LBNL-41750

ERNEST ORLANDO LAWRENCE
BERKELEY NATIONAL LABORATORY

On Effective Temperatures and Electron Spin Polarization in Storage Rings

J.D. Jackson

Physics Division

May 1998

Presented at the
Advanced ICFA Beam Dynamics Workshop on Quantum Aspects of Beam Physics,
Monterey, CA,
January 4–9, 1998,
and to be published in
the Proceedings

REFERENCE COPY
Does Not
Circulate
Bldg. 50 Library - Ref.
Lawrence Berkeley National Laboratory
Copy 1
LBNL-41750

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LBNL-41750

On Effective Temperatures and Electron Spin Polarization in Storage Rings

J.D. Jackson

Theoretical Physics Group
Physics Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

May 1998

This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

LBNL - 41750

ON EFFECTIVE TEMPERATURES AND ELECTRON SPIN POLARIZATION IN STORAGE RINGS

J. D. JACKSON
Theoretical Physics Group, Lawrence Berkeley National Laboratory
Berkeley, CA 94270, USA
E-mail: jdj@lbl.gov

I came to the Monterey meeting expecting to be a silent observer and student, but was drawn into the discussions of "Unruh radiation" and the "Unruh temperature." This note is a summary of my views on the subject of effective temperatures (and the Unruh temperature in particular) in accelerators.

Unruh[1] analyzes the response to vacuum fluctuations of a linearly accelerated two-level "atom" serving as a detector and shows that the relative populations of the two states are given by a Boltzmann factor with $kT = \hbar a/2\pi c$, where a is the acceleration. The inference is that the detector is immersed in a black-body spectrum of "Unruh radiation" at the "Unruh temperature." I will refer to these ideas as the "Unruh effect."

Because Bell and Leinaas[2] (BL) argue that the less-than-100% polarization in electron storage rings is a manifestation of an Unruh effect ("Electrons as Accelerated Thermometers"), some accelerator physicists have begun to see the Unruh effect everywhere. The identification of a significant frequency or acceleration with an effective temperature via $\omega_{sig} = kT_{eff}/\hbar$ is not a bad thing in itself. The question is, "Is it helpful?" As an illustration, consider ordinary synchrotron radiation by a relativistic particle. In the instantaneous rest frame of the particle, the acceleration is $a' = \gamma^2 c\omega_0$, where ω_0 is the orbital frequency in the laboratory. The Unruh temperature is then $kT' = \gamma^2\hbar\omega_0/2\pi$ in that frame. After boosting to the laboratory, we expect a roughly blackbody spectrum (if indeed it is observable) with a characteristic "cut-off" energy $\hbar\omega_c \approx \gamma^3\hbar\omega_0/2\pi$. This energy is actually close to the peak of Schwinger's synchrotron radiation spectrum, but Schwinger's detailed shape is far different from Planckian. The estimate of $\hbar\omega_c$ is, of course, obtainable in other elementary ways. It is a matter of taste whether it is helpful to detour through a thermodynamic concept in order to "understand" even crudely the Schwinger spectrum.

In that first (1983) paper, BL applied the basic Unruh formula to Boltzmann factors for the imagined spin-flip splitting of the relativistic

electron in a magnetic field. The elementary treatment of these energy levels in the instantaneous rest frame can be found in Jackson[3], Section II.A. The energy level splitting is $\Delta E' = g\gamma^2\hbar\omega_0/2$. With the kT' given above, the spin polarization is $P = \tanh(\pi g/2)$. For $g = 2$, the formula yields $P = 0.99627$, rather different from the standard result, $P = 8/5\sqrt{3} \approx 0.92376$. If we interpret the conventionally calculated (and observed) polarization in terms of Boltzmann factors with an effective temperature, we find $T'_{eff}/T'_{Unruh} \approx 1.95$. The failure of the simple Unruh recipe is far worse for other values of the g factor. Note that the BL formula give the sign of the polarization the same as the sign of g. For $0 < g < 1.2$, however, the conventional calculation[3] yields a

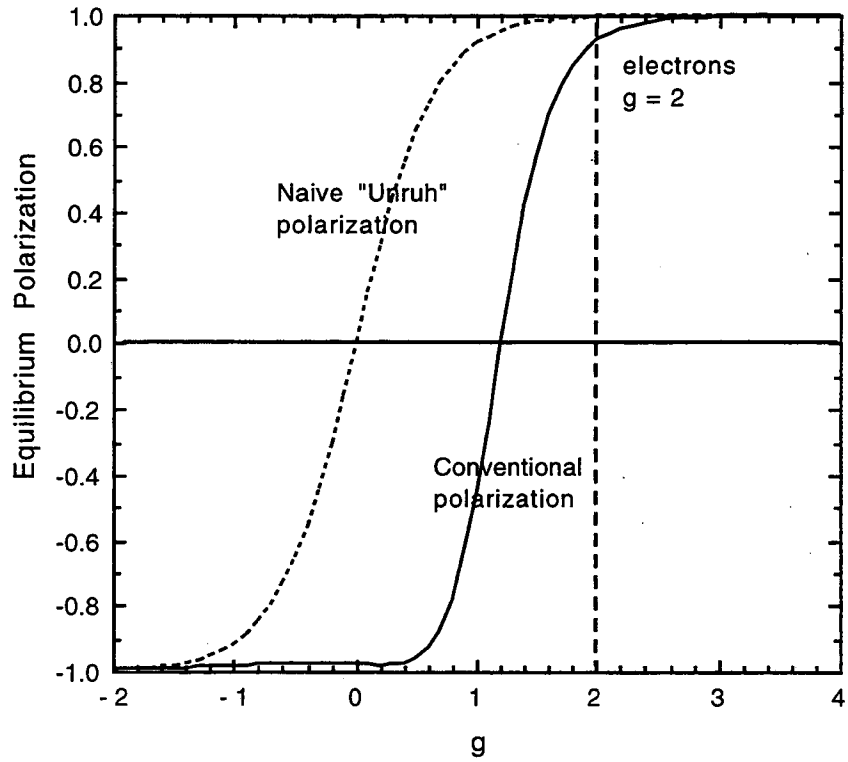

Figure 1. Equilibrium polarization in storage rings for arbitrary g-factor for a particle in a classical circular orbit. Solid curve, conventional result[3], dashed curve, $P = \tanh(\pi g/2)$, the naive BL result[2]. For the range, $0 < g < 1.2$, the effective temperature T_{eff} is negative ($gP < 0$).

negative polarization even though g is positive. The effective temperature is negative! The comparison is shown in Figure 1. BL speculate that the disparity is a consequence of the circular, not linear, acceleration.

In their second (1987) paper[4], Bell and Leinaas examine the quantum fluctuations of particle orbits in an idealized storage ring from the viewpoint of Unruh acceleration and also discuss the spin polarization. As far as the spin polarization goes, BL do a conventional calculation (their Eq. (46)) and in the subsequent discussion state that, if they treat the orbit as a classical circular path (i.e., without orbit fluctuations), they agree in every detail with the standard calculation[3]. They insist however that their full calculation, which includes the beam dynamics and quantum fluctuations, all calculated in the accelerated frames, includes effects not present otherwise. BL find that the mean square vertical orbit fluctuations are consistent with a "circular Unruh effect" with an effective temperature $T'_{eff}/T'_{Unruh} \approx 1.474$.

Bell and Leinaas were not the first to treat the effects of orbital dynamics on the equilibrium polarization in storage rings. Derbenev and Kondratenko[5, 6] (DK) presented formulas that include such depolarizing effects as resonances. In an illuminating paper, Barber and Mane[7] make a detailed comparison of the work of Bell and Leinaas and Derbenev and Kondratenko. While the two approaches are quite different, they deal with the same physics and find the same results. BL may claim an integrated approach and criticize DK for treating orbit dynamics as a separate depolarization mechanism, but Barber and Mane show the complete equivalence of the two methods. Correcting minor errors in both BL and DK, Barber and Mane give a generalized formula (their Eq.(42)) for the polarization, applicable to strong-focusing machines, derived in the conventional (DK) way. It contains the BL results as a special case. Evidently, it is a matter of taste, not substance, whether one stays in the laboratory to calculate or goes into an accelerated frame.

For spin polarization it is clear that no one has made an "Unruh effect" computation of the magical $P = 8/5\sqrt{3}$ of the conventional calculations. Application (by BL in their first paper) of the original Unruh formula leads to an expression that does not reproduce, even qualitatively, the polarization as a function of g-factor for a particle in a circular orbit. There is no "Unruh radiation" to be observed, just the (admittedly tiny) spin-flip radiation, first computed by Sokolov, Ternov, and collaborators.

On the issue of identifying an effective temperature with an important acceleration, I have no strong feelings. The examples cited show that, if we

force the "Unruh interpretation" on the phenomena, the effective temperatures differ from the simple Unruh temperature. The ratios are not huge in my examples, but exponentiation in Boltzmann factors can lead to large differences. The reasonable rule is to introduce the concept of an effective temperature if the physics is illuminated thereby. Avoid the indiscriminate appeal to Unruh in order to "understand" something amenable to a simpler explanation.

If someone insists on attempting to calculate spin-flip radiation and spin polarization using the Unruh acceleration technology, I challenge him or her to reproduce the photon distributions in Fig. 4 of reference 3 for various g-factors, or even the distributions in angle and energy for g = 2 (Fig. 7 of reference 3). If it can be done, I will be impressed, but will still believe that the roundabout achievement will be akin to scratching one's left ear with one's right hand.

Acknowledgments

I thank Dr. Pisin Chen for inviting me to attend the Monterey workshop, William G. Unruh for interesting conversations and exchanges of e-mail concerning his effect, and Andrew M. Sessler for reminding me of the Barber-Mane paper. This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, of the U.S. Department of Energy under Contract DE-AC03-76SF0098.

References

1. W. G. Unruh, *Phys. Rev. D* **14**, 870 (1976).
2. J. S. Bell and J. M. Leinaas, *Nucl. Phys. B* **212**, 131 (1983).
3. J. D. Jackson, *Rev. Mod. Phys.* **48**, 417 (1976).
4. J. S. Bell and J. M. Leinaas, *Nucl. Phys. B* **284**, 488 (1987).
5. Ya. S. Derbenev and A. M. Kondratenko, *Zh. Eksp. Teor. Fiz.* **62**, 430 (1972) [Sov. Phys. - JETP **35**, 230 (1972)].
6. Ya. S. Derbenev and A. M. Kondratenko, *Zh. Eksp. Teor. Fiz.* **64**, 1918 (1973) [Sov. Phys. - JETP **37**, 968 (1973)].
7. D. P. Barber and S. R. Mane, *Phys. Rev. A* **37**, 456 (1988).

ERNEST ORLANDO LAWRENCE BERKELEY NATIONAL LABORATORY
ONE CYCLOTRON ROAD | BERKELEY, CALIFORNIA 94720

Prepared for the U.S. Department of Energy under Contract No. DE-AC03-76SF00098