representations, it is a straightforward task to

determine the low-energy behavior of all $\mu < G$ theories as well.

The paper is organized as follows. In Section 2 we review the basic properties of the Seiberg-Witten solutions and their applications to $N = 1$ theories. In Section 3 we give our general arguments which help us classify all $N = 1$ Seiberg-Witten theories based on simple groups and no tree-level superpotential. In Section 4 we give the actual low-energy solutions of these theories, and we conclude in Section 5. The derivation of the curves for the new theories is explained in Appendices A and B.

2 Review of the Seiberg-Witten Solution and its Application to $N = 1$ Theories

Seiberg and Witten showed how to employ electric-magnetic duality to obtain exact solutions to the low-energy dynamics of $N = 2$ theories [1]. The original example of $SU(2)$ theory was subsequently generalized to other classical groups in Refs. [7-17]. An alternative derivation of these solutions using the confining phase superpotential method has been given in Ref. [18], while the connection to integrable systems has been described in [13, 14]. The dynamics of the low-energy theory following from these solutions has been analyzed in Refs. [19, 20]. Below we review the basic features of these solutions of the $N = 2$ theories, and the application of the Seiberg-Witten methods to $N = 1$ theories [21].

Since $N = 2$ theories contain scalar fields in the adjoint representation, their classical moduli space has a submanifold with unbroken $U(1)$ gauge symmetries. In general, in addition to the Coulomb submanifold, there is usually a subspace where the gauge group is completely broken. The Higgs branch does not receive perturbative or non-perturbative quantum corrections in $N = 2$, while the Coulomb branch is affected by both perturbative and non-perturbative effects.

The low-energy Lagrangian of $N = 2$ theories can be characterized in terms of a single holomorphic function, the prepotential $\mathcal{F}$. In the more interesting case of the Coulomb branch, the prepotential can be computed in terms of the original "electric" fields and their dual "magnetic" degrees of freedom. It turns out that both kinds of variables are necessary for a consistent description of the theory.

The classical pattern of symmetry breaking by a field in the adjoint representation, $G \to U(1)^r$, where r is the rank of the gauge group, persists in the quantum theory everywhere on the Coulomb branch. Classically, there are points with a larger unbroken subgroup, when the VEVs of the adjoint field happen to coincide. Due to quantum effects there are no points of enhanced gauge symmetry, however, some states become massless on certain submanifolds of the moduli space. These additional massless states are indicated by singularities in the effective description. Since these massless particles carry magnetic charges, magnetic variables are more suitable for description of the theory near singularities.

Another indication that there are singularities on the moduli space is the presence of nontrivial monodromies. At large expectation values, the monodromy can be calculated using perturbation theory. The remaining strong-coupling monodromies are guessed using symmetry arguments and the requirement that the product of strong-coupling monodromies has to equal the monodromy at infinity. Knowledge of all monodromies, that is the singularity structure and the behavior at infinity is sufficient to determine the full form of a holomorphic quantity, in this case the prepotential $\mathcal{F}$.

The low-energy solution is obtained by introducing an auxiliary Riemann surface of genus equal to the rank of the gauge group. All holomorphic quantities can be computed as line integrals over this surface. This surface will also encode the singularity structure of the theory. In most cases, this surface turns out to be a hyperelliptic surface, which can be defined by an equation of the form:

$$y^2 = P(x, u_i, \Lambda),$$

where P is a polynomial in x, u_i and Λ. The variables x and y are auxiliary parameters, while the u_i's are the coordinates on the moduli space and Λ is the dynamical scale of the theory. Therefore, finding the exact low-energy action is equivalent to finding the polynomial P. From the curve described by P one can extract information about the dynamics of the theory. In particular, the effective gauge coupling, the metric on the moduli space and the spectrum of the BPS states can be calculated.

For example, the simplest theory in the abelian Coulomb phase is $N = 2$ $SU(2)$ with no hypermultiplets. In that case

$$y^2 = (x^2 - u)^2 - 4\Lambda^4,$$

where $u = \frac{1}{2}\mathrm{Tr}\Phi^2$ and Φ is the adjoint superfield. This equation describes a torus, which is a surface of genus one. Any VEV of an adjoint field can be rotated into the Cartan subalgebra, which for $SU(2)$ means that $\langle\Phi\rangle = \mathrm{diag}(a, -a)$. One needs also to introduce a dual scalar field a_D. The above curve has three singularities, for $u = \pm\Lambda^2$ and $u = \infty$. From the curve one can calculate the monodromies of the (a_D, a) vector around the singularities. The monodromies are elements of the duality group, which acts on the vector (a_D, a). The singularity at infinity is the perturbative singularity, while the two singularities at $u = \pm\Lambda^2$ occur at strong coupling. The non-perturbative singularities arise because of a monopole or a dyon becoming massless at these points of the moduli space. The charges of the massless fields are the left eigenvectors of the respective monodromy matrices.

From the curve one can also compute a and a_D as functions of the parameter of the moduli space u. These are determined as integrals over the periods, $\gamma_{1,2}$, of the torus: $a_D = \oint_{\gamma_1} \lambda$, $a = \oint_{\gamma_2} \lambda$, where λ is the so-called Seiberg-Witten differential. In the case of $SU(2)$, $\lambda \propto \frac{x^2\,dx}{y}$. The gauge coupling $\tau = \frac{da_D/du}{da/du} = \frac{\partial^2\mathcal{F}}{\partial a^2}$, from which one can also establish the metric on the moduli space $\frac{\partial\mathcal{F}}{\partial a}$.

Many features of the Seiberg-Witten solution of $N = 2$ theories persist in $N = 1$ theories in the Coulomb phase. Intriligator and Seiberg pointed out that the $U(1)$ gauge coupling, which is a holomorphic quantity, can be described by the methods used in $N = 2$ [21]. The dependence of the gauge coupling on the parameters of the moduli space can be found once a curve describing the theory is established. The gauge-kinetic term

$$\mathcal{L} = \frac{1}{4\pi}\mathrm{Im}\int d^2\theta \tau_{ij} W^i_\alpha W^{\alpha,j},$$

where τ_{ij} is the effective gauge coupling matrix, whose eigenvalues are related to the effective gauge coupling and theta parameter of the kth $U(1)$ factor by $\tau_k = i\frac{4\pi}{g_k^2} + \frac{\theta_k}{2\pi}$. The effective gauge coupling function τ_{ij} is not related by supersymmetry to the Kähler potential. Hence, no information about the Kähler potential is provided by the $N = 1$ solution. Likewise, there is no central extension of the $N = 1$ supersymmetry algebra which would incorporate BPS particles. However, one can learn about the charges of massless states associated with singularities. The monodromy around a singularity still encodes the information about the charge. These methods have been used in Refs. [21, 22, 23] to obtain solutions to several $N = 1$ theories in the Coulomb phase or to Coulomb branches of $N = 1$ theories with tree-level superpotential terms.

3 Necessary Criteria for Seiberg-Witten Theories

In this section we will show how to systematically find all $N = 1$ supersymmetric theories based on a simple gauge group and no tree-level superpotential, which are in the Coulomb phase. We will call such theories the $N = 1$ Seiberg-Witten theories. First we show that such theories have to satisfy the index condition $\mu = G$ mentioned in the introduction, and then decide which $\mu = G$ theories are actually in the Coulomb phase.

3.1 The Index Condition

We have seen in the previous section that the effective gauge kinetic function τ_{ij} can be identified with the period matrix of an auxiliary Riemann surface, usually a hyperelliptic curve $y^2 = P(x, u_i, \Lambda)$. The curve obtained in the limit $\Lambda \to 0$ is singular everywhere on the moduli space reflecting the fact that turning off the gauge coupling will result in additional massless gauge bosons independently of the VEVs of the scalars, since there is no Higgs mechanism in the $\Lambda \to 0$ limit. This singularity must be smoothed out by effects proportional to Λ, except at a submanifold where the singularity persists indicating the existence of additional massless states. The lesson which should be learned from this is that the dynamical scale Λ has to appear as a parameter of the full Seiberg-Witten curve to smooth out the classical singularities. In the absence of a tree-level superpotential coupling this requirement will severely restrict the matter content of the theory.

We now discuss the constraint which arises from requiring that the dynamical scale Λ appears in the Seiberg-Witten curve. This curve must respect all symmetries of the original theory. In particular, one can consider a $U(1)_R$ symmetry under which all fields Φ_i carry zero charge. This symmetry is anomalous under the gauge group G, that is the $G^2U(1)_R$ anomaly is non-vanishing. One can however restore this symmetry by promoting the dynamical scale Λ to a background chiral superfield [24]. The reason for this is that an anomalous $U(1)$ rotation by an angle α acts like a shift

$$\theta \to \theta + \sum_i \mu_i q_i \alpha$$

on the θ parameter of the theory, where q_i is the charge of the ith fermion under the $U(1)$ transformation, and μ_i is the Dynkin index of the ith fermion[2]. This means that the dynamical scale

$$\Lambda^b = \mu^b e^{-\frac{8\pi^2}{g^2(\mu)}+i\theta}$$

of the theory has charge $\sum_i \mu_i q_i$ under the anomalous $U(1)$ symmetry, where b is the coefficient of the one-loop beta function. In the case of the $U(1)_R$ symmetry defined above, the fermions from the chiral superfield have charge -1, while the gauginos have charge $+1$, thus the charge of Λ^b is $G - \mu$, where G is the Dynkin index of the adjoint representation, and $\mu = \sum_i \mu_i$ is the sum of the Dynkin indices of the matter fields. After assigning this charge $G - \mu$ to the dynamical scale, one can require that the Seiberg-Witten curve is invariant (or at least covariant) under this anomalous $U(1)_R$ symmetry as well.

Let us now consider the $\Lambda \to 0$ limit again. In this limit we obtain the classical curve $P_{cl}(y, x, u_i) = 0$. Since we expect that the full curve defined by x and y describes a Riemann surface, we expect that P_{cl} is not a homogeneous polynomial in x and y (which is true for every known solution). However this implies that x and y have to have zero R-charge as well. But now considering the full curve $P(y, x, u_i, \Lambda) = 0$ one can see that Λ can appear in a non-trivial way in the curve only if its R-charge is zero as well, implying that $\mu = G$. Note that all pure $N = 2$ theories without matter fields satisfy this condition, however the $N = 2$ theories with hypermultiplets do not. The way these theories evade this constraint is that $N = 2$ supersymmetry automatically requires a tree-level superpotential coupling between the hypermultiplets and the adjoint from the $N = 2$ vector superfield, which explicitly breaks the $U(1)_R$ symmetry used above.

The $\mu = G$ condition is exactly the same as one gets for theories with a quantum deformed moduli space. The coincidence of these two conditions is not very surprising, since in both cases we are requiring that Λ appears in an equation involving the moduli in a non-trivial way. In the case of the quantum deformed moduli space we require that a term proportional to Λ appears on the right hand side of a classical constraint, while in the case of Seiberg-Witten

[2]The Dynkin index is defined by $\mathrm{Tr}\, T^a T^b = \mu \delta^{ab}$, where the T's are the generators of the gauge group in a given representation.

theory we require that Λ appears as a non-trivial modification of the classical Seiberg-Witten curve.

One can argue for the necessity of the $\mu = G$ condition in a different way as well. We are requiring that a Seiberg-Witten theory has unbroken $U(1)$'s on the generic point in the moduli space. It is however known [25], that for $\mu > G$ the gauge group is completely broken, thus the $\mu > G$ theories can not be in the Coulomb phase everywhere on the moduli space. If $\mu < G$, then a dynamically generated superpotential of the form $W_{dyn} \propto \frac{1}{(\Pi_i \Phi_i^{\mu_i})^{\frac{2}{G-\mu}}}$ is allowed by all symmetries of the theory, and such a superpotential is presumably generated either by instantons or by gaugino condensation. This superpotential pushes the fields to large expectation values and the theory has no stable vacuum. Thus, we expect the $\mu < G$ theories to be generically in the Higgs phase, or at least have a Higgs branch with a runaway vacuum. Thus we again conclude that the only possibility for a theory to be in the Coulomb phase everywhere on the moduli space is when the index condition $\mu = G$ is satisfied.

We have established that a necessary condition for Seiberg-Witten theories is that the index condition $\mu = G$ be satisfied. This restricts the number of candidate theories considerably. In the next subsection we show how to find the theories which are actually in the Coulomb phase.

3.2 Flows

We have seen in the previous section, that a necessary condition for a theory to be in the Coulomb phase is that it satisfies the index condition $\mu = G$. This requirement alone reduces the number of candidate theories to a finite set. In order to decide which of these theories is in the Coulomb phase we have to check whether there are unbroken $U(1)$'s on the generic point of the moduli space.

However, in order to exclude a given candidate theory from being in the Coulomb phase one doesn't always have to consider the most generic point on the moduli space. It is enough to find a flow which leads to a theory which is known not to be in the Coulomb phase. For example, consider the theory based on the exceptional group E_7 with three 56 dimensional representations. This theory satisfies the $\mu = G$ constraint. However, by giving a VEV to one of the **56**'s we get an E_6 theory with $2 \cdot (\mathbf{27} + \overline{\mathbf{27}})$. Giving an expectation value to the **27** breaks E_6 to F_4, with the remaining field content being three 26-dimensional representations. Further breaking F_4 by a VEV of **26** will give $SO(9)$ with two spinors and three vectors, $2 \cdot \mathbf{16} + 3 \cdot \mathbf{9}$. However, this $SO(9)$ theory is known to be confining with a quantum modified constraint [3], and is therefore not in the Coulomb phase. This argument implies that the whole chain of theories, with matter content such that $\mu = G$, given below is excluded from being in the Coulomb phase.

$$E_7 : \; [3 \cdot \mathbf{56}] \stackrel{\langle \mathbf{56} \rangle}{\rightarrow} E_6 : \; [2 \cdot \mathbf{27} + 2 \cdot \overline{\mathbf{27}}] \stackrel{\langle \mathbf{27} \rangle}{\rightarrow} F_4 : [3 \cdot \mathbf{26}] \stackrel{\langle \mathbf{26} \rangle}{\rightarrow} SO(9) \; [2 \cdot \mathbf{16} + 3 \cdot \mathbf{9}] \qquad (3.1)$$

Similarly, one could consider an E_6 theory with $n \cdot \mathbf{27} + (4-n) \cdot \overline{\mathbf{27}}$, where $n = 0, 1, 2, 3, 4$. All of these theories will also flow to F_4 with $3 \cdot \mathbf{26}$, so they are not in the Coulomb phase either. Indeed, it has been shown recently in Refs. [5, 6], that all of the above theories based on exceptional groups satisfying $\mu = G$ are confining with a quantum deformed moduli space.

In Tables 1-4 we list all theories that satisfy the $\mu = G$ constraint and give the phase of the given theory. The first column gives the gauge group, the second column the field content and the third column the phase of the theory. Finally, the fourth column contains a reference to where the actual low-energy solution of the given theory can be found. One can see from Tables 1-4 that finding the Seiberg-Witten curves for the remaining $N = 1$ theories in the Coulomb phase completes the study of all $\mu = G$ theories.

There are only two possibilities for the phase of the $\mu = G$ theories: confining phase or Coulomb phase. The confining theories all have a low-energy description in terms of composite gauge invariants satisfying a quantum-modified version of the classical constraints (the SU, Sp and SO theories in this class have been analyzed in Refs. [3, 4, 5], while the theories based on exceptional groups in Refs. [5, 6].) This solution is valid everywhere on the moduli space, and there is no phase boundary between the Higgs and the confining phases. This is possible, because the massless fermions are in a faithful representation of the gauge group, therefore any external source can be screened by the massless fields. One can also explicitly check that every $\mu = G$ theory with chiral superfields in a faithful representation breaks the gauge group completely at generic points on the moduli space. Since at large expectation values the theory can be described entirely in terms of gauge singlet fields and there is no invariant distinction between the Higgs and the confining phase, one indeed expects confinement at strong coupling. Contrary, if chiral superfields are not in a faithful representation of the gauge group, some external sources can not be screened by the massless quarks, and there can be points on the moduli space where additional massless fields appear. Indeed, we find that in every case where the matter fields are not in a faithful representation, at large expectation values there are unbroken $U(1)$ gauge factors and that the low-energy theory is in the Coulomb phase. One can see from Tables 1-4 that the only theories which are in the Coulomb phase besides the pure $N = 2$ theories (which are $N = 1$ theories with a chiral superfield in the adjoint representation) are $SO(N)$ with $N-2$ vectors, $SU(6)$ with $2\begin{smallmatrix}\square\\\square\end{smallmatrix}$ and $Sp(6)$ with $2\begin{smallmatrix}\square\\\square\end{smallmatrix}$. The other two theories in the Coulomb phase belong to the $SO(N)$ series with $N-2$ vectors, since $SU(4)$ with $4\begin{smallmatrix}\square\\\square\end{smallmatrix}$ is equivalent to $SO(6)$ with four vectors and $Sp(4)$ with $3\begin{smallmatrix}\square\\\square\end{smallmatrix}$ is equivalent to $SO(5)$ with three vectors.

One can easily see that these $N = 1$ theories are indeed in the Coulomb phase by considering the following flows:

$$
\begin{aligned}
&SO(N): \ (N-2)\square \to SU(2) \times SU(2): \ 2\,(\square,\square) \\
&SU(6): \ 2\begin{smallmatrix}\square\\\square\end{smallmatrix} \to SU(3) \times SU(3): \ (\square,\overline{\square}) + (\overline{\square},\square) \\
&Sp(6): \ 2\begin{smallmatrix}\square\\\square\end{smallmatrix} \to SU(2) \times SU(2) \times SU(2): \ (\square,\square,1) + (1,\square,\square) + (\square,1,\square).
\end{aligned}
\tag{3.2}
$$

$SU(N)$	$N(\square+\bar{\square})$	confining	[2]
$SU(N)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}+(N-1)\bar{\square}+3\square$	confining	[27]
$SU(N)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}+\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}+2(\square+\bar{\square})$	confining	[3]
$SU(N)$	Adj	Coulomb phase	[1, 7, 8]
$SU(4)$	$3\begin{smallmatrix}\square\\\square\end{smallmatrix}+(\square+\bar{\square})$	confining	[3, 4]
$SU(4)$	$4\begin{smallmatrix}\square\\\square\end{smallmatrix}$	Coulomb phase	[26]
$SU(5)$	$2\begin{smallmatrix}\square\\\square\end{smallmatrix}+\square+3\bar{\square}$	confining	[3]
$SU(5)$	$2\begin{smallmatrix}\square\\\square\end{smallmatrix}+\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}+\bar{\square}$	confining	[3]
$SU(6)$	$2\begin{smallmatrix}\square\\\square\end{smallmatrix}+4\bar{\square}$	confining	[3]
$SU(6)$	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}+3(\square+\bar{\square})$	confining	[3]
$SU(6)$	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}+\begin{smallmatrix}\square\\\square\end{smallmatrix}+2\bar{\square}$	confining	[4]
$SU(6)$	$2\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	Coulomb phase	Appendix B
$SU(7)$	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}+4\bar{\square}+2\square$	confining	[4]

Table 1: The SU theories satisfying the index constraint $\mu = G$. The first column gives the gauge group, the second column the field content and the third column gives the phase of the low-energy theory. The last column gives a reference to where the low-energy solution of the given theory can be found.

In the above flows the following fields have to get expectation values: in the $SO(N)$ theories $N-4$ vectors, in the $SU(6)$ theory one $\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$ and in the $Sp(6)$ theory one $\begin{smallmatrix}\square\\\square\end{smallmatrix}$. Such product group theories have been shown to be in the Coulomb phase in Refs. [21, 22]. The $SO(N)$ series in (3.2) has been described in Ref. [26], while the remaining flows will enable us to find the Seiberg-Witten curves for the $SU(6)$ and $Sp(6)$ theories. The results are summarized in the next section, while the detailed derivations are presented in Appendices A and B.

4 The $N = 1$ Seiberg-Witten Theories

In this section we give the low-energy solution of the $N = 1$ Seiberg-Witten theories. Since the solutions to the theories with one adjoint (the pure $N = 2$ theories) are well-known, we refer the reader interested in these theories to the references given in the last column of Tables 1-4. Below, we give only the solution to the $N = 1$ theories considered in (3.2), and do not list all $N = 2$ examples. For these theories, we first give the high-energy field content and the unbroken global symmetries, then list the low-energy degrees of freedom which satisfy the 't Hooft anomaly matching conditions. Finally we give the curve for every

$Sp(2N)$	$(2N+2)\square$	confining	[28]
$Sp(2N)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}+4\square$	confining	[29]
$Sp(2N)$	$\square\square = Adj$	Coulomb phase	[12, 14]
$Sp(4)$	$2\begin{smallmatrix}\square\\\square\end{smallmatrix}+2\square$	confining	[4]
$Sp(4)$	$3\begin{smallmatrix}\square\\\square\end{smallmatrix}$	Coulomb phase	[26]
$Sp(6)$	$2\begin{smallmatrix}\square\\\square\end{smallmatrix}$	Coulomb phase	Appendix A
$Sp(6)$	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}+3\square$	confining	[4]

Table 2: The Sp theories satisfying the index constraint $\mu = G$. The first column gives the gauge group, the second column the field content and the third column gives the phase of the low-energy theory. The last column gives a reference to where the low-energy solution of the given theory can be found.

theory which provides the low-energy solution for the effective $U(1)$ gauge coupling.

4.1 $SU(6)$ with $2\begin{smallmatrix}\square\\\square\end{smallmatrix}$

	$SU(6)$	$SU(2)$	$U(1)_R$	Z_{12}
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	0	1
$S = A^2$	1	1	0	2
$T = A^4$	1	$\square\square\square\square$	0	4
$U = A^6$	1	1	0	6

The Seiberg-Witten curve is

$$\begin{aligned} y^2 &= \Big[x^3 - \Big(\frac{9}{2}(T^2) + \frac{11}{3}S^4 - SU\Big)x - 2^7\Lambda^{12} - \\ &\quad \Big(2S^3U - \frac{107}{27}S^6 - \frac{1}{4}U^2 - \frac{3}{2}S^2(T^2) + 9(T^3)\Big)\Big]^2 - 2^{14}\Lambda^{24}. \end{aligned} \tag{4.1}$$

A detailed description of this theory is given in Appendix B.

4.2 $SO(N)$ with $(N-2)\square$ [26]

	$SO(N)$	$SU(N-2)$	$U(1)_R$	Z_{2N-4}
Q	$\square$	$\square$	0	1
$M = Q^2$	1	$\square\square$	0	2

$SO(N)$	$\boxminus = Adj$	Coulomb phase	[10, 11, 14]
$SO(N)$	$(0, N-2)$	Coulomb phase	[26]
$SO(7)$	$(1,4)$	confining	[31]
$SO(7)$	$(2,3)$	confining	[32]
$SO(7)$	$(3,2)$	confining	[3]
$SO(7)$	$(4,1)$	confining	[3]
$SO(7)$	$(5,0)$	confining	[3]
$SO(8)$	$(3,2,1)$	confining	[3]
$SO(8)$	$(2,2,2)$	confining	[3]
$SO(8)$	$(4,2,0)$	confining	[3]
$SO(8)$	$(4,1,1)$	confining	[32]
$SO(8)$	$(3,3,0)$	confining	[3]
$SO(8)$	$(5,1,0)$	confining	[32]
$SO(9)$	$(1,5)$	confining	[32]
$SO(9)$	$(2,3)$	confining	[3]
$SO(9)$	$(3,1)$	confining	[3]
$SO(10)$	$(1,0,6)$	confining	[32]
$SO(10)$	$(2,2,0)$	confining	[3]
$SO(10)$	$(1,1,4)$	confining	[3]
$SO(10)$	$(2,1,2)$	confining	[3]
$SO(10)$	$(3,1,0)$	confining	[3]
$SO(10)$	$(2,0,4)$	confining	[3]
$SO(10)$	$(3,0,2)$	confining	[3]
$SO(10)$	$(4,0,0)$	confining	[3]
$SO(11)$	$(2,1)$	confining	[3]
$SO(11)$	$(1,5)$	confining	[3]
$SO(12)$	$(1,1,2)$	confining	[3]
$SO(12)$	$(2,0,2)$	confining	[3]
$SO(12)$	$(1,0,6)$	confining	[3]
$SO(13)$	$(1,3)$	confining	[3]
$SO(14)$	$(1,0,4)$	confining	[3]

Table 3: The SO theories satisfying $\mu = G$. The first column gives the gauge group, the second one the field content and the third one the phase of the low-energy theory. The last column gives a reference to where the low-energy solution of the given theory is described. We use the following notation for the field content: for an $SO(N)$ group for N odd (s, v) denotes the number of spinors and the number of vectors, while for N even (s, s', v) denotes the number of the two inequivalent spinor representations and the number of vectors. We do not distinguish between $SO(N)$ and its covering group $Spin(N)$.

G_2	$4 \times \mathbf{7}$	confining	[31]
G_2	$\mathbf{14} = Adj$	Coulomb phase	[15, 14, 17]
F_4	$3 \times \mathbf{26}$	confining	[5, 6]
F_4	$\mathbf{52} = Adj$	Coulomb phase	[14, 16, 17]
E_6	$n \times \mathbf{27} + (4-n) \times \overline{\mathbf{27}},$	confining	[5, 6]
E_6	$\mathbf{78} = Adj$	Coulomb phase	[14, 16, 17]
E_7	$3 \times \mathbf{56}$	confining	[5, 6]
E_7	$\mathbf{133} = Adj$	Coulomb phase	[14, 16]
E_8	$\mathbf{248} = Adj$	Coulomb phase	[14, 16]

Table 4: The theories based on exceptional groups satisfying the index constraint $\mu = G$. The first column gives the gauge group, the second column the field content and the third column gives the phase of the low-energy theory. The last column gives a reference to where the low-energy solution of the given theory can be found.

Note that in this example the anomaly matching conditions at the origin are not satisfied by the meson field M itself. The reason is that a number of monopoles become massless exactly at the origin, and their contribution to the anomalies has to be taken into account. For details see Refs. [26, 30]. The Seiberg-Witten curve for this theory is:

$$y^2 = (x^2 - (\det M - 8\Lambda^{2N-4}))^2 - 64\Lambda^{4N-8}.$$

4.3 $Sp(6)$ with $2\square$

	$Sp(6)$	$SU(2)$	$U(1)_R$	Z_8
A_i	$\square$	$\square$	0	1
$S_{ij} = \mathrm{Tr}(JA_iJA_j)$		$\square\square$	0	2
$T_{ijk} = \mathrm{Tr}(JA_iJA_jJA_k)$		$\square\square\square$	0	3
$U = \mathrm{Tr}(JA_1JA_2JA_1JA_2)$		1	0	4

The Seiberg-Witten curve is given by

$$\begin{aligned} y^2 &= \Big[x^2 - (-72\Lambda^8\big((S^2) + 12U\big) + \big(-\frac{1}{2}(T^4) + 24U^3 + \frac{3}{2}U^2(S^2) + \\ &\quad +3U(ST^2) + \frac{1}{4}(S^3T^2)\big)\Big]^2 - 768\Lambda^{24}. \end{aligned} \tag{4.2}$$

A detailed description of this theory is given in Appendix A.

5 Conclusions

We have studied $N = 1$ supersymmetric gauge theories which are in the Coulomb phase on the entire moduli space. We have shown that theories based on a simple gauge group and no tree-level superpotential must satisfy the index condition $\mu = G$, which is exactly the same as for theories with a quantum deformed moduli space. One can find the theories which are actually in the Coulomb phase by studying the flows of the theory. It turns out that all $\mu = G$ theories are either confining with a quantum deformed moduli space if the matter content is in the faithful representation of the gauge group or in the Coulomb phase if the matter fields are not in a faithful representation. The Seiberg-Witten curves for the new theories in the Coulomb phase can be found by studying the flows to the product group theories of Ref. [22]. This work, together with the results on confining theories with quantum-deformed moduli spaces, completes the solutions to all $N = 1$ theories with $\mu \leq G$.

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Appendix A $Sp(6)$ with $2\begin{smallmatrix}\square\\\square\end{smallmatrix}$

In this appendix we outline the derivation of the Seiberg-Witten curve for the $Sp(6)$ theory with $2\begin{smallmatrix}\square\\\square\end{smallmatrix}$. As we already mentioned in Section 3, giving a VEV to one of antisymmetric tensors breaks $Sp(6)$ to $SU(2)^3$ with precisely the field content that was considered in Ref. [22]. The curve describing the $Sp(6)$ theory must therefore reduce to the curve for $SU(2)^3$ in the limit of large VEVs. It turns out that considering this limit is sufficient for determining the complete curve.

Lets us first describe the global symmetries of the theory and gauge invariant operators parameterizing the moduli space.

	$Sp(6)$	$SU(2)$	$U(1)_R$	Z_8
A_i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	0	1
$S_{ij} = \mathrm{Tr}(JA_iJA_j)$		$\square\square$	0	2
$T_{ijk} = \mathrm{Tr}(JA_iJA_jJA_k)$		$\square\square\square$	0	3
$U = \mathrm{Tr}(JA_1JA_2JA_1JA_2)$		1	0	4

, (A.1)

where J is the two-index antisymmetric invariant tensor of Sp. Indeed, with this choice of operators the 't Hooft anomaly matching conditions (including the discrete anomaly matching conditions of Ref. [30]) are satisfied at the origin, once a $U(1)$ vector field is included in the low-energy spectrum.

We give a VEV to A_1 of the form $\langle A_1 \rangle = iv\,\mathrm{diag}(\sigma_2, \omega\sigma_2, \omega^2\sigma_2)$, where $\omega = \exp(2\pi i/3)$. The remaining field, A_2, decomposes under $SU(2) \times SU(2) \times SU(2)$ as

$$A_2 \to Q_1(\Box,\Box,1) + Q_2(1,\Box,\Box) + Q_3(\Box,1,\Box) + s_{1,2}(1,1,1).$$

Next, we express the invariants of the $SU(2)^3$ theory [22], that is $M_i = Q_i^2$ and $T = Q_1Q_2Q_3$ in terms of the $Sp(6)$ gauge invariants keeping only leading terms in $1/v$. The curve for $SU(2)^3$

$$y^2 = [x^2 - (\Lambda_1^4 M_2 + \Lambda_2^4 M_3 + \Lambda_3^4 M_1 + T^2 - M_1M_2M_3)]^2 - 4\Lambda_1^4\Lambda_2^4\Lambda_3^4$$

contains two combinations of invariants. We will now write the two combinations, $\Lambda_1^4 M_2 + \Lambda_2^4 M_3 + \Lambda_3^4 M_1$ and $T^2 - M_1M_2M_3$, in terms of $Sp(6)$ invariants, where Λ_i is the characteristic scale of the i-th $SU(2)$ factor. With the above choice of the VEV for A_1, the scale matching relations between the $Sp(6)$ scale, Λ, and the scales of the $SU(2)$ factors are

$$\Lambda_1^4 = \frac{\Lambda^8}{v^4(1-\omega^2)(1-\omega^4)}, \quad \Lambda_2^4 = \frac{\Lambda^8}{v^4(\omega^2-1)(\omega^2-\omega^4)}, \quad \Lambda_3^4 = \frac{\Lambda^8}{v^4(\omega^4-1)(\omega^4-\omega^2)}.$$

Since a curve can depend only on the field combinations invariant under global symmetries, we need to introduce invariants under the global $SU(2)$ symmetry, which will be the variables of the full $Sp(6)$ curve. The connection between the global $SU(2)$ invariants and the parameters of the $SU(2)^3$ curve are

$$\begin{aligned}
&\Lambda_1^4\Lambda_2^4\Lambda_3^4 = \frac{1}{27v^{12}}\Lambda^{24},\\
&\Lambda_1^4 M_2 + \Lambda_2^4 M_3 + \Lambda_3^4 M_1 = -\frac{1}{72v^6}\Lambda^8\Big((S^2) + 12U\Big),\\
&T^2 - M_1M_2M_3 = \frac{1}{5184v^6}\Big(-\frac{1}{2}(T^4) + 24U^3 + \frac{3}{2}U^2(S^2) + 3U(ST^2) + \frac{1}{4}(S^3T^2)\Big),
\end{aligned}$$

where the $SU(2)$ contractions are defined as follows

$$\begin{aligned}
(S^2) &= S_{i_1i_2}S_{j_1j_2}\epsilon^{i_1j_1}\epsilon^{i_2j_2}\\
(T^4) &= T_{i_1i_2i_3}T_{j_1j_2j_3}T_{k_1k_2k_3}T_{l_1l_2l_3}\epsilon^{i_1j_1}\epsilon^{i_2j_2}\epsilon^{k_1l_1}\epsilon^{k_2l_2}\epsilon^{i_3k_3}\epsilon^{j_3l_3},\\
(ST^2) &= T_{i_1i_2i_3}T_{j_1j_2j_3}S_{k_1k_2}\epsilon^{i_1j_1}\epsilon^{i_2j_2}\epsilon^{i_3k_1}\epsilon^{j_3k_2},\\
(S^3T^2) &= T_{i_1i_2i_3}T_{j_1j_2j_3}S_{k_1k_2}S_{l_1l_2}S_{m_1m_2}\times\\
&\quad (\epsilon^{i_1j_1}\epsilon^{i_2j_2}\epsilon^{i_3k_1}\epsilon^{k_2l_1}\epsilon^{l_2m_1}\epsilon^{m_2j_3} - \frac{1}{3}\epsilon^{i_1k_1}\epsilon^{i_2l_1}\epsilon^{i_3m_1}\epsilon^{j_1k_2}\epsilon^{j_2l_2}\epsilon^{j_3m_2}).
\end{aligned}$$

Finally, we obtain the curve for $Sp(6)$ theory in the large VEV limit by substituting the above expression into the $SU(2)^3$ curve:

$$\begin{aligned} y^2 &= \Big[x^2 - (-\frac{1}{72v^6}\big((S^2)+12U\big)\Lambda^8 + \frac{1}{5184v^6}[\big(-\frac{1}{2}(T^4)+24U^3+\frac{3}{2}U^2(S^2)+ \\ &\quad 3U(ST^2)+\frac{1}{4}(S^3T^2)\big)\Big]^2 - \frac{4}{27v^{12}}\Lambda^{24}. \end{aligned} \tag{A.2}$$

After rescaling $x \to x/(72v^3)$ and $y \to y/(5184v^6)$ we obtain:

$$\begin{aligned} y^2 &= \Big[x^2 - (-72\Lambda^8\big((S^2)+12U\big) + \big(-\frac{1}{2}(T^4)+24U^3+\frac{3}{2}U^2(S^2)+ \\ &\quad 3U(ST^2)+\frac{1}{4}(S^3T^2)\big)\Big]^2 - 768\Lambda^{24}. \end{aligned} \tag{A.3}$$

It is easy to check that symmetries prohibit modifications to this form of the curve. Since only $SU(2)$ singlets can appear in the curve, any other allowed term would be of the same order in $1/v$ and therefore a coefficient of any such terms must be zero. Thus (A.3) is the final form of the full $Sp(6)$ curve.

Appendix B $SU(6)$ with $2\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$

We now present the derivation of the curve for the $SU(6)$ theory with $2\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$. Similarly to the derivation of the $Sp(6)$ case, we give a VEV to one of the tensors, which breaks $SU(6)$ to $SU(3)\times SU(3)$ with two bifundamental fields, which is the theory considered in Ref. [22]. The field content, global symmetries and independent gauge invariants are defined below:

	$SU(6)$	$SU(2)$	$U(1)_R$	Z_{12}
A_i	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	$\square$	0	1
$S = A_1A_2$		1	0	2
$T_{ijkl} = A_iA_jA_kA_l$		$\square\square\square\square$	0	4
$U = A_1^3A_2^3$		1	0	6

(B.1)

The anomaly matching is satisfied for the low-energy spectrum including the gauge invariants and two $U(1)$ vector multiplets. The $SU(6)$ gauge contractions for the invariants S, T and U are given by

$$\begin{aligned} S &= \frac{1}{6^2}A_1^{\alpha_1\alpha_2\alpha_3}A_2^{\beta_1\beta_2\beta_3}\epsilon_{\alpha_1\alpha_2\alpha_3\beta_1\beta_2\beta_3}, \\ T_{ijkl} &= \frac{1}{6^4}\left[A_i^{\alpha_1\alpha_2\alpha_3}A_j^{\beta_1\beta_2\beta_3}A_k^{\gamma_1\gamma_2\gamma_3}A_l^{\delta_1\delta_2\delta_3}\epsilon_{\alpha_1\alpha_2\alpha_3\beta_1\beta_2\gamma_1}\epsilon_{\beta_3\gamma_2\gamma_3\delta_1\delta_2\delta_3} - \frac{S^2}{9}(\epsilon_{ij}\epsilon_{kl}+\epsilon_{ik}\epsilon_{jl})\right], \\ U &= \frac{1}{144}A_1^{\alpha_1\alpha_2\alpha_3}A_1^{\beta_1\beta_2\beta_3}A_1^{\gamma_1\gamma_2\gamma_3}A_2^{\delta_1\delta_2\delta_3}A_2^{\zeta_1\zeta_2\zeta_3}A_2^{\eta_1\eta_2\eta_3}\epsilon_{\alpha_1\alpha_2\alpha_3\beta_1\beta_2\gamma_1}\epsilon_{\beta_3\gamma_2\gamma_3\delta_1\delta_2\zeta_1}\epsilon_{\delta_3\zeta_2\zeta_3\eta_1\eta_2\eta_3}. \end{aligned}$$

Note the additional subtlety in the definition of T. T is a four-index symmetric tensor under the global $SU(2)$. However, the gauge contraction defined above does not yield an irreducible tensor of $SU(2)$. This contraction also contains an $SU(2)$ singlet piece, which needs to be subtracted.

We give a VEV to A_1 of the form $\langle A_{123}\rangle = \langle A_{456}\rangle = v$ with zero VEVs for all other independent components. The field A_2 decomposes under $SU(3)\times SU(3)$ as

$$A_2 \to Q_1(\square,\bar{\square}) + Q_2(\bar{\square},\square) + s_{1,2}(1,1,1).$$

The $SU(3)^2$ theory has four invariants [22], $B_i = \det Q_i$ and $T_i = \mathrm{Tr}(Q_1Q_2)^i$, $i = 1,2$, which we now express in terms of $SU(6)$ invariants. The $SU(3)\times SU(3)$ curve is

$$y^2 = (x^3 - u_2x - u_3 - \Lambda_1^6 - \Lambda_2^6)^2 - 4\Lambda_1^6\Lambda_2^6,$$

where $\Lambda_{1,2}$ are the scales of the two $SU(3)$ gauge groups, while $u_2 = \frac{1}{2}(T_2 - \frac{1}{3}T_1^2)$ and $u_3 = \frac{1}{3}(3B_1B_2 + \frac{1}{2}T_2T_1 - \frac{5}{18}T_1^3)$. In terms of $SU(6)$ invariants we have

$$\begin{aligned} u_2 &= \frac{1}{16v^4}\left(\frac{9}{2}(T^2) + \frac{11}{3}S^4 - SU\right) \\ u_3 &= \frac{1}{64v^6}\left(2S^3U - \frac{107}{27}S^6 - \frac{1}{4}U^2 - \frac{3}{2}S^2(T^2) + 9(T^3)\right), \end{aligned}$$

where the invariants under the global $SU(2)$ are defined as follows:

$$\begin{aligned} (T^2) &= T_{i_1i_2i_3i_4}T_{j_1j_2j_3j_4}\epsilon^{i_1j_1}\epsilon^{i_2j_2}\epsilon^{i_3j_3}\epsilon^{i_4j_4}, \\ (T^3) &= T_{i_1i_2i_3i_4}T_{j_1j_2j_3j_4}T_{k_1k_2k_3k_4}\epsilon^{i_1j_1}\epsilon^{i_2j_2}\epsilon^{i_3k_1}\epsilon^{i_4k_2}\epsilon^{j_3k_3}\epsilon^{j_4k_4}. \end{aligned}$$

Using the matching relations for the $SU(6)$ scale, Λ, and the scales of the $SU(3)$ groups $\Lambda_1^6 = \Lambda_2^6 = \frac{\Lambda^{12}}{v^6}$ we obtain the $SU(6)$ curve in the large VEV limit:

$$\begin{aligned} y^2 &= \Big[x^3 - \frac{1}{16v^4}\Big(\frac{9}{2}(T^2) + \frac{11}{3}S^4 - SU\Big)x - 2\frac{\Lambda^{12}}{v^6} - \\ &\quad \frac{1}{64v^6}\Big(2S^3U - \frac{107}{27}S^6 - \frac{1}{4}U^2 - \frac{3}{2}S^2(T^2) + 9(T^3)\Big)\Big]^2 - 4\frac{\Lambda^{24}}{v^{12}}. \end{aligned} \tag{B.2}$$

After rescaling $x \to x/(4v^2)$ and $y \to y/(64v^6)$ the curve takes the form

$$\begin{aligned} y^2 &= \Big[x^3 - \Big(\frac{9}{2}(T^2) + \frac{11}{3}S^4 - SU\Big)x - 2^7\Lambda^{12} - \\ &\quad \Big(2S^3U - \frac{107}{27}S^6 - \frac{1}{4}U^2 - \frac{3}{2}S^2(T^2) + 9(T^3)\Big)\Big]^2 - 2^{14}\Lambda^{24}. \end{aligned} \tag{B.3}$$

As before, this is the final form of the full $SU(6)$ curve because any modification allowed by the global symmetries is of the same order in $1/v$ as the terms already present.

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