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Orbifolds of $AdS_5 \times S^5$ and $4d$ Conformal Field Theories

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Abstract

We study the relation between the large N limit of four dimensional $\mathcal{N} = 2, 1, 0$ conformal field theories and supergravity on orbifolds of $AdS_5 \times S^5$. We analyze the the Kaluza-Klein states of the supergravity theory and relate them to the spectrum of (chiral) primary operators of the (super) conformal field theories.

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1 Introduction

Recently a duality between superconformal field theories (SCFTs) in d dimensions and string or M theory compactified on anti-de Sitter (AdS) spaces of the form $AdS_{d+1} \times W$ has been proposed in [1] (see also [2–9]). Here W is a compact manifold which in the maximally supersymmetric cases is a sphere. A precise correspondence between the supergravity limit on the AdS_{d+1} side and an appropriate large N limit on the conformal field theory side has been formulated in [10, 11]*. According to [11] the correlation functions in the conformal field theory, which has as its spacetime M_d, the boundary at infinity of AdS_{d+1}, can be calculated via the dependence of the supergravity action on the asymptotic behavior of its fields at the boundary M_d. In particular, one can deduce the scaling dimensions of operators in the conformal field theory from the masses of particles in string theory (or M theory). Using this correspondence, the dimensions of chiral primary operators in four dimensional $\mathcal{N} = 4$ super-Yang-Mills (SYM) were matched with the masses of Kaluza-Klein states on $AdS_5 \times S^5$. Related works which appeared recently are [12-43].

In [17, 24] a relation between several classes of four dimensional $\mathcal{N} = 2, 1, 0$ conformal field theories and Type IIB supergravity (string) theory on orbifolds of $AdS_5 \times S^5$ was proposed. The orbifolds preserve the AdS_5 structure and its isometry group $SO(4,2)$ which becomes the conformal symmetry of the four dimensional theory. The orbifold action on S^5 breaks some or all the $\mathcal{N} = 4$ supersymmetry. When the orbifold group Γ acts freely on S^5 there is a limit where supergravity provides an applicable description. When the orbifold action is not free only the string theory description is reliable. Some of these $\mathcal{N} = 1, 2$ models were shown to be conformal [17]. The analysis of the one-loop and two-loop β functions of the general orbifold theories in [24] showed that they indeed vanish. Furthermore the analysis in [33] shows that these theories have indeed a fixed line (fixed hypersurface) at least in the large N limit. In the nonsupersymmetric models, the β functions need not vanish at finite N and they do not, as we will comment on in the discussion section.

In this paper we will study the proposed duality by analyzing the Kaluza-Klein states of the supergravity theory on the orbifolds of $AdS_5 \times S^5$ and relating them to the (chiral) primary operators of the (super) conformal field theories on the boundary. In the next section we will briefly review the relation between AdS supergravity (string) theory and SCFTs on the boundary of the AdS space. In particular we will review in detail the relation between the Kaluza-Klein harmonics of supergravity on $AdS_5 \times S^5$ and the chiral primary operators of $\mathcal{N} = 4$ SYM in four dimensions. In section 3 we will analyze the

*See also [9] for the relation between bulk fields and boundary composites.

Kaluza-Klein harmonics of supergravity on orbifolds $AdS_5 \times S^5/\Gamma$ where $\Gamma \subset SU(3)$ is a discrete subgroup. We will relate the Kaluza-Klein modes to the chiral primary operators of the $\mathcal{N} = 1$ theory on the boundary of the AdS space. In section 4 we will perform similar analysis when the orbifold group $\Gamma \subset SU(2)$. In this case one gets an $\mathcal{N} = 2$ theory on the boundary of the AdS space. However now the orbifold action is not free and one in general does not expect supergravity to provide an applicable description. Nevertheless we will still be able to relate the Kaluza-Klein modes to the chiral primary operators of the boundary $\mathcal{N} = 2$ theory. This means that chiral information is still reliably encoded in the supergravity description. In section 5 we study a possible relation between Kaluza-Klein states and primary operators of the boundary CFT when the orbifold group $\Gamma \subset SU(4)$ and the theory on the boundary is not supersymmetric. Section 6 is devoted to a summary of the results and discussion.

2 SCFT/AdS Relation: $\mathcal{N} = 4$ SYM In Four Dimensions

In this section we will briefly review the SCFT/AdS relation proposed in [10, 11]. One of the examples of this relation is between $\mathcal{N} = 4$ SYM in four dimensions and Type IIB supergravity (or string) theory on $AdS_5 \times S^5$. This example will be of particular importance for us since orbifolds of this relation will be studied in next sections.

The boundary M_d of AdS_{d+1} is a d-dimensional Minkowski space with points at infinity added. The isometry group of AdS_{d+1} is $SO(d, 2)$. It is also the conformal group on M_d. The proposed duality relates string theory (or M theory) on AdS_{d+1} to the large N limit of SCFTs on its boundary M_d. In the Euclidean version the boundary is S^d. Consider the maximally supersymmetric case, so that the internal space is also a sphere. Let ϕ be a scalar field on AdS_{d+1} and ϕ_0 its restriction to the boundary S^d. According to the SCFT/AdS relation ϕ_0 couples to a conformal operator $\mathcal{O}$ on the boundary via $\int_{S^d} \phi_0 \mathcal{O}$.

When ϕ has mass m the corresponding operator $\mathcal{O}$ has conformal dimension Δ given by

$$m^2 = \Delta(\Delta - d) \ . \tag{2.1}$$

Irrelevant, marginal, and relevant operators of the boundary theory correspond to massive, massless, and "tachyonic" modes in the supergravity theory. If a p-form C on AdS is coupled to a $d - p$ form operator $\mathcal{O}$ on the boundary, then the relation between the mass of C and the conformal dimension of $\mathcal{O}$ is given by

$$m^2 = (\Delta + p)(\Delta + p - d) \ . \tag{2.2}$$

The value of m^2 in this formula refers to the eigenvalue of the Laplace operator on the AdS space. In the supergravity literature, the values that are usually quoted for p-forms are the eigenvalues $\tilde{m}^2$ of the appropriate Maxwell-like operators. The relation of these to the dimension is given by

$$\tilde{m}^2 = (\Delta - p)(\Delta + p - d) \ . \tag{2.3}$$

Formula (2.3) can be derived by repeating the analysis of [11] using the Maxwell type equations for the p-forms. Alternatively, one can use the definition of the mass via the quadratic Casimir of the $SO(d,2)$ isometry group of AdS_{d+1} as was done for AdS_5 in [20].

The massless graviton in the AdS supergravity couples to the dimension d stress-energy tensor of the SCFT. When the internal space W has continuous rotational symmetry, there are also AdS massless vector fields in its adjoint representation which couple to the dimension $d-1$ R-symmetry currents of the SCFT.

Consider the Type IIB superstring theory on $AdS_5 \times S^5$ with a 5-form flux of N units on S^5 and radius of curvature $(g_{st}N)^{\frac{1}{4}}$. In the large N limit with $g_{YM}^2 N = g_{st}N$ fixed and large, string theory is weakly coupled and the supergravity description is applicable. The bosonic symmetry of this compactification of ten dimensional Type IIB supergravity is $SO(4,2) \times SO(6)$. In [1] it was proposed that $\mathcal{N} = 4$ SYM in four dimensions is dual to string theory on the above background. The $SO(4,2)$ part of the symmetry of the supergravity theory corresponds to the conformal symmetry of the $\mathcal{N} = 4$ superconformal theory. The $SO(6) \simeq SU(4)$ part of the symmetry, which is the isometry of S^5, is the the R-symmetry of the superconformal theory.

AdS supergravity has multiplet shortening due to the internal symmetry generators of its superalgebra [44, 45]. In the maximally supersymmetric case the Kaluza-Klein excitations of supergravity fall into short representations of supersymmetry with spins ≤ 2, and their mass formula is protected from quantum and string corrections. They couple to chiral primary operators of four dimensional $\mathcal{N} = 4$ SYM on the boundary. Chiral operators are in short representations of the superconformal algebra and their dimensions are determined in terms of the R-symmetry representation and cannot receive any corrections [46, 47]. An $\mathcal{N} = 1$ superconformal subalgebra of the $\mathcal{N} = 4$ superconformal algebra has a generator, R, of the $U(1)_R$ symmetry. The dimensions of chiral operators are determined by their R charges

$$\dim(\mathcal{O}) = \frac{3}{2}R. \tag{2.4}$$

Since a bulk field ϕ with boundary value ϕ_0 couples to a conformal field $\mathcal{O}$ on the boundary via $\int_{S^d} \phi_0 \mathcal{O}$, ϕ_0 carries opposite R-charge to that of $\mathcal{O}$. Multiplet shortening in AdS

supergravity is expected also with a reduced number of supersymmetries [44, 45] and we expect the mass formulas of the Kaluza-Klein excitations to be protected from quantum and string corrections.

The spectrum of Kaluza-Klein harmonics of supergravity on $AdS_5 \times S^5$ was derived in [48, 49]. The Kaluza-Klein harmonics fall into irreducible representations of $SU(4)$. We will now review the families that contain fields with negative or zero mass.

There is one family of spin-2 fields. The mass formula, in $1/\sqrt{\alpha'}$ units, is given by

$$m^2 = k(k+4), \qquad k \geq 0 \ . \tag{2.5}$$

The $SU(4)$ Dynkin labels of the representations are $(0, k, 0)$, and the corresponding $SU(4)$ irreducible representations are $\mathbf{1}, \mathbf{6}, \mathbf{20'}, \ldots$. The $k = 0$ particle is the graviton that couples to the dimension 4 stress-energy tensor operator of the $\mathcal{N} = 4$ SCFT theory.

There is one family of vector fields that contains massless states with mass formula given by

$$m^2 = (k-1)(k+1), \qquad k \geq 1 \ . \tag{2.6}$$

The Dynkin labels are $(1, k-1, 1)$ and the irreducible representations are $\mathbf{15}, \mathbf{64}, \mathbf{175}, \ldots$. The massless vector bosons at $k = 1$ transform in the adjoint of $SU(4)$ and couple to the $SU(4)$ R-symmetry currents of the $\mathcal{N} = 4$ SCFT theory.

There are three families of scalar fields that contain negative and massless states. The first family has mass formula

$$m^2 = k(k-4), \qquad k \geq 2 \ , \tag{2.7}$$

with Dynkin labels $(0, k, 0)$ corresponding to the irreducible representations $\mathbf{20'}, \mathbf{50}, \mathbf{105}, \ldots$. They couple to dimension $\Delta = k$ chiral primary operators of the $\mathcal{N} = 4$ SCFT theory* which were identified in [11] as the symmetrized traceless $\mathrm{Tr}(\Phi^{i_1} \ldots \Phi^{i_k})$ of the adjoint chiral superfields $\Phi^i, i = 1, 2, 3$. These operators indeed transform in the same symmetric traceless representations of $SU(4)$ as the Kaluza-Klein particles (2.7).

The second family has mass formula

$$m^2 = (k-1)(k+3), \qquad k \geq 0 \ , \tag{2.8}$$

*More precisely, since the $\mathbf{20'}$ representation of $SU(4)$ decomposes into representations of $SU(3) \times U(1)_R$ as $\mathbf{20'} = \mathbf{6}_{4/3} + \overline{\mathbf{6}}_{-4/3} + \mathbf{8}_0$ then the $\mathbf{6}_{4/3}$ Kaluza-Klein states couple to chiral primary operators, the $\mathbf{6}_{-4/3}$ couple to anti-chiral primary operator and the $\mathbf{8}_0$ couple to operators which are neither chiral nor anti-chiral. Nevertheless, since they all sit in the same $\mathcal{N} = 4$ multiplet they are all protected from quantum corrections.

with Dynkin labels $(0,k,2)$ corresponding to the irreducible representations $\mathbf{10}, \mathbf{45}, \mathbf{126}, \ldots$. They couple to dimension $\Delta = k+3$ chiral primary operators of the $\mathcal{N}=4$ SCFT theory $Tr(W_\alpha W^\alpha \Phi^{i_1} \ldots \Phi^{i_k})$ where W_α is the field strength chiral superfield [11]

The third family has mass formula

$$m^2 = k(k+4), \qquad k \geq 0 \ , \tag{2.9}$$

with Dynkin labels $(0,k,0)$ corresponding to the irreducible representations $\mathbf{1}, \mathbf{6}, \mathbf{20'}, \ldots$. The massless particle in this family ($k=0$) is the dilaton. It couples to TrF^2 in the $\mathcal{N}=4$ theory. The particles couple to dimension $\Delta = k+4$ chiral primary operators of the boundary theory $Tra^kF^2 + \ldots$ where a is the complex scalar in the $\mathcal{N}=2$ vector multiplet (when viewing $\mathcal{N}=4$ as $\mathcal{N}=2$ with a hypermultiplet in the adjoint) [11].

The different towers of Kaluza-Klein harmonics are related by the action of the supersymmetry generators and can be organized in an $\mathcal{N}=4$ supertower [20]. For instance, the graviton, the **15** massless vector bosons and the scalars in the above three families in the representations $\mathbf{20'}, \mathbf{10}, \mathbf{1}$ of $SU(4)$ are in the same multiplet.

3 $\mathcal{N}=1$ Supersymmetric Theories

In [17,24] $\mathcal{N}=1$ SCFTs were constructed by studying D3 branes at orbifold singularities of the form R^6/Γ, where $\Gamma \subset SU(3)$ is a discrete subgroup. The worldvolume theory is constructed by taking $N|\Gamma|$ D3 branes on the covering space and performing a projection Γ on the worldvolume fields and the Chan Paton factors [50–52]. Conformal field theories are expected when the representation of Γ acting on the Chan Paton factors is chosen to be the N-fold copy of the regular representation. In the framework of [1] this translates to the study of Type IIB string theory on an orbifold of $AdS_5 \times S^5$ where the orbifold group acts only on the S^5 factor. The $SO(4,2)$ isometry of AdS_5 is not broken and corresponds to the conformal symmetry of the SCFT on the D3 branes worldvolume. The $SO(6) \simeq SU(4)$ isometry of S^5 is broken to $U(1)_R \times \Gamma$. The $U(1)_R$ factor is the R-symmetry of the boundary $\mathcal{N}=1$ D3 brane theory. The Γ factor becomes a discrete global symmetry of the D3 brane theory*.

Consider first the Z_3 orbifold example of [17]

$$X^{1,2,3} \to e^{\frac{2\pi i}{3}} X^{1,2,3} \ , \tag{3.1}$$

where X^i parametrize the R^6 transverse to the D3 branes worldvolume. The gauge group, global symmetries and field content of the D3 brane theory is given in Table 1.

*Γ is a symmetry of the quiver diagram description.

	$SU(N)$	$SU(N)$	$SU(N)$	$U(1)_R$
U^i	$\square$	$\overline{\square}$	$\mathbf{1}$	$\frac{2}{3}$
V^i	$\mathbf{1}$	$\square$	$\overline{\square}$	$\frac{2}{3}$
W^i	$\overline{\square}$	$\mathbf{1}$	$\square$	$\frac{2}{3}$

Table 1: Field content of the $\mathcal{N} = 1$ theory, where $i = 1, 2, 3$. The $SU(3)$ global symmetry is broken by the superpotential.

The orbifold (3.1) has a fixed point at the origin. Since the volume of S^5 is not zero, the orbifold action is free and the resulting manifold is smooth. In the large N limit as specified in the $\mathcal{N} = 4$ SYM theory in the previous section the supergravity description is applicable. If the relation between supergravity and SCFT of [1, 11] holds also here we expect to find Kaluza-Klein harmonics of supergravity on $AdS_5 \times S^5/Z_3$ corresponding to the chiral primary operators in the $\mathcal{N} = 1$ theory. In the following we will study this correspondence. We will analyze the Kaluza-Klein harmonics of the supergravity theory and the relation to chiral primary operators of the boundary SCFT.

The Kaluza-Klein harmonics of supergravity on $AdS_5 \times S^5/Z_3$ are Z_3 invariant states and can be obtained by a Z_3 projection of the Kaluza-Klein harmonics on $AdS_5 \times S^5$ discussed in the previous section. Consider the scalar modes with masses given by (2.7). Let us explicitly check the relevant and marginal chiral primary operators in this family. The $k = 2$ Kaluza-Klein particle in (2.7) transforms in the $\mathbf{20'}$ of $SU(4)$. Decomposing [53] the $\mathbf{20'}$ into representations of $SU(3) \times U(1)_R$ gives:

$$\mathbf{20'} = \mathbf{6}_{4/3} + \overline{\mathbf{6}}_{-4/3} + \mathbf{8}_0 \ . \tag{3.2}$$

We now have to perform the Z_3 projection on (3.2). The $\mathbf{8}_0$ is invariant under the Z_3 projection†. However these Kaluza-Klein modes are expected to couple to dimension 2 operators. A dimension 2 chiral primary operator has R-charge‡ $\frac{4}{3}$ (2.4). Therefore $\mathbf{8}_0$ has the wrong R-charge to couple to a dimension 2 chiral operator and we do not expect any dimension 2 chiral primary operators in the boundary $N = 1$ SCFT. We expect dimension 2 operators which are not chiral primary to couple to the the Kaluza-Klein modes in the $\mathbf{8}_0$. In the $\mathcal{N} = 4$ case, the $\mathbf{6}_{4/3}, \overline{\mathbf{6}}_{-4/3}$, and $\mathbf{8}_0$ in (3.2) sit in the same supermultiplet and therefore the masses of the $\mathbf{8}_0$ Kaluza-Klein states were protected; here there is no such guarantee.

†The Z_3 acts on the $\mathbf{3}$ of $SU(3)$ as $(x^1, x^2, x^3) \to (e^{\frac{2\pi i}{3}} x^1, e^{\frac{2\pi i}{3}} x^2, e^{\frac{-4\pi i}{3}} x^3)$. The $\mathbf{8}$ is made of $\mathbf{3}$ and $\overline{\mathbf{3}}$.

‡The sign of the R-charge assignments in the decomposition is merely a convention.

The $k = 3$ Kaluza-Klein particle in (2.7) which transforms in the **50** of $SU(4)$ should couple to a dimension 3 chiral primary operator. Decomposing the **50** into representations of $SU(3) \times U(1)_R$ gives:

$$\mathbf{50} = \mathbf{10}_2 + \overline{\mathbf{10}}_{-2} + \mathbf{15}_{2/3} + \overline{\mathbf{15}}_{-2/3} \ . \tag{3.3}$$

The **10** is invariant under the Z_3 projection, and this is the only part in the decomposition (3.3) with the correct R-charge to couple to a dimension 3 chiral primary operator. We therefore expect 10 dimension 3 chiral primary operators in the boundary SCFT and we identify them with the ten independent operators $\mathrm{Tr}U^{i_1}V^{i_2}W^{i_3}$ symmetric in the i_k indices. The antisymmetric parts have to be removed in order to form primary operators since they appear in the superpotential and its derivatives.

The $k = 4$ massless Kaluza-Klein particle in (2.7) should couple to a marginal operator. It transforms in the **105** which decomposes as:

$$\mathbf{105} = \mathbf{15'}_{8/3} + \overline{\mathbf{15'}}_{-8/3} + \mathbf{24}_{-4/3} + \overline{\mathbf{24}}_{4/3} + \mathbf{27}_0 \ . \tag{3.4}$$

We see that the **15′** has the right R-charge to couple to a dimension 4 chiral primary operator, but it is not invariant under Z_3. The **27** is invariant under Z_3 but it's R-charge is not consistent with coupling to a dimension 4 chiral operator. So there are no Kaluza-Klein harmonics in this family that can couple to dimension 4 chiral primary operators and no such operators are expected in the boundary SCFT.

In general we expect the Z_3 invariance and the R-charge condition to restrict the value of k to be a multiple of 3. The Kaluza-Klein modes that survive in this family are

$$m^2 = 3n(3n-4), \qquad n = 1, 2, \dots \ , \tag{3.5}$$

and they couple to chiral primary operators of dimensions

$$\dim(\mathcal{O}) = \{3n, \ \ n = 1, 2, \dots\} \ , \tag{3.6}$$

in the boundary SCFT. We can identify these operators as symmetric operator $O_n = \mathrm{Tr}(UVW)^n$.

Consider next the scalar modes with masses given by (2.8). Again we will explicitly check the relevant and marginal chiral primary operators in this family. The Kaluza-Klein mode with $k = 0$ transforms in the **10** of $SU(4)$. We decompose the **10** into representations of $SU(3) \times U(1)_R$ as:

$$\mathbf{10} = \mathbf{1}_2 + \mathbf{3}_{2/3} + \mathbf{6}_{-2/3} \ . \tag{3.7}$$

The $\mathbf{1}_2$ is the only component that is invariant under the Z_3 projection, and it is also the only component with the correct R-charge to couple to a dimension 3 chiral operator. In fact the $\mathbf{1}$ component will be invariant under any projection that preserves $\mathcal{N} = 1$ supersymmetry. This Kaluza-Klein mode couples to the relevant operator $\sum_{i=1}^{3} \mathrm{Tr} W_\alpha^i W_i^\alpha$ where the index i enumerates the three gauge groups. This combination is dictated by the Z_3 global symmetry. This operator is a linear combination of the gaugino bilinears. As expected, the gaugino bilinears will be dimension 3 operators in any such theory derived from the $\mathcal{N} = 4$ by projection.

The $k = 1$ states in (2.8) transform in the $\mathbf{45}$, which decomposes[§] as:

$$\mathbf{45} = \mathbf{3}_{8/3} + \overline{\mathbf{3}}_{4/3} + \mathbf{6}_{4/3} + \mathbf{8}_0 + \mathbf{10}_0 + \mathbf{15}_{-4/3} \ . \tag{3.8}$$

The $\mathbf{3}$ has the correct R-charge to couple to a dimension 4 chiral primary operator, but it is not invariant under Z_3, while the $\mathbf{10}$ is invariant under Z_3 but it has the wrong R-charge, so we do not expect dimension 4 chiral primary operators in the boundary SCFT coupled to Kaluza-Klein modes of this family.

In general we expect Kaluza-Klein modes with

$$m^2 = (3n-1)(3n+3), \qquad n = 0, 1, 2, \ldots \tag{3.9}$$

coupled to chiral primary operators in the SCFT with dimensions

$$\dim(\mathcal{O}) = \{3n+3, \quad n = 0, 1, 2, \ldots\} \ . \tag{3.10}$$

These operators can be identified as $O_n = \mathrm{Tr} W_\alpha W^\alpha (UVW)^n$ where we suppressed the sum on the different gauge groups and the indices of the matter multiplets. For n bigger than zero these operators transform in the representation constructed from the product of $\mathbf{10}$ with the $3n$'th rank symmetric tensors.

Consider now the third family (2.9). The $k = 0$ state, the dilaton, transforms in the $\mathbf{1}$ which is invariant under the Z_3 projection. It couples to the marginal operator $\sum_{i=1}^{3} Tr F_i^2$. Evidently this result is independent of the choice of Γ, and the operator $\mathrm{Tr} F^2$ will be marginal in any theory obtained by Γ projection on the $\mathcal{N} = 4$ theory. We only find one marginal chiral primary operator in this family, since all higher values of k couple to irrelevant operators. As before we expect Kaluza-Klein harmonics with

$$m^2 = 3n(3n+4), \qquad n = 0, 1, 2, \ldots \tag{3.11}$$

[§]There is a typo in ref. [53] in the decomposition of the $\mathbf{45}$ in Table 27, the $\mathbf{6}$ is repeated.

coupled to chiral primary operators in the SCFT with dimensions

$$\dim(\mathcal{O}) = \{3n+4, \quad n = 0, 1, 2, \ldots\} \ . \tag{3.12}$$

The graviton with $k = 0$ in the spin 2 family (2.5) is in the **1** of $SU(4)$ and is invariant under the Z_3 projection and in general under any $\Gamma \subset SU(4)$ projection. As mentioned previously it couples to the stress-energy tensor.

The massless vector $k = 1$ in the spin-1 family (2.6) is in the **15** of $SU(4)$. Decomposing the **15** we find

$$\mathbf{15} = \mathbf{1}_0 + \mathbf{3}_{-4/3} + \overline{\mathbf{3}}_{4/3} + \mathbf{8}_0 \ . \tag{3.13}$$

The **1** is invariant under Z_3, and it has the correct charge to couple to the unbroken $U(1)_R$ current. The **1** component will be invariant under any projection that preserves $\mathcal{N} = 1$ supersymmetry. The $\mathbf{8}_0$ is also invariant under Z_3 and has the correct R-charge, these are the Z_3 remnants of the broken $SU(3)$. The currents to which they couple are not conserved, so there is no guarantee that the masses of these Kaluza-Klein states and the dimensions of the currents are protected from quantum corrections.

It is straightforward to extend the analysis to other projections that preserve $\mathcal{N} = 1$ supersymmetry. The discrete, non-Abelian subgroups of $SU(3)$ have been classified in [54]. Consider first the group $\Delta(3)$, the group of cyclic permutations on three objects. Since $\Delta(3)$ is a subgroup of the other non-Abelian subgroups of $SU(3)$, it is easy to see that using these more complicated projections can only further restrict the list of relevant and marginal chiral primary operators. We have seen that $\mathrm{Tr}F^2$ and $\mathrm{Tr}W_\alpha W^\alpha$ are always dimension 4 and dimension 3 operators in these theories since they transform in the **1** of $SU(3)$. The trilinear terms that we found in the decomposition of the **50** in eq. (3.3) are of particular interest. We saw that since the **10** of $SU(3)$ is invariant under Z_3 there were Kaluza-Klein states that were not projected out and coupled to chiral primary operators on the boundary. Under the group action of $\Delta(3)$ one component of the **10** will be invariant[¶]. A brief inspection of Table I of [54] shows that the all the subgroups $\Delta(3n^2)$ will preserve these Kaluza-Klein states, while the other non-Abelian groups ($\Delta(6n^2)$ and $\Sigma(m)$) will not.

[¶]This invariance is familiar from the $SU(3)_{\text{flavor}}$ symmetry of the quark model: the baryon octet has two states, Λ and Σ^0, that are invariant under cyclic permutations of the three flavors, while the baryon decuplet has one such state, Σ^0.

4 $\mathcal{N} = 2$ Supersymmetric Theories

In [17,24] $\mathcal{N} = 2$ SCFTs were constructed by studying D3 branes at orbifold singularities of the form R^4/Γ, where $\Gamma \subset SU(2)$ is a discrete group. The groups Γ fall into the ADE classification. As in the $\mathcal{N} = 1$ case, the worldvolume theory is constructed by taking $N|\Gamma|$ D3 branes on the covering space and performing projection Γ on the worldvolume fields and the Chan Paton factors. Again conformal field theories are expected when the representation of Γ acting on the Chan Paton factors is chosen to be the N-fold copy of the regular representation, which translates to the study of Type IIB string theory on an orbifold of $AdS_5 \times S^5$ where the orbifold group acts only on the S^5 factor. The $SO(4,2)$ isometry of AdS_5 is not broken and corresponds to the conformal symmetry of the SCFT on the D3 branes worldvolume. The $SU(4)$ isometry of the sphere is broken to $SU(2)_R \times U(1)_R \times \Gamma$. The $SU(2)_R \times U(1)_R$ factor is the R-symmetry of the boundary $\mathcal{N} = 2$ D3 brane theory. The Γ factor is a discrete global symmetry.

This orbifold acts only on four of the six coordinates transverse to the D3 branes worldvolume and there is a fixed plane of its action. This implies that the supergravity description is not valid even at large N. Nevertheless we will perform the analysis of the Kaluza-Klein spectrum and relate it to chiral primary operators of the boundary theory. The analysis suggests that chiral information is still encoded in the supergravity description.

Consider first the A_{n-1} case. The discrete group $\Gamma = Z_n$ acts as

$$\begin{aligned} X^1 &\to e^{\frac{2\pi i}{n}} X^1 \\ X^2 &\to e^{\frac{-2\pi i}{n}} X^2 \ . \end{aligned} \tag{4.1}$$

The gauge group, global symmetries and field content of the D3 brane theory is given in Table 2.

First we consider the particles in the family (2.7). We explicitly check the relevant and marginal operators for $k = 2, 3, 4$ below. For $k = 2$, decomposing the $\mathbf{20'}$ into representations of $SU(2) \times SU(2)_R \times U(1)_R$ gives:

$$\mathbf{20'} = (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{1})_4 + (\mathbf{1},\mathbf{1})_{-4} + (\mathbf{2},\mathbf{2})_2 + (\mathbf{2},\mathbf{2})_{-2} + (\mathbf{3},\mathbf{3})_0 \ . \tag{4.2}$$

If we now perform the Z_n projection* of the $SU(2)$ we find that the following components are invariant (labeled by $SU(2)_R \times U(1)_R$):

$$\mathbf{1}_0 + \mathbf{1}_4 + \mathbf{1}_{-4} + \mathbf{3}_0 \ . \tag{4.3}$$

*The Z_n acts on the $\mathbf{2}$ of $SU(2)$ as $(x^1, x^2) \to (e^{\frac{2\pi i}{n}} x^1, e^{-\frac{2\pi i}{n}} x^2)$.

	$SU(N)_1$	$SU(N)_2$	$SU(N)_3$	...	$SU(N)_n$	$U(1)_R$
Q_1	$\square$	$\bar{\square}$	$\mathbf{1}$	...	$\mathbf{1}$	0
$\tilde{Q}_1$	$\bar{\square}$	$\square$	$\mathbf{1}$	...	$\mathbf{1}$	0
Q_2	$\mathbf{1}$	$\square$	$\bar{\square}$	...	$\mathbf{1}$	0
$\tilde{Q}_2$	$\mathbf{1}$	$\bar{\square}$	$\square$	...	$\mathbf{1}$	0
$\vdots$						
Q_n	$\bar{\square}$	$\mathbf{1}$	$\mathbf{1}$	...	$\square$	0
$\tilde{Q}_n$	$\square$	$\mathbf{1}$	$\mathbf{1}$	...	$\bar{\square}$	0
Φ_1	Ad	$\mathbf{1}$	$\mathbf{1}$	...	$\mathbf{1}$	2
$\vdots$						
Φ_n	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	...	Ad	2

Table 2: Field content of the $\mathcal{N} = 2$ theory, the $SU(2)_R$ symmetry is not manifest in the $\mathcal{N} = 1$ notation used here.

At this point we need to identify the superconformal R-charge. To do this we make use of the fact that in $\mathcal{N} = 1$ language there is an additional R-symmetry, $U(1)_J$, which is a subgroup of $SU(2)_R$. Under this symmetry adjoint chiral multiplets, Φ, have charge 0, while fundamental chiral multiplets, $\tilde{Q}$ and Q, have charge 1. We can then identify the superconformal R-charge as:

$$R_{\rm sc} = \frac{1}{3}R + \frac{2}{3}J \ . \tag{4.4}$$

One can check that this gives the correct charge assignments to the gauginos and the scalars (charges 1 and 2/3 respectively). The corresponding $R_{\rm sc}$-charges of the components in (4.3) are (0,4/3,-4/3,4/3). Thus $\mathbf{1}_4$ and $\mathbf{3}_0$ have the the correct $R_{\rm sc}$-charges to couple to dimension 2 chiral primary operators. The $\mathbf{1}_4$ Kaluza-Klein mode can be be identified by its quantum numbers as the coupling to the Z_n invariant chiral primary operator $\sum_{i=1}^n \mathrm{Tr}\Phi_i^2$. The $\mathbf{3}_0$ Kaluza-Klein mode has to couple to a Z_n invariant chiral primary operator with charges like $\tilde{Q}Q$. The chiral primary operator is identified† as $\sum_{i=1}^n \tilde{Q}_i Q_i$. Note this chiral primary operator vanishes due to the F-term equations for $U(N)$ gauge groups. We can also see that since the $\mathbf{1}_4$ came from the $(\mathbf{1},\mathbf{1})_4$, it will be invariant under any projection Γ that preserves $\mathcal{N} = 2$ supersymmetry, so $\sum_i \mathrm{Tr}\Phi_i^2$ will be a dimension 2 chiral primary operator in any such theory.

†We would like to thank M. Schmaltz for pointing this out to us.

For $k=3$ decomposing the **50** into representations of $SU(2)\times SU(2)_R\times U(1)_R$ gives:

$$\begin{aligned}\mathbf{50} &= (\mathbf{1},\mathbf{1})_2+(\mathbf{1},\mathbf{1})_{-2}+(\mathbf{1},\mathbf{1})_6+(\mathbf{1},\mathbf{1})_{-6}+(\mathbf{2},\mathbf{2})_0+(\mathbf{2},\mathbf{2})_4+(\mathbf{2},\mathbf{2})_{-4}\\ &\quad+(\mathbf{3},\mathbf{3})_2+(\mathbf{3},\mathbf{3})_{-2}+(\mathbf{4},\mathbf{4})_0\ .\end{aligned} \tag{4.5}$$

The Z_n projection leaves invariant

$$\mathbf{1}_2+\mathbf{1}_{-2}+\mathbf{1}_6+\mathbf{1}_{-6}+\mathbf{3}_2+\mathbf{3}_{-2}\ . \tag{4.6}$$

The corresponding $R_{\rm sc}$-charges are (2/3,-2/3,2,-2,2,2/3), thus $\mathbf{1}_6$ and $\mathbf{3}_2$ have the correct $R_{\rm sc}$-charge to couple to a dimension 3 chiral primary operator. From their quantum numbers we see that they couple to $\sum_{i=1}^n \mathrm{Tr}\Phi_i^3$ and $\sum_{i=1}^n\left(\tilde{Q}_{i-1}\Phi_iQ_{i-1}-\tilde{Q}_i\Phi_iQ_i\right)$. The choice of the latter is dictated by the need to remove derivatives of the superpotential in order to get a primary operator. Since the $\mathbf{1}_6$ came from $(\mathbf{1},\mathbf{1})_6$ we see that $\sum_{i=1}^n \mathrm{Tr}\Phi_i^3$ will be a dimension 3 operator in any theory obtained by a projection that preserves $\mathcal{N}=2$.

For $k=4$ decomposing the **105** into representations of $SU(2)\times SU(2)_R\times U(1)_R$ gives:

$$\begin{aligned}\mathbf{105} &= (\mathbf{1},\mathbf{1})_0+(\mathbf{1},\mathbf{1})_4+(\mathbf{1},\mathbf{1})_{-4}+(\mathbf{1},\mathbf{1})_8+(\mathbf{1},\mathbf{1})_{-8}+(\mathbf{2},\mathbf{2})_2+(\mathbf{2},\mathbf{2})_{-2}\\ &\quad+(\mathbf{2},\mathbf{2})_6+(\mathbf{2},\mathbf{2})_{-6}+(\mathbf{3},\mathbf{3})_0+(\mathbf{3},\mathbf{3})_4+(\mathbf{3},\mathbf{3})_{-4}+(\mathbf{4},\mathbf{4})_2+(\mathbf{4},\mathbf{4})_{-2}\\ &\quad+(\mathbf{5},\mathbf{5})_0\ .\end{aligned} \tag{4.7}$$

The Z_n projection leaves invariant

$$\mathbf{1}_0+\mathbf{1}_4+\mathbf{1}_{-4}+\mathbf{1}_8+\mathbf{1}_{-8}+\mathbf{3}_0+\mathbf{3}_4+\mathbf{3}_{-4}+\mathbf{5}_0\ . \tag{4.8}$$

The corresponding $R_{\rm sc}$-charges are (0,4/3,-4/3,8/3,-8/3,4/3,8/3,0,8/3), thus $\mathbf{1}_8$, $\mathbf{3}_4$, and $\mathbf{5}_0$ have the correct $R_{\rm sc}$-charge to couple to a dimension 4 chiral primary operator. From their quantum numbers we see that they couple to $\sum_{i=1}^n \mathrm{Tr}\Phi_i^4$, $\sum_{i=1}^n\left(\tilde{Q}_{i-1}\Phi_i^2Q_{i-1}-\tilde{Q}_i\Phi_i^2Q_i\right)$ and $\sum_{i=1}^n\left(\tilde{Q}_{i-1}Q_{i-1}-\tilde{Q}_iQ_i\right)^2$ respectively[‡]. We see again that $\mathrm{Tr}\Phi^4$ will be a dimension 4 operator for any choice of projection that preserves $\mathcal{N}=2$. In general we expect Kaluza-Klein modes with masses (2.7) will couple to dimension k chiral primary operators in the boundary SCFT.

Next we consider particles in the family (2.8). To explicitly check the relevant chiral operator for $k=0$ we decompose the **10** into representations of $SU(2)\times SU(2)_R\times U(1)_R$:

$$\mathbf{10}=(\mathbf{2},\mathbf{2})_0+(\mathbf{3},\mathbf{1})_{-2}+(\mathbf{1},\mathbf{3})_2\ . \tag{4.9}$$

[‡]Note that one can recast the chiral primary operators in a different form by adding appropriate powers of lower dimension chiral primary operators.

The Z_n projection leaves invariant

$$\mathbf{1}_{-2} + \mathbf{3}_2 \ . \tag{4.10}$$

The corresponding $R_{\rm sc}$-charges are (-2/3,2), thus $\mathbf{3}_2$ has the correct $R_{\rm sc}$-charge to couple to a dimension 3 chiral primary operator. The corresponding operator is $\sum_{i=1}^{n} \mathrm{Tr} W^i_\alpha W^\alpha_i$, and we see that it will be dimension

3 with any projection that preserves $\mathcal{N} = 2$.

For $k = 1$ we decompose the **45** as:

$$\begin{aligned} \mathbf{45} \ &= \ (\mathbf{2},\mathbf{2})_2 + (\mathbf{2},\mathbf{2})_{-2} + (\mathbf{3},\mathbf{1})_{-4} + (\mathbf{1},\mathbf{3})_4 + (\mathbf{3},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{3},\mathbf{3})_0 \\ &\quad + (\mathbf{4},\mathbf{2})_{-2} + (\mathbf{2},\mathbf{4})_2 \ . \end{aligned} \tag{4.11}$$

The Z_n projection leaves invariant

$$\mathbf{1}_{-4} + \mathbf{3}_4 + \mathbf{1}_0 + \mathbf{3}_0 + \mathbf{3}_0 \ . \tag{4.12}$$

The corresponding $R_{\rm sc}$-charges are (-4/3,8/3,0,4/3,4/3), thus the $\mathbf{3}_4$ has the correct $R_{\rm sc}$-charge to couple to a dimension 4 chiral primary operator. The corresponding operator is $\sum_{i=1}^{n} \mathrm{Tr} W^i_\alpha W^\alpha_i \Phi_i$. Again this result is completely general as long as $\mathcal{N} = 2$ supersymmetry is preserved. In general we expect Kaluza-Klein modes with masses (2.8) to couple to dimension $k+3$ chiral primary operator in the boundary SCFT.

Now consider particles in the family (2.9). For $k = 0$ we get the relevant operator $\sum_{i=1}^{n} \mathrm{Tr} F_i^2$. Again the Z_n projection on the only relevant representation, **1**, is trivial, a result that holds independent of the projection Γ. We expect Kaluza-Klein modes with masses (2.9) to couple to dimension $k+3$ chiral primary operator in the boundary SCFT.

Finally consider the spin one family (2.6). The only relevant mode is the massless mode which is in the **15** of $SU(4)$. Decomposing the **15** we find

$$\mathbf{15} = (\mathbf{1},\mathbf{1})_0 + (\mathbf{2},\mathbf{2})_2 + (\mathbf{2},\mathbf{2})_{-2} + (\mathbf{3},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 \ . \tag{4.13}$$

The Z_n projection leaves invariant

$$\mathbf{1}_0 + \mathbf{1}_0 + \mathbf{3}_0 \ , \tag{4.14}$$

corresponding to the currents of the unbroken $SU(2)_R \times U(1)_R$ symmetry and one extra Z_n invariant current which is a remnant of the broken $SU(2)$ symmetry. As above the the currents corresponding to the unbroken R-symmetry will be dimension 3 under any projection that preserves $\mathcal{N} = 2$ supersymmetry.

It is straightforward to extend the analysis to the D_n and E_n orbifolds. Since Z_n is a subgroup of D_n, E_6, and E_7, we further restrict the list of relevant and marginal chiral primary operators. Also, the form of the generators of the E_8 singularity (Icosahedral group) [55] dictates the same list of operators as E_6 and E_7. In going from the A_{n-1} to D_n we find that the **3** no longer has an invariant component, while the **5** still does. So there are no analogs of the chiral primary operators: $\tilde{Q}_i\Phi_i Q_i - \tilde{Q}_{i-1}\Phi_i Q_{i-1}$, and $\tilde{Q}_i\Phi_i^2 Q_i - \tilde{Q}_{i-1}\Phi_i^2 Q_{i-1}$. Under the E_n projections neither the **3** nor the **5** have invariant components, so we also do not have an analog of the $\left(\tilde{Q}_{i-1}Q_{i-1} - \tilde{Q}_i Q_i\right)^2$ operator.

5 Nonsupersymmetric Theories

Examples of $\mathcal{N} = 0$ candidates for CFTs in the large N limit can be constructed by considering orbifold singularities of the form R^6/Γ, where $\Gamma \subset SU(4)$ is a discrete subgroup, and the orbifold group acts only on the S^5 factor of $AdS_5 \times S^5$. As before, the $SO(4,2)$ isometry of AdS_5 is not broken and corresponds to the conformal symmetry of the boundary CFT, while the the $SU(4)$ isometry of S^5 is broken to Γ which becomes a discrete global symmetry of the CFT.

Consider for example the Z_5 orbifold example of [17]

$$\begin{aligned} X^1 &\rightarrow e^{\frac{2\pi i}{5}} X^1 \\ X^2 &\rightarrow e^{\frac{6\pi i}{5}} X^2 \,. \end{aligned} \tag{5.1}$$

The gauge group, global symmetries and field content of the D3 brane theory is given in Table 3.

As in the $\mathcal{N} = 2$ case the orbifold group has a fixed plane and we do not expect the supergravity description to be valid even at large N. Unlike the supersymmetric cases we also do not a priori expect the masses of the Kaluza-Klein harmonics to be protected from quantum and string corrections. However, there is still the possibility that dimensions of certain primary operators in the the $\mathcal{N} = 0$ boundary CFT do not receive quantum corrections, and that the AdS/CFT correspondence generalized to the $\mathcal{N} = 0$ theories will imply that masses of certain Kaluza-Klein modes do not receive quantum corrections. With this in mind we will now carry out some examples of an analysis similar to that of the previous sections. In order to get the Kaluza-Klein harmonics on the supergravity side we will project those of $AdS_5 \times S^5$ on Z_5 invariant states*. The results are summarized in Table 4.

*The Z_5 acts on the **4** of $SU(4)$ as $(x^1, x^2, x^3, x^4) \rightarrow (e^{\frac{2\pi i}{5}}x^1, e^{\frac{-2\pi i}{5}}x^2, e^{\frac{4\pi i}{5}}x^3, e^{\frac{-4\pi i}{5}}x^4)$.

	$SU(N)_i$	$SU(N)_{i+1}$	$SU(N)_{i+2}$	$SU(N)_{i-2}$	$SU(N)_{i-1}$
$\phi_{i,i+1}$	$\square$	$\bar{\square}$	**1**	**1**	**1**
$\phi_{i,i+2}$	$\square$	**1**	$\bar{\square}$	**1**	**1**
ϕ_i	Ad	**1**	**1**	**1**	**1**
$\psi_{i,i+1}$	$\square$	$\bar{\square}$	**1**	**1**	**1**
$\psi_{i,i-1}$	$\square$	**1**	**1**	**1**	$\bar{\square}$
$\psi_{i,i+2}$	$\square$	**1**	$\bar{\square}$	**1**	**1**
$\psi_{i,i-2}$	$\square$	**1**	**1**	$\bar{\square}$	**1**

Table 3: Field content of the $\mathcal{N} = 0$ theory, where Z_5 cyclicly permutes the gauge groups. ϕ's are scalars and ψ's are fermions. We have not listed the conjugates of the bifundamental fields.

In order to construct the primary operators of the $\mathcal{N} = 0$ boundary CFT we have to remove dependences on the derivatives with respect to the fields of the Yukawa and quartic couplings. We expect Yukawa couplings for each triangle with two fermion lines and one scalar line in the quiver diagram description of the model, and quartic couplings for each square with four scalar lines [17, 24].

Consider the invariant Kaluza-Klein states. There are four invariant states from the $k = 2$ modes in (2.7) that transform in the **20′** of $SU(4)$ and should couple to dimension 2 primary operators. Since the **20′** is made of two **6**'s of $SU(4)$ we should construct primary operators bilinear in the scalars. We have $\mathrm{Tr}\phi_i^2$, $\mathrm{Tr}\phi_{i,i+1}\phi_{i+1,i}$, $\mathrm{Tr}\phi_{i,i+2}\phi_{i+2,i}$ where ϕ_i is the adjoint scalar of the i-th gauge group and $\phi_{i,j}$ is the scalar associated with the line connecting the i, j nodes of the quiver diagram. The above operators (and also in the other examples below) have an implicit summation on the different nodes i in order to make them Z_5 invariant. We do not seem to see a possible fourth primary operator of dimension 2 in the theory. This suggests that the number of invariant Kaluza-Klein states is larger than the number of primary operators in the boundary CFT. This is not too surprising in view of the fact that we lack here the chirality constraint which was important in the supersymmetric case. We will see more examples of this in the following.

There are 10 invariant states from the $k = 3$ modes in (2.7) that transform in the **50** of $SU(4)$ and should couple to dimension 3 primary operators. Since the **50** is made of three **6**'s of $SU(4)$ we should construct primary operators from three scalars. An obvious one is $\mathrm{Tr}\phi_i^3$. Others can are made of $\phi_{i,i+1}, \phi_{i+1,i}, \phi_{i,i+2}, \phi_{i+2,i}$. Also in this case we have more invariant Kaluza-Klein states than the number of primary operators in the boundary

CFT. Similarly there are 21 invariant states from the $k = 4$ modes in (2.7) that transform in the **105** of $SU(4)$ and should couple to dimension 4 primary operators. Since the **105** is made of four **6**'s of $SU(4)$ we should construct primary operators from four scalars. One of them is $\mathrm{Tr}\phi_i^4$ and as before the rest are made from the other scalars. Clearly we have more invariant Kaluza-Klein modes than primary operators.

Consider now the invariant Kaluza-Klein states of (2.8). There are two invariant states from the $k = 0$ modes that transform in the **10** of $SU(4)$ and should couple to dimension 3 primary operators. Since the **10** is made of two **4**'s of $SU(4)$ we should construct primary operators bilinear in the fermions. There are two such independent primary operators $\mathrm{Tr}\psi_{i,i+1}\psi_{i+1,i}, \mathrm{Tr}\psi_{i,i+2}\psi_{i+2,i}$ where $\psi_{i,j}$ is the fermion associated with the line connecting the i, j nodes of the quiver diagram. There are 9 invariant states from the $k = 1$ modes that transform in the **45** of $SU(4)$ and should couple to dimension 4 primary operators. Since the **45** is made of two **4**'s and a **6** of $SU(4)$ the primary operators are bilinear in the fermions and linear in the scalars and again we seem to see that there are more invariant Kaluza-Klein modes than primary operators.

The dilaton $k = 0$ in (2.9) and the graviton $k = 0$ in (2.5) are not projected out and couple to the relevant operators $\mathrm{Tr}F^2$ and $T_{\mu\nu}$.

6 Summary and Discussion

In this work we studied the relation between (chiral) primary operators of (super) conformal field theories in four dimensions constructed in [17, 24] and the Kaluza-Klein states of supergravity on orbifolds of $AdS_5 \times S^5$. This generalizes the relation between the chiral primary operators of $\mathcal{N} = 4$ SCFT and Kaluza-Klein states of supergravity on $AdS_5 \times S^5$ found in [11]. We obtained the Kaluza-Klein modes in the orbifold models by projecting those of supergravity on $AdS_5 \times S^5$ on Γ invariant states where Γ is the orbifold group. In Table 4 we summarize the results. In the $\mathcal{N} = 0$ we saw more Kaluza-Klein states than primary operators.

Note that even in cases where the supergravity description is not applicable we see that chiral information is still reliably encoded in this description. The fact that BPS information is obtained correctly even when the supergravity description is not valid is already a known phenomenon. For instance, when considering gauge theories via wrapping the fivebrane of eleven dimensional supergravity (M theory) on a Riemann surface in order to obtain $N = 2$ supersymmetric gauge theories in four dimensions [56] there are points in the $N = 2$ moduli space where the Riemann surface degenerates and the

		Γ	A_n	D_n	E_n	Z_3	$\Delta(3n^2)$	$\neq \Delta(3n^2)$	Z_5
		$\mathcal{N}$	2	2	2	1	1	1	0
$\mathcal{R}$	spin	Δ							
20′	0	2	$\mathbf{1}_{4/3},\mathbf{3}_{4/3}$	$\mathbf{1}_{4/3}$	$\mathbf{1}_{4/3}$	-	-	-	4
50	0	3	$\mathbf{1}_2,\mathbf{3}_2$	$\mathbf{1}_2$	$\mathbf{1}_2$	10_2	1_2	-	10
105	0	4	$\mathbf{1}_{8/3},\mathbf{3}_{8/3},\mathbf{5}_{8/3}$	$\mathbf{1}_{8/3},\mathbf{5}_{8/3}$	$\mathbf{1}_{8/3}$	-	-	-	21
10	0	3	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	1_2	1_2	1_2	2
45	0	4	$\mathbf{3}_{8/3}$	$\mathbf{3}_{8/3}$	$\mathbf{3}_{8/3}$	-	-	-	9
1	0	4	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	1_0	1_0	1_0	1
15	1	3	$\mathbf{1}_0,\mathbf{1}_0,\mathbf{3}_0$	$\mathbf{1}_0,\mathbf{3}_0$	$\mathbf{1}_0,\mathbf{3}_0$	1_0, 8_0	1_0, 1_0, 1_0	1_0	3

Table 4: Kaluza-Klein harmonic projections. The projections are labeled by Γ and $\mathcal{N}$, while for the $SU(4)$ representations, the spin and scaling dimensions are also listed. For $\mathcal{N} = 2$ the invariant components of the representations are labeled by $SU(2)_R \times U(1)_{R_{sc}}$, for $\mathcal{N} = 1$ by the number of components and $U(1)_{R_{sc}}$, and for $\mathcal{N} = 0$ by the number of components. The notation $\neq \Delta(3n^2)$ refers to the other non-Abelian subgroups of $SU(3)$, $\Delta(6n^2)$ and $\Sigma(m)$.

eleven dimensional supergravity description is not valid. Nevertheless the spectrum of BPS particles is obtained correctly. The basic reason is that the BPS spectrum is protected from quantum corrections and the BPS mass formula continues to hold even when extrapolated to regions where supergravity theory does not provide a good description. Similar phenomenon occurs for $N = 1$ supersymmetric gauge theories in four dimensions that are obtained via wrapping the fivebrane on a Riemann surface [57, 58]. In our case we see another example of this phenomenon, since the spectrum of chiral operators is protected from quantum corrections. A deeper analysis of this is still lacking.

The results of this work can be generalized in a straightforward way to orbifolds of $AdS_7 \times S^4$ and $AdS_4 \times S^7$ that lead to six and three dimensional SCFTs respectively [37, 39].

Note that in our analysis we have not seen Kaluza-Klein modes that correspond in the boundary SCFTs to the Yukawa couplings that arise from the superpotential of the $\mathcal{N} = 4$ theory upon projection. The reason being that these Yukawa couplings do not correspond to primary operators. They should however play an important role. The orbifold theories have a vanishing one-loop β-function [17, 24]. If the Yukawa couplings vanish then the two-loop β-function will not not be zero, in fact the two-loop β-function will be positive and these theories will be infrared free theories, that is, they will be conformal but trivial. Thus, it would be important to carefully analyze these couplings and their effect on the higher loop β-functions.

It is interesting to note that for the $\mathcal{N} = 0, 1$ cases we can easily see that the fixed line is only present in the large N limit unless we modify the Yukawa couplings with N dependent corrections that vanish as $N \to \infty$*. In the $N = 1$ cases where we have a Leigh-Strassler type argument [59, 17] such a modification is guaranteed to exist. Consider the $\mathcal{N} = 0$ case. The vanishing of the one-loop Yukawa β-function [60] works for a $U(N)$ gauge group because there is a cancelation between Yukawa contributions, which receive a counting factor N from the N fundamentals, and a gauge contribution which is proportional to the Casimir of the the fundamental, $N/2$. Of course the $U(1)$ sub-group is infrared-free, so the fixed point can only occur for the $SU(N)$ theory, but then the cancellation fails at next-to-leading order in $1/N$ since the Casimir of $SU(N)$ is $(N^2 - 1)/2N$. It would be important to show that a modification of the Yukawa couplings and a fixed line at finite N exist also in the $\mathcal{N} = 1$ theories.

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